Large N and confining flux tubes as strings
– a view from the lattice

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Zakopane '09

Lecture I

Large N and the lattice – brief overviews

Is $N = \infty$ physically relevant: i.e. is large-N confining and is $N = 3$ close to $N = \infty$?

What have we learned so far?

Lecture II

What string theory describes confining flux tubes in $D = 3$ and $D = 4$?

Large N – a brief overview

• QCD : value of $g^2 \leftrightarrow$ scale of physics,

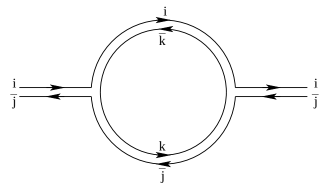
$\Rightarrow$ there is no obvious expansion parameter

$\Rightarrow$ try something much less obvious 't Hooft 1974:

expand SU(N) as a power series in $1/N$ around SU(∞)

$$\mathrm{SU}(N) \simeq \mathrm{SU}(\infty) + \mathrm{O}(1/N^2)$$

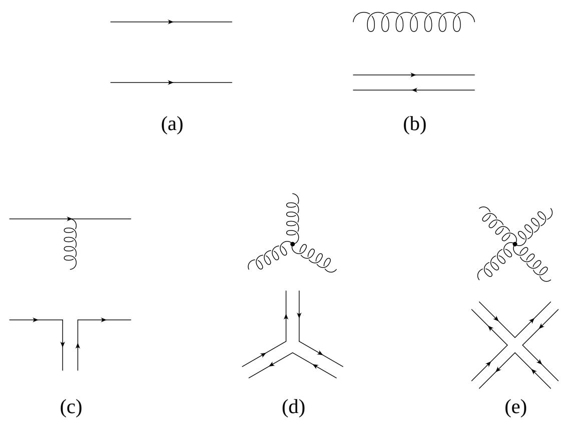
• now, in perturbation theory gluon loop is $O(g^2 N)$



so to have smooth large-N limit for perturbative physics on scale l

$$\Rightarrow \quad g^2(l)N = \text{const} \quad \text{at large } N$$

this uses 't Hooft's double line notation



Note that although the argument looks rooted in diagrams, it is more general than that:

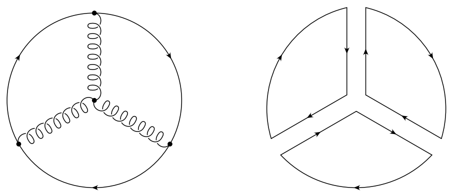
if on a typical physical scale $g^2(l)N \sim N^\epsilon$ then at $N = \infty$:

$\epsilon > 0 \Rightarrow$ no asymptotic freedom ; $\epsilon < 0 \Rightarrow$ the theory is free

examples of large-N counting

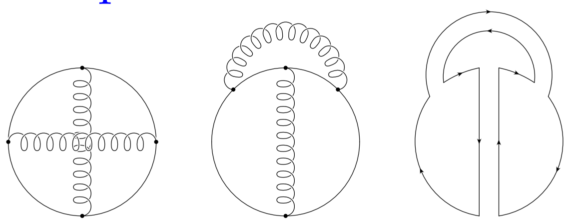
plagiarised from Manohar's '98 Les Houches lectures

planar vacuum bubble:



$$\sim g^4 N^3 \sim N(g^2 N)^2$$

non-planar vacuum bubble:



(a) (b) (c)

$$\sim g^4 N \sim N(g^2 N)^2 \times \frac{1}{N^2}$$

- $N \to \infty$ colour singlet phenomenology

't Hooft, Witten-Veneziano, Dashen-Manohar, ...

zero decay widths; no mixing; exact OZI, η'; SU(4) spin-flavour symmetry for baryons; chiral effective lagrangians, ...

- no scattering of colour singlets – integrability?

but strongly interacting bound states

- factorisation colour singlet operators: e.g.

$$\langle \Phi_1(x_1)\Phi_2(x_2)\rangle = \langle \Phi_1\rangle\langle \Phi_2\rangle \left\{1 + O\left(\frac{1}{N^2}\right)\right\}$$

$\Rightarrow$ Witten's Master Field $\to$ translation invariant $\to$ Eguchi-Kawai single point reduction

$\to$ large-N lattice calculations (rough!) in mid-80's

- the large N counting for hadrons follows from:

– coupling variation with N determined so as to control the 'non-confined'perturbative dynamics

– in the confined phase the probability for colour singlets from products of adjoints $\rightarrow 0$ as $N \rightarrow \infty$

- Feynman diagrams on 2D surfaces :

$g^2 N \rightarrow \infty \rightarrow$ vertices dense $\rightarrow$ stringy sheets

$\Rightarrow$

$N = \infty$ gauge theory $\sim$ a string theory   't Hooft, 1974

$N = \infty$ gauge theory $\sim$ dual to a string theory   Maldacena, 1997

• Since 1997 Maldacena new hope of a solution at $N = \infty$ has been provided by the strong-weak coupling gauge-gravity dualities and this has provided new motivation for numerical calculations at large N

• A trivial but effective strategy is to repeat the calculations for larger N and compare the results, i.e. SU(2), SU(3), SU(4), SU(5), SU(6), ...

• Since the leading correction in a theory with just adjoint fields is expected to be $O(1/N^2)$, going to say $N = 8$ should usually be sufficient to provide a range of N from which we can extrapolate using

$$\frac{m(N)}{\sqrt{\sigma(N)}} = \frac{m(\infty)}{\sqrt{\sigma(\infty)}} + \frac{c}{N^2} + O\left(\frac{1}{N^4}\right)$$

Lattice – overview

Wilson 1974 ; Creutz 1979-80

- Euclidean $R^4 \longrightarrow$ hypercubic lattice on T^4 : finite problem

- comparing colour:

continuum infinitesimal: $x_\mu \bullet - \bullet\, x_\mu + \hat{\mu}\delta x$: $A_\mu(x) \in$ SU(N) Lie Algebra

$\longrightarrow$

continuum finite: $x_\mu \bullet - - - \bullet\, x'_\mu$: $P\left\{e^{\int_x^{x'} A.dx}\right\} \in$ SU(N) group

$\xrightarrow{x_\mu = a n_\mu}$

finite on lattice: $a n_\mu \bullet - - - \bullet\, a n_\mu + a\hat{\mu}$: $U_\mu(n) \in$ SU(N) group

i.e. SU(N) matrices U_l on each link l

• gauge transformation: $U_\mu(n) \to g(n)U_\mu(n)g^\dagger(n+\hat{\mu})$

$\longrightarrow$

$\mathrm{Tr}\prod_{l\in\partial c} U_l$ gauge invariant for any closed curve c

$\longrightarrow$

so $\quad Z = \int \prod_l dU_l e^{-\beta S} \quad$ where $\quad S = \sum_p \left\{1 - \frac{1}{N}\mathrm{ReTr}u_p\right\}$

and u_p is product links around the plaquette p is a suitable, although not unique, SU(N) lattice gauge theory

• symmetries ensure that:

$\int \prod_l dU_l e^{-\beta S} \quad \stackrel{a\to 0}{\longrightarrow} \quad \int DA e^{-\frac{4}{g^2}\int d^4x TrF_{\mu\nu}F_{\mu\nu}}$ with $\beta = \frac{2N}{g^2(a)} \stackrel{a\to 0}{\longrightarrow} \infty$

so we vary the parameter β in order to vary the lattice spacing a

• Monte Carlo: $\quad Z^{-1}\int \prod_l dU_l \Phi(U) e^{-\beta S} = \frac{1}{n}\sum_{I=1}^{n} \Phi(U^I) + O(\frac{1}{\sqrt{n}})$

Calculating masses : Wilson, Coseners House, March 1981

• write the Euclidean correlator of an operator $\phi(t)$:

$$\langle \phi^\dagger(t = an_t)\phi(0)\rangle = \langle \phi^\dagger e^{-Han_t}\phi\rangle = \sum_i |c_i|^2 e^{-aE_i n_t} \stackrel{t\to\infty}{=} |c|^2 e^{-man_t}$$

where am is lightest mass (in lattice units) with quantum numbers of ϕ. In particular, take $\vec{p} = 0$, colour singlet, and some particular J^{PC}.

• in a numerical calculation, with finite errors, we need to be able to calculate am at small t before the 'signal' has become too small, so that we have *significant* evidence for the exponential $\propto e^{-man_t}$ over some range of n_t – i.e. we need (normalised) $|c|^2 \simeq 1 \quad \leftrightarrow \phi$ is a good wavefunctional for the desired ground state

Example of problem:

if we use simple Wilson loops of various sizes, then

the overlap onto ground state $\twoheadrightarrow 0$ as $a \longrightarrow 0$

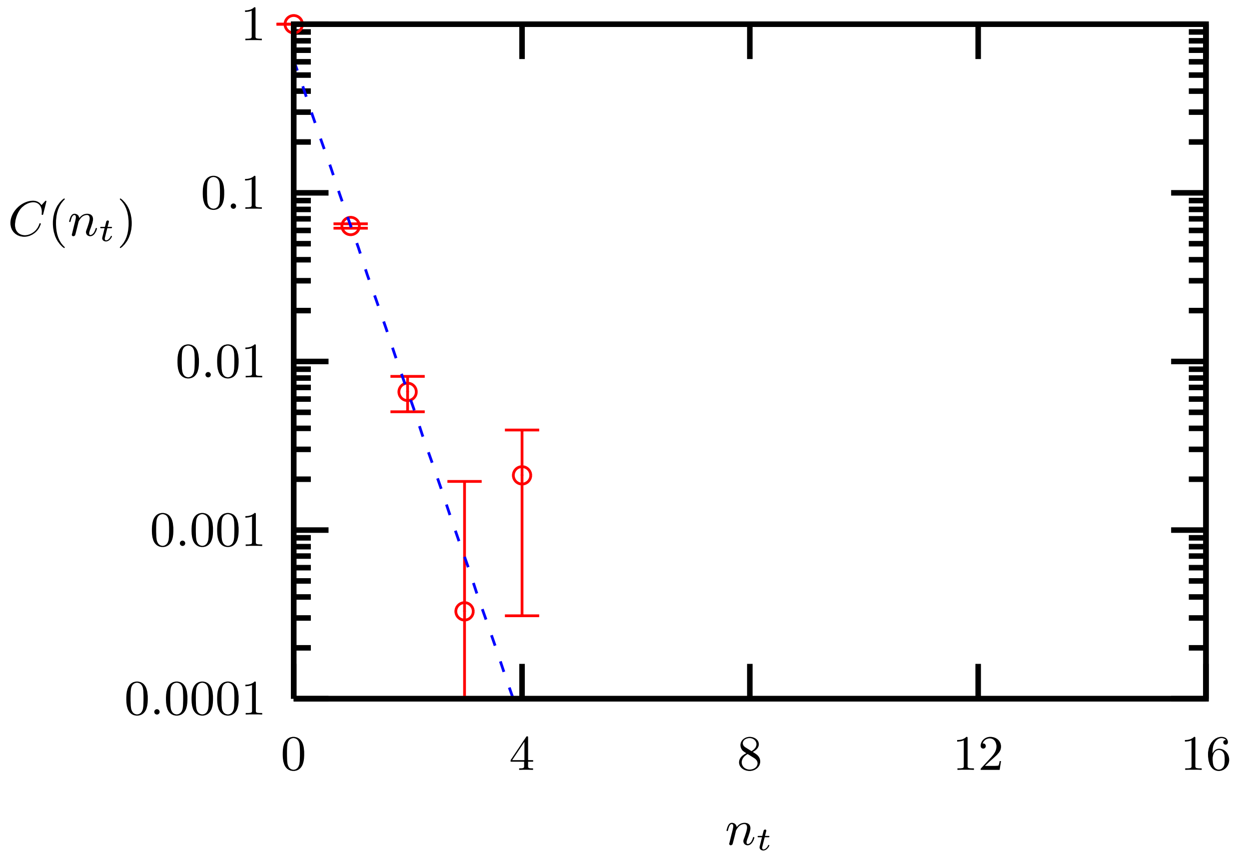
and the 'signal' is lost in the 'noise' before we can identify the mass of the ground state

e.g. take

SU(3), 32^4, $a \simeq 0.046$ 'fm'

and use the simple plaquette for the glueball operator

$\longrightarrow$



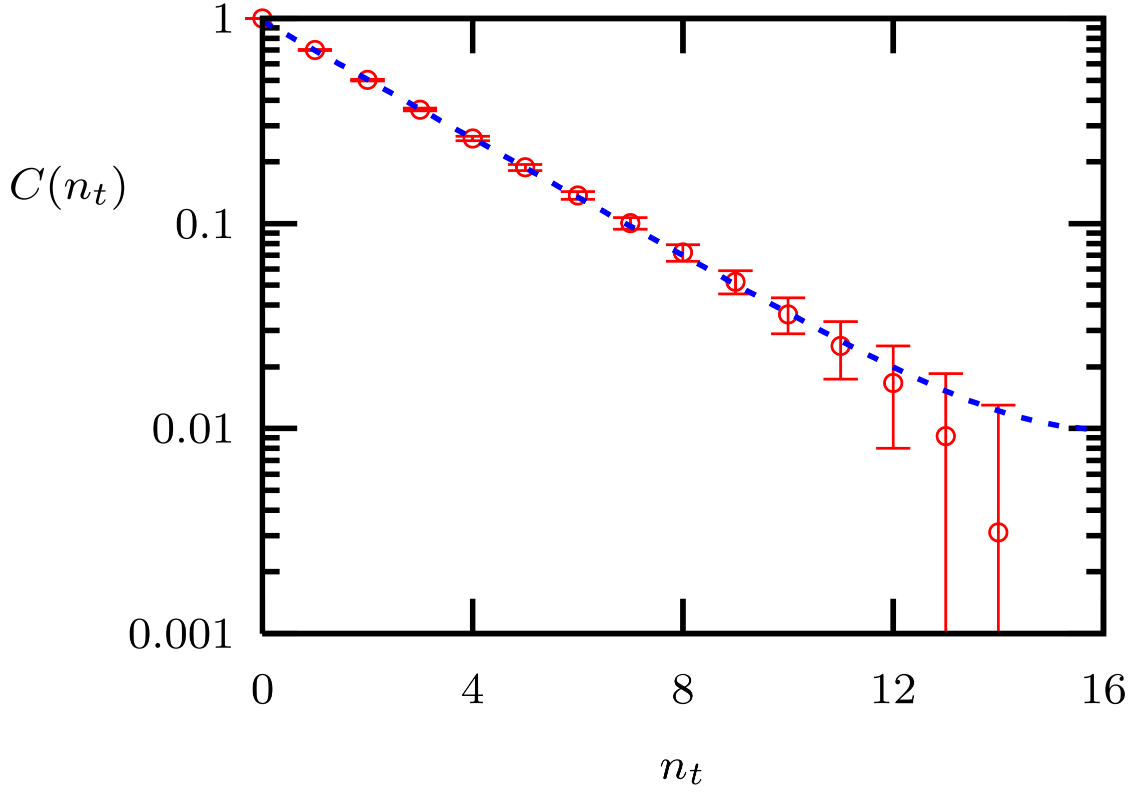
$C(t) \propto e^{-amn_t}$ at larger $t = an_t$?!?

The problem is that the plaquette is so local that it does not see the structure of a wave-function and will therefore have a roughly equal overlap on all states.
Since the number of states increases rapidly with decreasing a, the overlap onto the groundstate will decrease rapidly.
So what we need are operators that are 'smooth' on a scale $\sim$ 1fm

We can efficiently construct operators that are 'smooth' on physical length scales by the iterative spatial 'smearing' and 'blocking' of the lattice gauge fields, and then use these 'blocked' link matrices in constructing appropriate Wilson loops

$\longrightarrow$

best blocked/smeared glueball operator



$C(t = an_t) \overset{t\uparrow}{\simeq} |c|^2 e^{-man_t} \quad \Rightarrow \quad \text{fit}: \; am_{0^{++}} = 0.330(7)$ with $|c|^2 \simeq 0.97$

• here we have in addition generalised to a variational calculation over a vector space V_ϕ spanned by some convenient blocked/smeared operators $\{\phi_i; i = 1, ..., n\}$ of the desired quantum numbers:

$$\langle {\psi_0}^\dagger(t_0)\psi_0(0)\rangle = \max_{\phi\in V_\phi}\langle \phi^\dagger(t_0)\phi(0)\rangle = \max_{\phi\in V_\phi}\langle \phi^\dagger e^{-Ht_0}\phi\rangle$$

where t_0 is some convenient value of t. Then ψ_0 is our best variational estimate for the true eigenfunctional of the ground state (with these quantum numbers). We can now use $\langle {\psi_0}^\dagger(t)\psi_0(0)\rangle$ to obtain our best estimate of the ground state mass.

• generalise this in an obvious way to calculating excited state energies

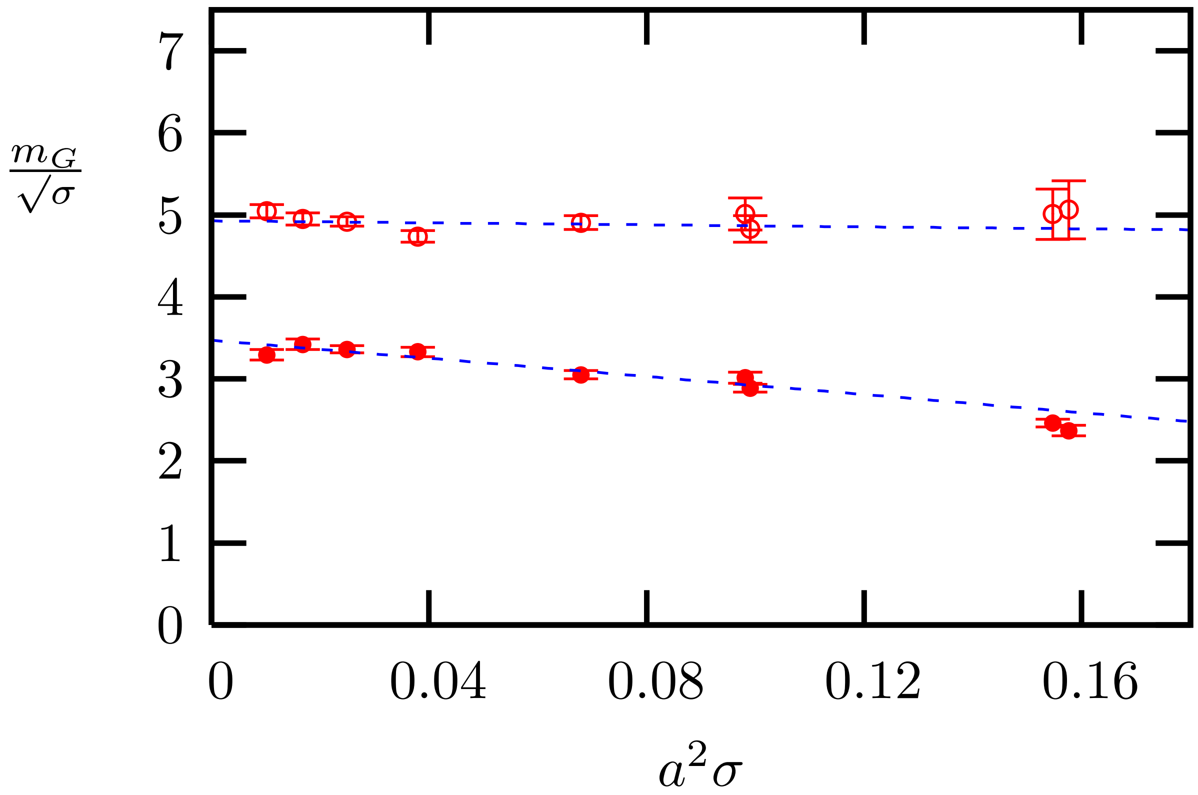
So, we are able to calculate masses accurately at a chosen value of a determined by the choice of $\beta = 6/g^2(a)$ in the lattice action.
But what we want is the continuum value – not some lattice value!

• to obtain the continuum limit from masses that are in lattice units and are distorted by the finite lattice cutoff, take dimensionless mass ratios and extrapolate with an $O(a^2)$ correction, Symanzik early-80's , e.g.

$$\frac{am(a)}{a\sqrt{\sigma(a)}} = \frac{m(a)}{\sqrt{\sigma(a)}} = \frac{m(0)}{\sqrt{\sigma(0)}} + c_0 a^2 \sigma + O(a^4)$$

where we choose to use the square root of the string tension σ as one of the masses (here c_0 is a power series in the bare coupling, but this logarithmic variation with a can usually be ignored)

SU(3) continuum limit: B.Lucini et al: hep-lat/0404008



(●) $\frac{m_{0^{++}}}{\sqrt{\sigma}}$; (○) $\frac{m_{2^{++}}}{\sqrt{\sigma}}$

$O(a^2)$ extrapolations to $a = 0$:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.47(4) - 5.52(75)a^2\sigma \qquad ; \qquad \frac{m_{2^{++}}}{\sqrt{\sigma}} = 4.93(5) - 0.61(1.36)a^2\sigma$$

$\longrightarrow$

$$m_{0^{++}} \simeq 3.5\sqrt{\sigma} \simeq 1.6 \text{ GeV}$$

which fits in with the three observed $J^{PC} = 0^{++}$ flavour 'singlet' states $f_0(1350), f_0(1500), f_0(1700)$ coming from mixing of nearby $u\overline{u} + d\overline{d}$, $s\overline{s}$ and glueball states

Aside : Full QCD

• there are now calculations with quark masses at their physical values i.e. $m_{u,d} \sim 5\text{MeV}$; $m_s \sim 90\text{MeV}$

e.g.

PACS-CS Collaboration:

Y. Kuramashi, Plenary Talk at Lattice 2008, arXiv:0811.2630

and

S. Durr et al., Science 322 (2008) 1224

• For delicate issues we now have good lattice fermions: Neuberger overlap fermions, Kaplan-Shamir domain wall fermions

Basic questions for the large-N limit

- Is $N = \infty$ confining?

- Is $N = 3$ close to $N = \infty$?

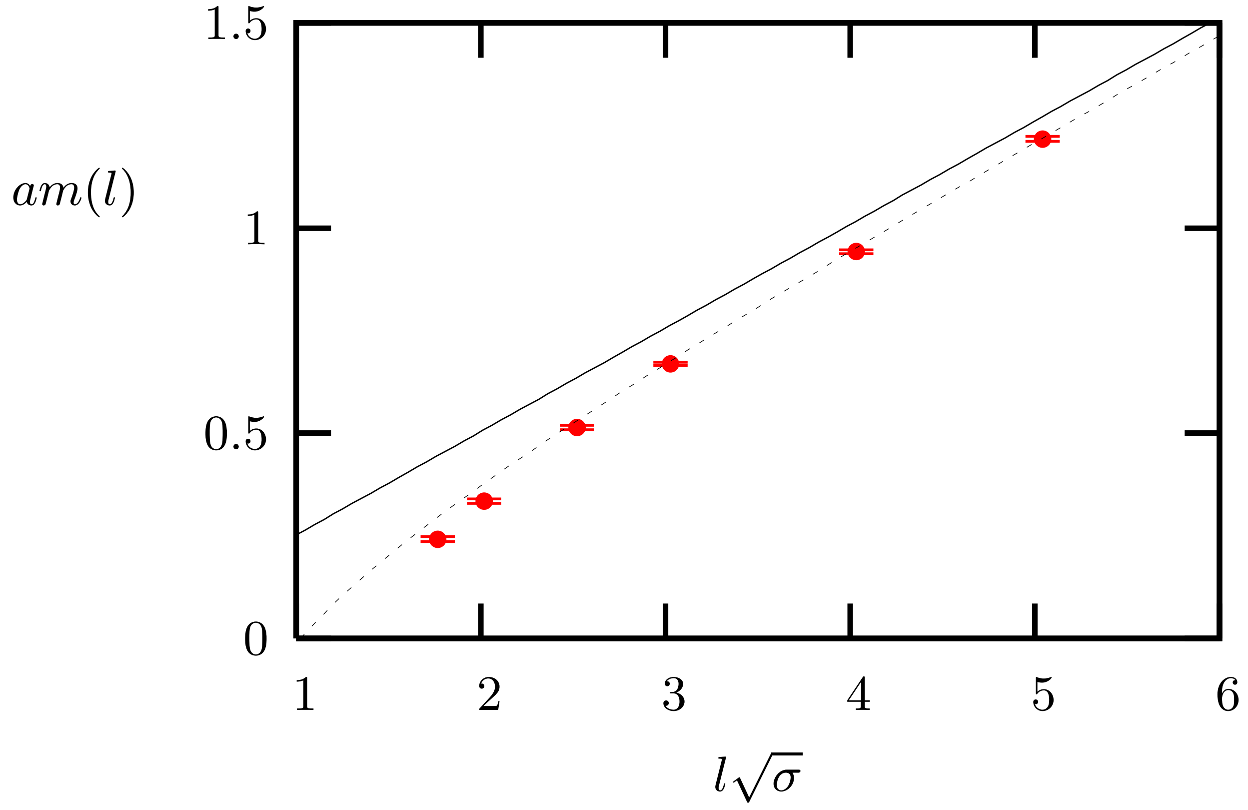
– glueball masses

– meson masses

- Non-perturbatively, do we find that the limit entails $g^2 N = \text{const}$?

SU(6) : energy of flux loop closed around a spatial torus

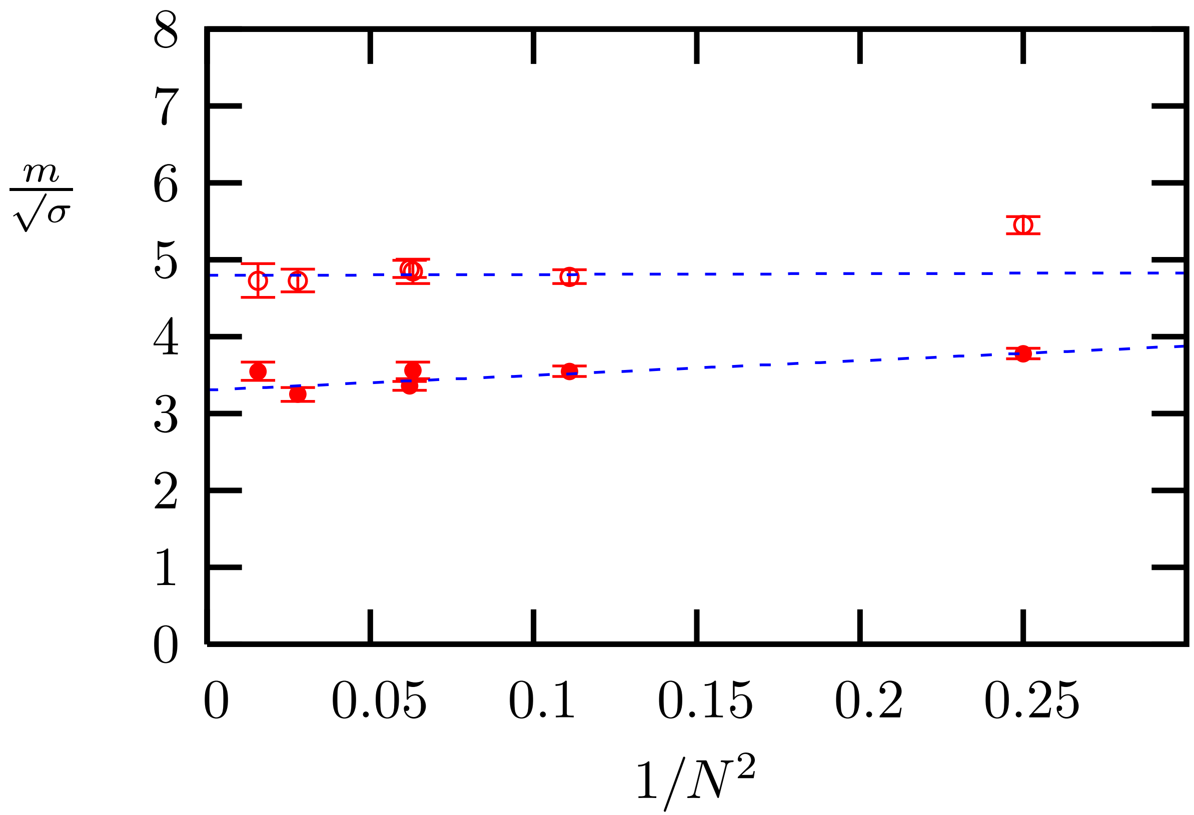
H. Meyer, M. Teper: hep-lat/0411039



⟶ linear confinement: $am(l) \simeq \sigma l - \frac{\pi}{3l}$ at large N

Glueball mass spectrum: large-N limit

B.Lucini, M.Teper, U.Wenger: hep-lat/0404008



($\bullet$) 0^{++}; ($\circ$) 2^{++} $\longrightarrow$ SU(3) is 'close to' SU(∞) for many quantities

QCD at $N = \infty$

Note : $\quad$ QCD $\stackrel{N=\infty}{\equiv}$ quenched QCD

- L. Del Debbio, B. Lucini, A. Patella and C. Pica: arXiv:0712:3036.
- G. Bali and F. Bursa, arXiv:0806:2278; arXiv:0708:3427.

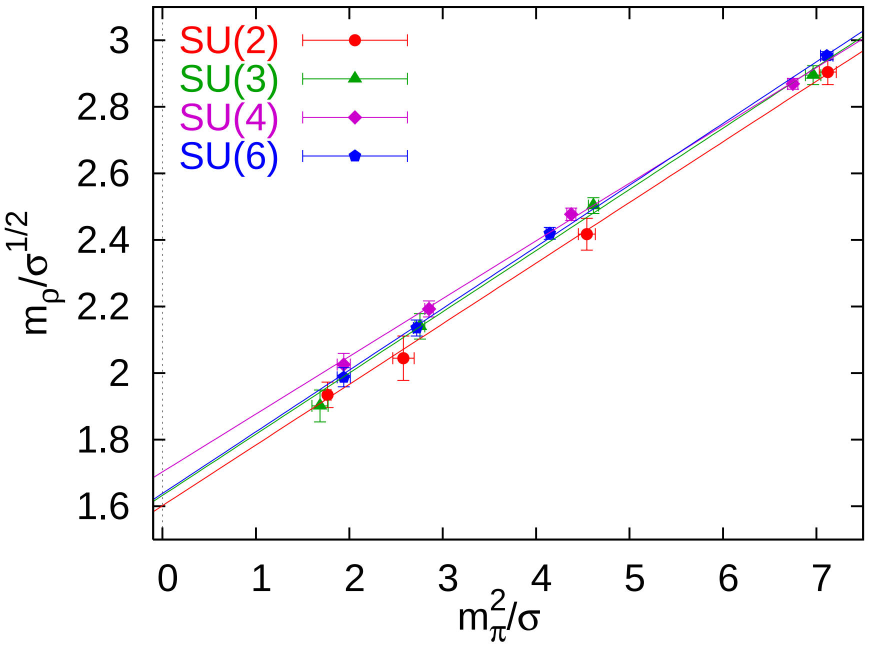
Strategy:

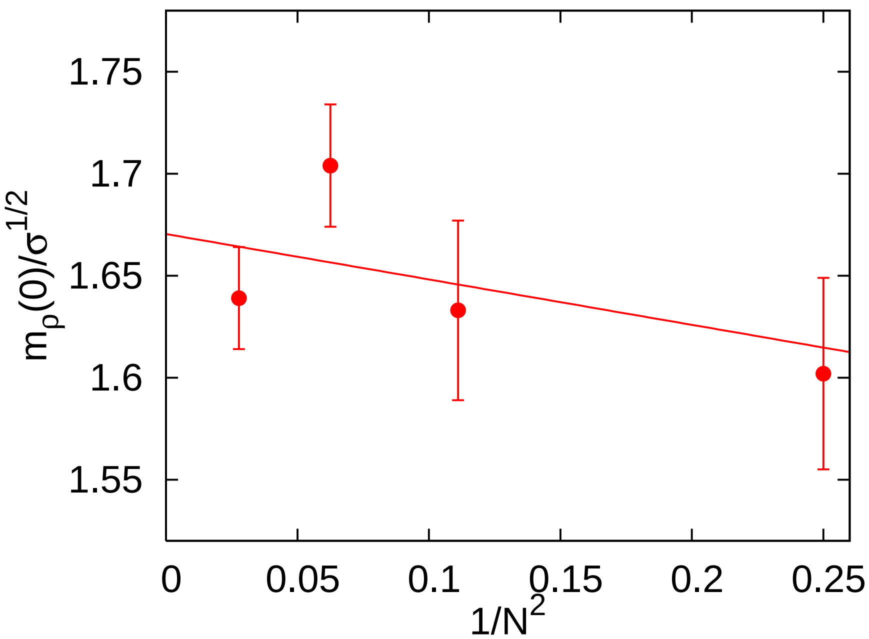
$$\text{quenched } QCD_N \stackrel{N\to\infty}{\longrightarrow} \text{full } QCD_{N=\infty}$$

- perform quenched QCD calculations at various N at a common value of a and various common values of m
- extrapolate at each fixed m to $N = \infty$, with $O(1/N^2)$ corrections
- now do conventional (full QCD) chiral extrapolation
- repeat for various a and extrapolate to continuum

- now compare to full QCD (or expt!) with SU(3)

G. Bali and F. Bursa, arXiv:0806:2278





m_ρ versus m_π (left); m_ρ for $m_q = 0$ versus $1/N^2$ (right).

Del Debbio et al: $\lim_{N\to\infty} \frac{m_\rho}{\sqrt{\sigma}} = 1.627(10) \quad ; \qquad a\sqrt{\sigma} = 0.335$

$+$

Bali and Bursa: $\lim_{N\to\infty} \frac{m_\rho}{\sqrt{\sigma}} = 1.688(25) \quad ; \qquad a\sqrt{\sigma} = 0.209$

$\longrightarrow$

$$\lim_{N\to\infty, a\to 0} \frac{m_\rho}{\sqrt{\sigma}} = 1.79(5)$$

versus, in the real world :

$$\frac{m_\rho}{\sqrt{\sigma}} \simeq \frac{770\mathrm{MeV}}{440\mathrm{MeV}} \simeq 1.75$$

$\longrightarrow$

$N = 3$ is 'close to' $N = \infty$ for full QCD ...

Some questions:

Scalar mesons as $N \to \infty$: do the $\leq$ 1GeV states disappear?

The scalar nonet and the place of lightest scalar glueball?

Flavour singlet tensor and pseudoscalar mesons and glueballs?

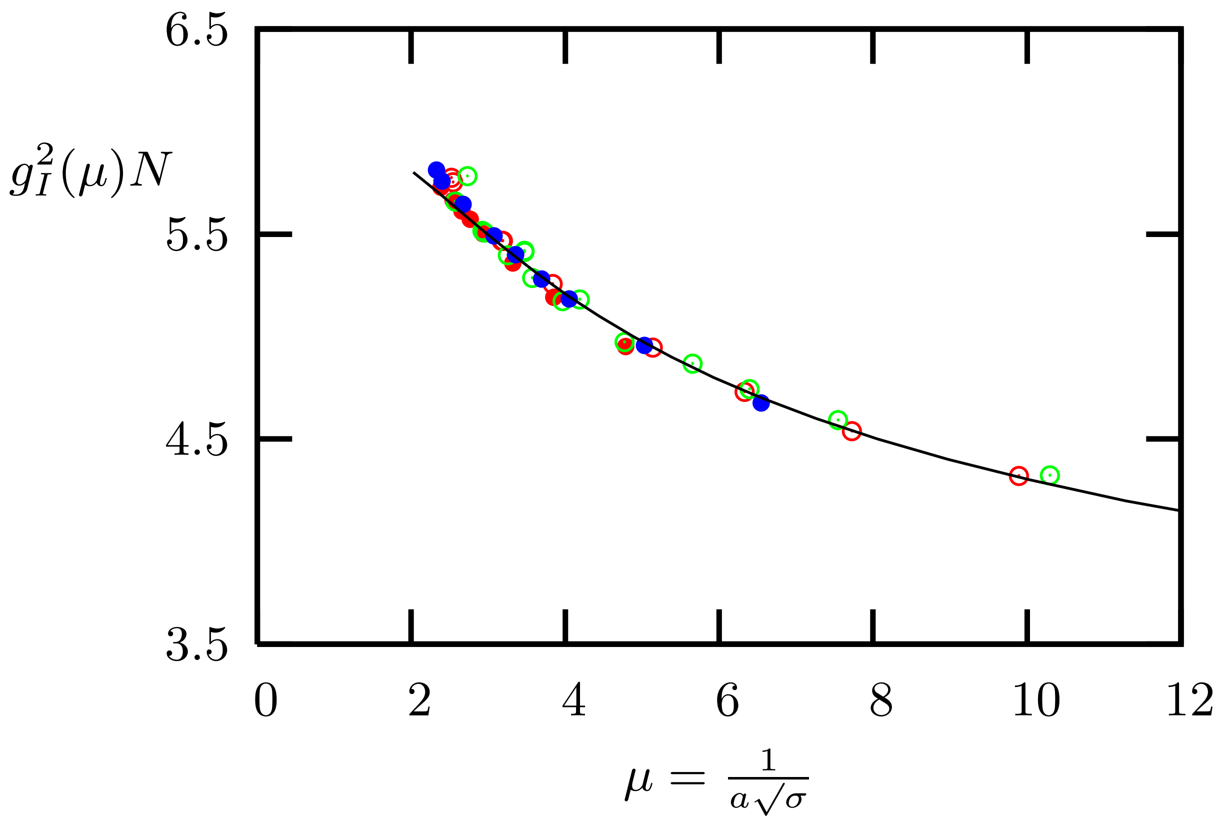
Excited states stable $\longrightarrow$ Regge trajectories?

Excited states stable $\longrightarrow$ clean meson excitation spectrum.

SU($2n_f$) baryon (Dashen-Manohar) symmetry as $N \to 3$.

$g^2 N$ fixed as $N \to \infty$? MT, Lat 08 , arXiv:0812.0085

bare coupling (Parisi): $g_I^2(a) = \frac{g^2}{u_p} = \frac{2N}{\beta}\frac{1}{u_p}$



SU(2) ○ ; SU(3) ○; SU(4) ● ; SU(6) ○ ; SU(8) ●

As well as its usual continuum running, the *bare* lattice coupling will also receive lattice spacing corrections : thus we fit the relation between the scale of the coupling, $a\sqrt{\sigma}(a)$ (the lattice spacing expressed in units of the calculated string tension), and the coupling $g^2(a)$ by:

$$\begin{aligned} a\sqrt{\sigma}(a) &= \text{lattice running} \times 3 - \text{loop continuum running} \\ &= \frac{\sqrt{\sigma}(0)}{\Lambda_s}\left(1+ca^2\sigma\right) e^{-\frac{1}{2\beta_0 g_s^2}} \left(\frac{\beta_1}{\beta_0^2}+\frac{1}{\beta_0 g_s^2}\right)^{\frac{\beta_1}{2\beta_0^2}} e^{-\frac{\beta_2^s}{2\beta_0^2} g_s^2} \end{aligned}$$

Now from the 2-loop β-function we see that:

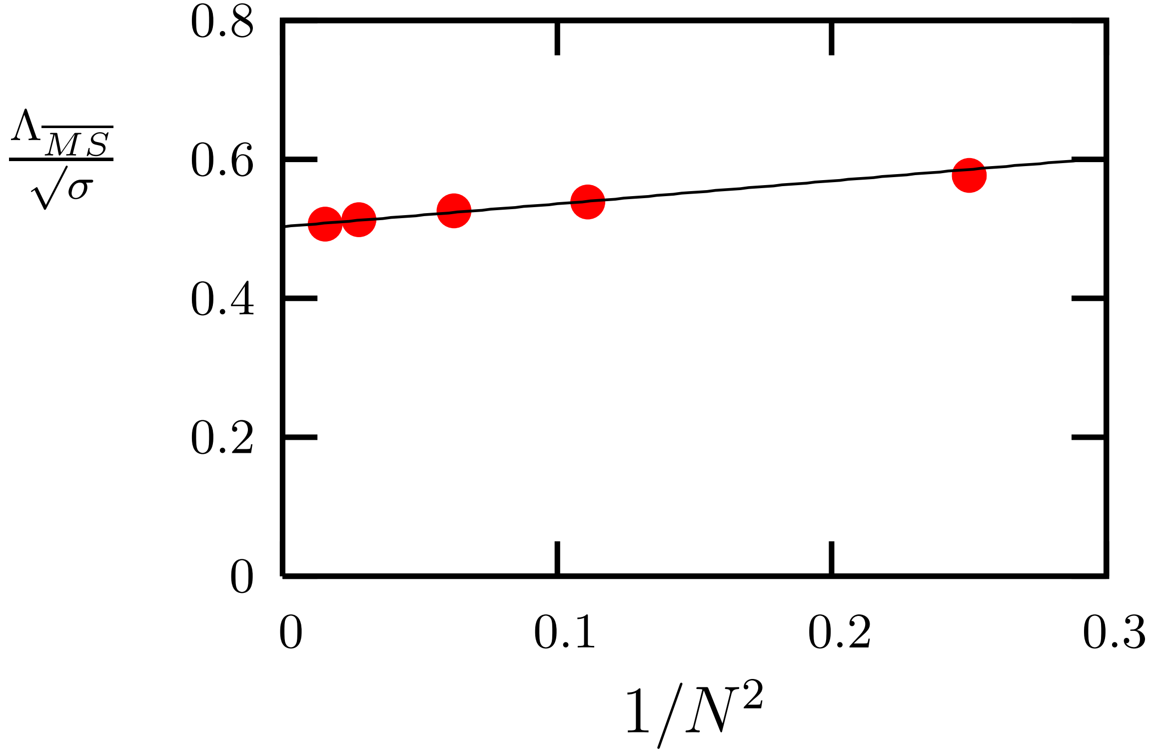
$$g^2 N = \text{constant} \quad \Leftrightarrow \quad \text{physics} = \text{ind of } N$$

$\equiv$

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = \text{ind of } N$$

Fitting the bare coupling at various N one finds:

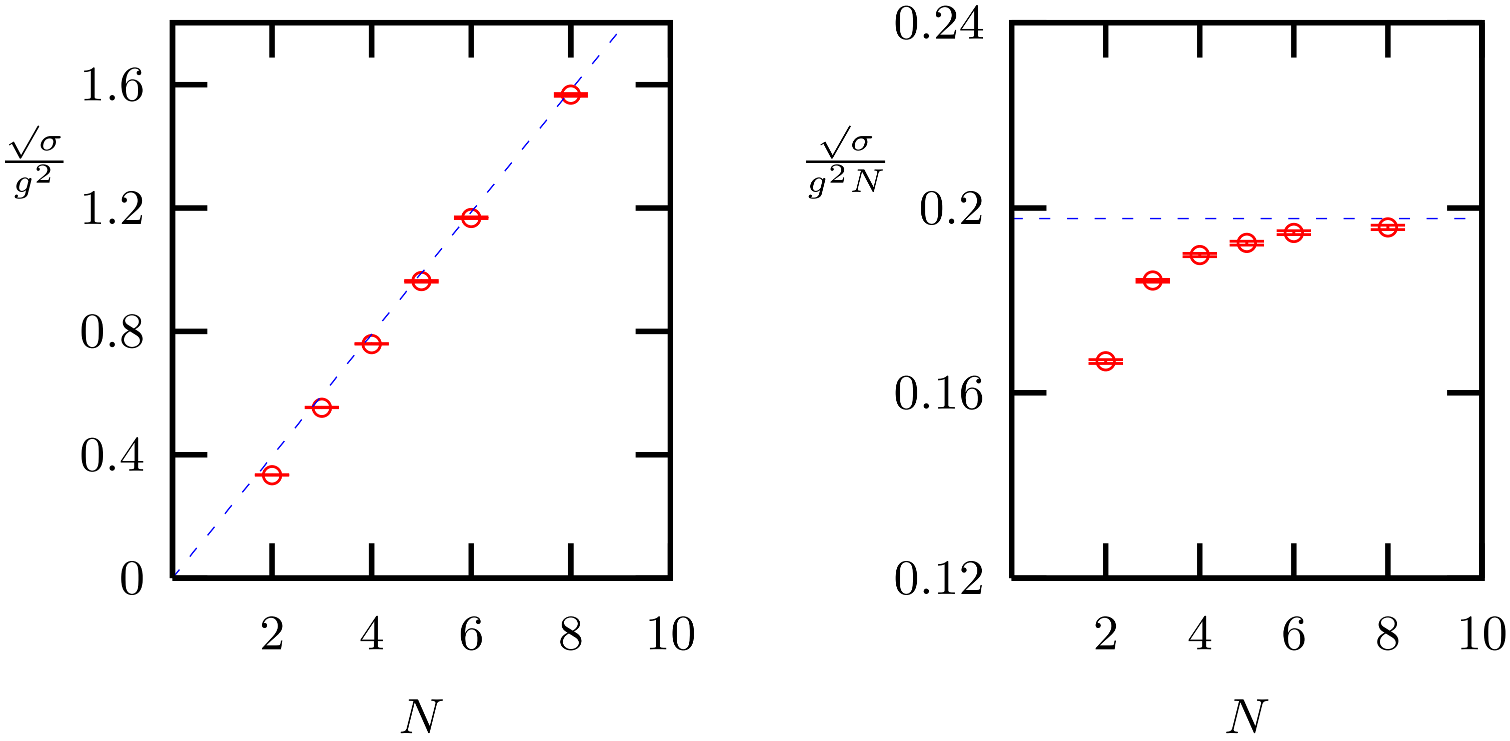
C. Allton, M. Teper, A. Trivini, arXiv:0803.1092



$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N^2} \qquad \text{QED}$$

same question in $D = 2 + 1$

g^2 dimensions of [m] : $\beta = 2N/ag^2$



smooth physics at large $N \to \quad g^2 N$ fixed

∘ So much for the cheap and dirty calculation ...

cheap $\leftrightarrow$ no extra work

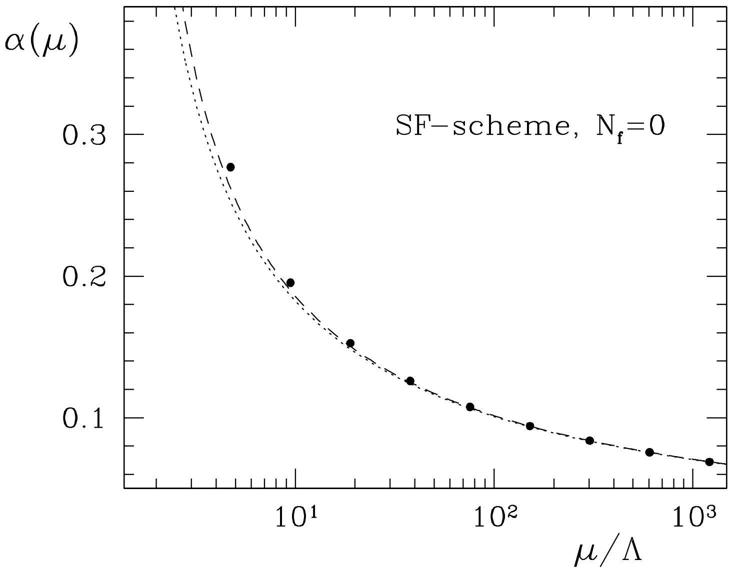
dirty $\leftrightarrow$ lattice corrections

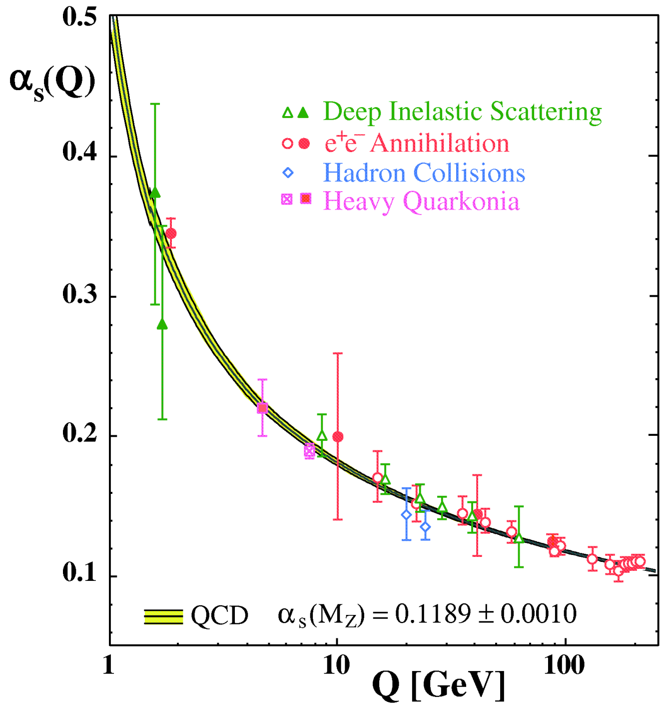
∘ clearly it would be better to calculate some coupling on a scale l at some a and then to send $a \to 0$ while keeping l fixed in 'physical units' e.g. keeping $l\sqrt{\sigma} = l/a \times a\sqrt{\sigma(a)}$ fixed.
That is: to extract a continuum running coupling from the lattice calculation.

∘ a nice calculation of this kind is the 'Schrodinger Functional scheme' of the *Alpha* Collaboration:

continuum SF coupling in SU(3)

Alpha collaboration , hep-lat/9810063 S. Bethke, hep-ex/0606035

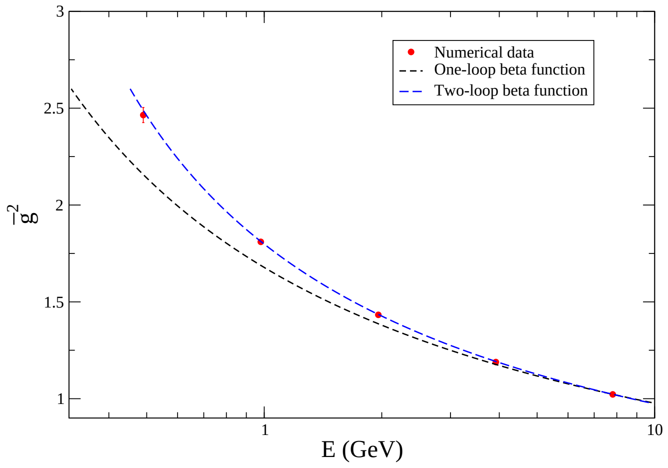


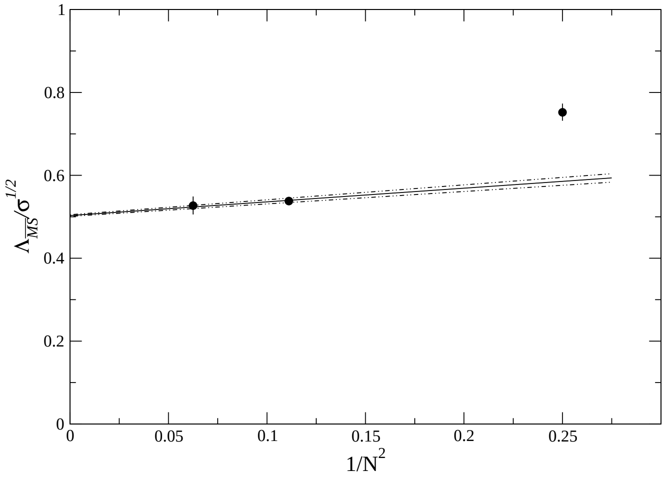


comparable range to (real) experiment ($\Lambda \sim 125\,\mathrm{MeV}$) and much more accurate!

recently for SU(4):

B. Lucini, G. Moraitis, arXiv:0805.2913, 0710.1533





new SU(4) plus older *Alpha* SU(2) and SU(3) →

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.528(40) + \frac{0.18(36)}{N^2}$$

which is consistent with bare coupling

QUESTION: does the SF coupling acquire non-perturbative jumps at the Narayanan-Neuberger $N = \infty$ phase transitions?

So:

- large N is confining

- $N = 3$ is close to $N = \infty$ for many physical quantities – so large N is phenomenononologically relevant

- $g^2(l)N =$ ind/of/N as $N \to \infty$

$\Rightarrow$

interesting and useful to investigate in detail the physics of large-N gauge theories

Calculating as $N \to \infty$: how much harder?

• $\propto N^3$ factor coming from matrix multiplication;
partly offset by smaller finite V corrections at larger N.

• We calculate masses from connected correlators i.e. correlations between fluctuations

but

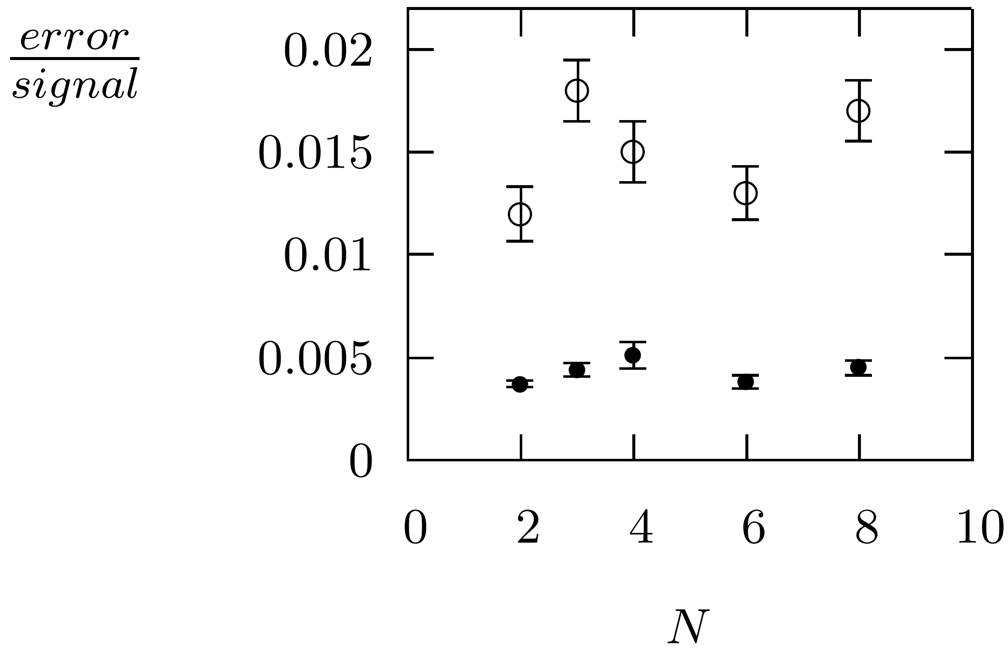
as $N \to \infty$ all fluctuations vanish

$\Rightarrow$

mass calculations become impossible as $N \to \infty$?

NO

the errors on the fluctuations are themselves determined by higher order correlators, which generically vanish at the same rate the observed ratio of error to signal for the same number of Monte Carlo field configurations is roughly independent of N



Error to signal ratio for $C_2(t)$ after 10^5 sweeps on 10^4 lattices at fixed lattice spacing, $a \simeq 1/5T_c$, and for $t = 0$ ($\bullet$) and $t = a$ ($\circ$).

• For QCD with quarks:
the most expensive part of current calculations is matrix *times* vector multiplication (e.g. in propagators) and this is $\propto N^2$;
in principle also *partly offset* by smaller finite V corrections at larger N.

• As for glueballs, we calculate masses from connected correlators

but

in this case the errors on the correlators are determined by higher order correlators, which generically vanish *not* at the same rate, but as $O(1/N)$ – which translates into an effective improvement of $\propto N^2$ in statistics

$\Rightarrow$

ideally, increasing N has no extra cost!

but

in practice things are not ideal and the cost grows as $\propto N$
Bali,Bursa

Most of the following large N topics we will not have time for ...

- space-time reduction : see Narayanan ; Neuberger

- confinement : see Blaizot ; Greensite

- instantons ; topology ; chiral symmetry breaking and topology; k-string tensions ; domain wall tensions ; intertwined θ-vacua ; glueball spectrum ; ....

Instead we focus on physics at finite temperature:

$$L_s^3 L_t \Rightarrow T = \frac{1}{a(\beta) L_t} \quad \text{if} \quad L_s \gg L_t$$

- RHIC and LHC experiments

- area of choice for AdS/CFT applications:

SUSY $\sim$ gauge theory : $T_c < T < \text{few} \times T_c$

since :

adjoint fermions acquire a Matsubara mass

scalars then unprotected and acquire mass

- AdS/CFT is at large N, so important to check what features of QCD at $T \geq T_c$ have small finite N corrections.

N counting of free energies: $F = E - TS$

At $T = T_c$ we have

$$F_c = F_g$$

but

$$F_g \sim N^2 \qquad \text{colour singlet spectrum, entropy} \sim N^0$$

So why does $T_c \not\to 0$ as $N \to \infty$?

Well

$$E_c = hadron\, masses + E_{vac}$$

and

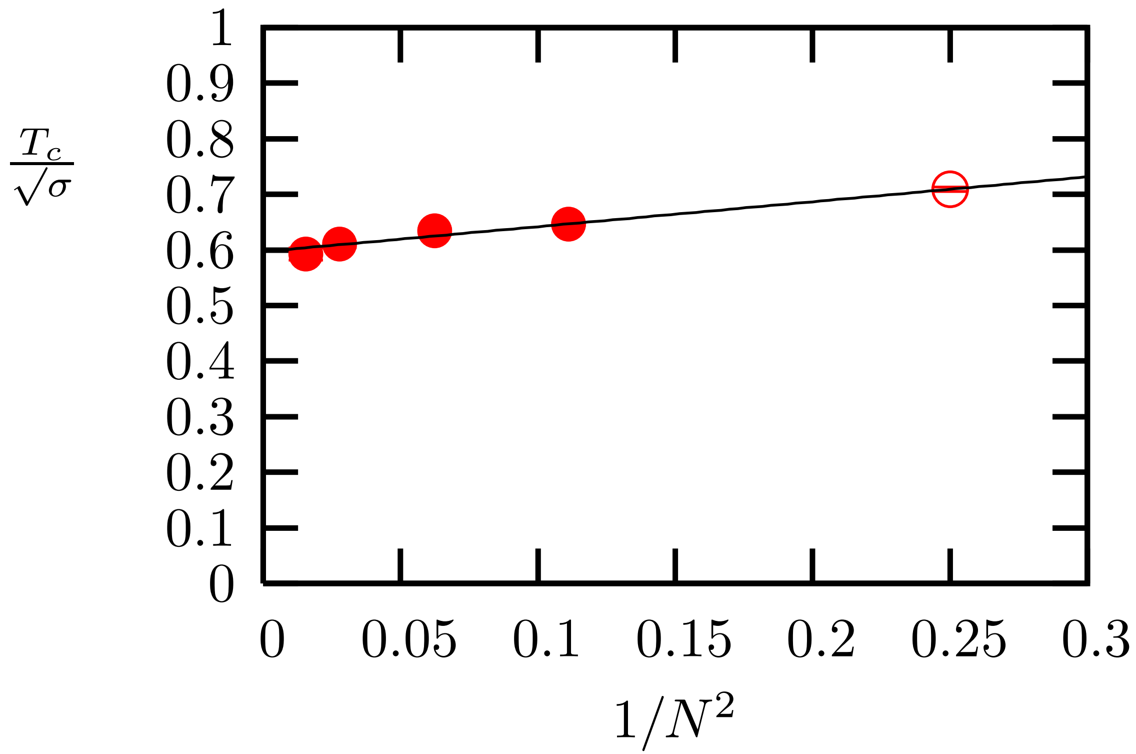
$$E_{vac} \sim -O(N^2) \sim gluon\, condensate$$

so

$$F_g = -E_{vac} \quad \text{at} \quad T = T_c$$

So if the gluon plasma above T_c was weakly coupled we could calculate T_c just from the gluon condensate

Deconfining temperature for all N B.Lucini et al: hep-lat/0307017,0502003

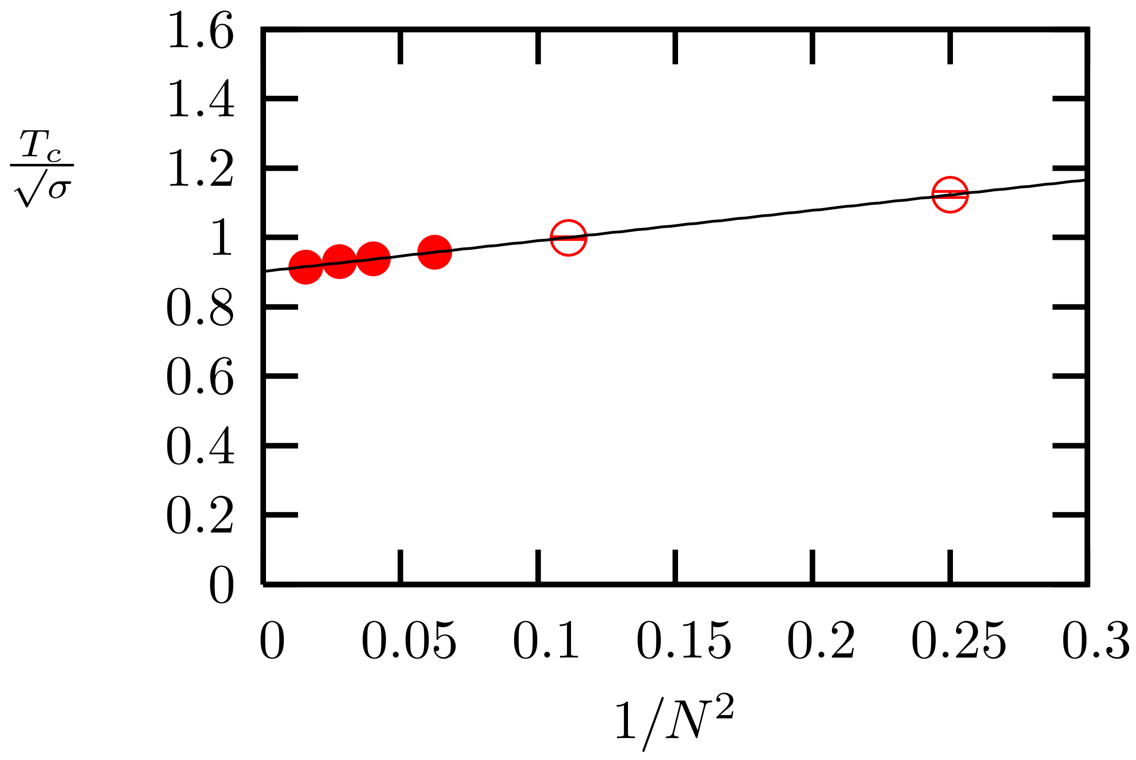


2nd order $\circ$; 1st order $\bullet$

$\Rightarrow$ $$\frac{T_c}{\sqrt{\sigma}} = 0.597(4) + \frac{0.45(3)}{N^2}$$

Aside : D=3+1 $\longrightarrow$ D=2+1

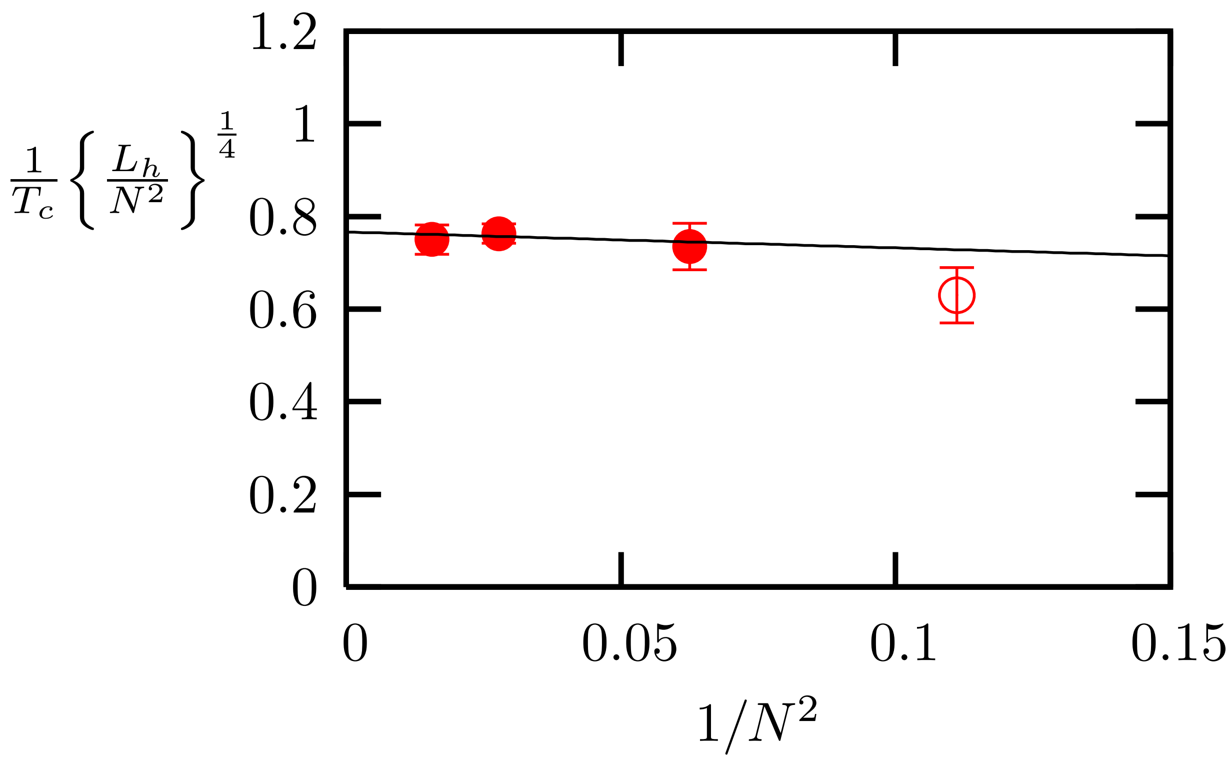
J. Liddle, M.Teper : hep-lat/0509082; arXiv:0803.2128 ; K. Holland : hep-lat/0509041 ; K. Holland, M. Pepe, U-J Wiese : arXiv:0712.1216



2nd order $\circ$; 1st order ($N = 4$ weak) $\bullet$ $\Rightarrow$ $\frac{T_c}{\sqrt{\sigma}} = 0.903(3) + \frac{0.88(5)}{N^2}$

Confinement-deconfinement latent heat

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003

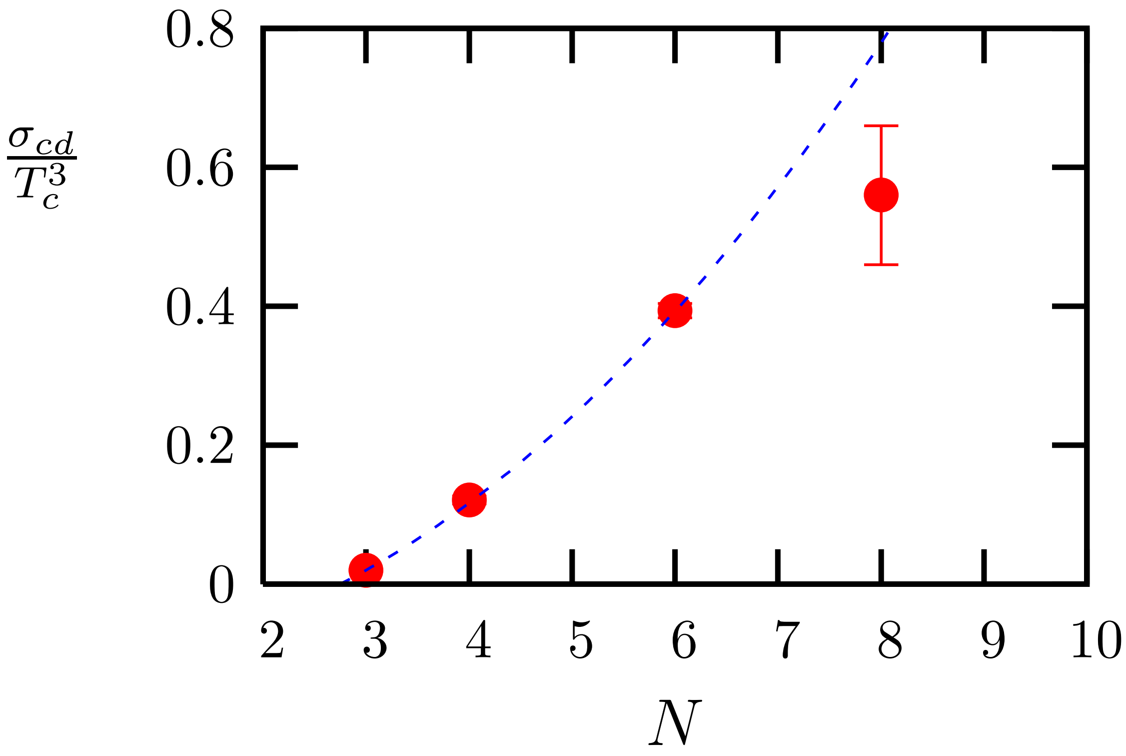


$\Rightarrow$

large-N deconfinement is 'normal' first order ; $N = 3$ 'weakly' first order

Confinement-deconfinement wall tension (aT=0.2)

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

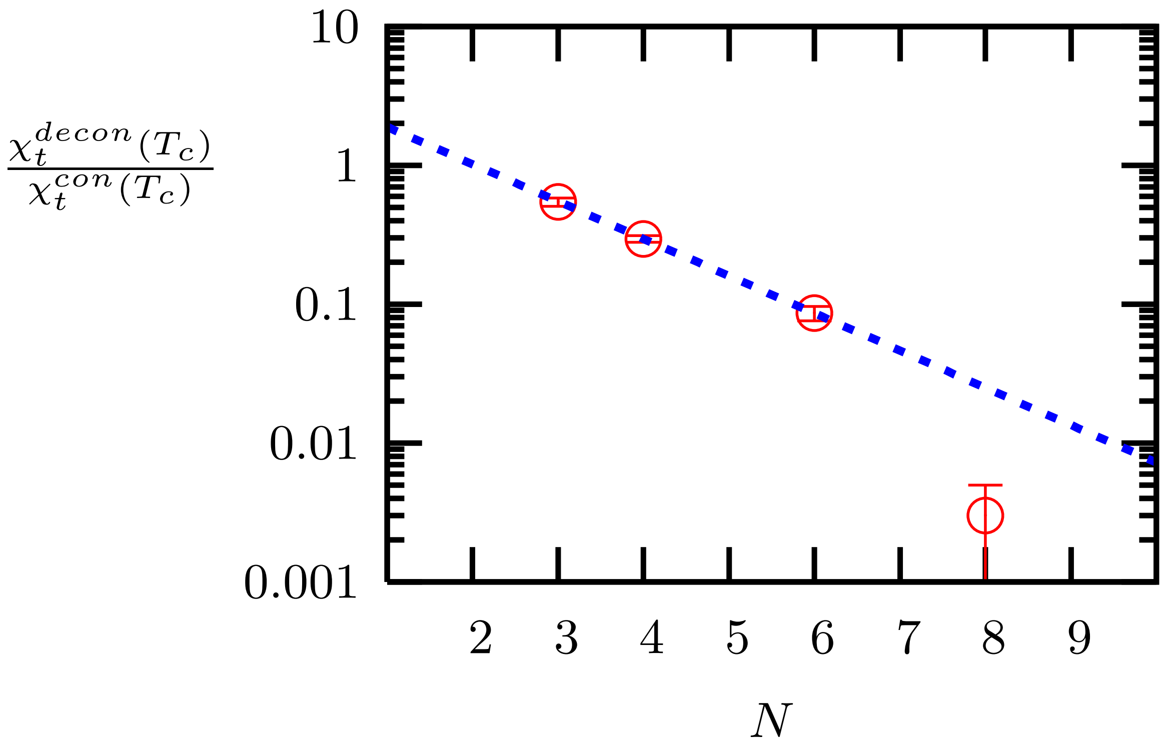


fit : $\frac{\sigma_{cd}}{T_c^3} = 0.0138N^2 - 0.104 = 0.0138N^2\left(1 - \frac{7.53}{N^2}\right)$

$\Rightarrow$ interface tension small and $O(1/N^2)$ corrections large

For $T > T_c$ topology disappears exponentially fast in N:

Lucini, Teper, Wenger, hep-th/0401028



$\Rightarrow$ the $U_A(1)$ symmetry is restored at large N – exponentially fast

Strongly Coupled Gluon Plasma - at large N?

Consider

$$Z(T,V) = \exp\left\{-\frac{F}{T}\right\} = \exp\left\{-\frac{fV}{T}\right\} = \int DU \exp\left(-\beta S_{\mathrm{W}}\right).$$

now $p = T\frac{\partial}{\partial V}\log Z(T,V) = \frac{T}{V}\log Z(T,V) = \frac{T}{V}\int_{\beta_0}^{\beta} d\beta' \frac{\partial \log Z}{\partial \beta'}$

but $\frac{\partial \log Z}{\partial \beta} = -\langle S_W \rangle = N_p \langle u_p \rangle$

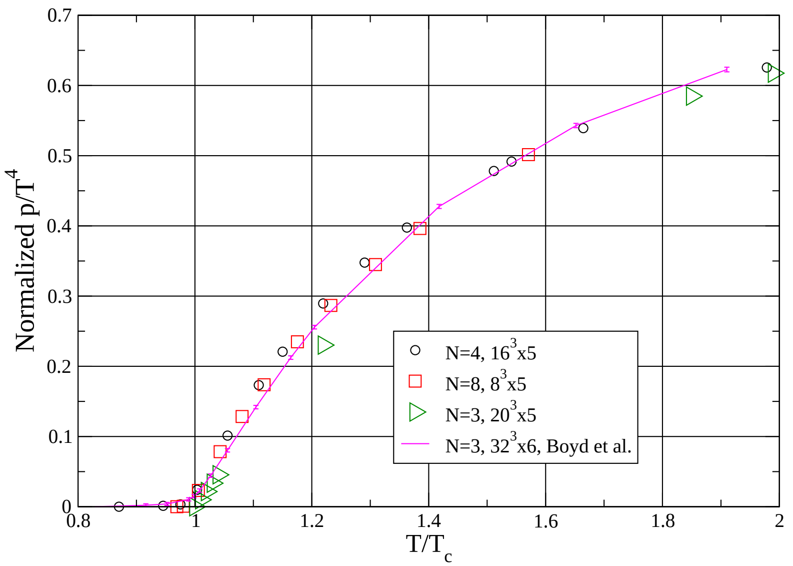
so $a^4[p(T) - p(0)] = 6\int_{\beta_0}^{\beta} d\beta' (\langle u_p\rangle_T - \langle u_p \rangle_0).$

i.e. $\frac{p(T)}{T^4} = 6L_t^4 \int_{\beta_0}^{\beta} d\beta' (\langle u_p\rangle_T - \langle u_p \rangle_0).$

similarly $(\epsilon - 3p)/T^4 = 6L_t^4(\langle u_p(\beta)\rangle_0 - \langle u_p(\beta)\rangle_T) \times \frac{\partial \beta}{\partial \log(a(\beta))}.$

Strong Gluon Plasma - high-T pressure anomaly

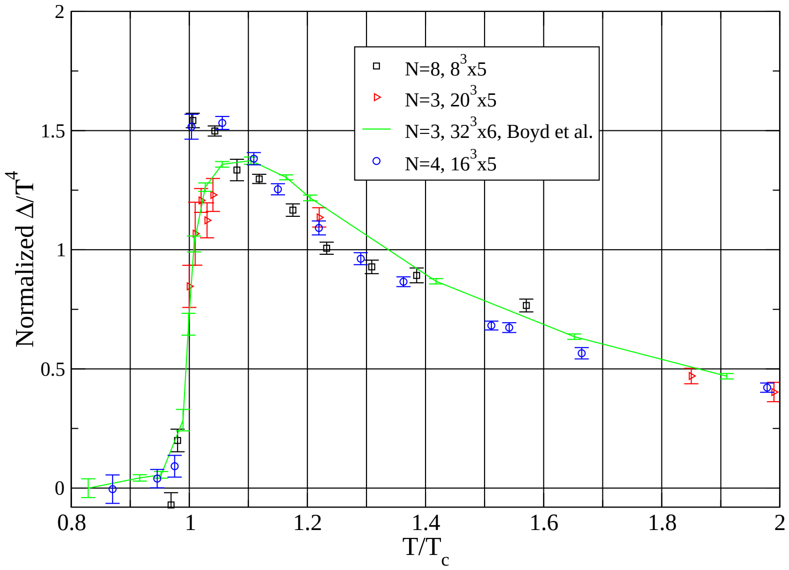
B. Bringoltz, M.Teper: hep-lat/0506034



$$\Delta \equiv \epsilon - 3p$$

B. Bringoltz, M.Teper: hep-lat/0506034

[$\Delta = 0$ in Stefan-Boltzman gas]



$\Rightarrow$

SGP is a large-N phenomenon: dynamics must survive at $N = \infty$

$\Rightarrow$

- not (colour singlet) hadrons above T_c
- not topology (instantons)
- good news for AdS/CFT !
- D=2+1? su3: Bialas,Daniel,Morel,Petersson, arXiv:08070855

now back to some topics at $T = 0$ ....

Interlaced θ-vacua in SU(N) gauge theories

Consider the gauge action with a θ term

$$S[g^2,\theta] = \frac{1}{4g^2}\int d^4x \mathrm{Tr} F_{\mu\nu}F^{\mu\nu} + \frac{i\theta}{16\pi^2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr} F_{\mu\nu}F_{\rho\sigma}$$

Since

$$\frac{1}{16\pi^2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr} F_{\mu\nu}F_{\rho\sigma} = Q = \text{integer}$$

we know that $\exp -S[\theta]$ and hence the vacuum energy density $E(\theta)$ are periodic in θ

$$E(\theta) = E(\theta + 2\pi) \qquad \forall N$$

On the other hand, we expect that for a smooth $N \to \infty$ limit, we need to factor N from S so that the couplings to keep fixed are $1/g^2 N$, θ/N, ... i.e.

$$E(\theta) = N^2 h(\theta/N)$$

How do we reconcile these two apparently irreconciliable demands?

E.Witten hep-th/9807109

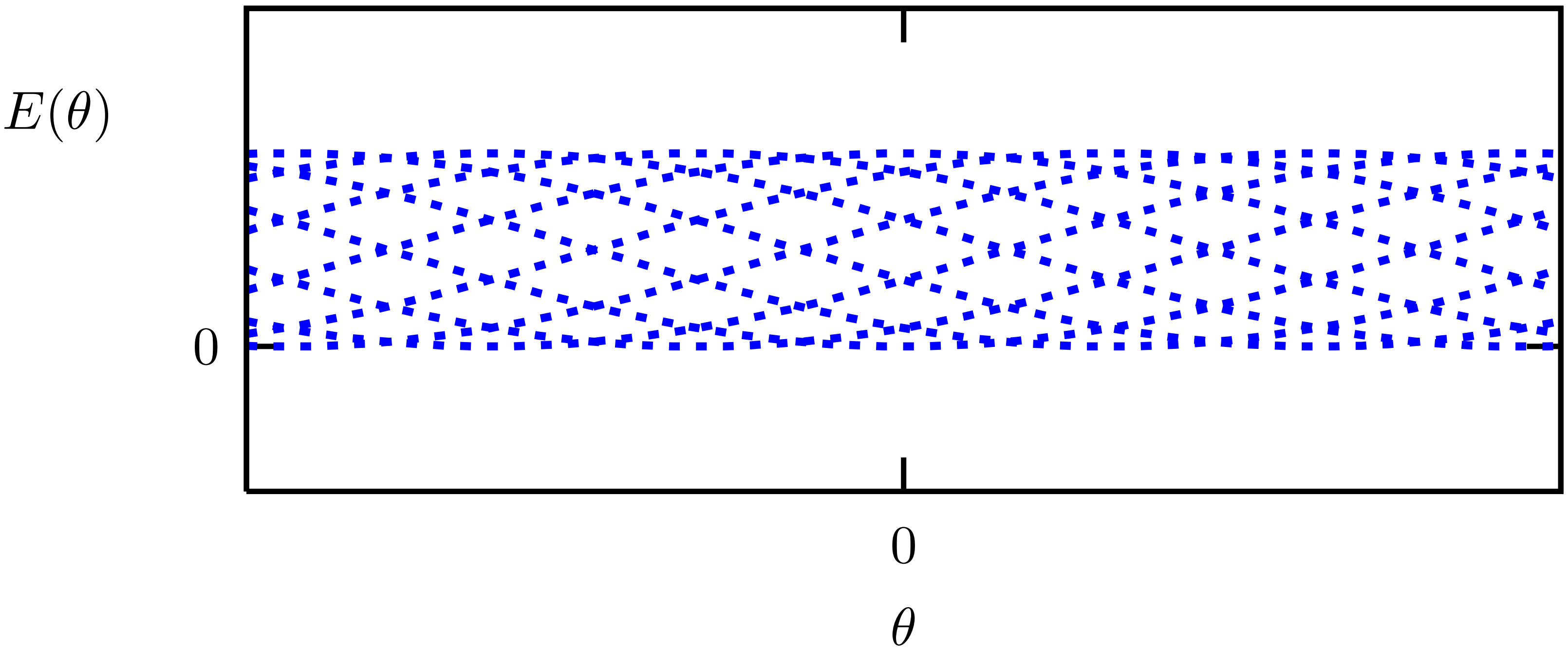
suggestion: E(θ) is a multi-branched function E.Witten hep-th/9807109

$$E_k(\theta) = N^2 h\left(\frac{\theta + 2\pi k}{N}\right) \quad ; \quad E(\theta) = \min_k E_k(\theta)$$

so that: $E(\theta) = E(\theta + 2\pi)$

while each $E_k(\theta)$ is periodic in $2\pi N$

e.g. N=10:



domain wall tension between different 'k-vacua' is $O(N)$ so as $N \to \infty$ these will all become stable ...

Witten: AdS/CFT ; Shifman: $\mathcal{N} = 1$ SUSY

So:

$T \leq T_c$

we have N vacua at any given θ

there is some nice lattice evidence for this scenario:

Del Debbio, Panagopoulos, Vicari, hep-th/0204125, arXiv:0706.1479

$T > T_c$

topology disappears exponentially in N:

so no interlaced vacua – just naive 2π periodicity in θ with exponentially small $E(\theta)$ variation

$\longrightarrow$

So as $N \to \infty$ there are N stable vacua at $\theta = 0$

Now

large N $\to$ lowest vacua are close to their minima $\to$ we can use a quadratic approximation for $E_k(\theta)$ if $k \ll N$, $\to$

$$E_k(\theta = 0) = E_0(\theta = 0) + \frac{1}{2}\chi_t(2\pi k)^2$$

where χ_t is the topological susceptibility
E.g.

$$E_{k=1} - E_0 = 2\pi^2\chi_t \sim (360MeV)^4$$

in contrast to the $E_0 \sim -O(N^2)$ vacuum energy
So:

these near-stable vacua should appear as quasistable in computer time when using local Monte Carlo updates and we should be able to find them!

$\longrightarrow$

Some properties of these k-vacua:

- in a $k \neq 0$ vacuum

$$\langle Q \rangle \neq 0$$

since $k \stackrel{CP}{\rightarrow} N - k$; degeneracy, mixing?

- string tension $\downarrow$ as $k \uparrow$ since vacuum energy decreases, perhaps Zhitnitsky

$$\sigma(k) \simeq \sigma(k = 0) \cos\{\frac{2\pi k}{N}\}$$

- upper half of vacua, $N/4 \leq k \leq 3N/4$, are unstable?

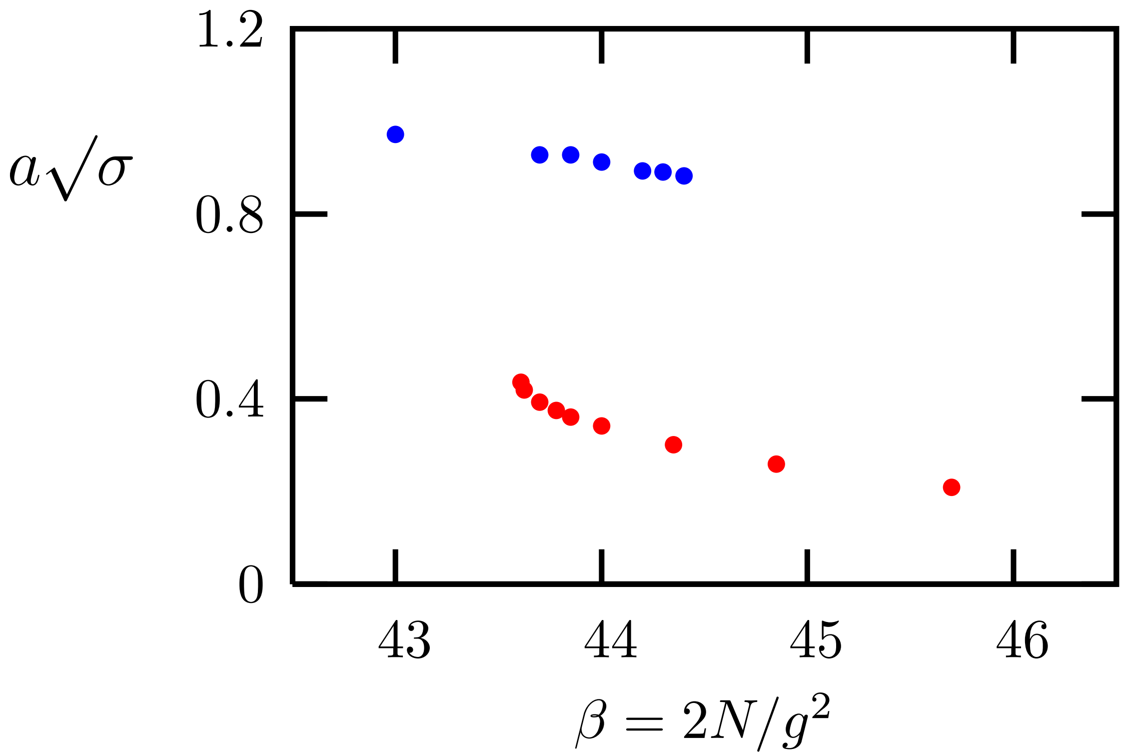
- since the vacuum energy increases with k,$E(k) = E(k = 0) + O(k^2)$, perhaps such a state deconfines at a lower temperature

$$T_c(k \neq 0) < T_c?$$

How to find?

- quench β across bulk transition ? has not worked for me
- quench β from high T to low T ?

SU(8) in 3+1 dimensions



1st order bulk transition for $N \geq 5$ ensures clean weak-coupling physics on weak-coupling branch

$\Rightarrow$

Is there a ‘physical’ lattice strong coupling regime?

- Why ask? AdS/CFT addresses $g^2 N \to \infty$.

- usual lattice

$g^2 N \to \infty \quad \leftrightarrow \quad \beta \to 0$

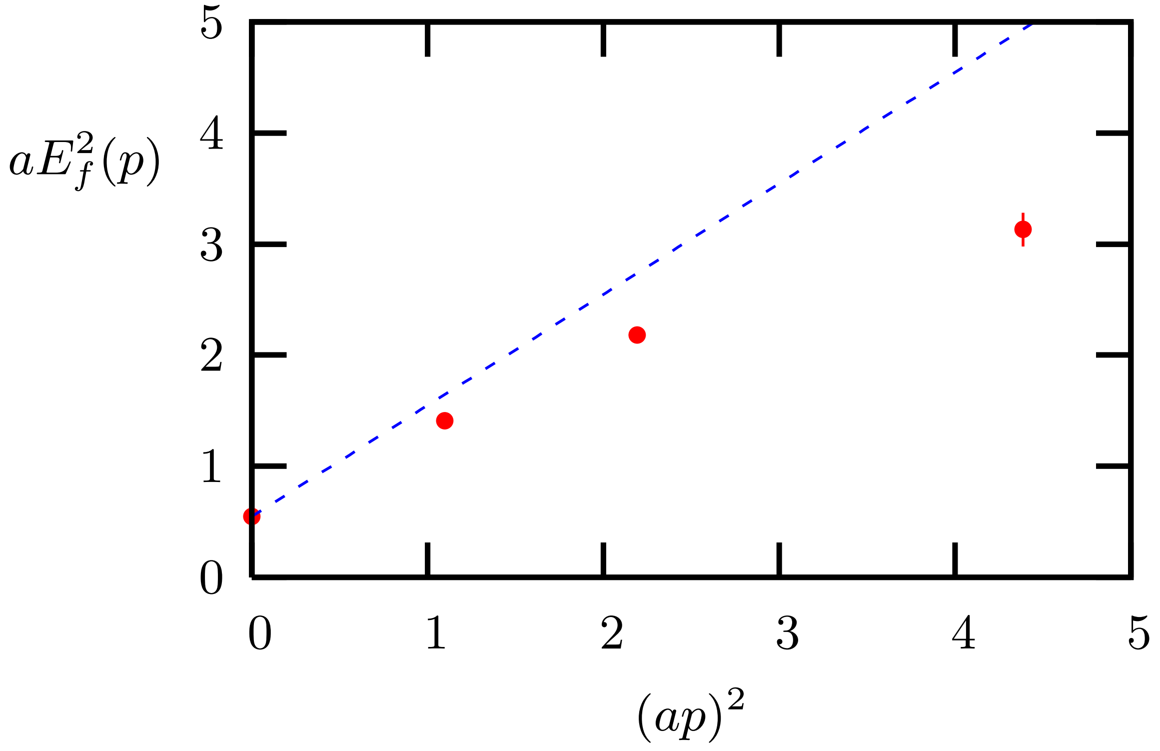
is presumably not relevant as essential space-time symmetries badly broken in that limit

- but at large N we can use the metastability of the strong-to-weak coupling bulk transition to go to smaller β while remaining still in a strong coupling phase

- if we are past the ‘roughening transition’ then the space-time symmetries will begin to be restored and we might be in a strong coupling phase that has some physical features – even if it does not have an asymptotically free UV completion

there is some numerical evidence for this:

SU(8) , $\beta = 44.3$, $2 \times 6^2 \times 8$



$- - -$ $E^2 = m^2 + p^2$ with $p^2 \to 4\sin^2 p/2$ better

Some core Conclusions

∘ large N gauge theories are linearly confining at low T

∘ moreover for many basic physical quantities,
e.g. the lightest glueball masses, m_ρ, the deconfinement temperature, the string tension, $\Lambda_{\overline{MS}}$,
one finds $SU(3) \simeq SU(\infty)$

∘ this is good news for applications of gauge-gravity duality, where what one calculates is at $N = \infty$

∘ in particular, it appears that for simple thermodynamic quantities in the region $T_c < T <$ few $\times\, T_c$, which is currently the favourite area for applying AdS/CFT methods, we also have $SU(3) \simeq SU(\infty)$