Flux tubes as strings

$\longleftrightarrow$

Confining flux tubes in SU(N) gauge theories and their effective string theory description

- Veneziano amplitude
- 't Hooft large-N – genus diagram expansion
- Polyakov action
- Maldacena ... AdS/CFT/QCD ...

at large N, flux tubes and perhaps the whole gauge theory can be described by a weakly-coupled string theory

Numerical calculations – a long history:

Euclidean $D = 3, 4$

• potential between static sources e.g. in $D = 3 + 1$

$$V(r) = -\frac{c_f \alpha_s(r)}{r} \qquad r \ll \frac{1}{\sqrt{\sigma}}$$

$$V(r) = \sigma r - \frac{\pi(D-2)}{24r} + O\left(\frac{1}{r^3}\right) \qquad r \gg \frac{1}{\sqrt{\sigma}}$$

and corresponding excitations.

• flux tubes wound around a spatial torus

$$E(l) = \sigma l - \frac{\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right) \qquad l \geq l_c = \frac{1}{T_c}$$

and corresponding excitations.

• ratios of Wilson loops vs Nambu-Goto

recent work :
e.g. Luscher, Weisz, Caselle, Gliozzi, Sommer, Necco, Kuti, Meyer, Bringoltz, Majumdar , Lucini, MT, ... and collaborators

older work – from early 80's :
e.g. Creutz, Copenhagen group, Michael, Schierholz, Bali, ... and collaborators

Here focus on the spectrum of flux tubes

closed around a spatial torus of length l

— mainly D=2+1, but also D=3+1

- flux localised in 'tubes' $\forall l \geq l_c = 1/T_c$
- at $l = l_c$ there is a phase transition: first order for $N \geq 3$ in $D = 4$ and for $N \geq 4$ in $D = 3$
- so may have a simple string description of the closed string spectrum for all possible lengths
- most plausible at $N \to \infty$ where complications such as mixing, e.g string $\to$ string + glueball, go away

Note: the static potential $V(r)$ describes the transition in r between UV (Coulomb potential) and IF (flux tubes) physics; potentially of great interest as $N \to \infty$.

based on recent work in collaboration with:

∘ Spectrum of fundamental and $k > 1$ strings in $D = 3 + 1$ SU(N) gauge theories:
A. Athenodorou, B. Bringoltz, M.Teper : in progress

∘ Spectrum of fundamental and $k = 2$ strings in $D = 2 + 1$ SU(N) gauge theories:
A. Athenodorou, B. Bringoltz, M.Teper : arXiv:0812.0334 , arXiv:0709.0693

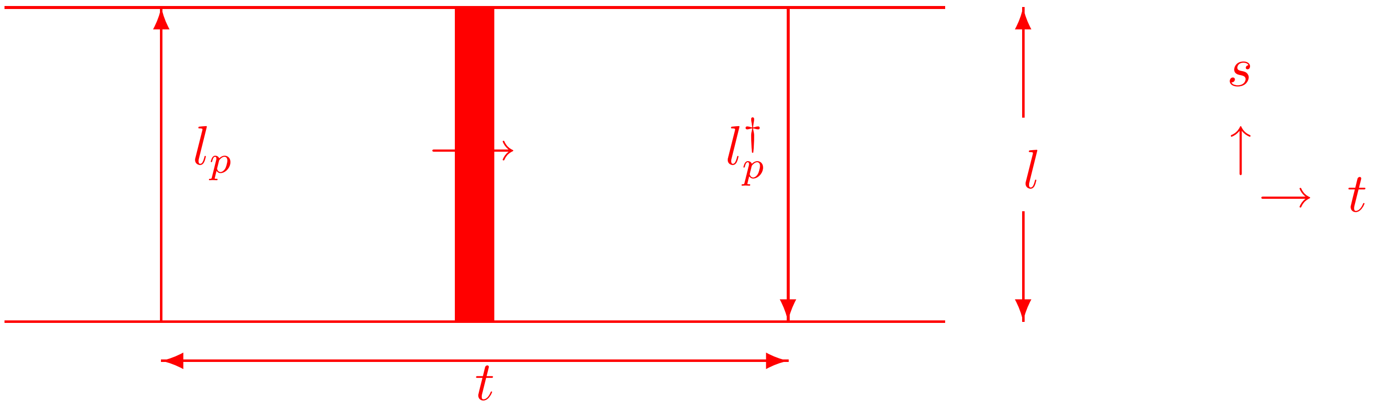
∘ Ground state fundamental and $k > 1$ strings in $D = 2 + 1$ SU(N) gauge theories:
B. Bringoltz, M.Teper : hep-th/0611286 arXiv:0802.1490

— also earlier work with Lucini, Meyer, Wenger

Calculate the mass of a confining flux tube winding around a spatial torus of length l, using correlators of Polyakov loops:

$$\langle l_p^\dagger(t) l_p(0)\rangle \stackrel{t\to\infty}{\propto} \exp\{-m_p(l)t\}$$

in pictures



where we expect, for linear confinement,

$$m_p(l) \stackrel{l\to\infty}{=} \sigma l - c\frac{\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right)$$

where $c = 1$ if only massless modes are from transverse translations

$\circ$ linear confinement

$\Rightarrow \sigma l$

$\circ$ spontaneous breaking of (transverse) translation invariance

$\Rightarrow -\frac{\pi(D-2)}{6l}$

from the sum of zero-point energies of the massless (Goldstone) transverse oscillations – the Luscher correction

$\circ$ any other massless modes on the flux tube

$\Rightarrow$ further $O(1/l)$ contributions

$\Longrightarrow$

determine the coefficient of the $O(1/l)$ Luscher correction in order to determine whether a long flux tube is described by an effective bosonic string theory simplest example is Nambu-Goto in flat space-time; this is a free string theory and it is 'sick' outside $D = 26$

but

the diseases are invisible if we focus on the sector of states built on a single long string

e.g. P. Olesen, PLB160 (1985) 144; J. Polchinski, A. Strominger, PRL67 (1991) 1681.

$\Longrightarrow$

the gound state energy: J. Arvis, PLB 127 (1983) 127

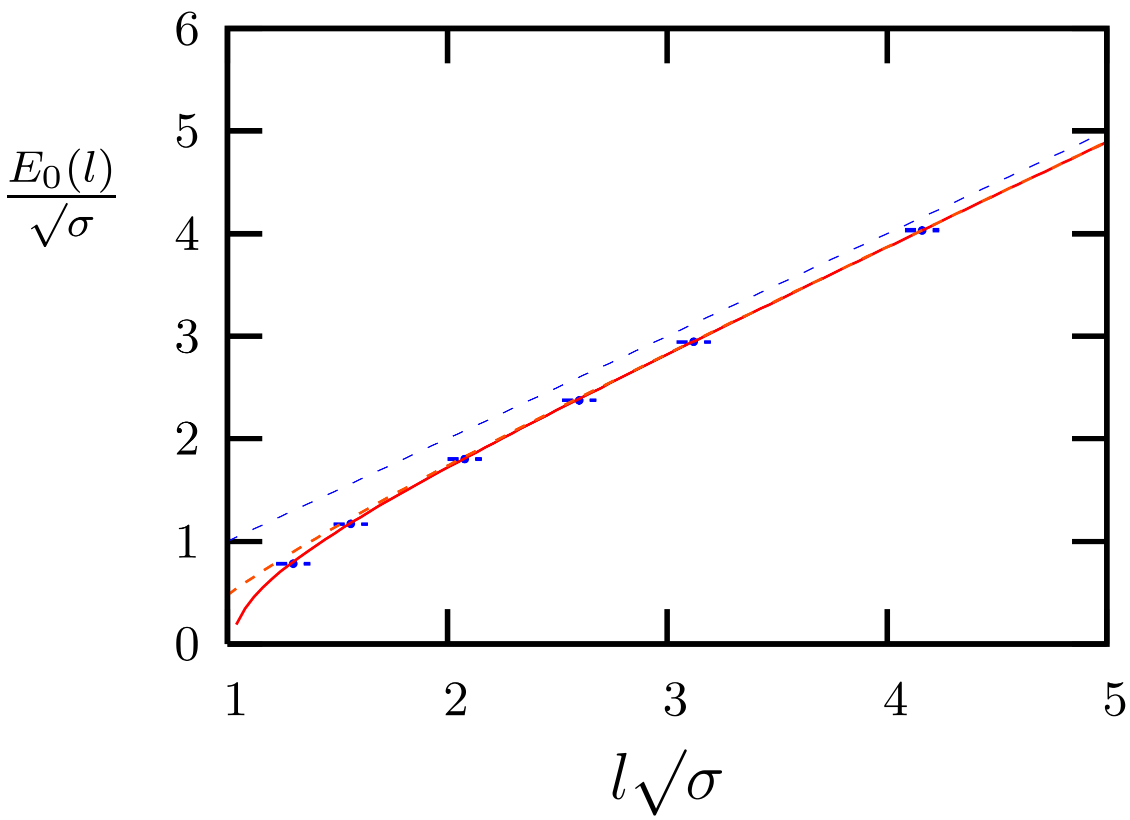
$$\begin{aligned} E(l) &= \sigma l \left(1 - \frac{\pi(D-2)}{3\sigma l^2}\right)^{\frac{1}{2}} \\ &= \sigma l - \frac{\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right) \quad : \quad l \geq \sqrt{\frac{\pi(D-2)}{3\sigma}} \end{aligned}$$

$\Longrightarrow$

so how well does this describe the actual $E(l)$?

D=2+1 ; SU(5)

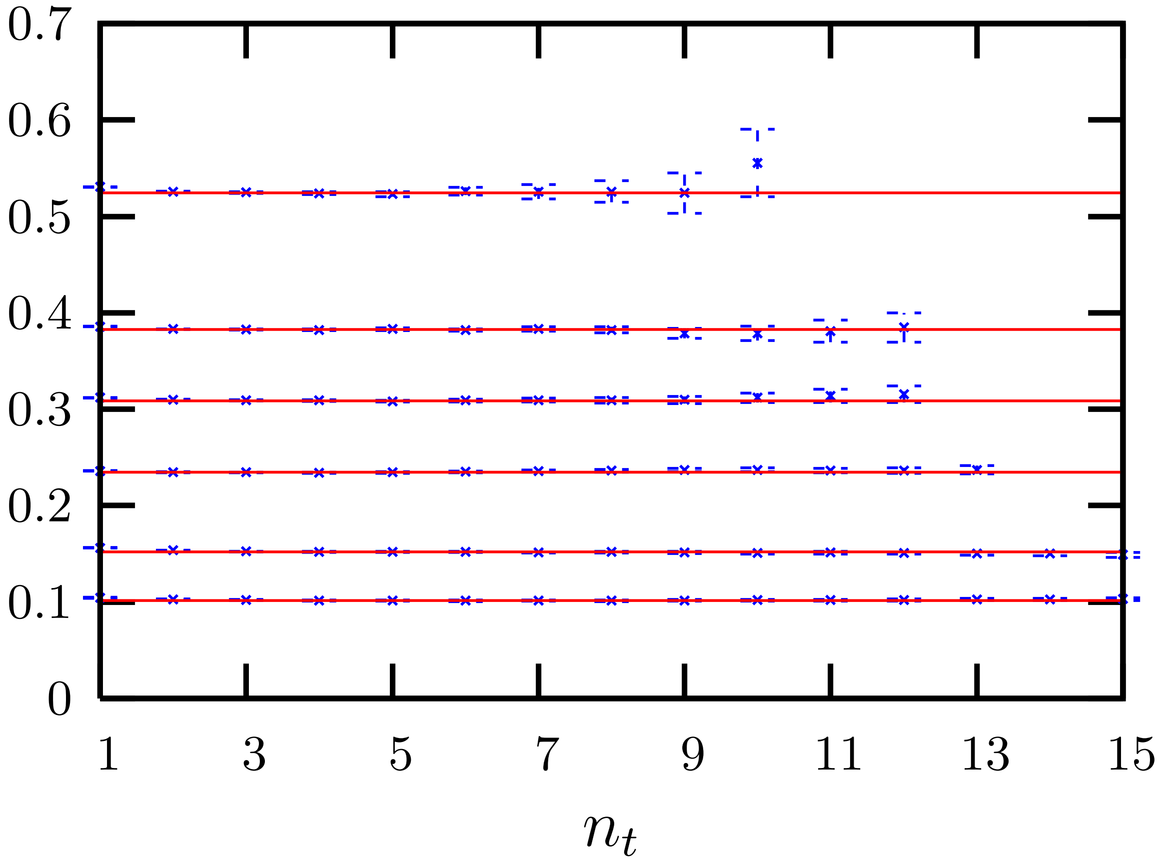
$a\sqrt{\sigma} \simeq 0.130 \quad ; \quad l_c\sqrt{\sigma} \simeq 1.07$



...Luscher: $E_0(l) = \sigma l - \frac{\pi}{6l}$; -Nambu-Goto:- $E_0(l) = \sigma l \left(1 - \frac{\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$

how good are the energies?

$aE_{eff}(t) = -\ln C(t)/C(t-1)$



lattice sizes and MC 'statistics'

transverse and longitudinal sizes need to be large enough in units of the loop mass i.e. increase as $l \downarrow$

l	lattice	MC sweeps
10	$10 \times 40 \times 120$	$0.5M$
12	$12 \times 32 \times 80$	$1.0M$
16	$16 \times 32 \times 56$	$1.5M$
20	$20 \times 32 \times 40$	$2.0M$
24	$24 \times 24 \times 32$	$2.0M$
32	$32 \times 32 \times 32$	$1.5M$

effective string theory – universality class?

central charge appears in the string 'Casimir' energy

$$E_0(l) \stackrel{l\to\infty}{=} \sigma l - \frac{c\pi(D-2)}{6l}$$

where e.g.

$$c = 1,\ 1.5,\ 0$$

for bosonic, Neveu-Schwartz, Ramond strings respectively

to determine the central charge numerically, fit c to neighbouring values of l obtaining a $c_{eff}(l)$ such that

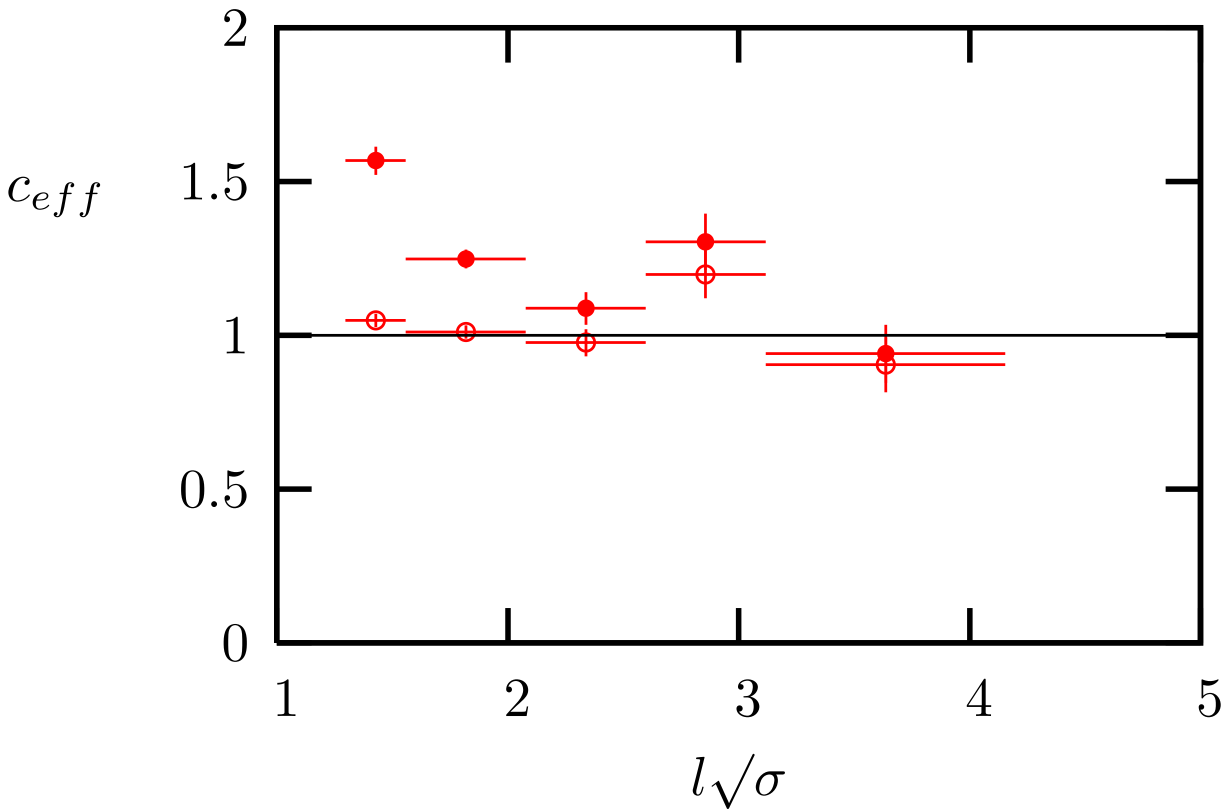
$$c_{eff}(l) \stackrel{l\to\infty}{\longrightarrow} c$$

and do the same for Nambu-Goto using:

$$E_0(l) = \sigma l \left(1 - c_{eff}\frac{\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$$

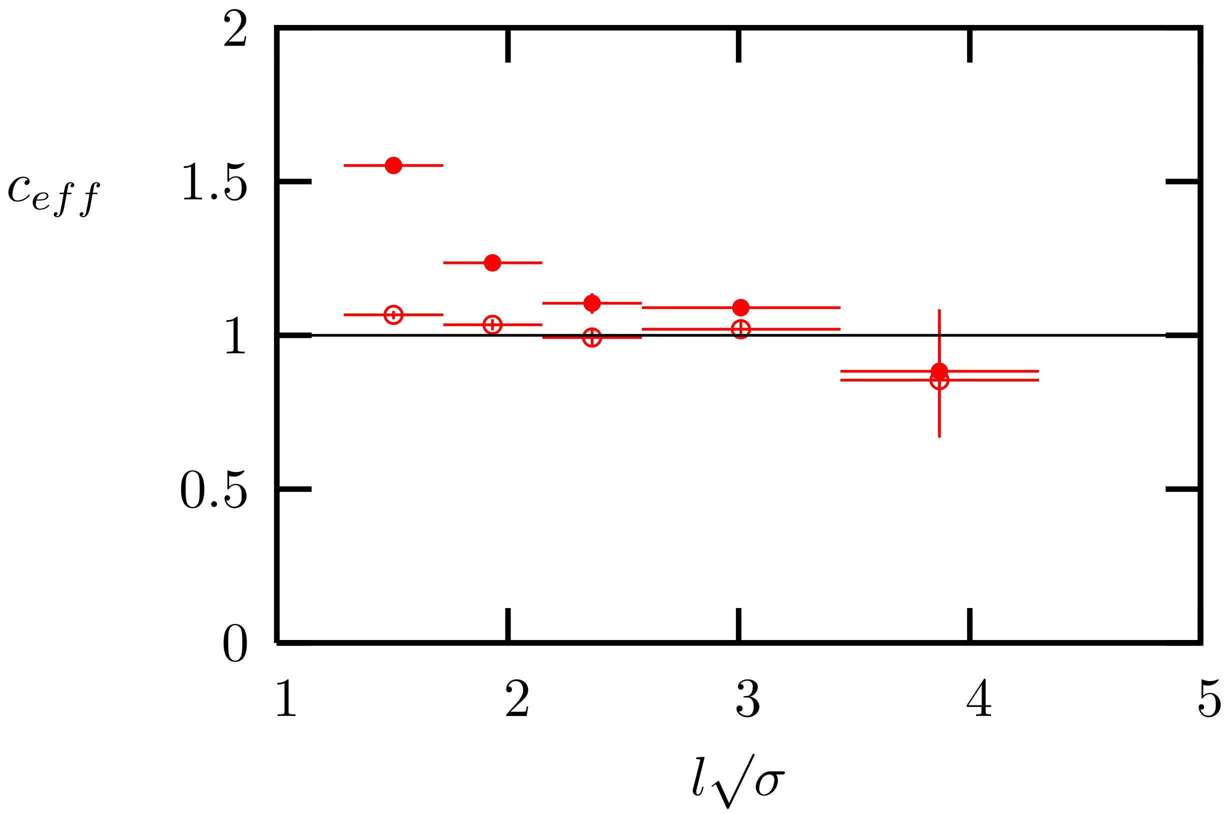
we refer to these as Fit 1 and Fit 2 respectively

SU(5) : $l_c\sqrt{\sigma} \simeq 1.07$



c_{eff} : from Luscher $\bullet$, and from Nambu-Goto $\circ$

similarly for SU(4) : $l_c\sqrt{\sigma} \simeq 1.08$



c_{eff} : from Luscher ● , and from Nambu-Goto ○

$\Rightarrow$

• fitting the ground state energy of the flux tube with just the Luscher correction, we see good evidence that

$$c_{eff}(l \to \infty) \to 1$$

i.e. that the only massless mode of the confining flux tube is the massless mode associated with the spontaneous breaking of translations transverse to the string

but the significant deviation of $c_{eff}(l)$ from unity at smaller l, suggests substantial higher order corrections to the Luscher term

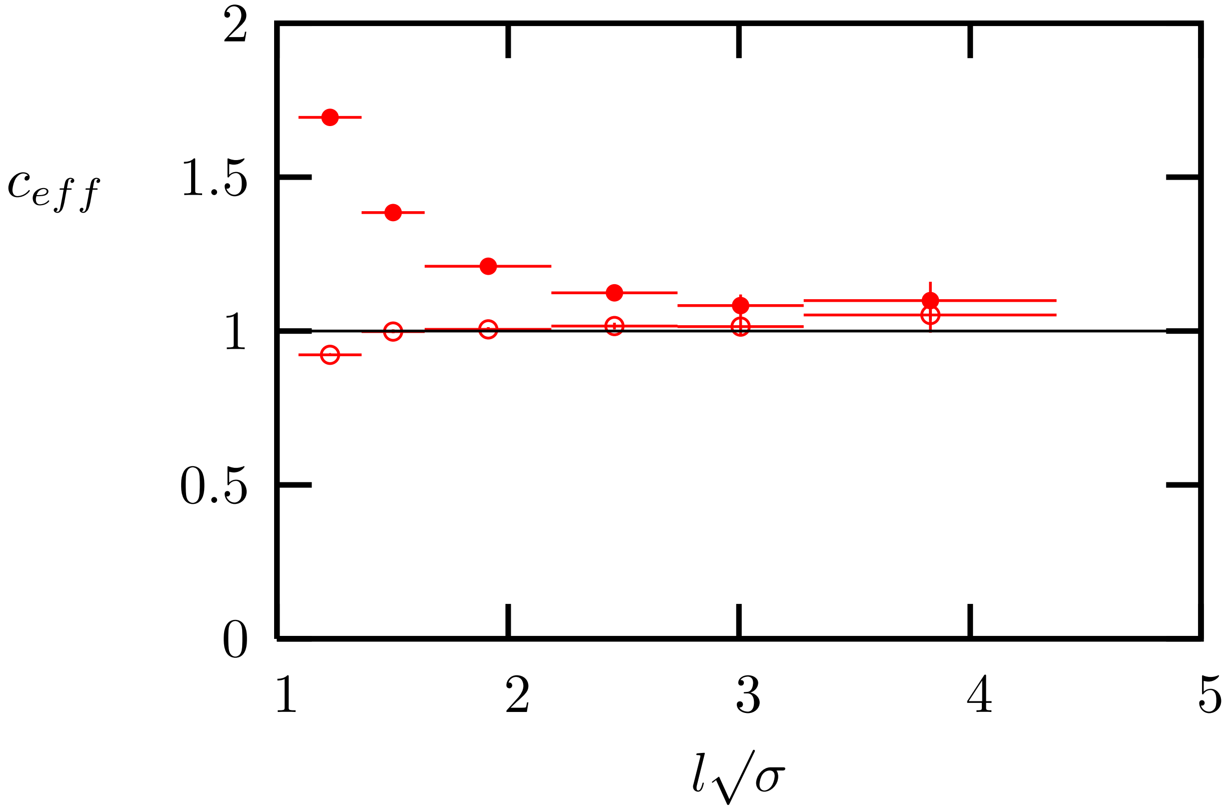
• by contrast we see that the Nambu-Goto expression is almost exact all the way down to $l \sim l_c$

$$c^{NG}_{eff}(l) \simeq 1 \quad \forall l$$

i.e. the confining flux tube behaves almost exactly like an ideal free bosonic string, even when it is hardly longer than it is wide

$\Rightarrow$ is this a large N effect?

small $N \longrightarrow$ SU(2) : $l_c\sqrt{\sigma} \simeq 0.95$



c_{eff} : from Luscher $\bullet$, and from Nambu-Goto $\circ$

So it appears that the ground state flux tube energy, $E_0(l)$, is very close to that of a free bosonic string theory $\forall N$.

∘ Are there theoretical reasons to think that some of the corrections beyond the Luscher term are just like Nambu-Goto?

∘ What do the excited states look like?

But first two asides ...

First Aside : $\quad l \to l_c$

for small N the phase transition to the deconfined phase with no winding flux loops, becomes 2nd order, and then

$$E_0(l) \overset{l\to l_c^+}{\propto} (l - l_c)^\gamma$$

where γ is determined by the critical exponents, which will be the same as that of the Z_N spin model in $D-1$ dimensions Svetitsky-Yaffe

e.g. $\gamma = 1.0$ for SU(2) in D=2+1 and Ising in $D = 2$

in contrast for Nambu-Goto we have

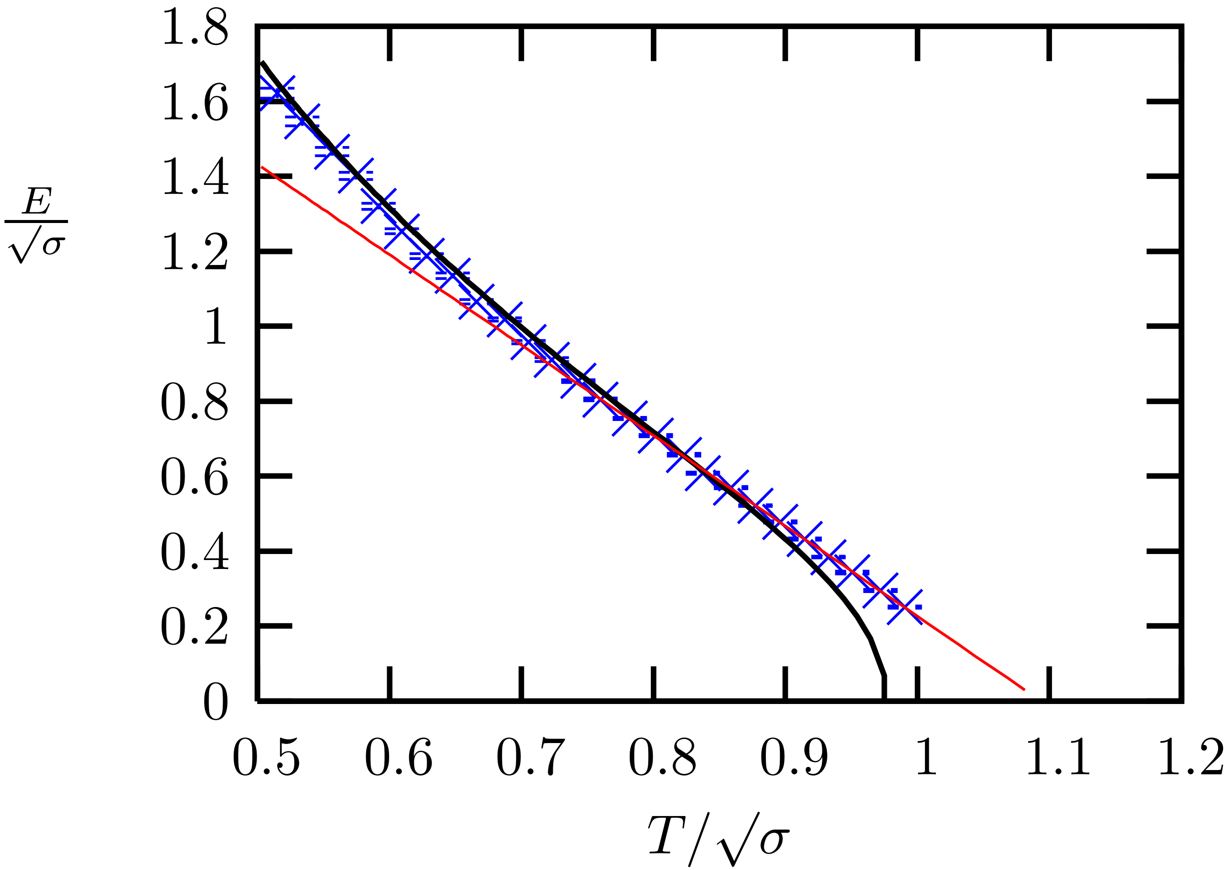
$$E_0(l) \overset{l\to l_c^+}{\propto} (l - l_c)^{\frac{1}{2}}$$

so at some point the Nambu-Goto fit *must* break down for small N

E.g.

SU(2) D=2+1

$l_t = 4 \leftrightarrow T = \frac{1}{4a}$



$---$: $\alpha(T - T_c)$; $---$: Nambu-Goto

Second Aside:

SU(N) string tension: Karabali-Nair prediction

Barak Bringoltz, MT: hep-th/0611286

Karabali and Nair analytic Hamiltonian formalism: e.g. hep-th/0309061, arXiv:0705.2898, 0705.0394

(also: Freidel, Leigh, Minic hep-th/0604184)

$\Rightarrow$

$$\frac{\sqrt{\sigma}}{g^2 N} = \sqrt{\frac{1-\frac{1}{N^2}}{8\pi}}$$

within $\sim 3\%$ of 'old' lattice calculations for all N

$\Rightarrow$

need calculations where *all* systematic (=hard) as well as statistical (= easy) errors are controlled to $\ll 1\%$.

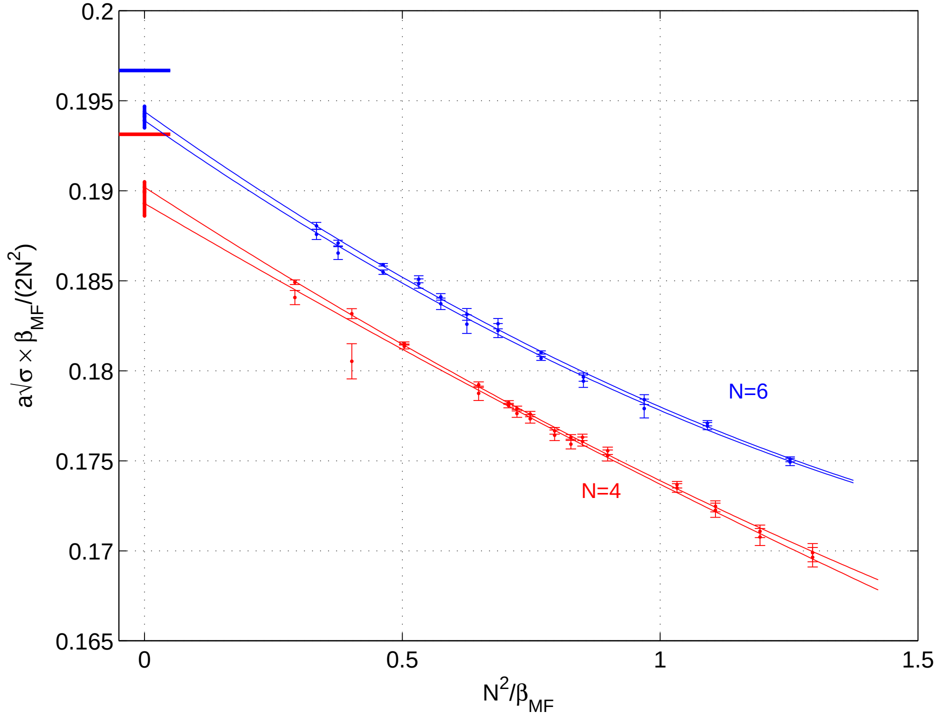
no screening in KN $\Rightarrow$ if it is to be exact, then can only be so for $N \to \infty$

and indeed the 'old' lattice results approached KN as $N \uparrow$

no $O(1/\sigma l^2)$ corrections to $E_0(l)$ so can only possibly be exact as $l \to \infty$
KN as $N \uparrow$

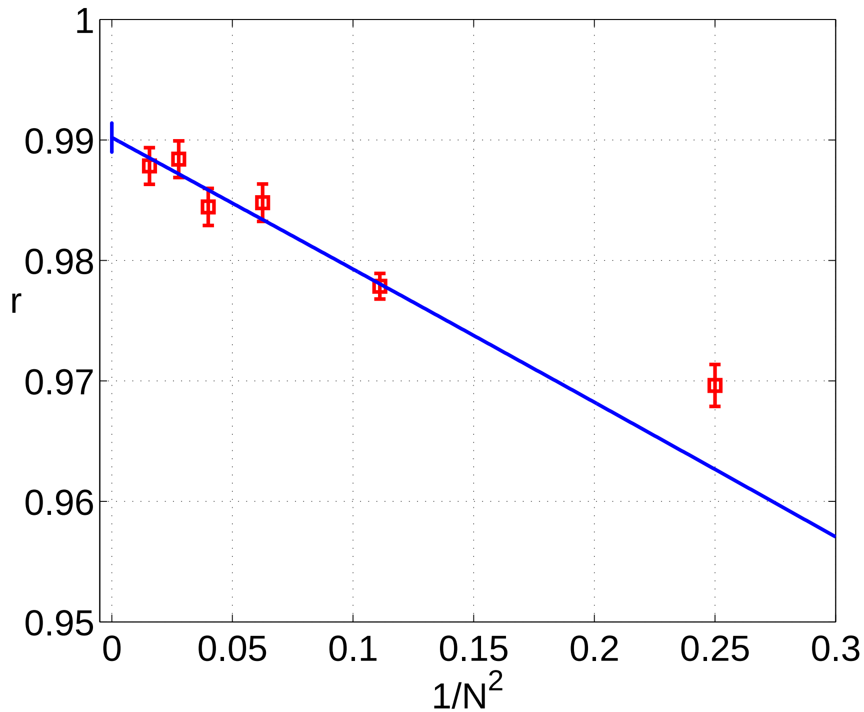
some continuum limits :

$$\frac{\sqrt{\sigma(a)}}{g^2 N} \text{ vs. } ag^2 N$$



$$N \to \infty :$$

$$r \equiv \frac{\left(\sqrt{\sigma}/g^2N\right)_{\mathrm{Lattice}}}{\left(\sqrt{\sigma}/g^2N\right)_{\mathrm{KKN}}}$$



$$\Rightarrow \lim_{N\to\infty} \frac{\sqrt{\sigma}}{\sqrt{\sigma_{\mathrm{KKN}}}} = 0.9901 \pm 0.0010 - 0.0025 \quad \text{convincing} \sim 6 \text{ sd}$$
discrepancy

(bosonic) strings in D=4?

e.g. J. Polchinski: hep-th/9210045

can long confining flux tubes in $D = 4$ SU(N) gauge theories be described by some string theory?

but

there are no consistent string theories in D=4 : we need D=26 for bosonic, and D=10 for SUSY strings

yes, but

while the properties of confining flux tubes are not that well known, because the physics is strongly coupled, there exist weakly coupled examples, such as Nielsen-Olesen vortices in the Abelian-Higgs model, that provide explicit examples of string like objects in D=4

so

effective string theories, for long strings, should certainly exist in $D = 4$

indeed

a typical inconsistency in quantising a free bosonic string of length R in D=4 is a breakdown of Lorentz covariance: e.g. generators of rotations are anomalous

J. Arvis, Phys. Lett. 127B(1983)106

$$[L^i, L^j] = -L^{ij} + F(R)$$

but one sees that

$$F(R) \propto 1/R^2 \stackrel{R\to\infty}{\longrightarrow} 0$$

so that the inconsistencies disappear in any D for long enough strings

P. Olesen, Phys. Lett. 160B(1985)144

$\star$same for $D = 3$$\star$

analysing effective string theories

• field theory approach (non-covariant ‘gauge fixing’ of the string theory) ; low-energy effective Lagrangian for the transverse displacement
M. Luscher, K. Symanzik, P. Weisz : Nucl. Phys. B173 (1980) 365; M. Luscher : Nucl. Phys. B180 (1981) 317

• covariant effective string approach; low-energy effective Lagrangian for a long string
J. Polchinski, A. Strominger : Phys. Rev. Lett. 67 (1991) 1681

In both approaches the starting point is to consider a long (open or closed) string of length l and to consider those corrections allowed by symmetry arguments (different in the two approaches) ordered in powers of $1/l$

lowest order (‘Luscher correction’) $\Rightarrow E_n = \sigma l + \frac{\pi}{l}\left(n - \frac{D-2}{6}\right)$
i.e. identical to Nambu-Goto to this order in $1/l$ for both $D = 2+1$ *and* $D = 3+1$

at the next order ...

• field theory approach

M. Luscher, P. Weisz : hep-th/0406205

• covariant effective string approach

J. Drummond : hep-th/0411017; N. Hari Dass, P. Matlock : hep-th/0612291

$\Rightarrow$

$$E_n = \sigma l + \frac{\pi}{l}\left(n - \frac{D-2}{6}\right) - \frac{\pi^2}{2\sigma l^3}\left(n - \frac{D-2}{6}\right)^2 + O(l^{-4})$$

i.e. still identical to Nambu-Goto to this order in $1/l$!

for $D = 2+1$ in both approaches *and* $D = 3+1$ in the Polchinski framework and very recently ...

O. Aharony, E. Karzbrun : arXiv:0903.1927

• extends field theory approach to a further order in $1/\sigma l^2$

i.e. if we write

$$E_n(l) = \sigma l \left(1 + \frac{c_{1,n}}{\sigma l^2} + \frac{c_{2,n}}{(\sigma l^2)^2} + \frac{c_{3,n}}{(\sigma l^2)^3} + O\left(\frac{1}{l^8}\right)\right)$$

then in addition to older result that

- ○ $c_{1,n} = \mathrm{Nambu-Goto} \quad \forall D$
- ○ $c_{2,n} = \mathrm{Nambu-Goto} \quad D = 2+1$

they show that:

- $c_{2,n} = \mathrm{Nambu-Goto} \quad \forall D$
- $c_{2,n} = \mathrm{Nambu-Goto} \quad \forall D$
- $c_{3,n} = \mathrm{Nambu-Goto} \quad D = 2+1$
- $c_{3,0} = \mathrm{Nambu-Goto} \quad \forall D$
- $\sum_n c_{3,n} = \mathrm{Nambu-Goto} \quad \forall D$

and also something else that is very interesting ...

• calculate explicitly in a class of confining theories with a dual string theory description in a weakly curved background – *leading dependence* on curvature of background $\leftrightarrow$ to a certain order in the inverse 't Hooft coupling

$\Rightarrow$

∘ all above contraints satisfied

and also

∘ $c_{3,n} = \mathrm{Nambu-Goto} \qquad \forall D$

$\Rightarrow$

there may indeed be extra constraints not captured in the effective field theory approach.

So Nambu-Goto interesting even if no-one expects it to be the whole story!

Nambu-Goto free string theory

$$\int \mathcal{D}X e^{-\frac{i}{\sigma}\times \mathrm{Area}}$$

Spectrum:

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma\left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 .$$

$2\pi q/l$ = total momentum along string;
N_L, N_R = sum left and right oscillators ('phonons');
state $= \prod_k a_k^{n_k} \prod_{k'} \tilde{a}_{,}^{n_{k'}} k|0\rangle$

$$N_L = \sum_{k>0} n_L(k)\, k, \quad N_R = \sum_{k'>0} n_R(k')\, k', \quad N_L - N_R = q$$

J. Arvis, Phys. Lett. 127B(1983)106

Nambu-Goto ground state $N_L = N_R = 0$:

$$E^2(l) = (\sigma\, l)^2 - \tfrac{\pi(D-2)\sigma}{6}$$

where the second term is the 'Luscher term', from zero-point energies of oscillators - the Casimir energy of a (periodic) string

$\Rightarrow$ tachyon of mass $\mu^2 = -\frac{\pi(D-2)\sigma}{6}$

$\leftrightarrow$ 'Hagedorn' deconfinement

$\Rightarrow$ string theory does not exist for $l \leq l_c = \sqrt{\frac{\pi(D-2)}{6\sigma}}$

$\Rightarrow$ spectrum exists and well-behaved for $l > \sqrt{\frac{\pi(D-2)}{6\sigma}}$

field-theoretic approach

- string of length r with fixed ends and world sheet coords

$$z = (z_0, z_1), \qquad 0 \le z_1 \le r \; ; \; 0 \le z_0 \le T$$

and displacement vector $h(z)$ with effective action (D=2)

$$S = \sigma r T + \mu T + S_0 + S_1 + \cdots \,, \qquad S_0 = \frac{1}{2} \int d^D z \, (\partial_a h \partial_a h)$$

one finds

$$S_1 = \tfrac{1}{4} b \int d^{D-1} z \left\{ (\partial_1 h \partial_1 h)_{z_1=0} + (\partial_1 h \partial_1 h)_{z_1=r} \right\}, \qquad b = [l]^1$$

$$S_2 = \tfrac{1}{4} c_2 \int d^D z \, (\partial_a h \partial_a h) \, (\partial_b h \partial_b h) \,, \qquad c_2 = [l]^2$$

$$S_3 = \tfrac{1}{4} c_3 \int d^D z \, (\partial_a h \partial_b h) \, (\partial_a h \partial_b h) \,, \qquad c_3 = [l]^2$$

since the couplings of S_i are increasing powers of length, they will contribute with increasing powers of $1/r$

• lowest $O(\{\partial h\}^2) \Rightarrow$ linear + Luscher correction
– exact expression (standard functions) for partition function.

• impose open-closed duality at higher orders (Luscher) – or open-open duality (Aharony) i.e. cylinder $\rightarrow$ torus, using

$$Z = \sum_n e^{-E_n(r)T} = \sum_n{}' e^{-E_{n'}(T)r}$$

calculate Z in powers of S_i and this fixes corresponding powers of $1/r$ or $1/T$: the consistency of the latter then fixes some of the free coefficients.
$\Rightarrow$
accurate calculation of the energy spectrum as a function of l provides a *direct* way to determine the effective string action

covariant effective string theory approach

J. Polchinski, A. Strominger : Phys. Rev. Lett. 67 (1991) 1681

imagine integrating out all massive modes leaving an integration over the massless $D-2$ transverse oscillators with action

$S_o = \frac{1}{\sigma}\int d\tau^+ d\tau^- \partial_+ X^\mu \partial_- X_\mu$

and some determinants from the massive modes. These should be made out of the induced metric

$h_{ab} = \partial_a X^\mu \partial_b X_\mu$

and suppose the determinant is as for Polyakov but replacing the intrinsic metric e^ϕ with the induced metric h_{+-}. Then the determinant (in conformal gauge) is e^{iS_L} where

$S_o = \frac{26-D}{48\pi}\int d\tau^+ d\tau^- \partial_+ \phi \partial_- \phi$

becomes

$S_L = \frac{26-D}{48\pi}\int d\tau^+ d\tau^- \frac{\partial_+^2 X\cdot\partial_- X \partial_+ X\cdot\partial_-^2 X}{(\partial_+ X\cdot\partial_- X)^2}$

Instead of pursuing this, encode the determinants in an effective string

action, with conformal invariance as the sole restriction.
Since we are quantising around a long string, we do not mind non-polynomial terms since the denominator will always be large. One finds:

$$S = \int d\tau^+ d\tau^- \frac{1}{\sigma}\partial_+ X^\mu \partial_- X_\mu + \frac{\beta}{4\pi}\frac{\partial_+^2 X\cdot\partial_- X \partial_+ X\cdot\partial_-^2 X}{(\partial_+ X\cdot\partial_- X)^2} + O(1/l^3)$$

coordinate invariance not anomalous

$$\Rightarrow \qquad \beta = \frac{D-26}{12}$$

which translates into the usual expression for the Luscher string correction

we can now continue to higher orders imposing the same anomaly constraint

Note:
the effective action is only valid for very long strings – $l\sqrt{\sigma} \gg 1$ – as is obvious from the denominators in the effective action.
$\Rightarrow$ it tells us nothing about light glueballs since these are composed of small closed loops

it tells us nothing about k-strings or other multiple strings, since the interaction between these (at the origin of their binding) will in general involve the exchange of small closed loops

$\Rightarrow$

what we learn about confining flux tubes with $l\sqrt{\sigma} \not\gg 1$ will tell us whether what we have is just an effective string theory for very long flux tubes or possibly an effective string theory on all scales ...

Strings in D=2+1 : quantum numbers

- $\circ$ length of string

- $\circ$ non-zero momentum $p = 2\pi q/l$ along string

$\longrightarrow$ requires a deformation along the string

$\longrightarrow$ so need non-trivial phonon excitation: $q = N_L - N_R$

- $\circ$ parity: $h(x) \to -h(x) \leftrightarrow a_k \to -a_k, \tilde{a}_k \to -\tilde{a}_k$

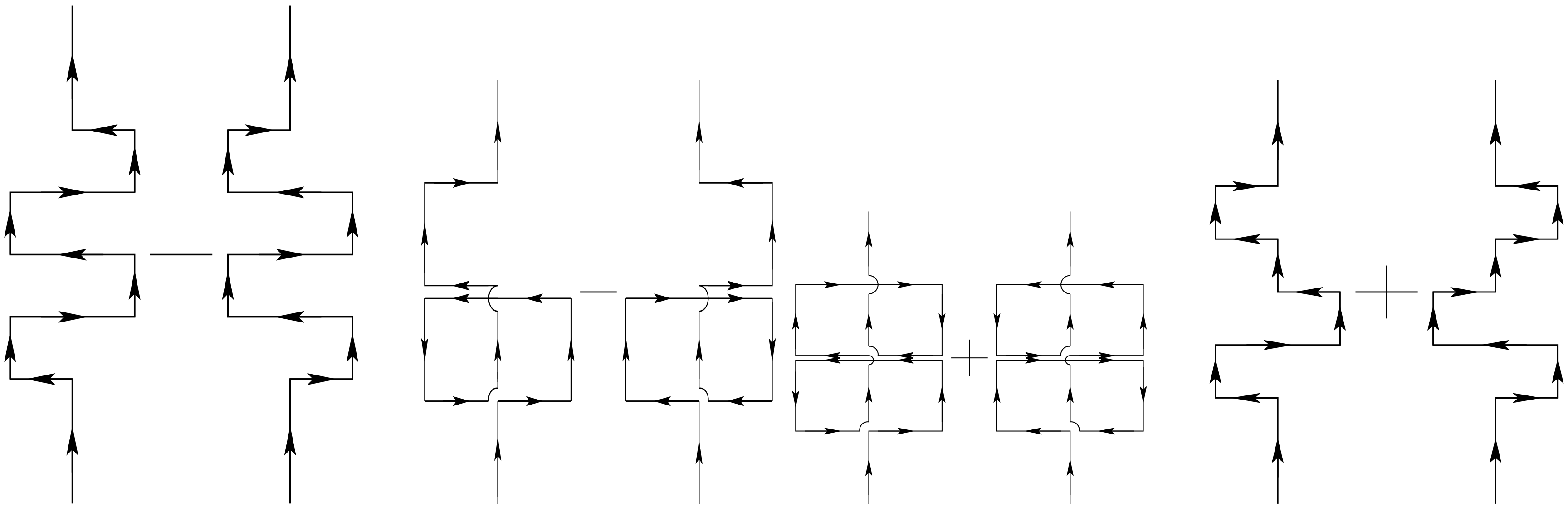
$\longrightarrow P = (-1)^{\sum_{k>0} n_L(k) + \sum_{k'>0} n_R(k')}$

- $\circ$ no rotations (2 space dimensions); transverse momentum uninteresting;

$C = \pm$ sectors degenerate, so charge conjugation uninteresting.

Excited States

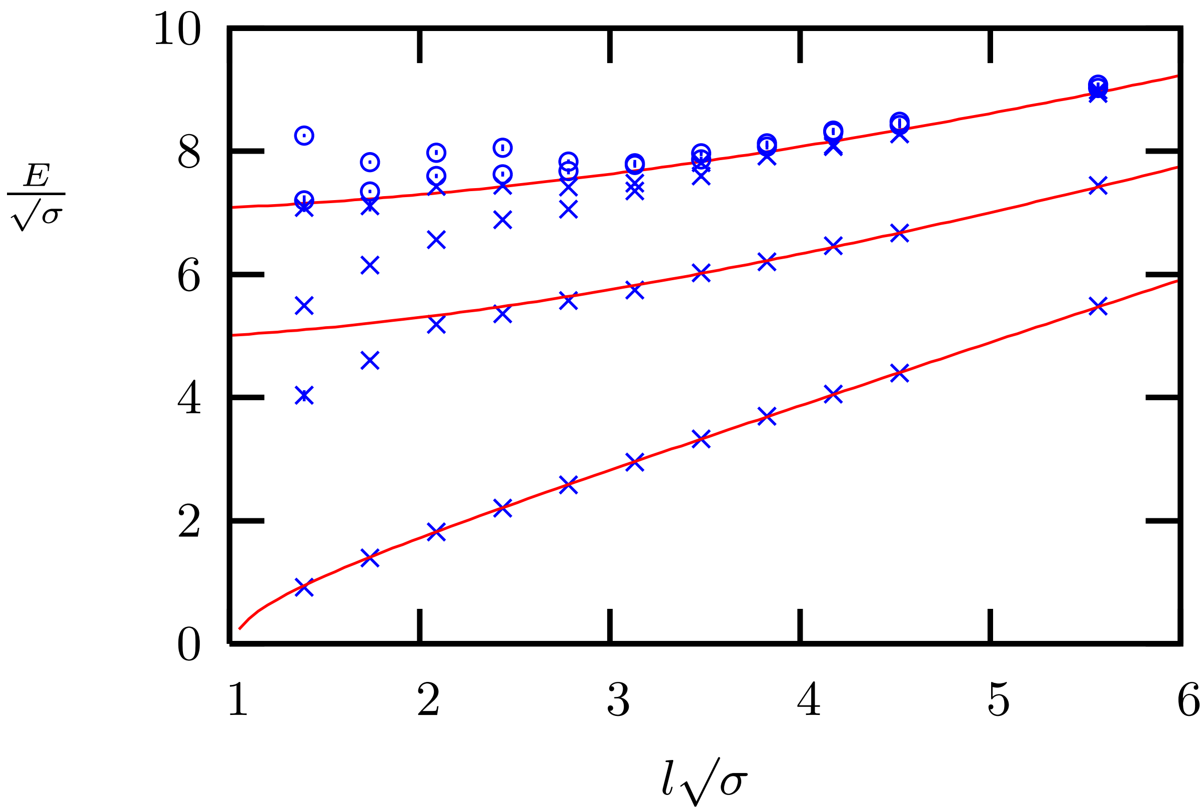
to have good overlaps onto excited string states, we need to include many more operators in our variational basis – in particular operators that ‘look’ excited and ones that have an intrinsic handedness so that we can construct $P = -$ as well as $P = +$, e.g.



typically we have 100-200 operators in our basis ...

SU(3) : $q = 0$ closed string spectrum

$a\sqrt{\sigma} = 0.17395(7) \quad ; \quad l_c\sqrt{\sigma} \simeq 1.0$

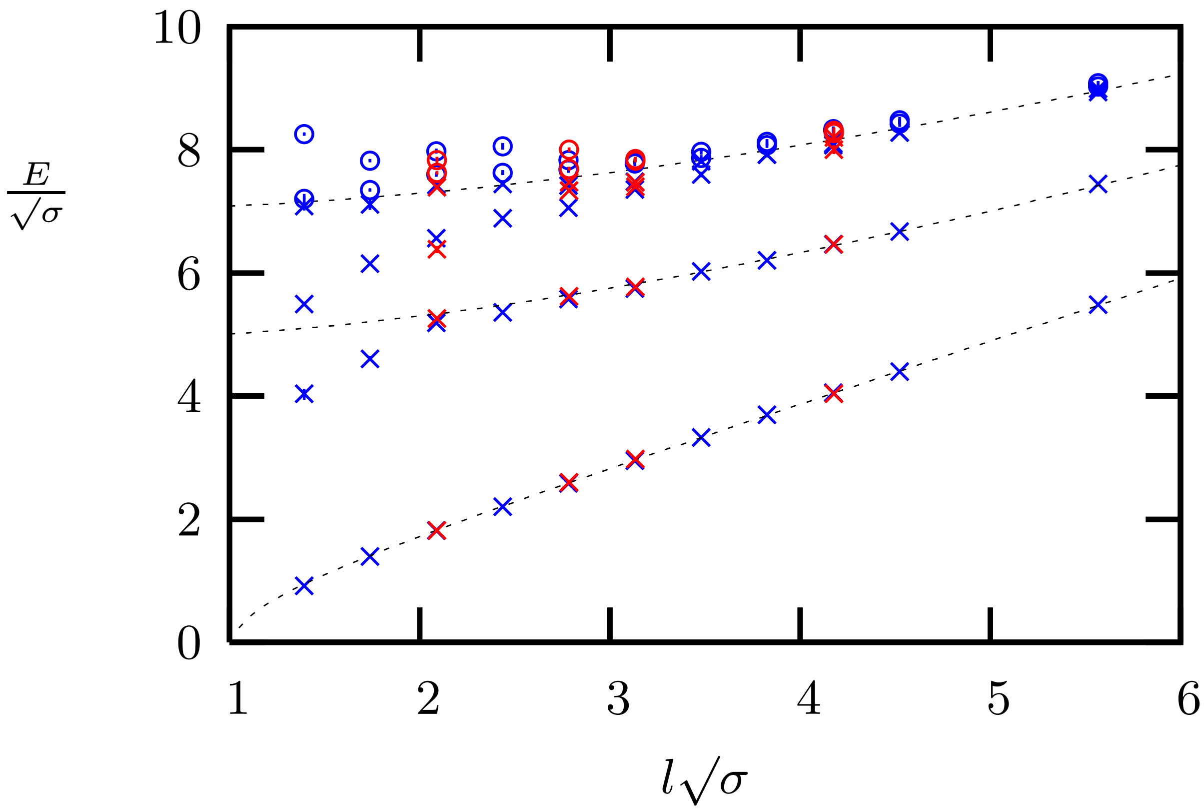


— : Nambu-Goto : σ from ground state

$\times$: +ve parity $\circ$: -ve parity

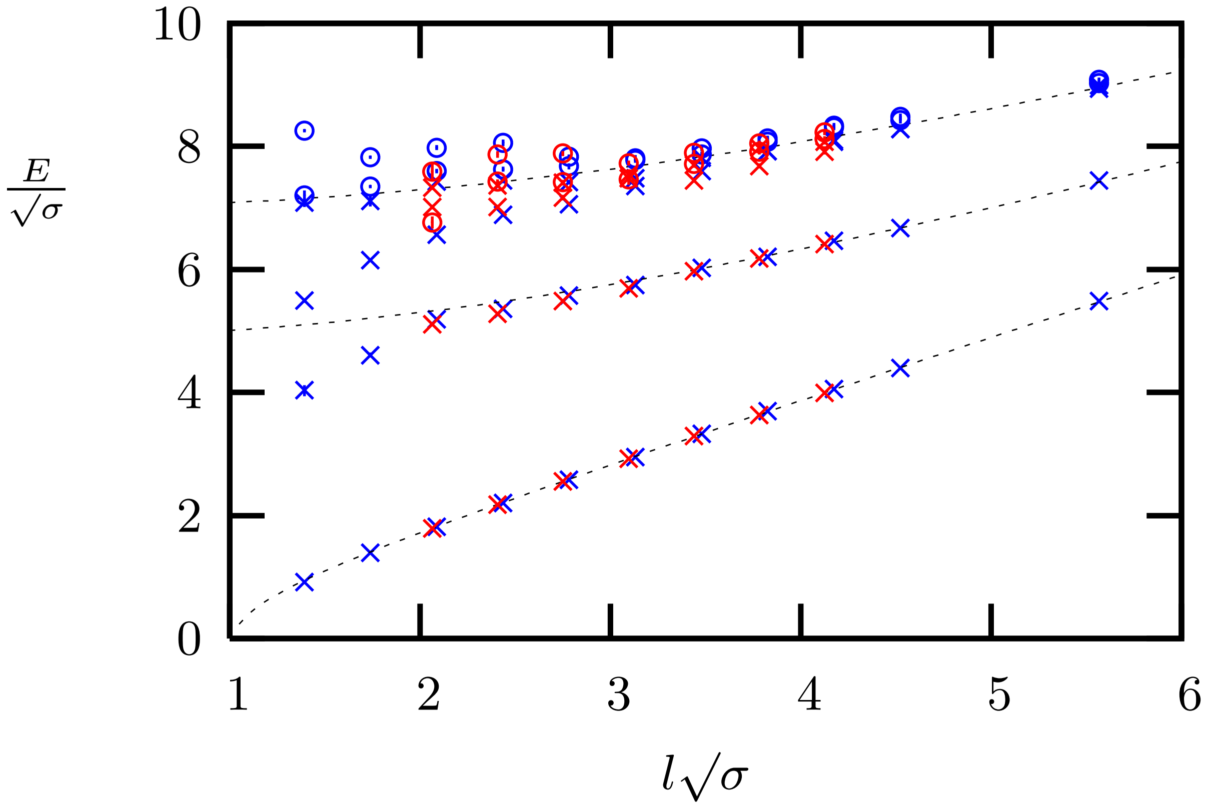
SU(3) : how close to continuum limit?

compare $a\sqrt{\sigma} \simeq 0.174 \quad vs \quad a\sqrt{\sigma} \simeq 0.087$



no significant difference as $a \to a/2 \Rightarrow$ we have ‘continuum’ physics

SU(3) vs SU(6) : same a



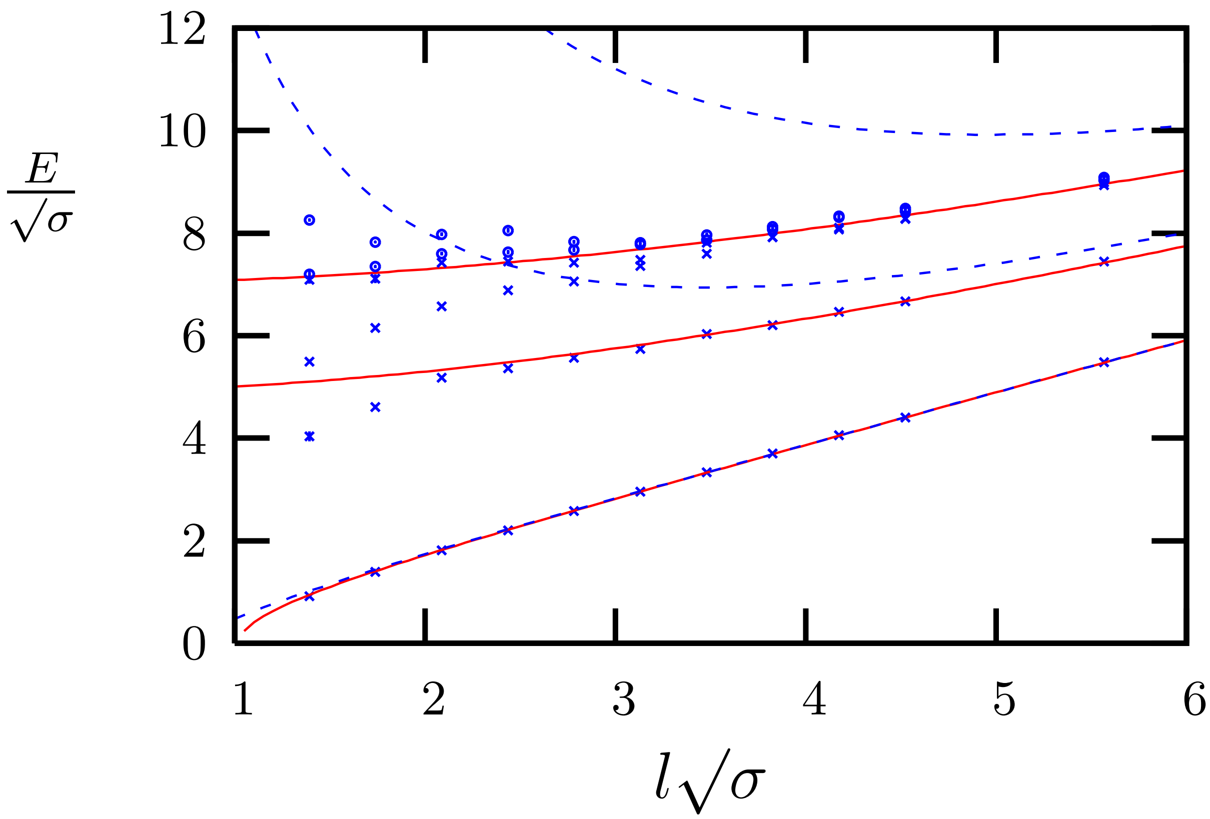
$\Rightarrow \quad SU(3) \simeq SU(\infty)$

$\diamond$ Striking agreement with free string model, down to $l\sqrt{\sigma} \simeq 2$.

$\diamond$ Remarkable since $l\sqrt{\sigma} \simeq 2 \Rightarrow$ the flux tube is maybe only twice as long as it is wide – hardly an ideal 'string'.

$\diamond$ Is this just a manifestation of the fact that the first 3 or 4 terms in an expansion of $E_n(l)$ in powers of $1/\sigma l^2$ must be the same as Nambu-Goto?

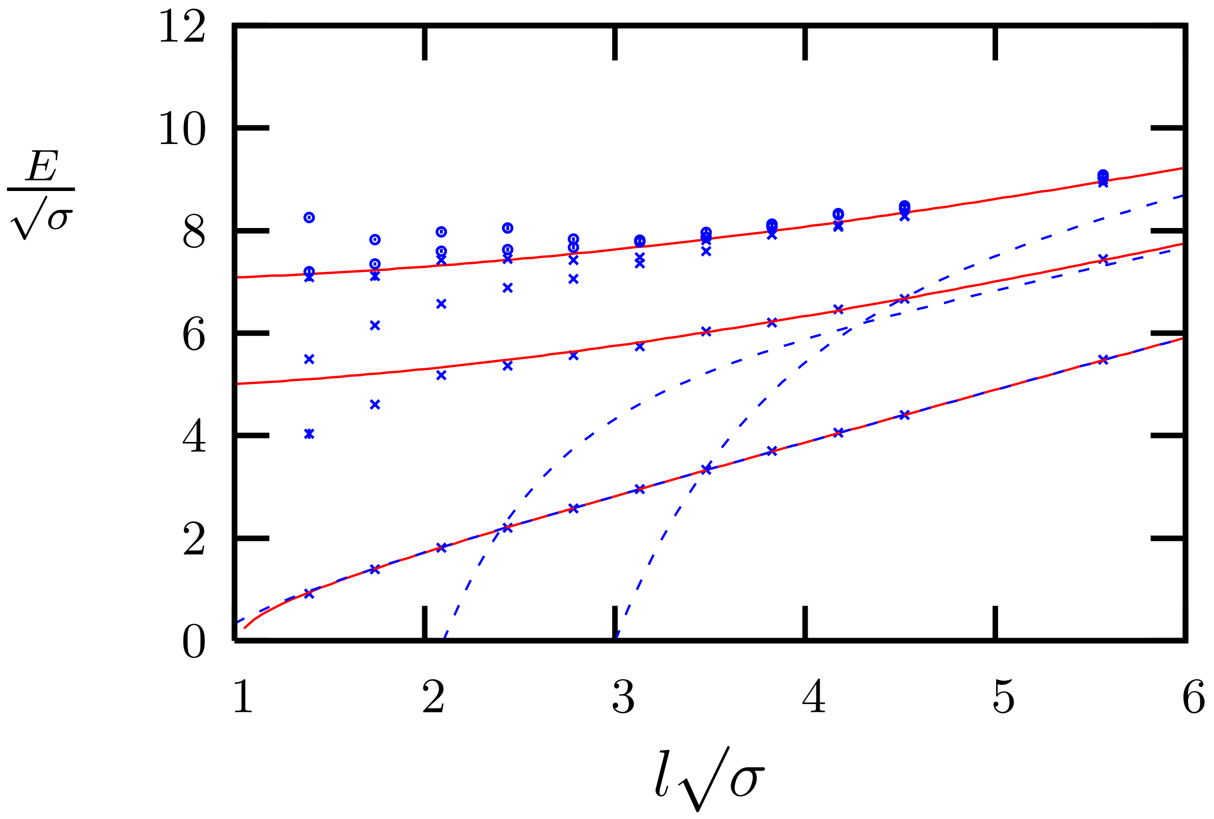
Nambu-Goto vs Luscher-Symanzik-Weisz



— Nambu-Goto : $E_n = \sigma l \sqrt{1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right)}$

... Luscher 1980: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{D-2}{24}\right)$

Nambu-Goto vs Luscher-Weisz, Drummond



— Nambu-Goto : $E_n = \sigma l \sqrt{1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right)}$

... MLPW,JD 2004: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{24}\right) - \frac{8\pi^2}{\sigma\, l^3}\left(n - \frac{1}{24}\right)^2$

Why ?

the covariant Nambu-Goto expression e.g. for $q = 0$,

$$E(l) = \sigma l \left(1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{D-2}{24}\right)\right)^{\frac{1}{2}}$$

can only be expanded as a power series in $1/l\sqrt{\sigma}$ when

$$\frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right) \leq 1 \quad \leftrightarrow \quad l\sqrt{\sigma} \geq \sqrt{8\pi n} \sim 5\sqrt{n}$$

whereas in practice we have a very good fit by Nambu-Goto even down to

$$l\sqrt{\sigma} \sim 2 \ , \ n = 1, 2$$

which is well outside its radius of convergence

$\Rightarrow$

the agreement with NG that we see goes well beyond the range of valdity of an expansion of $\mathcal{L}_{eff}$ in powers of derivatives: it makes a statement about $\mathcal{L}_{eff}$ to *all* orders in $1/\sigma l^2$

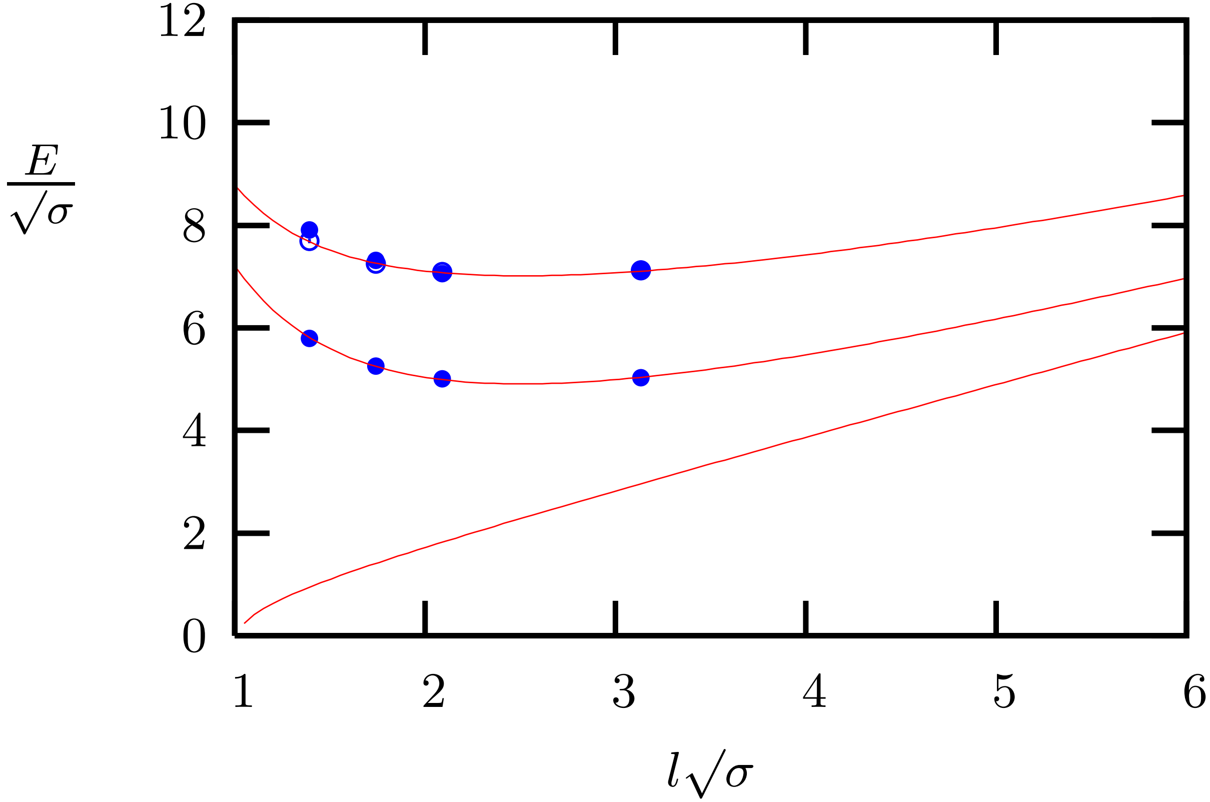
content of lightest $q = 0$ NG states:

$\|0\rangle$	P=+, q=0
$a^R(k=1)a^L(k=1)\|0\rangle$	P=+, q=0
$a^R(k=2)a^L(k=2)\|0\rangle$	P=+, q=0
$a^R(k=1)a^R(k=1)a^L(k=1)a^L(k=1)\|0\rangle$	P=+, q=0
$a^R(k=2)a^L(k=1)a^L(k=1)\|0\rangle$	P=−, q=0
$a^R(k=1)a^R(k=1)a^L(k=2)\|0\rangle$	P=−, q=0

Since our lightest states have energies and degeneracies as in Nambu-Goto down to $l\sqrt{\sigma} \sim 2$, they are well-described by the above states.

$\Rightarrow$ also the case for $q \neq 0$:

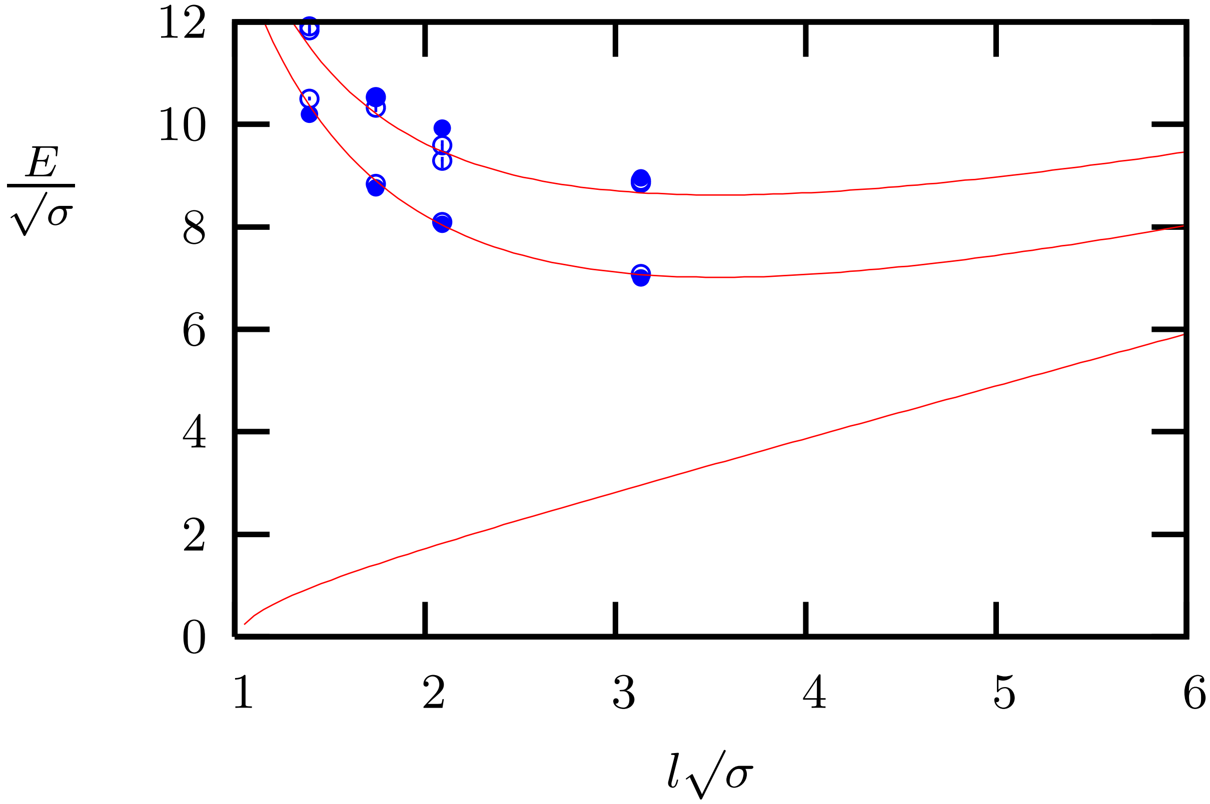
$q = 1$ spectrum : SU(3) at smaller a



$\bullet$ $P = -$; $\circ$ $P = +$

curves: NG predictions for $q = 1$ and also $q = 0$ ground state

$q = 2$ spectrum : SU(3) at smaller a



$\bullet\ P = -$; $\circ\ P = +$

curves: NG predictions for $q = 1$ and also $q = 0$ ground state content of $q = 1, 2$ NG states:

$a^R(k=1)|0\rangle$ P=−, q=1

$a^R(k=2)|0\rangle$ P=−, q=2

$a^R(k=1)a^R(k=1)|0\rangle$ P=+, q=2

$a^R(k=2)a^L(k=1)|0\rangle$ P=+, q=1

$a^R(k=1)a^R(k=1)a^L(k=1)|0\rangle$ P=−, q=1

$a^R(k=3)a^L(k=1)|0\rangle$ P=+, q=2

$a^R(k=2)a^R(k=1)a^L(k=1)|0\rangle$ P=−, q=2

$a^R(k=1)a^R(k=1)a^R(k=1)a^L(k=1)|0\rangle$ P=+, q=2

$q \neq 0$ requires a deformation of the flux tube (otherwise it is translation invariant and hence $q = 0$) so in NG the ground state will have at least one phonon

• in D=2+1 SU(N) gauge theories, confining flux tubes belong to the universality class of a simple bosonic string theory

• more than that, the Nambu-Goto covariant free string spectrum

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma \left(\frac{N_L + N_R}{2} - \frac{D-2}{24} \right) + \left(\frac{2\pi q}{l} \right)^2 .$$

very accurately describes the spectrum down to values of $l\sqrt{\sigma}$ where an effective string theory expansion in $x = l\sqrt{\sigma}$,

$$\frac{E_n}{\sqrt{\sigma}} = x \left(1 + \frac{c}{x^2} \right)^{\frac{1}{2}} = x + \frac{c}{2x} - \frac{c}{8x^3} + \cdots$$

makes no sense (i.e. is far past its range of convergence)

• This suggests that we take the energy eigenstates to be those of the free NG string theory, as our starting point, and treat the deviations as a perturbation induced by some weak interactions between the phonons. There should be relations in the interaction energies within different NG eigenstates that depend on the nature of the interactions in this $D = 1 + 1$ phonon field theory

• why is a simple bosonic string theory so good?

• usually we think of the flux tube as

either

some non-Abelian dual Nielsen-Olesen vortex, with a finite intrinsic width $\sim 1/\sqrt{\sigma}$;

and/or

a string in some '5D' gravity dual, dangling near some 'horizon' where the metric will have a highly non-trivial curvature, so that it projects tp a flux tube of non-zero width on our '4D' boundary

• in either scenario:

– where are the excited states due to excitations of the massive degrees of freedom generating the finite width?

– and the corrections to the stringy states from these massive degrees of freedom.

• very naively we might expect the mass scale of the lightest such extra states to be

$E(l) \sim E_0(l) + m_G \sim E_0(l) + 4\sqrt{\sigma}$

or maybe

$E(l) \sim E_0(l) + \Delta m_G \sim E_0(l) + 2\sqrt{\sigma}$

at low $l\sqrt{\sigma}$ this should be one of the lightest excitations – but we do not see it in our spectra ...

$\Rightarrow$ k-strings

k-strings

source = product of k fundamental sources

flux tube between such static sources = k-string

source may be screened by gluons from vacuum

gluons = adjoint so transform trivially under centre $\Rightarrow$ the screened source, always transforms under $z \in Z_N$ as
$\phi_k \longrightarrow z^k \phi_k$

Thus k is a good quantum number.

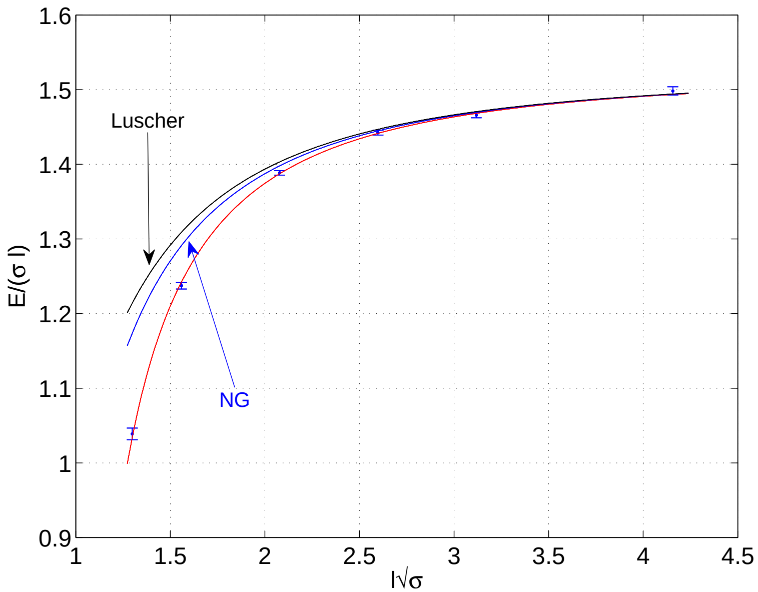
Typically a source will be screened to give the lightest string of given k.

One finds:

The lightest k-string is not composed of k separate fundamental strings, $\sigma_k = k\sigma_{k=1}$, but is a bound state with $\sigma_k < k\sigma_{k=1}$
e.g. for $k = 2$ in SU(4) one finds $\sigma_k \simeq 1.35\sigma_{k=1}$

$k = 2$ string corrections

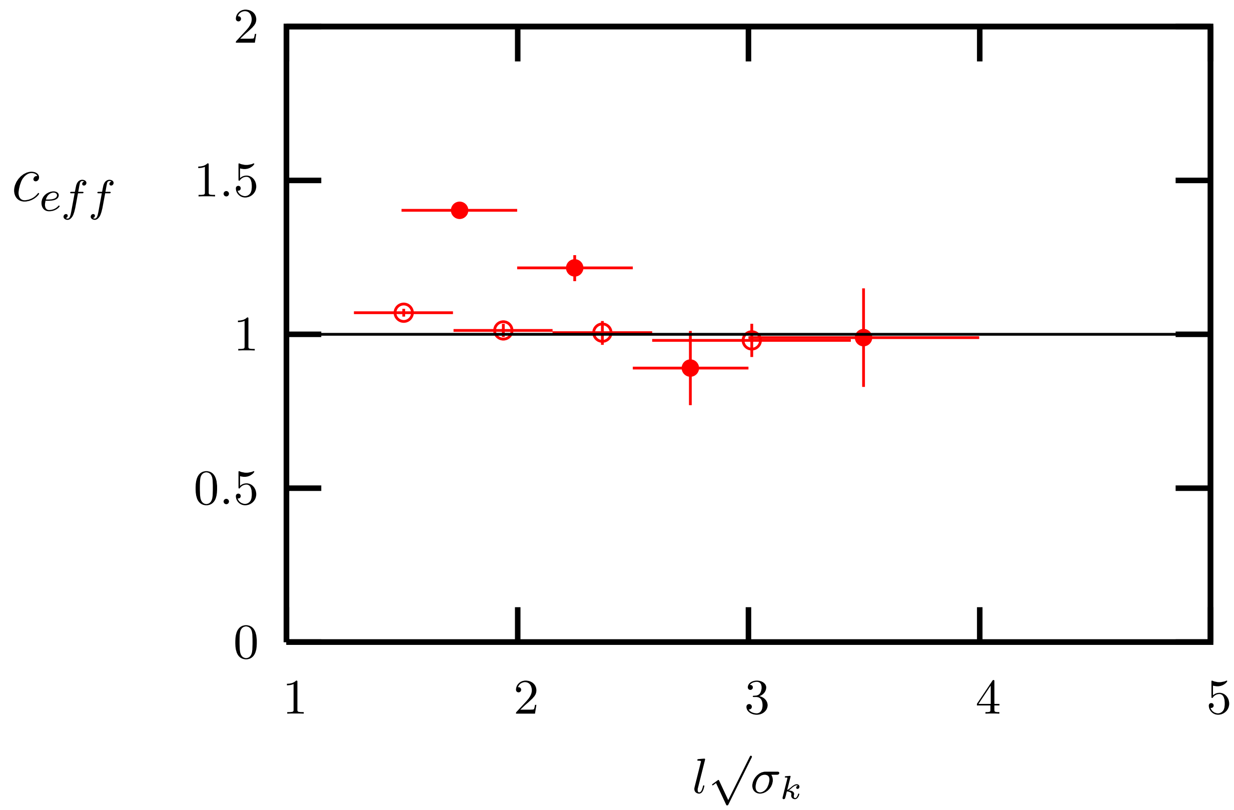
Nambu-Goto: $E_k(l) = \sigma_k l \left(1 - \frac{\pi(D-2)}{3\sigma_k l^2}\right)^{\frac{1}{2}}$ and Luscher $E_k(l) = \sigma_k l - \frac{\pi(D-2)}{6l}$



$\Rightarrow$

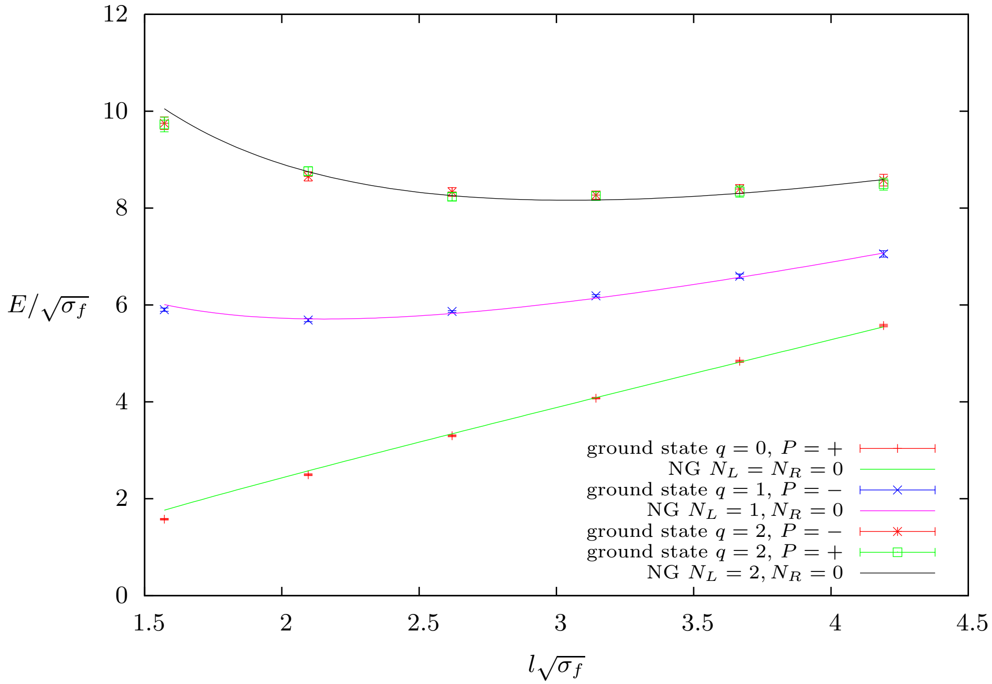
Much larger deviations at smaller l than for $k = 1$ flux tube : Nambu-Goto not much better than Luscher

SU(4) : Nambu-Goto effective charge, $\circ k = 1$ $\bullet k = 2$



$\Rightarrow$ $k = 2$ ground state flux tube is in the bosonic string universality class, but has larger corrections than for the $k = 1$ flux tube for $l\sqrt{\sigma_k} \leq 2$

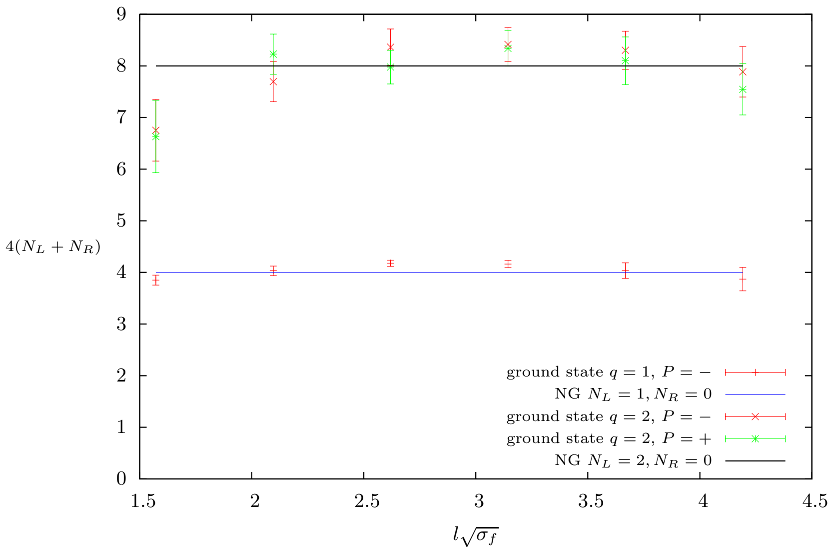
lightest $k = 2$ (antisymmetric) states with $q = 0, 1, 2$, for SU(4)



Lines are NG predictions with σ_k obtained by fitting the ground state.

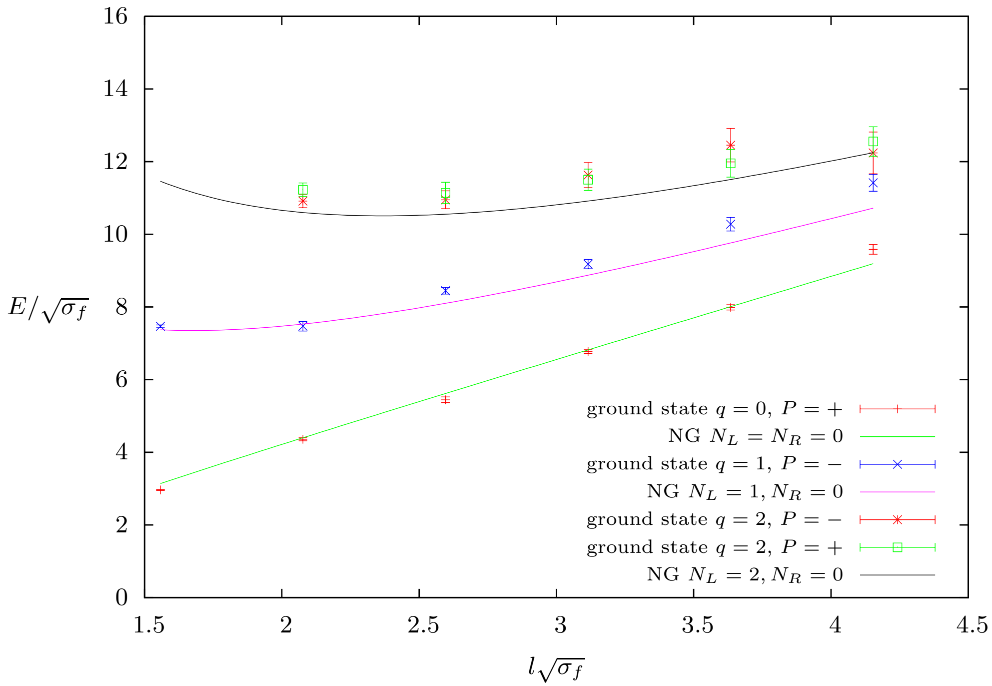
effective excitation numbers for $q = 1, 2$ using

$$\pi\sigma_k\{4(N_l + N_R)\}_{eff} = E^2_{gs}(q;l) - E^2_{gs}(0;l) - \left(\frac{2\pi q}{l}\right)^2$$



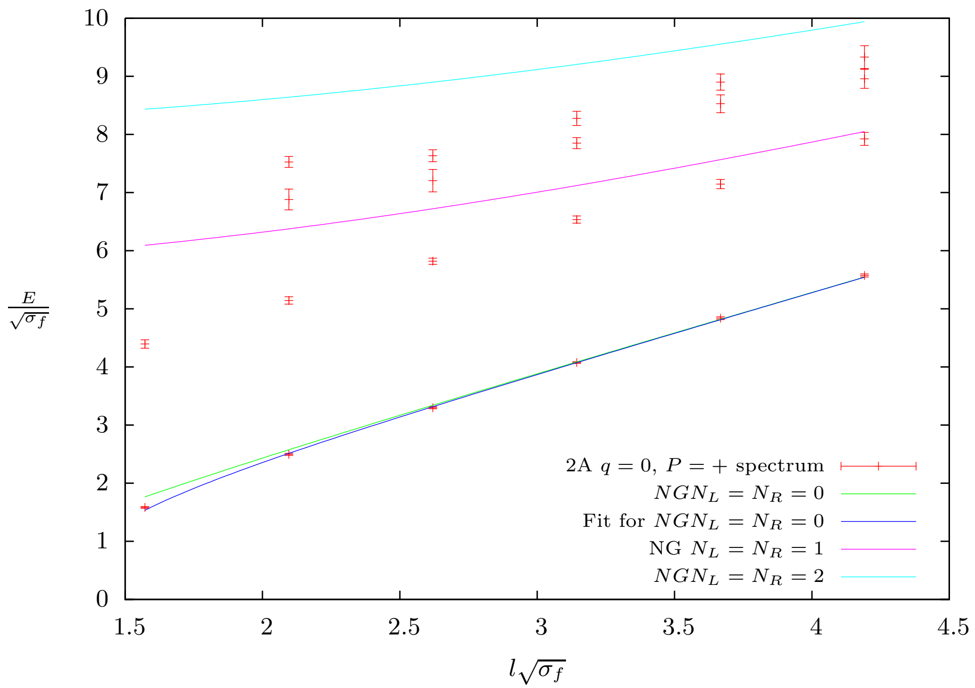
lines are NG

ASIDE: lightest $k = 2$ symmetric states with $q = 0, 1, 2$ for SU(4)



Lines are NG predictions with $\sigma_{k=2S}$ obtained by fitting the ground state.

spectrum light $k = 2$ (antisymmetric) states with $q = 0$ and $P = +$ for SU(4)



Lines are NG predictions with σ_k obtained by fitting the ground state.

• $q = 0$ excited states are far from NG – not even clear that they approach (rather than cross) NG

• indeed, is the first excited state maybe not NG, but a 'breather' mode ? comparing the $k = 1$ and $k = 2$ wave-functionals it looks like it is a stringy NG mode

• So:

some states very close to NG (e.g. q=0,1,2 ground states) while other states (such as these) have large deviations
– this is ideal in a sense : it allows us to try and draw structural conclusions about the dynamics

e.g.

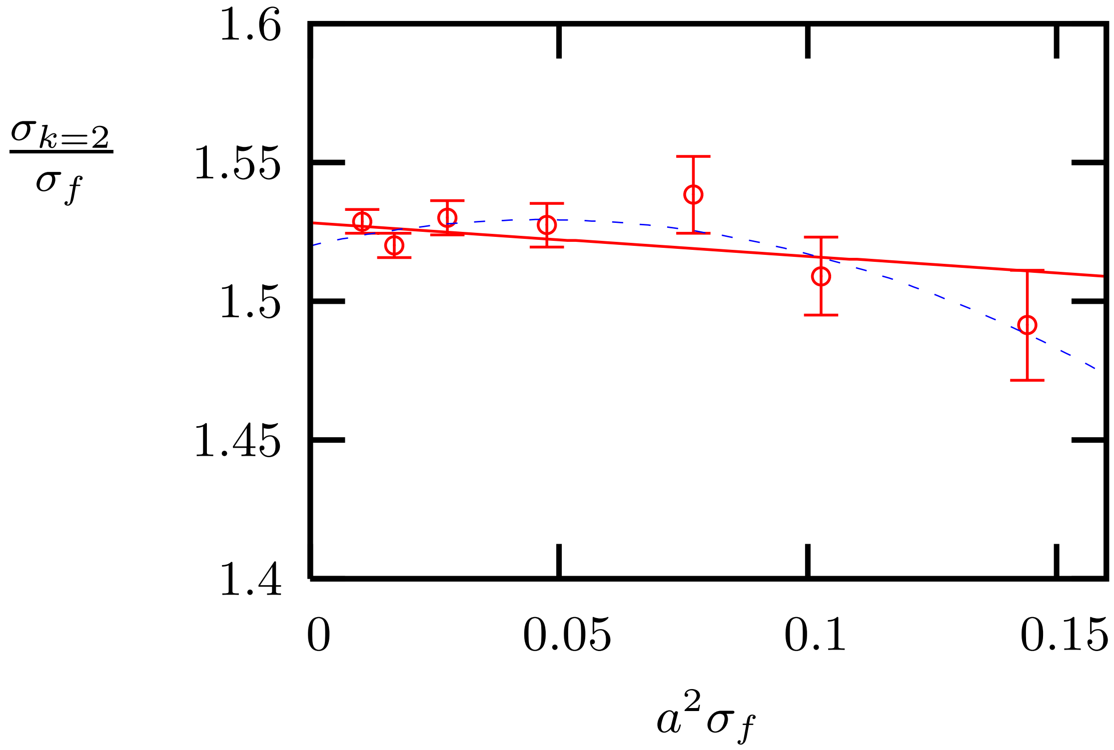
both the $q = 2$ $P++$ ground state and the $q = 0$ first excited states have 2 lowest momentum phonons: the only difference is that for $q = 0$ they have opposite momenta while for $q = 2$ they have the same momentum
$\Rightarrow$ interactions between 2 phonons on a $k = 2$ string are small near threshold and large at 'high energies'.

ASIDE: k-strings in $D = 3$

Bringoltz and Teper, arXiv:0802.1490

continuum limit:

$O(a^2)$ — ;$O(a^4)$ - - -



Casimir scaling: $$\frac{\sigma_k}{\sigma_f} = \frac{k(N-k)}{N-1}$$

'MQCD' Sine scaling: $$\frac{\sigma_k}{\sigma_f} = \frac{\sin\frac{k\pi}{N}}{\sin\frac{\pi}{N}}$$

Karabali-Nair: $$\frac{\sigma_k}{(g^2N)^2} = \frac{1}{8\pi}k\left(1-\frac{k}{N}\right)\left(1-\frac{1}{N}\right)$$

k	N	σ_k/σ_f	Casimir	Sine	Nair
2	4	1.3552(22)	1.333..	1.414..	1.361..
2	5	1.5275(26)	1.5	1.618..	1.529..
2	6	1.6234(66)	1.6	1.732..	1.629..
2	8	1.7524(51)	1.714..	1.848..	1.741..
3	6	1.8522(48)	1.8	2.0	1.832..
3	8	2.174(19)	2.143..	2.414..	2.177..
4	8	2.366(11)	2.286..	2.613..	2.322..

$\Rightarrow$

Casimir scaling:$\star\star$; 'MQCD' Sine scaling:$(\star)$; Karabali-Nair:$\star\star\star$

Corrections: $O(1/N)$ or $O(1/N^2)$?

e.g.

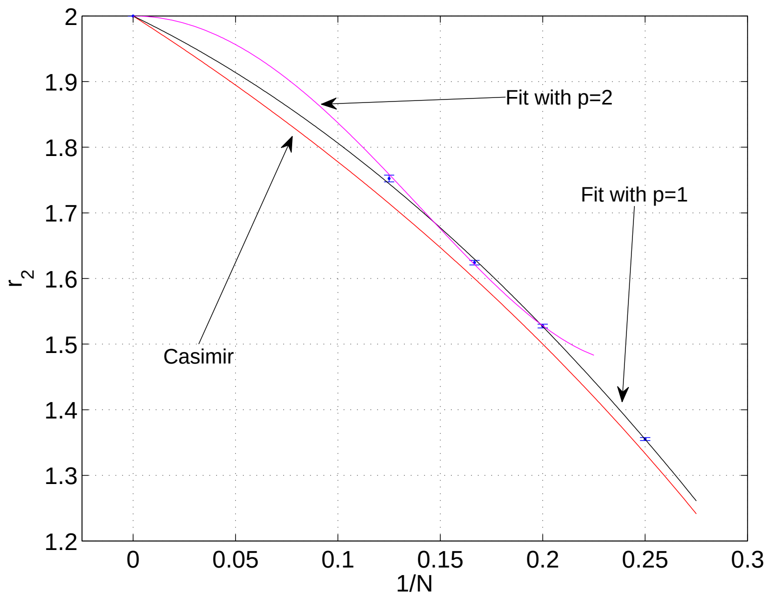
Casimir scaling gives an $O(1/N)$ correction:

$\frac{\sigma_k}{\sigma} = \frac{k(N-k)}{N-1} = k - \frac{k(k-1)}{N-1}$

while the (MQCD) sine formula gives a more conventional $O(1/N^2)$ correction.

since an $O(1/N)$ correction can feed into an $O(1/N)$ correction to glueball masses – think of excited glueballs composed of closed k-strings – this is an interesting issue ...

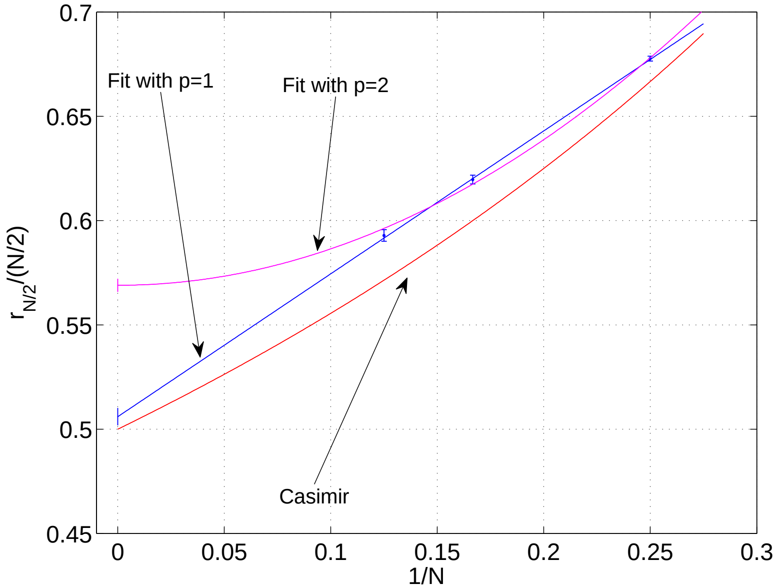
fit $$\frac{\sigma_{k=2}}{\sigma} = 2 - \frac{a}{N^p} - \frac{b}{N^{2p}}$$



$\Rightarrow$

$p = 1$ fit OK ; and gives $\sim$1,0 for N =3,2

$p = 2$ fit only possible if we drop SU(4) fit $$\frac{\sigma_{k=\frac{N}{2}}}{\sigma} = \frac{N}{2}\left\{a + \frac{b}{N^p}\right\}$$



$\Rightarrow$

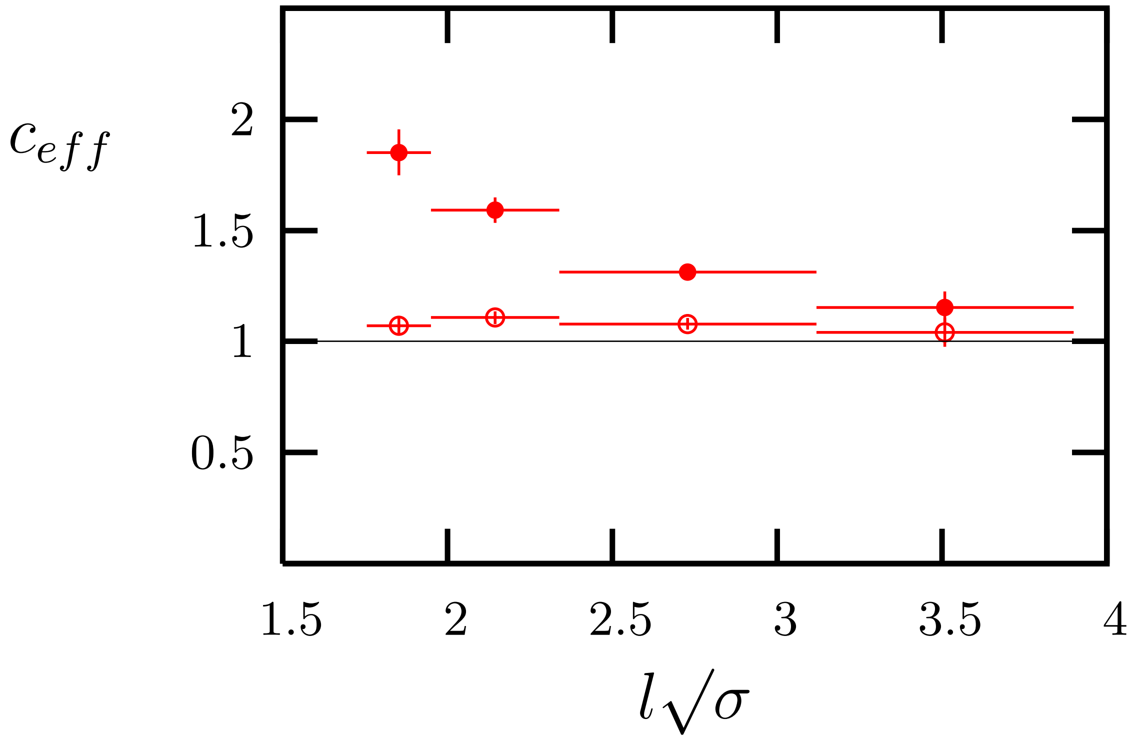
$p = 1$ fit OK, and $a = 0.501(4)$ consistent with Casimir

$p = 2$ fit less good

$\Rightarrow$ overall evidence is for $O(1/N)$ corrections

What about D=3+1?

[SU(3) ; $\beta = 6.0625$; $l_c\sqrt{\sigma} \sim 1.6$]



○ Nambu-Goto; ● Luscher

relevant string quantum numbers in 3+1 dimensions:

∘ length, l.

∘ momentum along string, $p = 2\pi q/l$.

∘ angular momentum around string axis, $J = 0, 1, 2...$

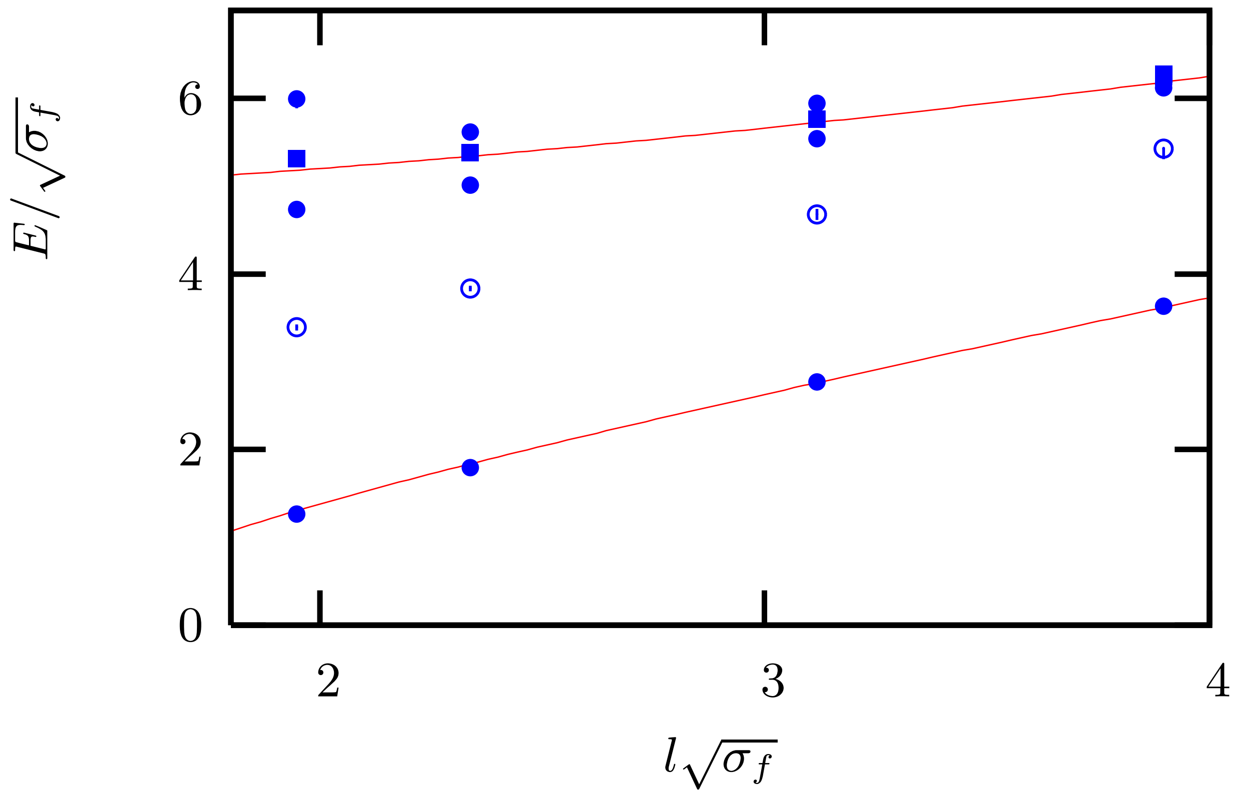
∘ $D = 2 + 1$ parity in plane orthogonal to string axis, P_ρ

∘ reflection 'parity' across this same plane, P_r

$\Rightarrow$

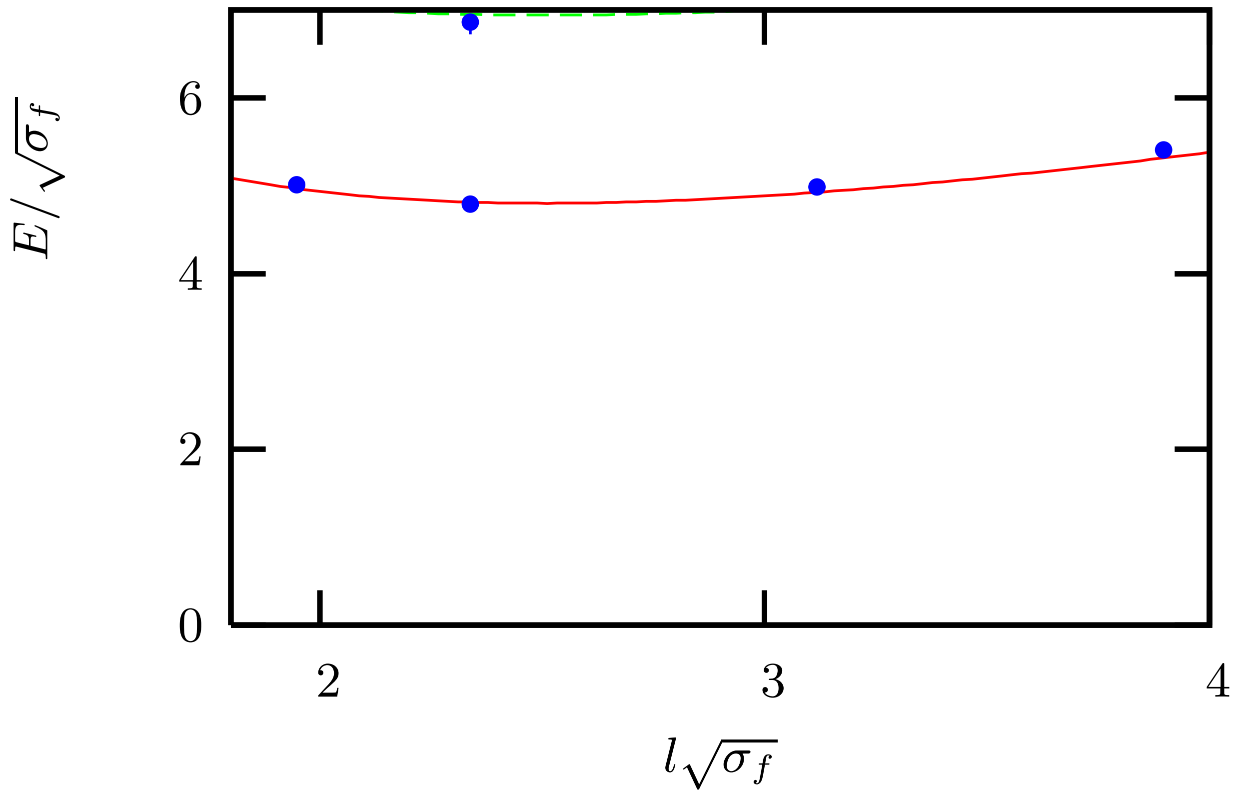
excitation spectrum – in progress

SU(3) D=3+1 ; $q = 0$ spectrum



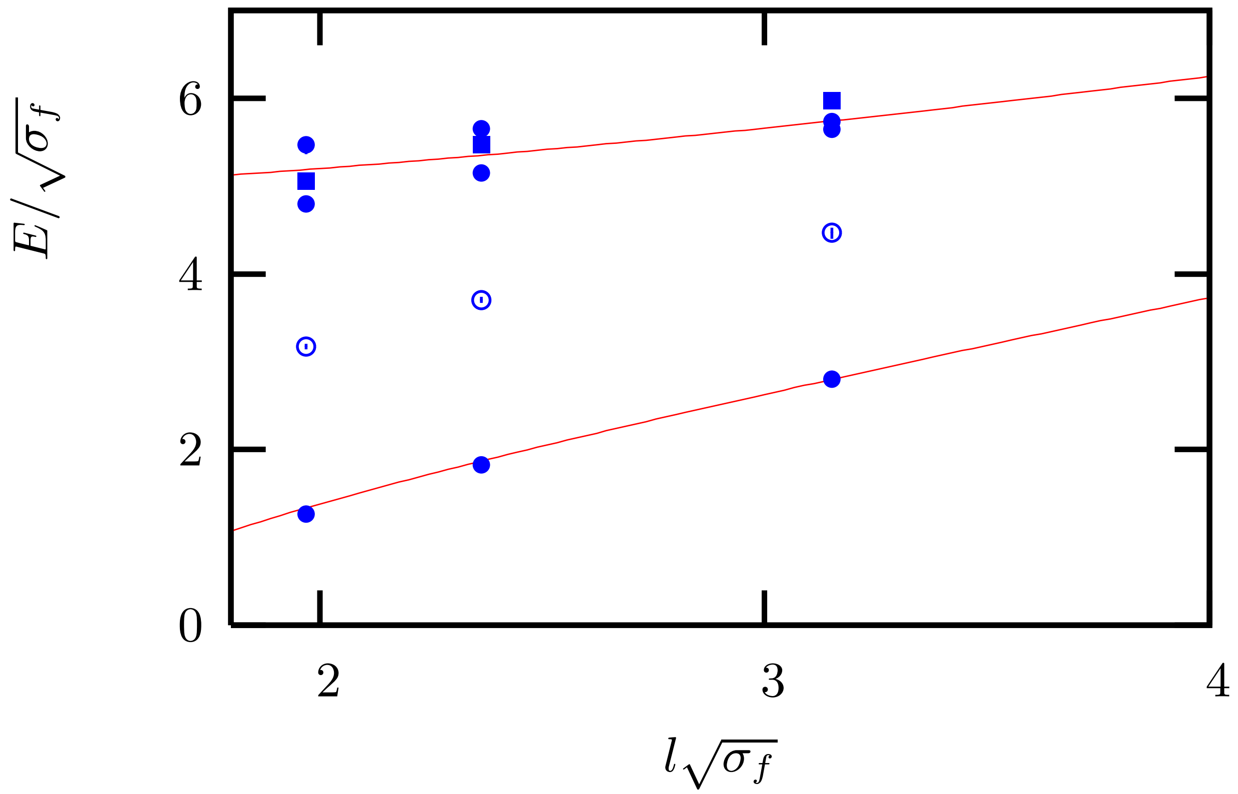
$\bullet$: $J = 0, P_\rho = P_r = +$. $\circ$: $J = 0, P_\rho = P_r = -$. $\square$: $J = 2, P_\rho = P_r = +$. $\bullet$: $J = 0, P_\rho = -, P_r = +$.

SU(3) D=3+1 ; $q = 1$ ground state



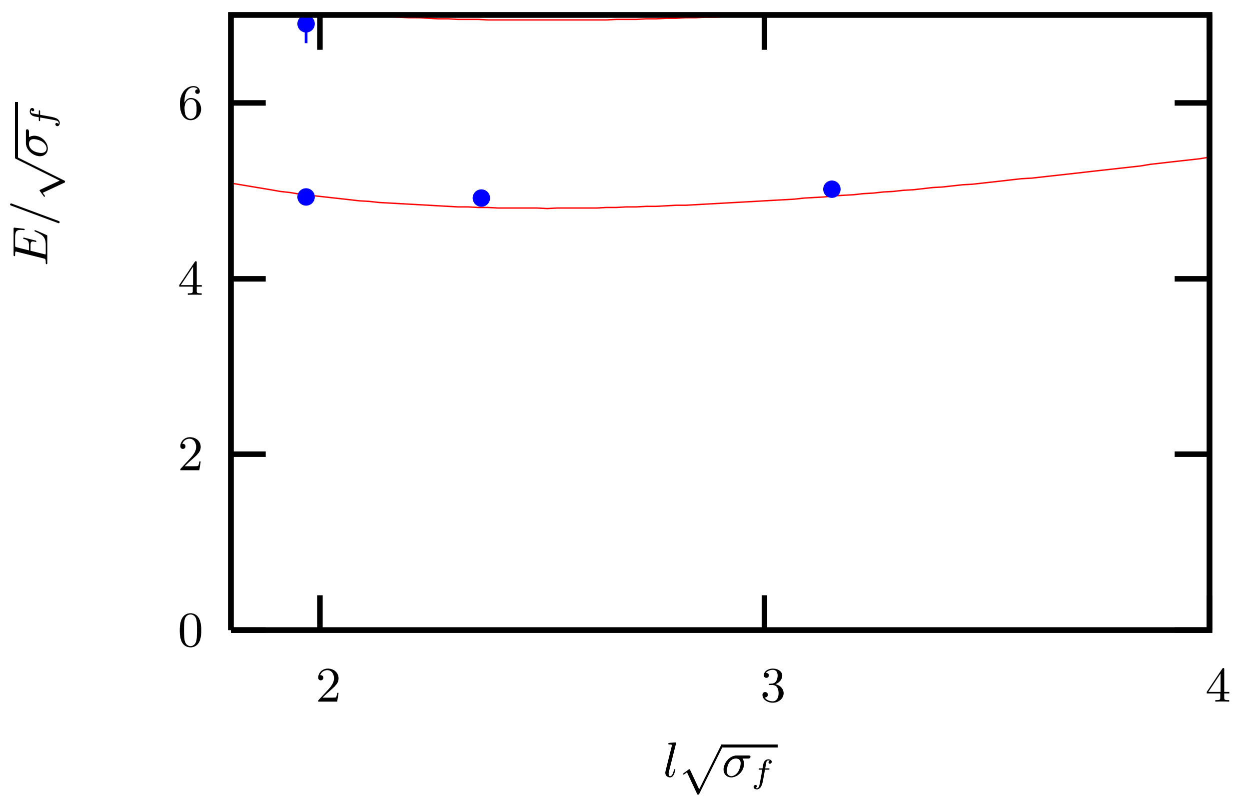
● : J=1

SU(5) D=3+1 ; $q = 0$ spectrum



$\bullet$: $J = 0, P_\rho = P_r = +$. $\circ$: $J = 0, P_\rho = P_r = -$. $\square$: $J = 2, P_\rho = P_r = +$. $\bullet$: $J = 0, P_\rho = -, P_r = +$.

SU(5) D=3+1 ; $q = 1$ ground state



● : J=1

$\Rightarrow$

in $D = 3 + 1$ many states remarkably close to NG;
and some very far from NG

– reminiscent of $k > 1$ in $D = 2 + 1$

as in $D = 2 + 1$, there is very little N-dependence

Some Conclusions

• The flux tube spectrum in $D = 2 + 1$ is very close to Nambu-Goto $\forall N$
– down to such small string lengths, $l\sqrt{\sigma} \sim 2$, that an expansion in $1/\sigma l^2$ no longer converges
$\rightarrow$ this striking feature cannot be readily attacked in the usual approach where one considers the low-energy effective string action, order-by-order in derivatives of transverse fluctuations

• It is also hard to see, within a traditional, bottom-up 'dual non-Abelian Nielsen-Olesen vortex' picture, why the fat flux-tube blob should have a phonon-like spectrum of a non-interacting thin string.

• Even a short fat flux tube appears to know that it is really a string: evidence for a stringy dual?
– although this does not in itself resolve this puzzle.

• In $D = 3 + 1$ corrections are larger but still modest, and again many states are remarkably close to NG down to very small $l\sqrt{\sigma}$, again $\forall N$; but there are other states that have large corrections.

• This is also the case for $k > 1$ strings in $D = 2 + 1$.

• In some sense this situation is ideal – far better than just having a variety of corrections all over the place.
It provides a potentially useful focus on the structure of the dynamics e.g our 2-phonon scattering example.

• But where are the massive modes?