# Boost-invariant early times dynamics – far from equilibrium physics and AdS/CFT.

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Based on ... [hep-th] (to be posted soon)

# Idea

- heavy ion collisions @ RHIC - strongly coupled quark-gluon plasma ( QGP )
- fully dynamical process - need for a new tool
- idea: exchange

  $$\text{QCD in favor of } \mathcal{N} = 4 \text{ SYM}$$

  and use the gravity dual
- there are differences
  - SUSY
  - conformal symmetry at the quantum level
  - no confinement...
- ... but not very important at high temperature

- RHIC suggests that QGP behaves as an almost perfect fluid
- there has been an enormous progress in understanding

  QGP hydrodynamics with the AdS/CFT

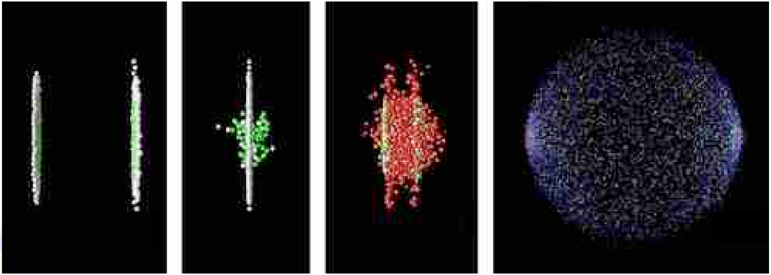
- can the AdS/CFT be used to shed light on

  far from equilibrium part of the QGP dynamics?

- maybe, but only at $\lambda \gg 1$!
- let's focus on

  the boost-invariant flow

  and use the AdS/CFT to grab some non-equilibrium physics.



- one-dimensional expansion along the collision axis $x^1$
- natural coordinates
  - proper time $\tau$ and rapidity $y$
  - $x^0 = \tau \cosh y$, $x^1 = \tau \sinh y$
- boost invariance (no rapidity dependence)

# AdS/CFT correspondence

**Gauge-gravity duality is an equivalence between**

$\mathcal{N} = 4$ **Supersymmetric Yang-Mills in** $\mathbb{R}^{1,3}$

- strong coupling
- non-perturbative results
- gauge theory operators

**Superstrings in curved** $\mathbf{AdS_5 \times S^5}$ **10D spacetime**

- (super)gravity regime
- classical behavior
- supergravity fields

AdS/CFT dictionary relates
**energy-momentum tensor** of $\mathcal{N} = 4$ SYM to 5D **AdS metric**

# Holographic reconstruction of spacetime

- $AdS_5$ metric in Fefferman-Graham gauge takes the form

$$ds^2 = m_{AB}\, dx^A dx^B = \frac{dz^2 + g_{\mu\nu}\, dx^\mu dx^\nu}{z^2}$$

where $z = 0$ corresponds to the boundary of AdS

- Einstein equations

$$\mathcal{G}_{AB} = \mathcal{R}_{AB} - \frac{1}{2}\mathcal{R} \cdot m_{AB} - 6\, m_{AB} = 0$$

can be solved near boundary given the boundary metric (here assumed to be $\mathbb{R}^{1,3}$) and any traceless and conserved $\langle T_{\mu\nu} \rangle$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} \left\{ = \eta_{\mu\nu} \right\} + z^4 g^{(4)}_{\mu\nu} \left\{ = \frac{2\pi^2}{N_c^2} \langle T_{\mu\nu} \rangle \right\} + g^{(6)}_{\mu\nu} \left( \langle T_{\alpha\beta} \rangle \right) z^6 + \ldots$$

- however most $\langle T_{\mu\nu} \rangle$ will lead to singularities in the bulk

# Gravity dual to the boost-invariant flow

- the energy-momentum tensor is specified by $\epsilon(\tau)$

$$T^{\mu\nu} = \mathrm{diag}\left\{\epsilon(\tau), -\frac{1}{\tau^2}\epsilon(\tau) - \frac{1}{\tau}\epsilon'(\tau), \epsilon(\tau) + \frac{1}{2}\tau\epsilon'(\tau)_\perp\right\}$$

- this suggests the metric Ansatz for the gravity dual

$$\mathrm{d}s^2 = \frac{-e^{a(\tau,z)}\mathrm{d}\tau^2 + \tau^2 e^{b(\tau,z)}\mathrm{d}y^2 + e^{c(\tau,z)}\mathrm{d}\mathbf{x}_\perp^2 + \mathrm{d}z^2}{z^2}$$

- Einstein equations

$$\mathcal{G}_{AB} = \mathcal{R}_{AB} - \frac{1}{2}\mathcal{R}\cdot m_{AB} - 6\, m_{AB} = 0$$

cannot be solved exactly ($\rightarrow$ numerics)

- however there are two regimes

$\tau \gg 1$ or $\tau \approx 0$

where analytic calculations can be done

# $\tau \gg 1$ and hydrodynamics [hep-th/0512162]

- asymptotic energy density is given by the ideal hydrodynamics

$$\epsilon'(\tau) = -\frac{\epsilon(\tau) + p(\tau)}{\tau}$$

which leads to the behavior (as $\tau \to \infty$)

$$\epsilon(\tau) \sim \frac{1}{\tau^{4/3}} + \ldots$$

- this result has been obtained using AdS/CFT assuming

$$\epsilon(\tau) \sim \frac{1}{\tau^{s}} + \ldots$$

and fixing $s$ from the nonsingularity of the dual geometry

- recovering large-$\tau$ behavior in the bulk requires taking

the scaling limit $\tau \to \infty$ while $v = z \cdot \tau^{-s/4}$ fixed

# Scaling variable doesn't work @ $\tau \approx 0$

- let's start with $\epsilon(\tau) \sim \frac{1}{\tau^s}$ and solve $\mathcal{G}_{AB}$ near the boundary

$$a(\tau, z) = \tilde{a}_0(\tau)\, z^4 + \tilde{a}_2(\tau)\, z^6 + \tilde{a}_4(\tau)\, z^8 + \dots$$

- for $\tau \gg 1$ certain terms dominate at each $z^{4+2k}$ and picking them gives

$$a(\tau, z) = f\left(\frac{z}{\tau^{s/4}}\right)$$

- for $\tau \approx 0$ other terms dominate leading to

$$a(\tau, z) = \frac{z^4}{\tau^s} \cdot \tilde{f}\left(\frac{z}{\tau}\right)$$

- `arXiv:0705.1234` argued that $s$ has to be 0 in this case
- this is wrong, since each term in this scaling expansion is

multiplied by $s$, thus vanishes identically

# Initial conditions and early times expansion of $\epsilon(\tau)$

- warp factors can be solved near the boundary given $\epsilon(\tau)$

$$a(\tau, z) = -\epsilon(\tau) \cdot z^4 + \left\{ -\frac{\epsilon'(\tau)}{4\tau} - \frac{\epsilon''(\tau)}{12} \right\} \cdot z^6 + \dots$$

- for $\epsilon(\tau) = \epsilon_0 + \epsilon_1 \tau + \epsilon_2 \tau^2 + \epsilon_3 \tau^3 + \epsilon_4 \tau^4 + \epsilon_5 \tau^5 + \dots$
  all $\epsilon_{2k+1}$ must vanish, otherwise $a(0, z) \to \infty$
- setting $\tau$ to zero in $a(\tau, z)$ for

$$\epsilon(\tau) = \epsilon_0 + \epsilon_2 \tau^2 + \epsilon_4 \tau^4 + \dots$$

  gives

$$a(0, z) = a_0(z) = \epsilon_0 \cdot z^4 + \frac{2}{3}\epsilon_2 \cdot z^6 + \left( \frac{\epsilon_4}{2} - \frac{\epsilon_0^2}{6} \right) \cdot z^8 + \dots$$

- it defines map between initial profiles in the bulk and $\epsilon(\tau)$

- warp factors $a, b$ and $c(\tau, z)$ have $\tau \approx 0$ expansion

$$a(\tau, z) = a_0(z) + \tau^2 a_2(z) + \tau^4 a_4(z) \dots$$

- both $\mathcal{G}_{\tau z}$ and $\mathcal{G}_{zz}$ at $\tau = 0$ are constraints equations
- $\mathcal{G}_{\tau z}$ forces $b_0(z) = a_0(z)$ whereas $\mathcal{G}_{zz}$ takes the form

$$v'(z) + w'(z) + 2z\left\{v(z)^2 + w(z)^2\right\} = 0$$

where $v(z) = \frac{1}{4z} a_0'(z)$ and $w(z) = \frac{1}{4z} c_0'(z)$

- this equation does not have any regular solution

$$\int_0^\infty (v' + w')\,\mathrm{d}z + 2\int_0^\infty (v^2 + w^2) z\,\mathrm{d}z = \int_0^\infty (v^2 + w^2) z\,\mathrm{d}z = 0$$

what is then the allowed set of initial data?

# Allowed initial conditions

- contraint equation

$$v'(z) + w'(z) + 2\,z\left\{v(z)^2 + w(z)^2\right\} = 0$$

can be solved using $v_+ = -w - v$ and $v_- = w - v$

$$v_-\left(u = z^2\right) = \sqrt{2v_+'(u) - v_+(u)^2}$$

- the regularity of $\mathcal{R}_{ABCD}\mathcal{R}^{ABCD}$ @ $\tau = 0$ fixes $v_+(u)$ to be

$$v_+(u) = \frac{2\epsilon_0\,u_0}{3}\cdot\frac{u^3}{u_0 - u}f(u)$$

where $f(0) = 1,\ f(u_0) = \frac{3}{2u_0^4\epsilon_0}$ and otherwise just regular

- the space of allowed initial data is parametrized by

all $v_+(u)$ satisfying above conditions

# Resummation of the energy density

- energy density power series @ $\tau = 0$

$$\epsilon(\tau) = \epsilon_0 + \epsilon_2 \tau^2 + \ldots + \epsilon_{2N_{cut}} \tau^{2N_{cut}} + \ldots$$

has a finite radius of convergence and thus

a resummation is needed

- presumably the simplest can be given by Pade approximation

$$\epsilon_{\text{approx}}(\tau)^3 = \frac{\epsilon_U^{(0)} + \epsilon_U^{(2)} \tau^2 + \ldots + \epsilon_U^{(N_{cut}-2)} \tau^{N_{cut}-2}}{\epsilon_D^{(0)} + \epsilon_D^{(2)} \tau^2 + \ldots + \epsilon_D^{(N_{cut}-2)} \tau^{N_{cut}+2}}$$

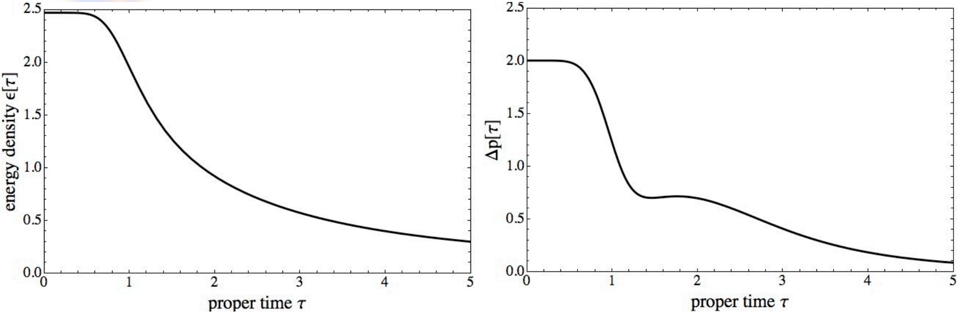
which uses the uniqueness of the asymptotic behavior

$$\epsilon \sim \frac{1}{\tau^{4/3}}$$

- nice example of allowed initial profile is given by

$$v_+(u) = \frac{\pi}{2}\tan\left(\frac{\pi}{2}u\right) - \frac{\pi}{2}\tanh\left(\frac{\pi}{2}u\right)$$

leading to the following $\epsilon(\tau)$ and $\Delta p(\tau) = 1 - \frac{p_{\parallel}(\tau)}{p_{\perp}(\tau)}$



**Results:**

- AdS/CFT is indispensable not only near equilibrium
- early time dynamics is not governed by the scaling limit
- gravity dual at $\tau = 0$ sets $\epsilon(\tau) = \epsilon_0 + \epsilon_2 \cdot \tau^2 + \ldots$
- simple resummation recovers reach dynamics

**Open questions:**

- numerics starting from some initial data
- towards colliding shock-waves