Coupling procedure for the Poincaré Gauge Theory of Gravity

Marcin Kaźmierczak



Chair of Theory of Relativity and Gravitation Institute of Theoretical Physics Faculty of Physics, University of Warsaw Poland

Zakopane, June 7, 2009

Marcin Kaźmierczak

University of Warsaw

1 Introduction — The Poincaré Gauge Theory

- Yang–Mills theories
 - The Poincaré group as a gauge group

2 The ambiguity of MCP in the presence of torsion

3 Removing the ambiguity by modifying coupling procedure

<ロト <回ト < 国ト < 国ト < 国ト = 国

Standard Yang–Mills theories

◆□▶ ◆□▶ ◆三▶ ◆三▶ ●□ ● ●

Marcin Kaźmierczak

University of Warsaw

Standard Yang–Mills theories

•
$$S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$$

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Standard Yang–Mills theories

- $S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$
- $\phi: M \to \mathcal{V}$, M – Minkowski space, \mathcal{V} – a linear space

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Standard Yang–Mills theories

- $S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$
- $\phi: M \to \mathcal{V},$ M – Minkowski space, \mathcal{V} – a linear space
- G a Lie group, $\pi : Lie(G) \to End(\mathcal{V})$,

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Standard Yang-Mills theories

- $S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$
- $\phi: M \to \mathcal{V},$ M – Minkowski space, \mathcal{V} – a linear space
- G a Lie group, $\pi: Lie(G) \to End(\mathcal{V})$,
- $\rho(\exp(\mathfrak{g})) = \exp(\pi(\mathfrak{g}))$

University of Warsaw

<ロ> <同> <同> <同> < 同> < 同>

Marcin Kaźmierczak

Standard Yang–Mills theories

- $S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$
- $\phi: M \to \mathcal{V},$ M – Minkowski space, \mathcal{V} – a linear space
- G a Lie group, $\pi : Lie(G) \to End(\mathcal{V})$,
- $\rho(\exp(\mathfrak{g})) = \exp(\pi(\mathfrak{g}))$
- $\mathfrak{L}_m\left(\rho(g)\phi, d\left(\rho(g)\phi\right)\right) = \mathfrak{L}_m\left(\phi, d\phi\right), \quad \forall g \in G$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

An interaction associated to G

Allow the group element to depend on space–time point and demand the Lagrangian four–form to be invariant under local action of G. In order to achieve this, replace the differentials by covariant differentials:

• $d\phi \rightarrow D\phi = d\phi + \mathbb{A}\phi$,

University of Warsaw

< ロ > < 同 > < 回 > < 回 >

Marcin Kaźmierczak

An interaction associated to G

Allow the group element to depend on space–time point and demand the Lagrangian four–form to be invariant under local action of G. In order to achieve this, replace the differentials by covariant differentials:

- $d\phi \rightarrow D\phi = d\phi + \mathbb{A}\phi$,
- $\mathbb{A} \mathbf{a} \pi (Lie(G))$ -valued one-form field on M,

Marcin Kaźmierczak

An interaction associated to G

Allow the group element to depend on space–time point and demand the Lagrangian four–form to be invariant under local action of G. In order to achieve this, replace the differentials by covariant differentials:

- $d\phi \rightarrow D\phi = d\phi + \mathbb{A}\phi$,
- $\mathbb{A} \mathbf{a} \pi (Lie(G))$ -valued one-form field on M,
- $\mathbb{A} \to \mathbb{A}' = \rho(g)\mathbb{A}\rho^{-1}(g) d\rho(g)\rho^{-1}(g).$

University of Warsaw

< ロ > < 同 > < 回 > < 回 >

Marcin Kaźmierczak

An interaction associated to G

Allow the group element to depend on space–time point and demand the Lagrangian four–form to be invariant under local action of G. In order to achieve this, replace the differentials by covariant differentials:

- $d\phi \rightarrow D\phi = d\phi + \mathbb{A}\phi$,
- $\mathbb{A} \mathbf{a} \pi (Lie(G))$ -valued one-form field on M,
- $\mathbb{A} \to \mathbb{A}' = \rho(g)\mathbb{A}\rho^{-1}(g) d\rho(g)\rho^{-1}(g).$
- $D'\phi' = \rho(g)D\phi \Rightarrow \tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A}) := \mathfrak{L}_m(\phi, D\phi)$ invariant under local transformations.

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

・ロト ・回ト ・ヨト ・ヨト

University of Warsaw

Yang–Mills theories

An interaction associated to G

Allow the group element to depend on space–time point and demand the Lagrangian four–form to be invariant under local action of G. In order to achieve this, replace the differentials by covariant differentials:

- $d\phi \rightarrow D\phi = d\phi + \mathbb{A}\phi$,
- $\mathbb{A} a \pi (Lie(G))$ -valued one-form field on M,
- $\mathbb{A} \to \mathbb{A}' = \rho(g)\mathbb{A}\rho^{-1}(g) d\rho(g)\rho^{-1}(g).$
- $D'\phi' = \rho(g)D\phi \Rightarrow \tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A}) := \mathfrak{L}_m(\phi, D\phi)$ invariant under local transformations.

MCP- the Minimal Coupling Procedure

Physical meaning of $\mathbb A$

components of A – Yang–Mills fields,

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Physical meaning of A

- components of A Yang–Mills fields,
- $\mathbb{F} := d\mathbb{A} + \mathbb{A} \wedge \mathbb{A}$ the field–strength two–form,

Marcin Kaźmierczak

University of Warsaw

.

Physical meaning of $\mathbb A$

- components of A Yang–Mills fields,
- $\mathbb{F} := d\mathbb{A} + \mathbb{A} \wedge \mathbb{A}$ the field–strength two–form,
- $\bullet \ \mathbb{F}' = \rho(g) \mathbb{F} \rho^{-1}(g),$

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Physical meaning of $\mathbb A$

- components of A Yang–Mills fields,
- $\mathbb{F} := d\mathbb{A} + \mathbb{A} \wedge \mathbb{A}$ the field–strength two–form,
- $\mathbb{F}' = \rho(g)\mathbb{F}\rho^{-1}(g),$

•
$$d\mathbb{F} + \mathbb{A} \wedge \mathbb{F} - \mathbb{F} \wedge \mathbb{A} = 0.$$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak



Marcin Kaźmierczak

University of Warsaw

- $\mathfrak{L} = \tilde{\mathfrak{L}}_m + \mathfrak{L}_G$,
- 𝔅_G gauge–field part, built of 𝔅, invariant under gauge transformations.

University of Warsaw

Poincaré Gauge Theory of Gravity

Marcin Kaźmierczak

•
$$\mathfrak{L} = \tilde{\mathfrak{L}}_m + \mathfrak{L}_G$$
,

- 𝔅_G gauge–field part, built of 𝔅, invariant under gauge transformations.
- $\mathfrak{L}_G \sim tr\left(\mathbb{F} \wedge \star \mathbb{F}\right)$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

- $\mathfrak{L} = \tilde{\mathfrak{L}}_m + \mathfrak{L}_G$,
- 𝔅_G gauge–field part, built of 𝔅, invariant under gauge transformations.
- $\mathfrak{L}_G \sim tr \left(\mathbb{F} \land \star \mathbb{F} \right)$
- (will not work for gravity!)

University of Warsaw

< ロ > < 同 > < 臣 > < 臣

Marcin Kaźmierczak

The Poincaré group as a gauge group

Marcin Kaźmierczak

University of Warsaw

The Poincaré group as a gauge group

• The Poincaré group \mathcal{P} consists of all isometries of M,

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ = 臣 = 釣�??

Marcin Kaźmierczak

University of Warsaw

The Poincaré group as a gauge group

- The Poincaré group \mathcal{P} consists of all isometries of M,
- $(\Lambda, a)x = \Lambda x + a, \quad x, a \in M, \quad \Lambda \in O(1,3),$

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

The Poincaré group as a gauge group

- The Poincaré group \mathcal{P} consists of all isometries of M,
- $\bullet \ (\Lambda,a)x = \Lambda x + a, \quad x,a \in M, \quad \Lambda \in O(1,3),$
- $(\Lambda_1, a_1)(\Lambda_2, a_2) = (\Lambda_1 \Lambda_2, \Lambda_1 a_2 + a_1).$

Marcin Kaźmierczak

(I) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1))

The Poincaré group as a gauge group

• The Poincaré group \mathcal{P} consists of all isometries of M,

•
$$(\Lambda, a)x = \Lambda x + a, \quad x, a \in M, \quad \Lambda \in O(1, 3),$$

•
$$(\Lambda_1, a_1)(\Lambda_2, a_2) = (\Lambda_1 \Lambda_2, \Lambda_1 a_2 + a_1).$$

Consider a representation

$$\begin{split} \rho(\Lambda, a) &:= \rho(a)\rho(\Lambda), \\ \rho(a) &:= \exp\left(a_a P^a\right), \quad \rho\left(\Lambda(\varepsilon)\right) := \exp\left(\frac{1}{2}\varepsilon_{ab}J^{ab}\right) \\ \text{where } P^a, J^{ab} \in \pi\left(Lie(\mathcal{P})\right). \end{split}$$

University of Warsaw

.

Marcin Kaźmierczak

•
$$\rho(\Lambda_1, a_1)\rho(\Lambda_2, a_2) = \rho(\Lambda_1\Lambda_2, \Lambda_1a_2 + a_1)$$

◆□ > ◆□ > ◆三 > ◆三 > 三 ・ のへで

Marcin Kaźmierczak

University of Warsaw

Introduction

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Poincaré group as a gauge group

•
$$\rho(\Lambda_1, a_1)\rho(\Lambda_2, a_2) = \rho(\Lambda_1\Lambda_2, \Lambda_1a_2 + a_1) \Rightarrow$$

Marcin Kaźmierczak

University of Warsaw

•
$$\rho(\Lambda_1, a_1)\rho(\Lambda_2, a_2) = \rho(\Lambda_1\Lambda_2, \Lambda_1a_2 + a_1) \Rightarrow$$

$$\rho(\Lambda, a)P^{a}\rho^{-1}(\Lambda, a) = \Lambda_{c}^{a}P^{c},$$

$$\rho(\Lambda, a)J^{ab}\rho^{-1}(\Lambda, a) = \Lambda_{c}^{a}\Lambda_{d}^{b}\left(J^{cd} + a^{c}P^{d} - a^{d}P^{c}\right),$$

$$[P^{a}, P^{b}] = 0,$$

$$[P^{a}, J^{cd}] = \eta^{ac}P^{d} - \eta^{ad}P^{c},$$

$$[J^{ab}, J^{cd}] = \eta^{ad}J^{bc} + \eta^{bc}J^{ad} - \eta^{bd}J^{ac} - \eta^{ac}J^{bd}.$$
(1)

< ≣ ► ≣ ৩ ৭ ৫ University of Warsaw

<ロ> <同> <同> < 同> < 同>

Marcin Kaźmierczak

•
$$\mathbb{A} = \frac{1}{2}\omega_{ab}J^{ab} + \Gamma_a P^a$$
 $\pi (Lie(\mathcal{P}))$ -valued one-form on M ,

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ▼ ○ ◇ ◇

Marcin Kaźmierczak

University of Warsaw

•
$$\mathbb{A} = \frac{1}{2}\omega_{ab}J^{ab} + \Gamma_a P^a \quad \pi (Lie(\mathcal{P}))$$
-valued one–form on M ,

•
$$\omega_{ab} = -\omega_{ba}$$



Marcin Kaźmierczak

University of Warsaw

• $\mathbb{A} = \frac{1}{2}\omega_{ab}J^{ab} + \Gamma_a P^a \quad \pi (Lie(\mathcal{P}))$ -valued one–form on M,

•
$$\omega_{ab} = -\omega_{ba}$$

 $\bullet \ \omega = \left(\omega^a{}_b \right), \quad \Gamma = \left(\Gamma^a \right),$

▲□ > ▲□ > ▲目 > ▲目 > ▲目 > ● ●

Marcin Kaźmierczak

University of Warsaw

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Poincaré group as a gauge group

•
$$\mathbb{A} = \frac{1}{2}\omega_{ab}J^{ab} + \Gamma_a P^a$$
 $\pi (Lie(\mathcal{P}))$ -valued one-form on M ,

•
$$\omega_{ab} = -\omega_{ba}$$

• $\omega = (\omega^a{}_b), \quad \Gamma = (\Gamma^a),$
• $\mathbb{A}' = \rho(g)\mathbb{A}\rho^{-1}(g) - d\rho(g)\rho^{-1}(g)$ and (1) imply

Marcin Kaźmierczak

< ≣ ► ≣ ৩ ৭ ৫ University of Warsaw

<ロ> <同> <同> < 同> < 同>

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Poincaré group as a gauge group

•
$$\mathbb{A} = \frac{1}{2}\omega_{ab}J^{ab} + \Gamma_a P^a \quad \pi (Lie(\mathcal{P}))$$
-valued one–form on M ,

•
$$\omega_{ab} = -\omega_{ba}$$

• $\omega = (\omega^a{}_b), \quad \Gamma = (\Gamma^a),$
• $\mathbb{A}' = \rho(g)\mathbb{A}\rho^{-1}(g) - d\rho(g)\rho^{-1}(g)$ and (1) imply

$$\omega' = \Lambda \omega \Lambda^{-1} - d\Lambda \Lambda^{-1}, \quad \Gamma' = \Lambda \Gamma - \omega' a - da \quad .$$
 (2)

<ロ > < 日 > < 日 > < 日 > < 日 > < 日 > < 日 > < 日 > < 日 > < 日 > の へ ○ University of Warsaw

Marcin Kaźmierczak

Introduction

The Poincaré group as a gauge group

General Relativity as a theory of gravitation

• The space-time is a manifold \mathcal{M} with a Lorentzian metric g.



Marcin Kaźmierczak

University of Warsaw

Introduction Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Poincaré group as a gauge group

General Relativity as a theory of gravitation

- The space-time is a manifold \mathcal{M} with a Lorentzian metric g.
- tetrad: $\tilde{e}_a = \tilde{e}^{\mu}_a \partial_{\mu}$, $g(\tilde{e}_a, \tilde{e}_b) = \eta_{ab}$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak
Modified coupling procedure

The Poincaré group as a gauge group

General Relativity as a theory of gravitation

- The space-time is a manifold \mathcal{M} with a Lorentzian metric g.
- tetrad: $\tilde{e}_a = \tilde{e}^{\mu}_a \partial_{\mu}$, $g(\tilde{e}_a, \tilde{e}_b) = \eta_{ab}$
- cotetrad: $e^a = e^a_\mu dx^\mu$, $e^a(\tilde{e}_b) = \delta^a_b$

Marcin Kaźmierczak

University of Warsaw

ヘロト ヘ回ト ヘヨト ヘヨト

Modified coupling procedure

The Poincaré group as a gauge group

General Relativity as a theory of gravitation

- The space-time is a manifold ${\mathcal M}$ with a Lorentzian metric g.
- tetrad: $\tilde{e}_a = \tilde{e}^{\mu}_a \partial_{\mu}$, $g(\tilde{e}_a, \tilde{e}_b) = \eta_{ab}$
- cotetrad: $e^a = e^a_\mu dx^\mu$, $e^a(\tilde{e}_b) = \delta^a_b$

•
$$g = \eta_{ab} e^a \otimes e^b$$

ヘロト ヘ回ト ヘヨト ヘヨト

Poincaré Gauge Theory of Gravity

Marcin Kaźmierczak

Modified coupling procedure

The Poincaré group as a gauge group

General Relativity as a theory of gravitation

- The space-time is a manifold ${\mathcal M}$ with a Lorentzian metric g.
- tetrad: $\tilde{e}_a = \tilde{e}^{\mu}_a \partial_{\mu}$, $g(\tilde{e}_a, \tilde{e}_b) = \eta_{ab}$
- cotetrad: $e^a = e^a_\mu dx^\mu$, $e^a(\tilde{e}_b) = \delta^a_b$

•
$$g = \eta_{ab} e^a \otimes e^b$$

$$e^{\prime a} = \Lambda^a{}_b e^b \equiv e^\prime = \Lambda e, \quad \Lambda \in O(1,3)$$
 (3)

ヘロト ヘ回ト ヘヨト ヘヨト

Marcin Kaźmierczak

Modified coupling procedure

The Poincaré group as a gauge group

General Relativity as a theory of gravitation

- The space-time is a manifold \mathcal{M} with a Lorentzian metric g.
- tetrad: $\tilde{e}_a = \tilde{e}^{\mu}_a \partial_{\mu}, \quad g\left(\tilde{e}_a, \tilde{e}_b\right) = \eta_{ab}$
- cotetrad: $e^a = e^a_\mu dx^\mu$, $e^a(\tilde{e}_b) = \delta^a_b$

•
$$g = \eta_{ab} e^a \otimes e^b$$

$$e'^a = \Lambda^a{}_b e^b \equiv e' = \Lambda e, \quad \Lambda \in O(1,3)$$
 (3)

• connection one–forms: $\omega^a{}_b = \Gamma^a_{bc} e^c$, $\nabla_{\tilde{e}_c} \tilde{e}_b = \Gamma^a_{bc} \tilde{e}_a$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Modified coupling procedure

The Poincaré group as a gauge group

General Relativity as a theory of gravitation

- The space-time is a manifold \mathcal{M} with a Lorentzian metric g.
- tetrad: $\tilde{e}_a = \tilde{e}_a^{\mu} \partial_{\mu}, \quad g(\tilde{e}_a, \tilde{e}_b) = \eta_{ab}$
- cotetrad: $e^a = e^a_\mu dx^\mu$, $e^a(\tilde{e}_b) = \delta^a_b$

•
$$g = \eta_{ab} e^a \otimes e^b$$

$$e^{\prime a} = \Lambda^a{}_b e^b \equiv e^\prime = \Lambda e, \quad \Lambda \in O(1,3)$$
 (3)

• connection one-forms: $\omega^a{}_b = \Gamma^a_{bc} e^c$, $\nabla_{\tilde{e}_c} \tilde{e}_b = \Gamma^a_{bc} \tilde{e}_a$ • $\omega' = \Lambda \omega \Lambda^{-1} - d\Lambda \Lambda^{-1}$,

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Modified coupling procedure

The Poincaré group as a gauge group

General Relativity as a theory of gravitation

- The space-time is a manifold \mathcal{M} with a Lorentzian metric g.
- tetrad: $\tilde{e}_a = \tilde{e}^{\mu}_a \partial_{\mu}, \quad g(\tilde{e}_a, \tilde{e}_b) = \eta_{ab}$
- cotetrad: $e^a = e^a_\mu dx^\mu$, $e^a(\tilde{e}_b) = \delta^a_b$

•
$$g = \eta_{ab} e^a \otimes e^b$$

$$e'^a = \Lambda^a{}_b e^b \equiv e' = \Lambda e, \quad \Lambda \in O(1,3)$$
 (3)

- connection one-forms: $\omega^a{}_b = \Gamma^a_{bc} e^c$, $\nabla_{\tilde{e}_c} \tilde{e}_b = \Gamma^a_{bc} \tilde{e}_a$
- $\omega' = \Lambda \omega \Lambda^{-1} d\Lambda \Lambda^{-1}$,

•
$$\omega_{ab} = -\omega_{ba} \quad \Leftrightarrow \quad \nabla g = 0.$$

イロト イヨト イヨト イヨト

Poincaré Gauge Theory of Gravity

Marcin Kaźmierczak

In order to construct a cotetrad within the framework of the Poincaré gauge theory, one has to introduce a vector–valued zero–form y^a on \mathcal{M} transforming under the gauge transformations as

• $y' = \Lambda y + a$.

・ロト・西ト・ヨト・ヨト ヨー ろくぐ

Marcin Kaźmierczak

University of Warsaw

In order to construct a cotetrad within the framework of the Poincaré gauge theory, one has to introduce a vector–valued zero–form y^a on \mathcal{M} transforming under the gauge transformations as

- $y' = \Lambda y + a$.
- $e := \Gamma + Dy$, $Dy = dy + \omega y \Rightarrow e' = \Lambda e$.

University of Warsaw

(I) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1))

Marcin Kaźmierczak

In order to construct a cotetrad within the framework of the Poincaré gauge theory, one has to introduce a vector–valued zero–form y^a on \mathcal{M} transforming under the gauge transformations as

- $y' = \Lambda y + a$.
- $e := \Gamma + Dy$, $Dy = dy + \omega y \Rightarrow e' = \Lambda e$.
- If $\mathfrak{L} = \mathfrak{L}(e(\Gamma, y, \omega), \omega, \phi)$, then it is justified to acknowledge e as a fundamental field.

ヘロト ヘ回ト ヘヨト ヘヨト

Marcin Kaźmierczak

In order to construct a cotetrad within the framework of the Poincaré gauge theory, one has to introduce a vector–valued zero–form y^a on \mathcal{M} transforming under the gauge transformations as

- $y' = \Lambda y + a$.
- $\bullet \ e := \Gamma + Dy, \quad Dy = dy + \omega y \quad \Rightarrow \quad e' = \Lambda e.$
- If $\mathfrak{L} = \mathfrak{L}(e(\Gamma, y, \omega), \omega, \phi)$, then it is justified to acknowledge e as a fundamental field.

•
$$\mathfrak{L} = \mathfrak{L}_G + \tilde{\mathfrak{L}}_m, \quad \mathfrak{L}_G = -\frac{1}{4k} \epsilon_{abcd} e^a \wedge e^b \wedge \Omega^{cd},$$

 $\Omega^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b.$

ヘロト ヘ回ト ヘヨト ヘヨト

Marcin Kaźmierczak

In order to construct a cotetrad within the framework of the Poincaré gauge theory, one has to introduce a vector–valued zero–form y^a on \mathcal{M} transforming under the gauge transformations as

•
$$y' = \Lambda y + a$$
.

- $e := \Gamma + Dy$, $Dy = dy + \omega y \Rightarrow e' = \Lambda e$.
- If $\mathfrak{L} = \mathfrak{L}(e(\Gamma, y, \omega), \omega, \phi)$, then it is justified to acknowledge e as a fundamental field.
- $\mathfrak{L} = \mathfrak{L}_G + \tilde{\mathfrak{L}}_m, \quad \mathfrak{L}_G = -\frac{1}{4k} \epsilon_{abcd} e^a \wedge e^b \wedge \Omega^{cd},$ $\Omega^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b.$
- GR: $\frac{1}{2}T^a{}_{bc}e^b \wedge e^c := de^a + \omega^a{}_b \wedge e^b = 0$

University of Warsaw

ヘロト ヘヨト ヘヨト ヘヨト

Marcin Kaźmierczak

In order to construct a cotetrad within the framework of the Poincaré gauge theory, one has to introduce a vector–valued zero–form y^a on \mathcal{M} transforming under the gauge transformations as

- $y' = \Lambda y + a$.
- $\bullet \ e := \Gamma + Dy, \quad Dy = dy + \omega y \quad \Rightarrow \quad e' = \Lambda e.$
- If $\mathfrak{L} = \mathfrak{L}(e(\Gamma, y, \omega), \omega, \phi)$, then it is justified to acknowledge e as a fundamental field.
- $\mathfrak{L} = \mathfrak{L}_G + \tilde{\mathfrak{L}}_m, \quad \mathfrak{L}_G = -\frac{1}{4k} \epsilon_{abcd} e^a \wedge e^b \wedge \Omega^{cd},$ $\Omega^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b.$

• GR:
$$\frac{1}{2}T^a{}_{bc}e^b \wedge e^c := de^a + \omega^a{}_b \wedge e^b = 0$$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Poincaré group as a gauge group

The field equations of the Einstein–Cartan theory

$$\begin{split} \frac{\delta \mathfrak{L}_G}{\delta e^a} &+ \frac{\delta \tilde{\mathfrak{L}}_m}{\delta e^a} = 0 \quad \Leftrightarrow \qquad G^a{}_b := R^a{}_b - \frac{1}{2}R\delta^a_b = k\,t_b{}^a \\ \frac{\delta \mathfrak{L}_G}{\delta \omega^{ab}} &+ \frac{\delta \tilde{\mathfrak{L}}_m}{\delta \omega^{ab}} = 0 \quad \Leftrightarrow \quad T^{cab} - T^a\eta^{bc} + T^b\eta^{ac} = kS^{abc} \\ \frac{\delta \tilde{\mathfrak{L}}_m}{\delta \phi} &= 0 \end{split}$$

where $R^a{}_b := \eta^{ac} R^d{}_{cdb}$, $R := R^a{}_a$, $T^a := T^{ba}{}_b$ and the dynamical definitions of energy–momentum and spin density tensors on Riemann–Cartan space are

$$t_{ab}e^b := -\star \frac{\delta \tilde{\mathfrak{L}}_m}{\delta e^a}, \quad S^{abc}e_c := 2 \star \frac{\delta \tilde{\mathfrak{L}}_m}{\delta \omega_{ab}}$$

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

1 Introduction — The Poincaré Gauge Theory

- 2 The ambiguity of MCP in the presence of torsion
 - Constructing a matter part of a Lagrangian
 - The Dirac field

3 Removing the ambiguity by modifying coupling procedure

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

Constructing a matter part of a Lagrangian

The addition of a divergence to the flat–space Lagrangian density is a symmetry transformation

 $\mathfrak{L}_m = \mathcal{L}_m d^4 x,$



University of Warsaw

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory ••••••• Modified coupling procedure

Constructing a matter part of a Lagrangian

The addition of a divergence to the flat–space Lagrangian density is a symmetry transformation

$$\mathfrak{L}_m = \mathcal{L}_m d^4 x,$$

$$\mathcal{L}_m \to \mathcal{L}'_m = \mathcal{L}_m + \partial_\mu V^\mu,$$
 (4)



University of Warsaw

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory ••••••• Modified coupling procedure

Constructing a matter part of a Lagrangian

The addition of a divergence to the flat–space Lagrangian density is a symmetry transformation

$$\mathfrak{L}_m = \mathcal{L}_m d^4 x,$$

$$\mathcal{L}_m \to \mathcal{L}'_m = \mathcal{L}_m + \partial_\mu V^\mu,$$
 (4)

$$\partial_{\mu}V^{\mu}\,d^4x = \pounds_V\,d^4x = d(V \lrcorner\,d^4x),$$

University of Warsaw

イロン イヨン イヨン イヨン

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

Constructing a matter part of a Lagrangian

The addition of a divergence to the flat–space Lagrangian density is a symmetry transformation

$$\mathfrak{L}_m = \mathcal{L}_m d^4 x,$$

$$\mathcal{L}_m \to \mathcal{L}'_m = \mathcal{L}_m + \partial_\mu V^\mu,$$
 (4)

$$\partial_{\mu}V^{\mu} d^4x = \pounds_V d^4x = d(V \lrcorner d^4x),$$

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

Constructing a matter part of a Lagrangian

The addition of a divergence to the flat–space Lagrangian density is a symmetry transformation

$$\mathfrak{L}_m = \mathcal{L}_m d^4 x,$$

$$\mathcal{L}_m \to \mathcal{L}'_m = \mathcal{L}_m + \partial_\mu V^\mu,$$
 (4)

< ロ > < 同 > < 回 > < 回 >

University of Warsaw

$$\partial_{\mu}V^{\mu} d^4x = \pounds_V d^4x = d(V \lrcorner d^4x),$$

- *£* denotes the Lie derivative,
 ⊥ the internal product,
 ★ the Hodge star of Minkowski metric.
- The field equations, as well as integrated Noether energy and momenta, remain unchanged.

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

Constructing a matter part of a Lagrangian

Introducing gravity

• How to construct $\tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A})$ from $\mathfrak{L}_m(\phi, d\phi)$?

Marcin Kaźmierczak

University of Warsaw

イロン イヨン イヨン イヨン

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

Constructing a matter part of a Lagrangian

Introducing gravity

- How to construct $\tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A})$ from $\mathfrak{L}_m(\phi, d\phi)$?
- A consistency requirement the transformation (4) is a symmetry of the resulting theory with gravity.

Marcin Kaźmierczak

University of Warsaw

A D > <
 A +
 A +
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

Constructing a matter part of a Lagrangian

Introducing gravity

- How to construct $\tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A})$ from $\mathfrak{L}_m(\phi, d\phi)$?
- A consistency requirement the transformation (4) is a symmetry of the resulting theory with gravity.
- MCP does not satisfy this requirement!

Image: A matrix

Poincaré Gauge Theory of Gravity

Marcin Kaźmierczak

Constructing a matter part of a Lagrangian

Introducing gravity

- How to construct $\tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A})$ from $\mathfrak{L}_m(\phi, d\phi)$?
- A consistency requirement the transformation (4) is a symmetry of the resulting theory with gravity.
- MCP does not satisfy this requirement!

•
$$\partial_{\mu}V^{\mu}d^{4}x \xrightarrow{MCP} d(V \lrcorner \epsilon) - T_{a}V^{a}\epsilon, \qquad T_{a} := T^{b}{}_{ab}.$$

Image: A matrix

Marcin Kaźmierczak

Constructing a matter part of a Lagrangian

Introducing gravity

- How to construct $\tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A})$ from $\mathfrak{L}_m(\phi, d\phi)$?
- A consistency requirement the transformation (4) is a symmetry of the resulting theory with gravity.
- MCP does not satisfy this requirement!

•
$$\partial_{\mu}V^{\mu}d^{4}x \xrightarrow{MCP} d(V \lrcorner \epsilon) - T_{a}V^{a}\epsilon, \qquad T_{a} := T^{b}{}_{ab}.$$

Image: A matrix

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

Constructing a matter part of a Lagrangian

Introducing gravity

- How to construct $\tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A})$ from $\mathfrak{L}_m(\phi, d\phi)$?
- A consistency requirement the transformation (4) is a symmetry of the resulting theory with gravity.
- MCP does not satisfy this requirement!
- $\partial_{\mu}V^{\mu}d^{4}x \xrightarrow{MCP} d(V \lrcorner \epsilon) T_{a}V^{a}\epsilon, \qquad T_{a} := T^{b}{}_{ab}.$

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

Constructing a matter part of a Lagrangian

Introducing gravity

- How to construct $\tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A})$ from $\mathfrak{L}_m(\phi, d\phi)$?
- A consistency requirement the transformation (4) is a symmetry of the resulting theory with gravity.
- MCP does not satisfy this requirement!

•
$$\partial_{\mu}V^{\mu}d^{4}x \xrightarrow{MCP} d(V \lrcorner \epsilon) - T_{a}V^{a}\epsilon, \qquad T_{a} := T^{b}{}_{ab}.$$

Image: A matrix

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

Constructing a matter part of a Lagrangian

Introducing gravity

- How to construct $\tilde{\mathfrak{L}}_m(\phi, d\phi, \mathbb{A})$ from $\mathfrak{L}_m(\phi, d\phi)$?
- A consistency requirement the transformation (4) is a symmetry of the resulting theory with gravity.
- MCP does not satisfy this requirement!
- $\partial_{\mu}V^{\mu}d^{4}x \xrightarrow{MCP} d(V \lrcorner \epsilon) T_{a}V^{a}\epsilon, \qquad T_{a} := T^{b}{}_{ab}.$

Any physical consequences?

< ロ > < 同 > < 回 > < 回 >

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

An example – the Dirac field

• $\mathfrak{L}_{F0} = -i (\star dx_{\mu}) \wedge \overline{\psi} \gamma^{\mu} d\psi - m \overline{\psi} \psi d^4 x$

▲口▶▲圖▶▲臣▶▲臣▶ 臣 のQC

Marcin Kaźmierczak

University of Warsaw

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

An example – the Dirac field

•
$$\mathfrak{L}_{F0} = -i (\star dx_{\mu}) \wedge \overline{\psi} \gamma^{\mu} d\psi - m \overline{\psi} \psi d^4 x$$

• $\gamma^{\mu} \gamma^{\nu} + \gamma^{\nu} \gamma^{\mu} = 2 \eta^{\mu\nu}, \quad \overline{\psi} := \psi^{\dagger} \gamma^0.$

Marcin Kaźmierczak

University of Warsaw

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

An example – the Dirac field

- $\mathfrak{L}_{F0} = -i (\star dx_{\mu}) \wedge \overline{\psi} \gamma^{\mu} d\psi m \overline{\psi} \psi d^4 x$
- $\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}, \quad \overline{\psi} := \psi^{\dagger}\gamma^{0}.$
- Invariant under the global action of $\ensuremath{\mathcal{P}}$

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

An example – the Dirac field

- $\mathfrak{L}_{F0} = -i (\star dx_{\mu}) \wedge \overline{\psi} \gamma^{\mu} d\psi m \overline{\psi} \psi d^4 x$
- $\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}, \quad \overline{\psi} := \psi^{\dagger}\gamma^{0}.$
- $\bullet\,$ Invariant under the global action of ${\cal P}\,$

$$\begin{aligned} x^{\mu} \to x^{\prime \mu} &= \Lambda^{\mu}{}_{\nu}x^{\nu} + a^{\mu}, \quad \psi \to \psi^{\prime} = S(\Lambda)\psi, \\ S(\Lambda(\varepsilon)) &:= \exp\left(-\frac{i}{4}\varepsilon_{\mu\nu}\Sigma^{\mu\nu}\right), \ \Sigma^{\mu\nu} &:= \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]. \end{aligned}$$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

An example – the Dirac field

•
$$\mathfrak{L}_{F0} = -i (\star dx_{\mu}) \wedge \overline{\psi} \gamma^{\mu} d\psi - m \overline{\psi} \psi d^4 x$$

•
$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}, \quad \overline{\psi} := \psi^{\dagger}\gamma^{0}.$$

• Invariant under the global action of $\ensuremath{\mathcal{P}}$

$$\begin{aligned} x^{\mu} \to x'^{\mu} &= \Lambda^{\mu}{}_{\nu}x^{\nu} + a^{\mu}, \quad \psi \to \psi' = S(\Lambda)\psi, \\ S(\Lambda(\varepsilon)) &:= \exp\left(-\frac{i}{4}\varepsilon_{\mu\nu}\Sigma^{\mu\nu}\right), \ \Sigma^{\mu\nu} &:= \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]. \end{aligned}$$

$$\bullet \ \mathfrak{L}_{F0} \quad \stackrel{MCP}{\longrightarrow} \quad \mathfrak{\tilde{L}}_{F0} = -i\left(\star e_{a}\right) \wedge \overline{\psi}\gamma^{a}D\psi - m\overline{\psi}\psi \epsilon$$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

An example – the Dirac field

•
$$\mathfrak{L}_{F0} = -i (\star dx_{\mu}) \wedge \overline{\psi} \gamma^{\mu} d\psi - m \overline{\psi} \psi d^4 x$$

•
$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}, \quad \overline{\psi} := \psi^{\dagger}\gamma^{0}.$$

 $\bullet\,$ Invariant under the global action of ${\cal P}\,$

$$x^{\mu} \to x'^{\mu} = \Lambda^{\mu}{}_{\nu}x^{\nu} + a^{\mu}, \quad \psi \to \psi' = S(\Lambda)\psi,$$
$$S(\Lambda(\varepsilon)) := \exp\left(-\frac{i}{4}\varepsilon_{\mu\nu}\Sigma^{\mu\nu}\right), \ \Sigma^{\mu\nu} := \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}].$$

•
$$\mathfrak{L}_{F0} \xrightarrow{MCP} \mathfrak{L}_{F0} = -i(\star e_a) \wedge \overline{\psi} \gamma^a D \psi - m \overline{\psi} \psi \epsilon$$

• $D\psi = d\psi - \frac{i}{4} \omega_{ab} \Sigma^{ab} \psi, \quad \epsilon = e^0 \wedge e^1 \wedge e^2 \wedge e^3.$

University of Warsaw

・ロト ・回 ト ・ヨト ・ヨ

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

• $\frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \psi} = 0 \quad \Leftrightarrow \quad \frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \overline{\psi}} = 0$



Marcin Kaźmierczak

University of Warsaw

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

•
$$\frac{\delta \tilde{\mathcal{L}}_{F0}}{\delta \psi} = 0 \quad \Leftrightarrow \quad \frac{\delta \tilde{\mathcal{L}}_{F0}}{\delta \overline{\psi}} = 0$$

Commonly accepted solution:



Marcin Kaźmierczak

University of Warsaw

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

•
$$\frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \psi} = 0 \quad \Leftrightarrow \quad \frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \overline{\psi}} = 0$$

Commonly accepted solution:

•
$$\mathfrak{L}_{FR} = -\frac{i}{2} (\star dx_{\mu}) \wedge \left(\overline{\psi}\gamma^{\mu}d\psi - \overline{d\psi}\gamma^{\mu}\psi\right) - m\overline{\psi}\psi d^{4}x$$

Marcin Kaźmierczak

University of Warsaw

(< ≥) < ≥)</p>

 $\langle \Box \rangle \langle \Box \rangle$
Minimal coupling procedure in Poincaré Gauge Theory 000000000

Modified coupling procedure

The Dirac field

•
$$\frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \psi} = 0 \quad \Leftrightarrow \quad \frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \overline{\psi}} = 0$$

- Commonly accepted solution:
- $\mathfrak{L}_{FR} = -\frac{i}{2} (\star dx_{\mu}) \wedge (\overline{\psi}\gamma^{\mu}d\psi \overline{d\psi}\gamma^{\mu}\psi) m\overline{\psi}\psi d^{4}x$ justified, since $\mathcal{L}_{FR} = \mathcal{L}_{F0} + \partial_{\mu}V^{\mu}, \quad V^{\mu} = -\frac{i}{2}\overline{\psi}\gamma^{\mu}\psi$



University of Warsaw

イロト イヨト イヨト イヨト

Minimal coupling procedure in Poincaré Gauge Theory 000000000

Modified coupling procedure

The Dirac field

•
$$\frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \psi} = 0 \quad \Leftrightarrow \quad \frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \overline{\psi}} = 0$$

- Commonly accepted solution:
- $\mathfrak{L}_{FR} = -\frac{i}{2} (\star dx_{\mu}) \wedge (\overline{\psi}\gamma^{\mu}d\psi \overline{d\psi}\gamma^{\mu}\psi) m\overline{\psi}\psi d^{4}x$ justified, since $\mathcal{L}_{FR} = \mathcal{L}_{F0} + \partial_{\mu}V^{\mu}, \quad V^{\mu} = -\frac{i}{2}\overline{\psi}\gamma^{\mu}\psi$

•
$$MCP \Rightarrow \tilde{\mathfrak{L}}_{FR} = -\frac{i}{2} (\star e_a) \wedge \left(\overline{\psi}\gamma^a D\psi - \overline{D\psi}\gamma^a\psi\right) - m\overline{\psi}\psi\epsilon.$$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

•
$$\frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \psi} = 0 \quad \Leftrightarrow \quad \frac{\delta \tilde{\mathfrak{L}}_{F0}}{\delta \overline{\psi}} = 0$$

- Commonly accepted solution:
- $\mathfrak{L}_{FR} = -\frac{i}{2} (\star dx_{\mu}) \wedge \left(\overline{\psi}\gamma^{\mu}d\psi \overline{d\psi}\gamma^{\mu}\psi\right) m\overline{\psi}\psi d^{4}x$
- justified, since $\mathcal{L}_{FR} = \mathcal{L}_{F0} + \partial_{\mu}V^{\mu}$, $V^{\mu} = -\frac{i}{2}\overline{\psi}\gamma^{\mu}\psi$
- $MCP \Rightarrow \tilde{\mathfrak{L}}_{FR} = -\frac{i}{2} (\star e_a) \wedge \left(\overline{\psi}\gamma^a D\psi \overline{D\psi}\gamma^a\psi\right) m\overline{\psi}\psi\epsilon.$
- $\tilde{\mathfrak{L}}_{FR}$ is real and the theory is well defined. But

< ロ > < 同 > < 回 > < 回 >

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

•
$$\frac{\delta \tilde{\mathcal{L}}_{F0}}{\delta \psi} = 0 \quad \Leftrightarrow \quad \frac{\delta \tilde{\mathcal{L}}_{F0}}{\delta \overline{\psi}} = 0$$

- Commonly accepted solution:
- $\mathfrak{L}_{FR} = -\frac{i}{2} (\star dx_{\mu}) \wedge \left(\overline{\psi}\gamma^{\mu}d\psi \overline{d\psi}\gamma^{\mu}\psi\right) m\overline{\psi}\psi d^{4}x$
- justified, since $\mathcal{L}_{FR} = \mathcal{L}_{F0} + \partial_{\mu}V^{\mu}$, $V^{\mu} = -\frac{i}{2}\overline{\psi}\gamma^{\mu}\psi$
- $MCP \Rightarrow \tilde{\mathfrak{L}}_{FR} = -\frac{i}{2} (\star e_a) \wedge \left(\overline{\psi} \gamma^a D \psi \overline{D} \overline{\psi} \gamma^a \psi\right) m \overline{\psi} \psi \epsilon.$
- $\tilde{\mathfrak{L}}_{FR}$ is real and the theory is well defined. But
- $\mathcal{L}_{FR} \longrightarrow \mathcal{L}_{FR} + \partial_{\mu} V^{\mu}$,

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

•
$$\frac{\delta \tilde{\mathcal{L}}_{F0}}{\delta \psi} = 0 \quad \Leftrightarrow \quad \frac{\delta \tilde{\mathcal{L}}_{F0}}{\delta \overline{\psi}} = 0$$

- Commonly accepted solution:
- $\mathfrak{L}_{FR} = -\frac{i}{2} (\star dx_{\mu}) \wedge (\overline{\psi}\gamma^{\mu}d\psi \overline{d\psi}\gamma^{\mu}\psi) m\overline{\psi}\psi d^{4}x$
- justified, since $\mathcal{L}_{FR} = \mathcal{L}_{F0} + \partial_{\mu}V^{\mu}$, $V^{\mu} = -\frac{i}{2}\overline{\psi}\gamma^{\mu}\psi$
- $MCP \Rightarrow \tilde{\mathfrak{L}}_{FR} = -\frac{i}{2} (\star e_a) \wedge (\overline{\psi} \gamma^a D \psi \overline{D} \overline{\psi} \gamma^a \psi) m \overline{\psi} \psi \epsilon.$
- $\tilde{\mathfrak{L}}_{FR}$ is real and the theory is well defined. But
- $\mathcal{L}_{FR} \longrightarrow \mathcal{L}_{FR} + \partial_{\mu} V^{\mu}$,
- $V^{\mu} = a J^{\mu}_{(V)} + b J^{\mu}_{(A)}, \quad a, b \in \mathbb{R},$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

•
$$\frac{\delta \tilde{\mathcal{L}}_{F0}}{\delta \psi} = 0 \quad \Leftrightarrow \quad \frac{\delta \tilde{\mathcal{L}}_{F0}}{\delta \overline{\psi}} = 0$$

- Commonly accepted solution:
- $\mathfrak{L}_{FR} = -\frac{i}{2} (\star dx_{\mu}) \wedge \left(\overline{\psi}\gamma^{\mu}d\psi \overline{d\psi}\gamma^{\mu}\psi\right) m\overline{\psi}\psi d^{4}x$
- justified, since $\mathcal{L}_{FR} = \mathcal{L}_{F0} + \partial_{\mu}V^{\mu}$, $V^{\mu} = -\frac{i}{2}\overline{\psi}\gamma^{\mu}\psi$

•
$$MCP \Rightarrow \tilde{\mathfrak{L}}_{FR} = -\frac{i}{2} (\star e_a) \wedge \left(\overline{\psi}\gamma^a D\psi - \overline{D\psi}\gamma^a\psi\right) - m\overline{\psi}\psi\epsilon.$$

• $\tilde{\mathfrak{L}}_{FR}$ is real and the theory is well defined. But

•
$$\mathcal{L}_{FR} \longrightarrow \mathcal{L}_{FR} + \partial_{\mu} V^{\mu}$$

•
$$V^{\mu} = a J^{\mu}_{(V)} + b J^{\mu}_{(A)}, \quad a, b \in \mathbb{R},$$

 $J^{\mu}_{(V)} = \overline{\psi} \gamma^{\mu} \psi, \quad J^{\mu}_{(A)} = \overline{\psi} \gamma^{\mu} \gamma^{5} \psi$

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

Effective Lagrangian for Einstein–Cartan theory with fermions

Exploiting the algebraic invertible relation between spin and torsion $T^{cab}-T^a\eta^{bc}+T^b\eta^{ac}=kS^{abc}, \qquad S^{abc}e_c:=2\star\frac{\delta\tilde{z}_m}{\delta\omega_{ab}}, \qquad \text{one obtaines}$

 $\mathfrak{L}_{eff} \! = \! \overset{\circ}{\mathfrak{L}}_{G} \! + \! \overset{\circ}{\mathfrak{L}}_{FR} \! + \! \left(C_{AA} \, J^{(A)}_{a} J^{a}_{(A)} \! + \! C_{AV} \, J^{(A)}_{a} J^{a}_{(V)} \! + \! C_{VV} \, J^{(V)}_{a} J^{a}_{(V)} \right) \! \epsilon \; ,$



Marcin Kaźmierczak

University of Warsaw

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

Effective Lagrangian for Einstein–Cartan theory with fermions

Exploiting the algebraic invertible relation between spin and torsion $T^{cab}-T^a\eta^{bc}+T^b\eta^{ac}=kS^{abc}, \qquad S^{abc}e_c:=2\star\frac{\delta\tilde{\Sigma}_m}{\delta\omega_{ab}}, \qquad \text{one obtaines}$

$$\mathfrak{L}_{eff} = \overset{\circ}{\mathfrak{L}}_{G} + \overset{\circ}{\mathfrak{L}}_{FR} + \left(C_{AA} J_{a}^{(A)} J_{(A)}^{a} + C_{AV} J_{a}^{(A)} J_{(V)}^{a} + C_{VV} J_{a}^{(V)} J_{(V)}^{a} \right) \epsilon ,$$

$$C_{AA} = \frac{3k}{16} \left(1 - 4b^2 \right) , \qquad C_{AV} = -\frac{3k}{2} ab , \qquad C_{VV} = -\frac{3k}{4} a^2 ,$$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

The Dirac field

Effective Lagrangian for Einstein–Cartan theory with fermions

Exploiting the algebraic invertible relation between spin and torsion $T^{cab} - T^a \eta^{bc} + T^b \eta^{ac} = kS^{abc}, \qquad S^{abc} e_c := 2 \star \frac{\delta \tilde{\Sigma}_m}{\delta \omega_{ab}}, \qquad \text{one obtaines}$ $\mathfrak{L}_{eff} = \overset{\circ}{\mathfrak{L}}_G + \overset{\circ}{\tilde{\mathfrak{L}}}_{FR} + \left(C_{AA} J_a^{(A)} J_{(A)}^a + C_{AV} J_a^{(A)} J_{(V)}^a + C_{VV} J_a^{(V)} J_{(V)}^a \right) \epsilon ,$ $C_{AA} = \frac{3k}{16} \left(1 - 4b^2 \right) , \qquad C_{AV} = -\frac{3k}{2} ab , \qquad C_{VV} = -\frac{3k}{4} a^2 ,$ $\left(i\gamma^a \overset{\circ}{\nabla}_a - m \right) \psi + \left[-2C_{AA} J_a^{(A)} \gamma^5 + C_{AV} \left(J_a^{(A)} - J_a^{(V)} \gamma^5 \right) + 2C_{VV} J_a^{(V)} \right] \gamma^a \psi = 0 .$

> <ロト < 部 > < 国 > < 国 > < 国 > < 国 > < 国 > の Q @ University of Warsaw

Marcin Kaźmierczak

Effective Lagrangian for Einstein–Cartan theory with fermions

 $\begin{array}{ll} \text{Exploiting the algebraic invertible relation between spin and torsion} \\ T^{cab}-T^a\eta^{bc}+T^b\eta^{ac}=kS^{abc}, \qquad S^{abc}e_c:=2\star\frac{\delta\tilde{\Sigma}_m}{\delta\omega_{ab}}, \qquad \text{one obtaines} \end{array}$

$$\mathfrak{L}_{eff} = \overset{\circ}{\mathfrak{L}}_{G} + \overset{\circ}{\mathfrak{L}}_{FR} + \left(C_{AA} J_{a}^{(A)} J_{(A)}^{a} + C_{AV} J_{a}^{(A)} J_{(V)}^{a} + C_{VV} J_{a}^{(V)} J_{(V)}^{a} \right) \epsilon ,$$

$$C_{AA} {=} \frac{3k}{16} \Bigl(1 {-} 4b^2 \Bigr) \ , \qquad C_{AV} {=} {-} \frac{3k}{2} ab \ , \qquad C_{VV} {=} {-} \frac{3k}{4} a^2 \ ,$$

$$\left(i \gamma^a \overset{\circ}{\nabla}_a - m \right) \psi + \left[-2 C_{AA} J_a^{(A)} \gamma^5 + C_{AV} \left(J_a^{(A)} - J_a^{(V)} \gamma^5 \right) + 2 C_{VV} J_a^{(V)} \right] \gamma^a \psi = 0 \ . \label{eq:eq:phi_a}$$

 $\mbox{Loop Quantum Gravity:} \ \ \mathfrak{L}_{G}{=}-\tfrac{1}{4k}\epsilon_{abcd}e^{a}\wedge e^{b}\wedge\Omega^{cd}+\tfrac{1}{2k\beta}e^{a}\wedge e^{b}\wedge\Omega_{ab} \ ,$

$$C_{AA} = \frac{3k\beta}{16(1+\beta^2)} \Big[4b + \beta(1-4b^2) \Big], \quad C_{AV} = \frac{3k\beta}{4(1+\beta^2)} a(1-2\beta b), \quad C_{VV} = -\frac{3k\beta^2}{4(1+\beta^2)} a^2.$$

<ロ> <回> <三> <三> <三> <三> <三> <三> <三> <三> <三</p>

University of Warsaw

Marcin Kaźmierczak

- 1 Introduction The Poincaré Gauge Theory
- 2 The ambiguity of MCP in the presence of torsion
- 3 Removing the ambiguity by modifying coupling procedure

- The general idea
- The Dirac field
- Conclusions

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The general idea

The generalisation of the YM construction

•
$$S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$$

▲□ > ▲□ > ▲目 > ▲目 > ▲目 > ● ○ ○ ○ ○

Marcin Kaźmierczak

University of Warsaw

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The general idea

The generalisation of the YM construction

- $S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$
- $d\phi \rightarrow \mathcal{D}\phi = d\phi + \mathcal{A}\phi$, where \mathcal{A} is a $Lin(\mathcal{V})$ -valued one-form field on M,

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The general idea

The generalisation of the YM construction

•
$$S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$$

•
$$d\phi \rightarrow \mathcal{D}\phi = d\phi + \mathcal{A}\phi$$
,
where \mathcal{A} is a $Lin(\mathcal{V})$ -valued one-form field on M ,

•
$$\mathcal{A} \to \mathcal{A}' = \rho(g)\mathcal{A}\rho^{-1}(g) - d\rho(g)\rho^{-1}(g).$$

University of Warsaw

イロト イヨト イヨト イヨト

Poincaré Gauge Theory of Gravity

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The general idea

The generalisation of the YM construction

- $S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$
- $d\phi \rightarrow \mathcal{D}\phi = d\phi + \mathcal{A}\phi$, where \mathcal{A} is a $Lin(\mathcal{V})$ -valued one-form field on M,

•
$$\mathcal{A} \to \mathcal{A}' = \rho(g)\mathcal{A}\rho^{-1}(g) - d\rho(g)\rho^{-1}(g).$$

• $\mathcal{A} = \mathbb{A} + \mathbb{B}(\mathbb{A}, e)$, where \mathbb{A} is $Ran(\pi)$ -valued and $\mathbb{B}(\mathbb{A}, e)$ is $Ran(\pi)^{\perp}$ -valued.

イロト イヨト イヨト イヨト

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The general idea

The generalisation of the YM construction

- $S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$
- $d\phi \rightarrow \mathcal{D}\phi = d\phi + \mathcal{A}\phi$, where \mathcal{A} is a $Lin(\mathcal{V})$ -valued one-form field on M,

•
$$\mathcal{A} \to \mathcal{A}' = \rho(g)\mathcal{A}\rho^{-1}(g) - d\rho(g)\rho^{-1}(g).$$

• $\mathcal{A} = \mathbb{A} + \mathbb{B}(\mathbb{A}, e)$, where \mathbb{A} is $Ran(\pi)$ -valued and $\mathbb{B}(\mathbb{A}, e)$ is $Ran(\pi)^{\perp}$ -valued.

•
$$\mathbb{A}' = \rho(g)\mathbb{A}\rho^{-1}(g) - d\rho(g)\rho^{-1}(g), \qquad e' = \Lambda e,$$

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The general idea

The generalisation of the YM construction

- $S_m[\phi] = \int \mathcal{L}_m(\phi, \partial_\mu \phi) d^4 x = \int \mathfrak{L}_m(\phi, d\phi)$
- $d\phi \rightarrow \mathcal{D}\phi = d\phi + \mathcal{A}\phi$, where \mathcal{A} is a $Lin(\mathcal{V})$ -valued one-form field on M,

•
$$\mathcal{A} \to \mathcal{A}' = \rho(g)\mathcal{A}\rho^{-1}(g) - d\rho(g)\rho^{-1}(g).$$

• $\mathcal{A} = \mathbb{A} + \mathbb{B}(\mathbb{A}, e)$, where \mathbb{A} is $Ran(\pi)$ -valued and $\mathbb{B}(\mathbb{A}, e)$ is $Ran(\pi)^{\perp}$ -valued.

•
$$\mathbb{A}' = \rho(g)\mathbb{A}\rho^{-1}(g) - d\rho(g)\rho^{-1}(g), \qquad e' = \Lambda e,$$

• $\mathbb{B}(\mathbb{A}', e') = \rho(g)\mathbb{B}(\mathbb{A}, e)\rho^{-1}(g).$

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The general idea

What is $Ran(\pi)^{\perp}$?

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Marcin Kaźmierczak

University of Warsaw

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The general idea

What is $Ran(\pi)^{\perp}$?

• $Lin(\mathcal{V}) = Ran(\pi) \oplus Ran(\pi)^{\perp} \Rightarrow \mathbb{A} \text{ and } \mathbb{B}(\mathbb{A}, e) \text{ are }$ uniquely determined by \mathcal{A} .

◆□ > ◆□ > ◆豆 > ◆豆 > ̄豆 − ����

Marcin Kaźmierczak

University of Warsaw

What is $Ran(\pi)^{\perp}$?

- $Lin(\mathcal{V}) = Ran(\pi) \oplus Ran(\pi)^{\perp} \Rightarrow \mathbb{A} \text{ and } \mathbb{B}(\mathbb{A}, e) \text{ are }$ uniquely determined by \mathcal{A} .
- $Ran(\pi)^{\perp}$ is not uniquely determined by this requirement.

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

What is $Ran(\pi)^{\perp}$?

- $Lin(\mathcal{V}) = Ran(\pi) \oplus Ran(\pi)^{\perp} \Rightarrow \mathbb{A} \text{ and } \mathbb{B}(\mathbb{A}, e) \text{ are }$ uniquely determined by \mathcal{A} .
- $Ran(\pi)^{\perp}$ is not uniquely determined by this requirement.
- if \mathcal{V} admits a ρ -invariant scalar product \langle, \rangle_{ρ} , such that $\forall v, w \in \mathcal{V}, \quad g \in G, \quad \langle \rho(g)v, \rho(g)w \rangle_{\rho} = \langle v, w \rangle_{\rho},$

Marcin Kaźmierczak

University of Warsaw

< ロ > < 同 > < 回 > < 回 >

What is $Ran(\pi)^{\perp}$?

- $Lin(\mathcal{V}) = Ran(\pi) \oplus Ran(\pi)^{\perp} \Rightarrow \mathbb{A} \text{ and } \mathbb{B}(\mathbb{A}, e) \text{ are uniquely determined by } \mathcal{A}.$
- $Ran(\pi)^{\perp}$ is not uniquely determined by this requirement.
- if \mathcal{V} admits a ρ -invariant scalar product \langle, \rangle_{ρ} , such that $\forall v, w \in \mathcal{V}, g \in G, \langle \rho(g)v, \rho(g)w \rangle_{\rho} = \langle v, w \rangle_{\rho}$, (e.g. $\overline{\psi}\gamma^{0}\phi$ for Dirac representation, $\eta_{\mu\nu}V^{\mu}W^{\nu}$ for vector representation etc.)

Marcin Kaźmierczak

ヘロト ヘ回ト ヘヨト ヘヨト

What is $Ran(\pi)^{\perp}$?

- $Lin(\mathcal{V}) = Ran(\pi) \oplus Ran(\pi)^{\perp} \Rightarrow A \text{ and } \mathbb{B}(\mathbb{A}, e) \text{ are uniquely determined by } \mathcal{A}.$
- $Ran(\pi)^{\perp}$ is not uniquely determined by this requirement.
- if \mathcal{V} admits a ρ -invariant scalar product \langle, \rangle_{ρ} , such that $\forall v, w \in \mathcal{V}, g \in G, \langle \rho(g)v, \rho(g)w \rangle_{\rho} = \langle v, w \rangle_{\rho},$ (e.g. $\overline{\psi}\gamma^{0}\phi$ for Dirac representation, $\eta_{\mu\nu}V^{\mu}W^{\nu}$ for vector representation etc.)

then the induced product $\langle\!\!\langle \,,\,\rangle\!\!\rangle_{\rho}$ on $Lin(\mathcal{V})$ satisfying $\langle\!\!\langle \,\rho(g)X\rho^{-1}(g),\rho(g)Y\rho^{-1}(g)\,\rangle\!\!\rangle_{\rho} = \langle\!\!\langle \,X,Y\,\rangle\!\!\rangle_{\rho}$ can be used.

イロン イヨン イヨン ・

Marcin Kaźmierczak

What is $Ran(\pi)^{\perp}$?

- $Lin(\mathcal{V}) = Ran(\pi) \oplus Ran(\pi)^{\perp} \Rightarrow \mathbb{A} \text{ and } \mathbb{B}(\mathbb{A}, e) \text{ are uniquely determined by } \mathcal{A}.$
- $Ran(\pi)^{\perp}$ is not uniquely determined by this requirement.
- if \mathcal{V} admits a ρ -invariant scalar product \langle, \rangle_{ρ} , such that $\forall v, w \in \mathcal{V}, g \in G, \langle \rho(g)v, \rho(g)w \rangle_{\rho} = \langle v, w \rangle_{\rho},$ (e.g. $\overline{\psi}\gamma^{0}\phi$ for Dirac representation, $\eta_{\mu\nu}V^{\mu}W^{\nu}$ for vector representation etc.)

then the induced product $\langle\!\!\langle \,, \,\rangle\!\!\rangle_{\rho}$ on $Lin(\mathcal{V})$ satisfying $\langle\!\!\langle \, \rho(g) X \rho^{-1}(g), \rho(g) Y \rho^{-1}(g) \,\rangle\!\!\rangle_{\rho} = \langle\!\!\langle \, X, Y \,\rangle\!\!\rangle_{\rho}$ can be used.

• if the subspace $Ran(\pi) \subset Lin(\mathcal{V})$ is nondegenerate with respect to $\langle\!\langle, \rangle\!\rangle_{o}$, then $Lin(\mathcal{V}) = Ran(\pi) \oplus Ran(\pi)^{\perp}$.

< ロ > < 同 > < 回 > < 回 >

◆□> ◆□> ◆目> ◆目> ◆日> ◆□>

Marcin Kaźmierczak

University of Warsaw

The Dirac field

• $\mathcal{V} = \mathbb{C}^4$, $Ran(\pi)$ is spanned by $\Sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b]$,

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

The Dirac field

- $\mathcal{V} = \mathbb{C}^4$, $Ran(\pi)$ is spanned by $\Sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b]$,
- $\langle\!\langle X, Y \rangle\!\rangle_{\rho} = trace \left(\gamma^0 X^{\dagger} \gamma^0 Y\right), \ Ran(\pi)^{\perp}$ is spanned by $\mathbf{1}, \gamma^5, \gamma^a, \gamma^5 \gamma^a.$



Marcin Kaźmierczak

University of Warsaw

٠

The Dirac field

The Dirac field

• $\mathcal{V} = \mathbb{C}^4$, $Ran(\pi)$ is spanned by $\Sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b]$, • $\langle\!\langle X, Y \rangle\!\rangle_{\rho} = trace \left(\gamma^0 X^{\dagger} \gamma^0 Y\right)$, $Ran(\pi)^{\perp}$ is spanned by $\mathbf{1}, \gamma^5, \gamma^a, \gamma^5 \gamma^a$.

$$\begin{aligned} \mathcal{D}\psi &= d\psi + \mathcal{A}\psi, \quad \mathcal{A} = \mathbb{A} + \mathbb{B}, \\ \mathbb{A} &= -\frac{i}{4}\omega_{ab}\Sigma^{ab}, \quad \mathbb{B} &= \chi \mathbf{1} + \kappa \gamma^5 + \tau_a \gamma^a + \rho_a \gamma^5 \gamma^a, \end{aligned}$$

where χ , κ , τ_a , ρ_a are complex–valued one–forms on space–time.

Marcin Kaźmierczak

University of Warsaw

イロト イヨト イヨト イヨト

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

We will require that the Leibniz rule hold

Marcin Kaźmierczak

University of Warsaw

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

We will require that the Leibniz rule hold

۲

$$\begin{aligned} (\mathcal{D}\overline{\psi})\gamma^{a}\psi + \overline{\psi}\gamma^{a}\mathcal{D}\psi &= dJ^{a}_{(V)} + \tilde{\omega}^{a}{}_{b}J^{b}_{(V)}, \\ (\mathcal{D}\overline{\psi})\gamma^{a}\gamma^{5}\psi + \overline{\psi}\gamma^{a}\gamma^{5}\mathcal{D}\psi &= dJ^{a}_{(A)} + \tilde{\omega}^{a}{}_{b}J^{b}_{(A)}, \end{aligned}$$

where $D\overline{\psi} := (D\psi)^{\dagger}\gamma^0$ and $\tilde{\omega}^a{}_b$ is a modified connection on space–time. The solution:

Marcin Kaźmierczak

University of Warsaw

٥

۲

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

The Dirac field

We will require that the Leibniz rule hold

$$\begin{split} (\mathcal{D}\overline{\psi})\gamma^{a}\psi + \overline{\psi}\gamma^{a}\mathcal{D}\psi &= dJ^{a}_{(V)} + \tilde{\omega}^{a}{}_{b}J^{b}_{(V)}, \\ (\mathcal{D}\overline{\psi})\gamma^{a}\gamma^{5}\psi + \overline{\psi}\gamma^{a}\gamma^{5}\mathcal{D}\psi &= dJ^{a}_{(A)} + \tilde{\omega}^{a}{}_{b}J^{b}_{(A)}, \end{split}$$

where $D\overline{\psi} := (D\psi)^{\dagger}\gamma^0$ and $\tilde{\omega}^a{}_b$ is a modified connection on space–time. The solution:

$\tilde{\omega}^a{}_b = \omega^a{}_b + \lambda \delta^a_b, \quad \mathbb{B} = \frac{1}{2}\lambda \mathbf{1} + i\mu_1 \mathbf{1} + i\mu_2 \gamma^5,$

where λ, μ_1, μ_2 are real–valued one–forms.

University of Warsaw

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

٥

The Dirac field

Minimal coupling procedure in Poincaré Gauge Theory

Modified coupling procedure

We will require that the Leibniz rule hold

$$\begin{aligned} (\mathcal{D}\overline{\psi})\gamma^a\psi + \overline{\psi}\gamma^a\mathcal{D}\psi &= dJ^a_{(V)} + \tilde{\omega}^a{}_bJ^b_{(V)},\\ (\mathcal{D}\overline{\psi})\gamma^a\gamma^5\psi + \overline{\psi}\gamma^a\gamma^5\mathcal{D}\psi &= dJ^a_{(A)} + \tilde{\omega}^a{}_bJ^b_{(A)}, \end{aligned}$$

where $D\overline{\psi} := (D\psi)^{\dagger}\gamma^0$ and $\tilde{\omega}^a{}_b$ is a modified connection on space–time. The solution:

$$\tilde{\omega}^a{}_b = \omega^a{}_b + \lambda \delta^a_b, \quad \mathbb{B} = \frac{1}{2}\lambda \mathbf{1} + i\mu_1 \mathbf{1} + i\mu_2 \gamma^5,$$

where λ, μ_1, μ_2 are real–valued one–forms.

• μ_1 and μ_2 do not influence $\tilde{\omega}$. They could be hidden in the gauge fields corresponding to the localization of the global symmetry $\psi \rightarrow e^{i\alpha}\psi$ and the approximate symmetry $\psi \rightarrow e^{i\alpha\gamma^5}\psi$. We set $\mu_1 = \mu_2 = 0$.

・ロト ・回ト ・ヨト ・ヨト



• The procedure would be free of the ambiguity iff $\lambda = \mathbb{T}$, where $\mathbb{T} = T_a e^a$ is the torsion-trace-one-form.

Marcin Kaźmierczak

University of Warsaw

- E - M



- The procedure would be free of the ambiguity iff $\lambda = \mathbb{T}$, where $\mathbb{T} = T_a e^a$ is the torsion-trace-one-form.
- $\tilde{\omega}^{a}{}_{b} = \omega^{a}{}_{b} + \mathbb{T}\delta^{a}_{b}$ a modified connection on $\mathcal{M}(\omega, e)$,

イロト イヨト イヨト イヨト



- The procedure would be free of the ambiguity iff $\lambda = \mathbb{T}$, where $\mathbb{T} = T_a e^a$ is the torsion-trace-one-form.
- $\tilde{\omega}^{a}{}_{b} = \omega^{a}{}_{b} + \mathbb{T}\delta^{a}_{b}$ a modified connection on $\mathcal{M}(\omega, e)$,

•
$$\mathcal{D}\psi = d\psi - \frac{i}{4}\omega_{ab}\Sigma^{ab} + \frac{1}{2}\mathbb{T}.$$

イロト イヨト イヨト イヨト

Marcin Kaźmierczak

Introduction	Minimal coupling procedure in Poincaré Gauge Theory	Modified coupling procedure
The Dirac field		

• The procedure would be free of the ambiguity iff $\lambda = \mathbb{T}$, where $\mathbb{T} = T_a e^a$ is the torsion-trace-one-form.

$$\mathfrak{L}_{eff} = \overset{\circ}{\mathfrak{L}}_G + \overset{\circ}{\tilde{\mathfrak{L}}}_{FR} + \frac{3k}{16} J^{(A)}_a J^a_{(A)} \epsilon \; .$$

University of Warsaw

Marcin Kaźmierczak
Conclusions

< ロ ト < 団 ト < 臣 ト < 臣 ト ○

Marcin Kaźmierczak

University of Warsaw

Conclusions

 The modified coupling procedure provides a consistent method for coupling gravity to other field theories within the framework of the Poincaré gauge theory of gravity.

University of Warsaw

Introduction	
000000000000000000000000000000000000000	

Conclusions

- The modified coupling procedure provides a consistent method for coupling gravity to other field theories within the framework of the Poincaré gauge theory of gravity.
- As opposed to MCP, the results obtained do not depend on the choice of flat space Lagrangian from the class of equivalence corresponding to the possibility of adding divergence.

Marcin Kaźmierczak

University of Warsaw

Conclusions

- The modified coupling procedure provides a consistent method for coupling gravity to other field theories within the framework of the Poincaré gauge theory of gravity.
- As opposed to MCP, the results obtained do not depend on the choice of flat space Lagrangian from the class of equivalence corresponding to the possibility of adding divergence.
- In particular, the predictions of EC theory with fermions are made unique – they agree with those derived in earlier accounts for a particular choice of fermionic Lagrangian. The same concerns the predictions of EC theory modified by the presence of the Holst term.

< ロ > < 同 > < 回 > < 画

Marcin Kaźmierczak

M.K. "Nonuniqueness of gravity induced fermion interaction in the Einstein-Cartan theory",

Phys. Rev. D 78, 124025 (2008) arXiv:0811.1932v3.

M.K. "Einstein–Cartan gravity with Holst term and fermions", Phys. Rev. D **79**, 064029 (2009) arXiv:0812.1298v4.

M.K. "Modified coupling procedure for the Poincaré gauge theory of gravity"

(to be published in Phys. Rev. D).

University of Warsaw

.

< <p>O > < <p>O >

Marcin Kaźmierczak