Conformal Dynamics from LHC to Cosmology II

Francesco Sannino

Zakopane 2009

F.S. - 0804.0182

CP^3-Origins
Particle Physics & Origins of Mass

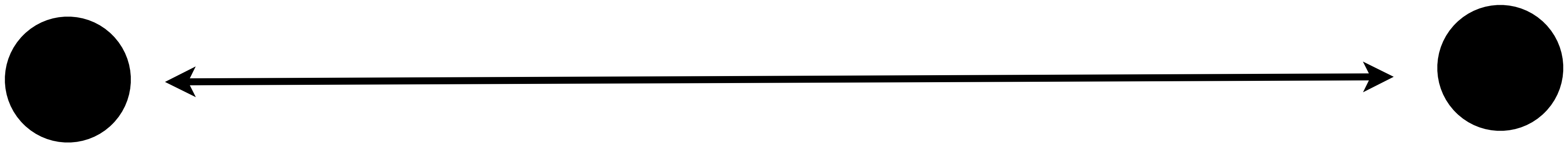
Phases of Gauge Theories

Why ?

Origin of Bright and Dark Mass

Phases of Gauge Theories

$$V(r)$$



Coulomb : $V(r) \propto \frac{1}{r}$

Free electric : $V(r) \propto \frac{1}{r \log(r)}$

Free magnetic : $V(r) \propto \frac{\log(r)}{r}$

$$\textbf{Higgs}: \qquad V(r) \propto \text{constant}$$

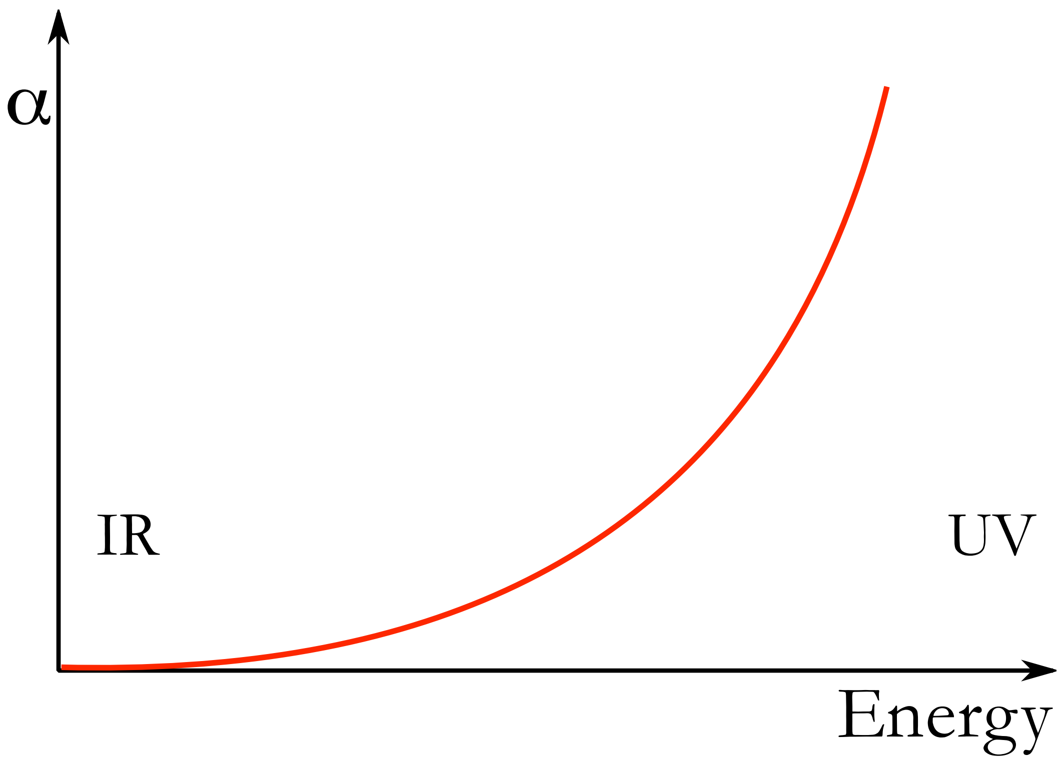
$$\textbf{Confining}: \qquad V(r) \propto \sigma r\ .$$

Free Electric (QED)

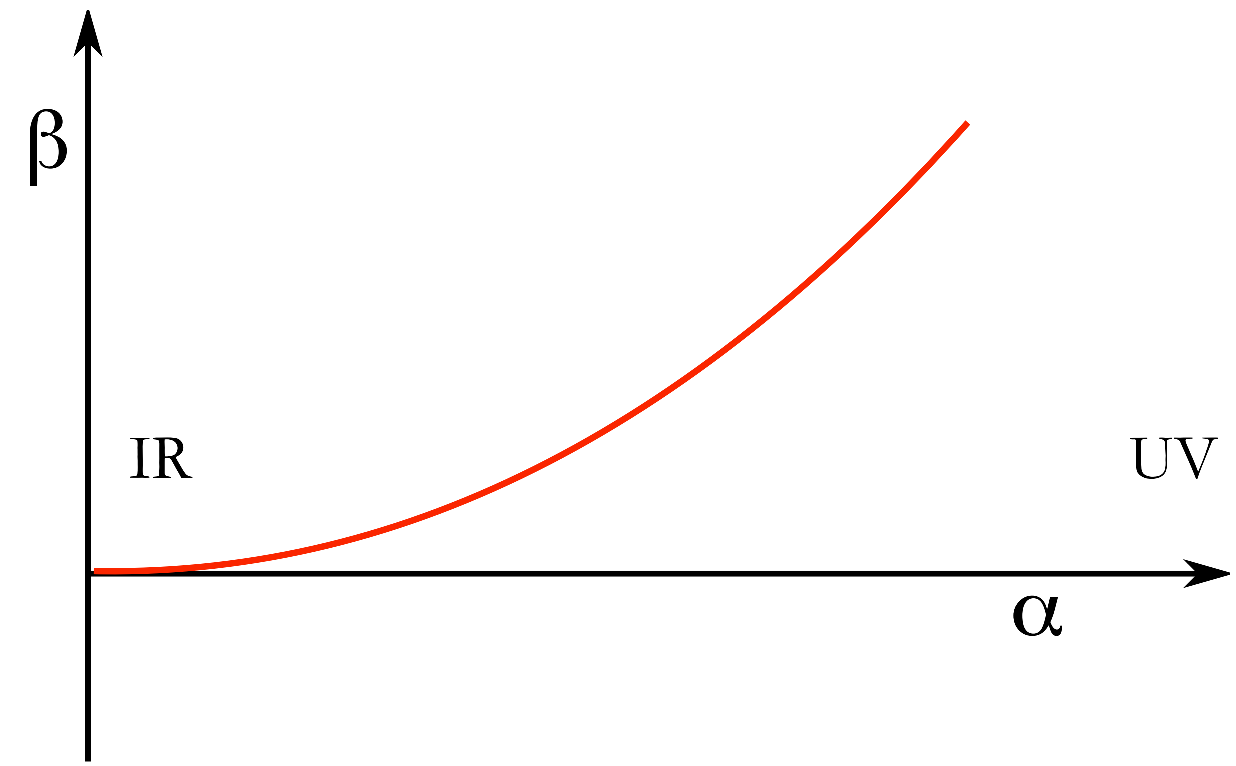
$$V(r) \propto \frac{1}{r \log(r)}$$

$$\alpha(r) \to \frac{1}{\log(r)}$$

Free Electric



$$\alpha(r) \to \frac{1}{\log(r)}$$



$$\alpha = \frac{g^2}{4\pi} \qquad \beta = \frac{d\,g}{d\ln\mu}$$

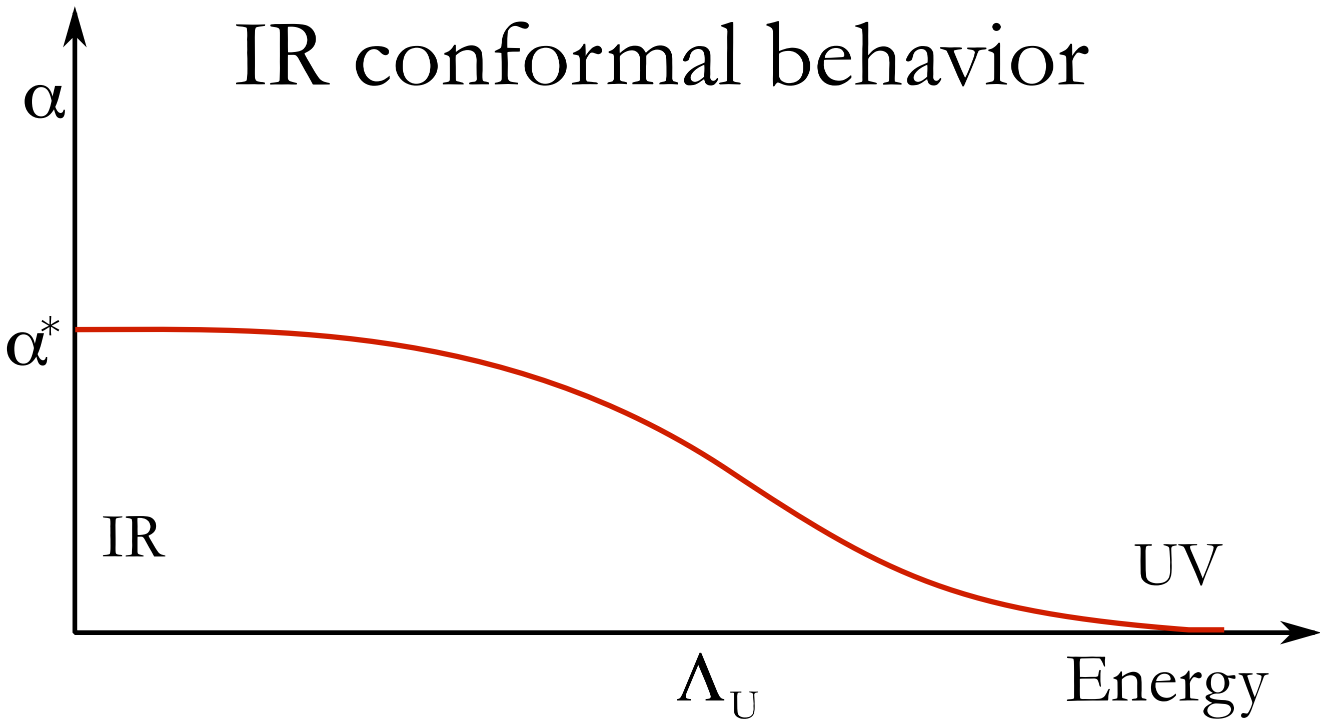
Coulomb

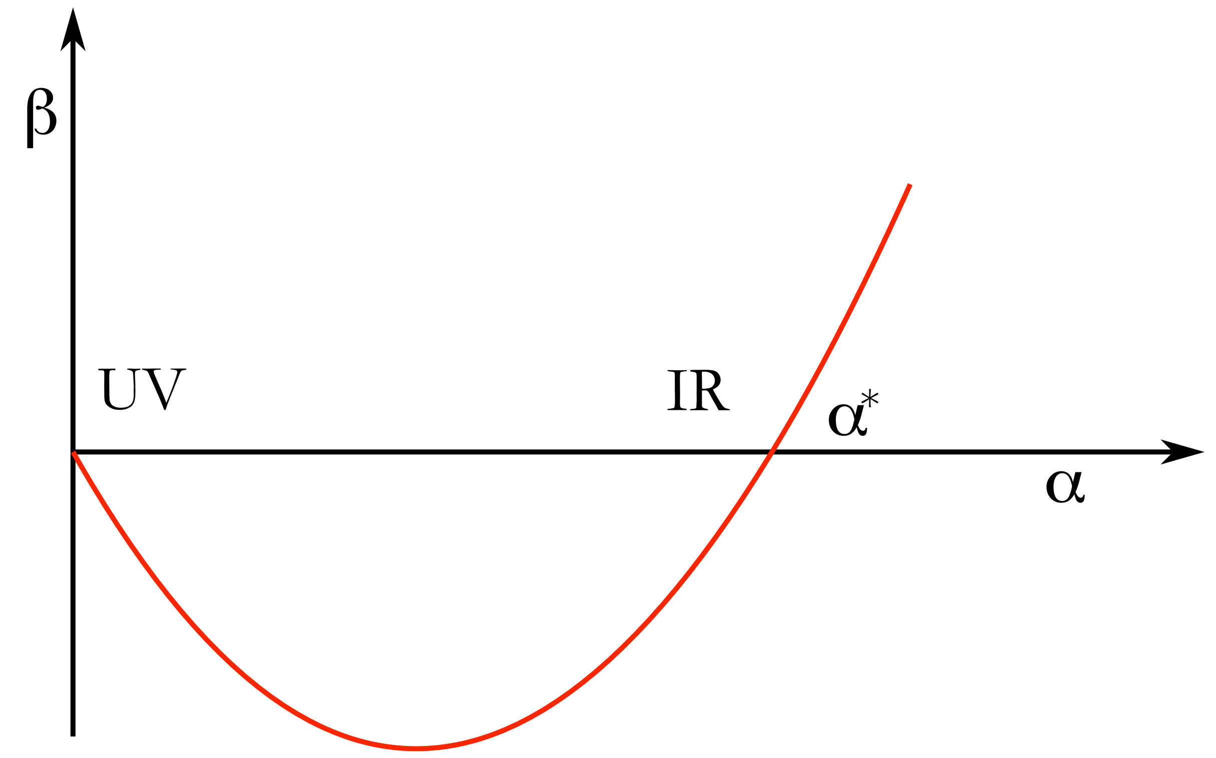
$$V(r) \propto \frac{1}{r}$$

$$\alpha \rightarrow \alpha^*$$

$$\beta = 0$$

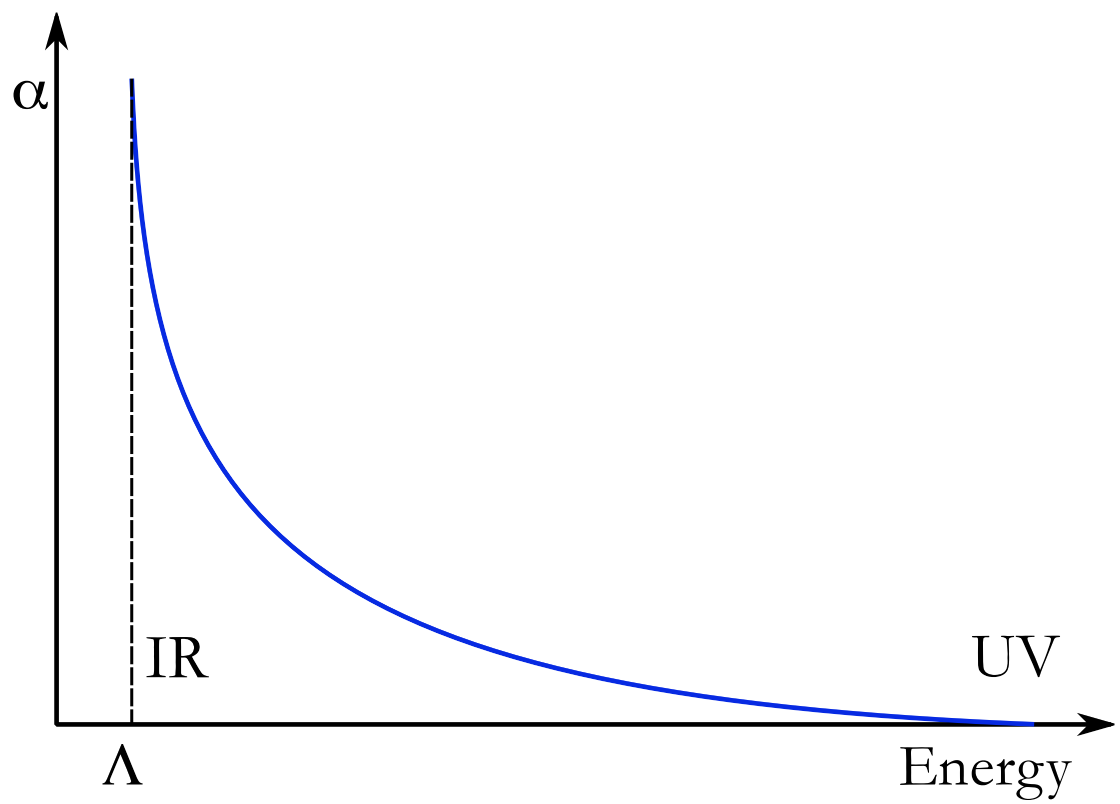
Coulomb

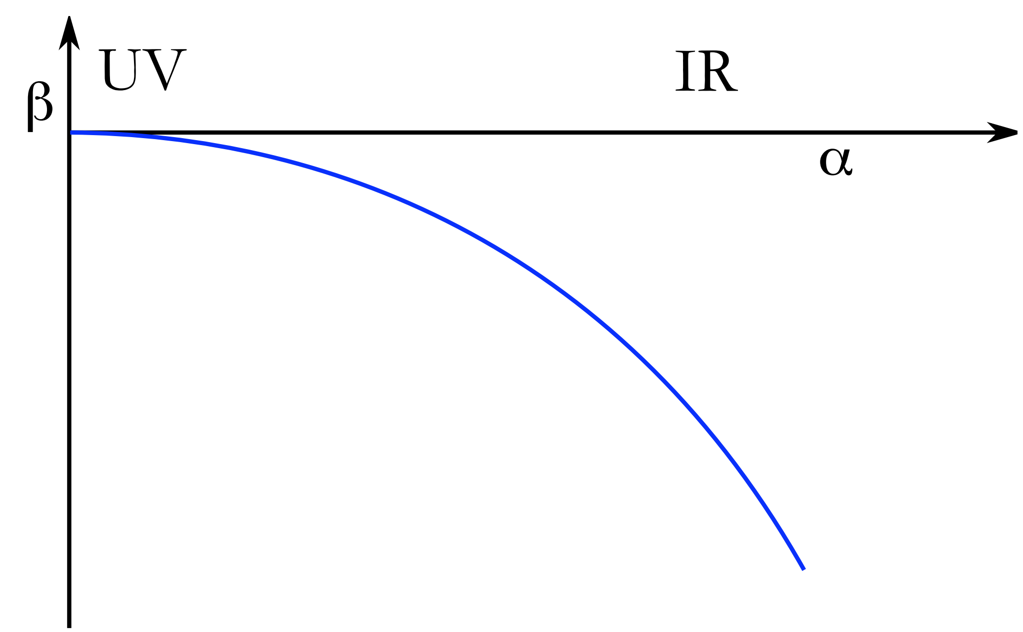




IR Conformal Phase

QCD-like Theory





Gauge Theory Knobs

Gauge Group, i.e. SU, SO, Exceptional

Matter Representation

# of Flavors per Representation

Temperature

Matter Density (i.e. Chemical Potential)

Example

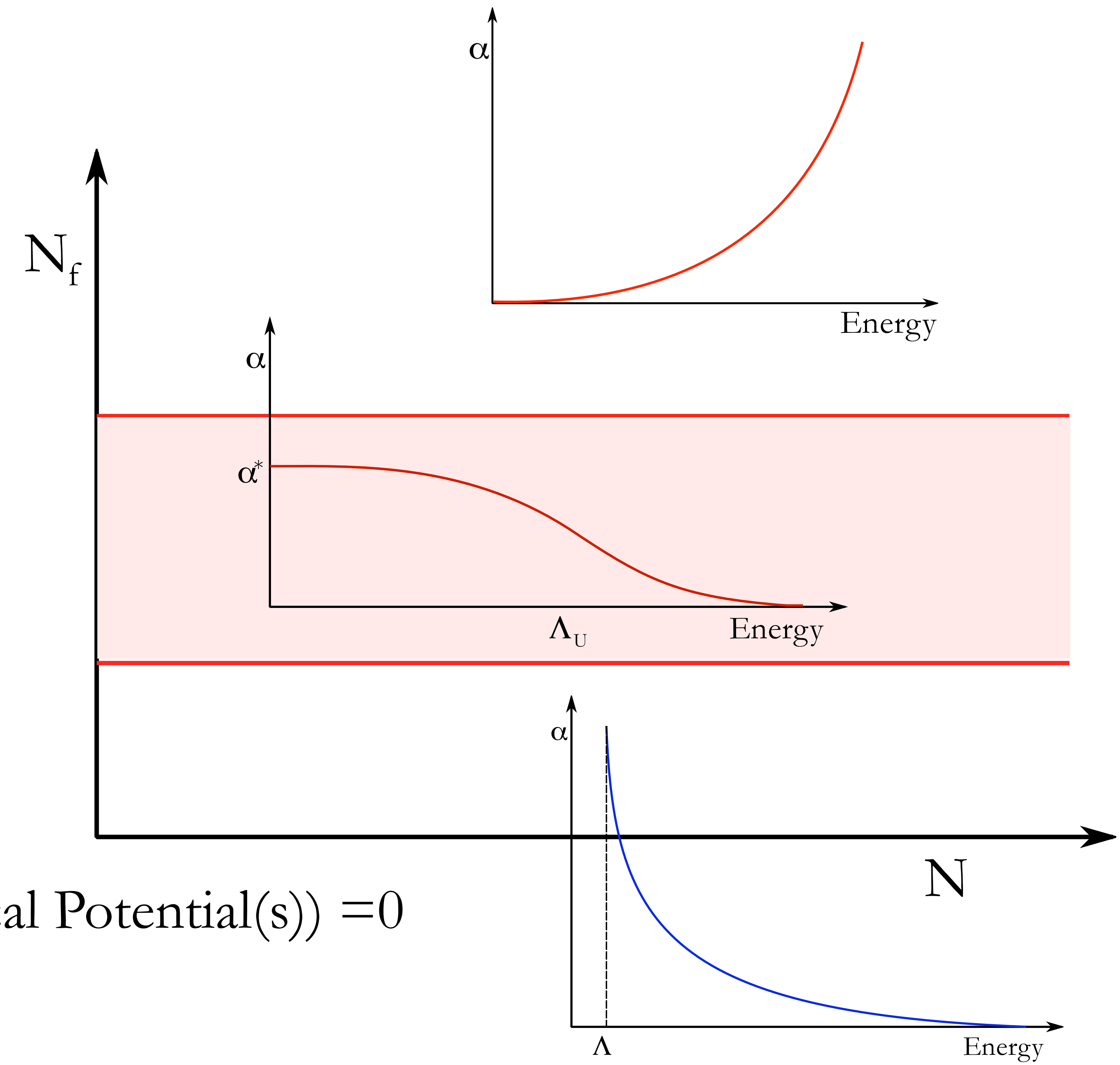
SU(N)

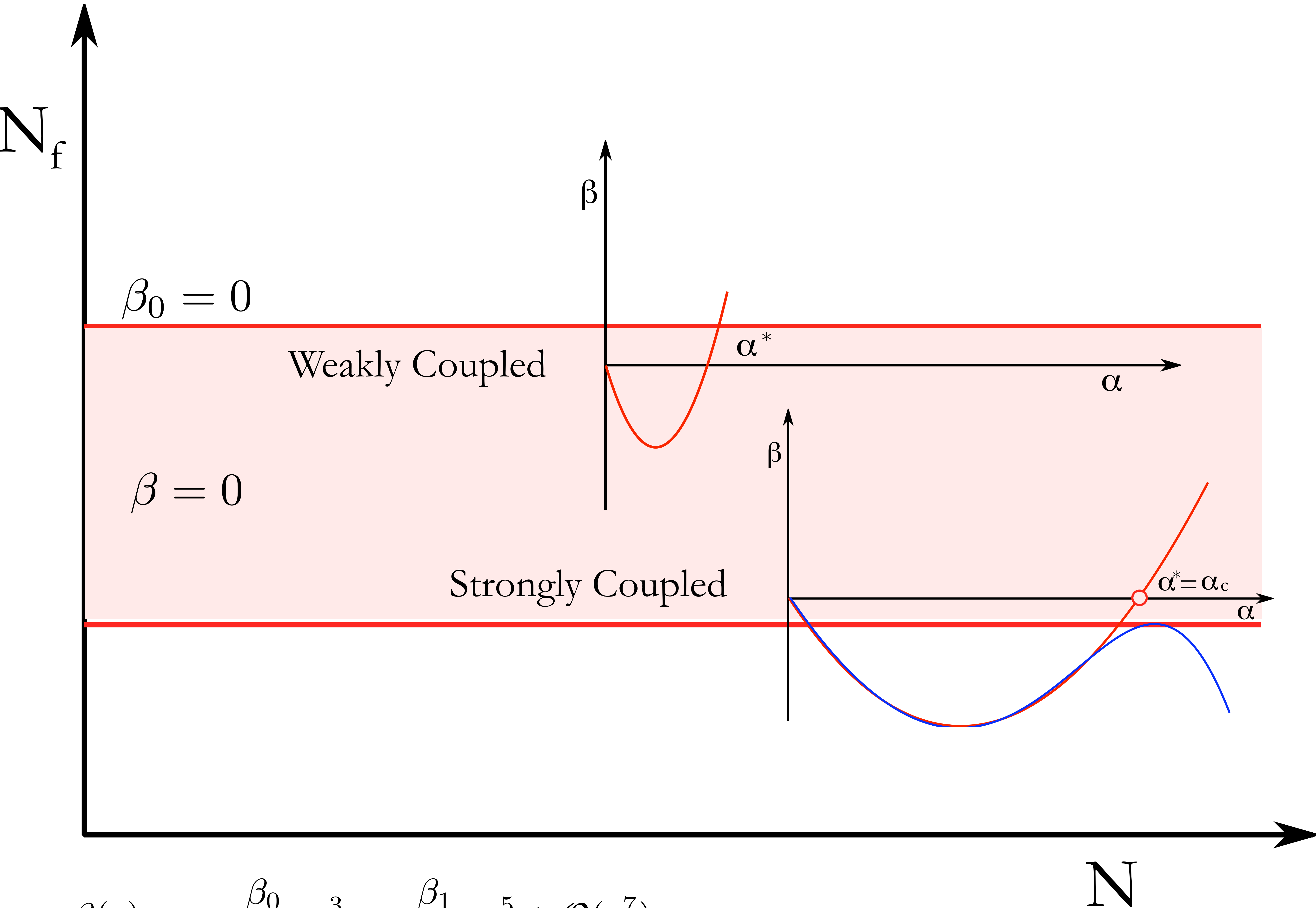
Adjoint Dirac Matter

Nf

Temperature = 0

Matter Density (i.e. Chemical Potential(s)) =0





$$\beta(g) = -\frac{\beta_0}{(4\pi)^2} g^3 - \frac{\beta_1}{(4\pi)^4} g^5 + \mathcal{O}(g^7)$$

Information Needed!

Tools to Plot the Phase Diagram

SUSY

The all orders beta function of NSVZ

Unitarity Bounds for Conformal Theories

Non-Renormalization of Superpotentials

‘t Hoofts Anomaly Matching Conditions

a-Maximization

Instanton Calculus

....

Non SUSY

Different Methods:

Schwinger - Dyson and any variations of it

Instanton Inspired Calculus

't Hooft Anomaly Matching Conditions

Thermal degrees of freedom count

Exact Renormalization Methods

Appelquist, Bowick, Chivukula, Cohen, Dobrescu, Eichten, Gies, Hill, Holdom, Karabali, Jaeckel, Fisher, Lane, Litim, Mahanta, Miransky, Pawlowski, Percacci, Shrock, Simmons, Terning, Wijewardhana....

Non SUSY

Also:

The all orders beta function conjecture

Unitarity of the Operators for Conformal Theories

First Principle Lattice Computations

Non SUSY

Different Methods:

Schwinger - Dyson and any variations of it *

Instanton Inspired Calculus

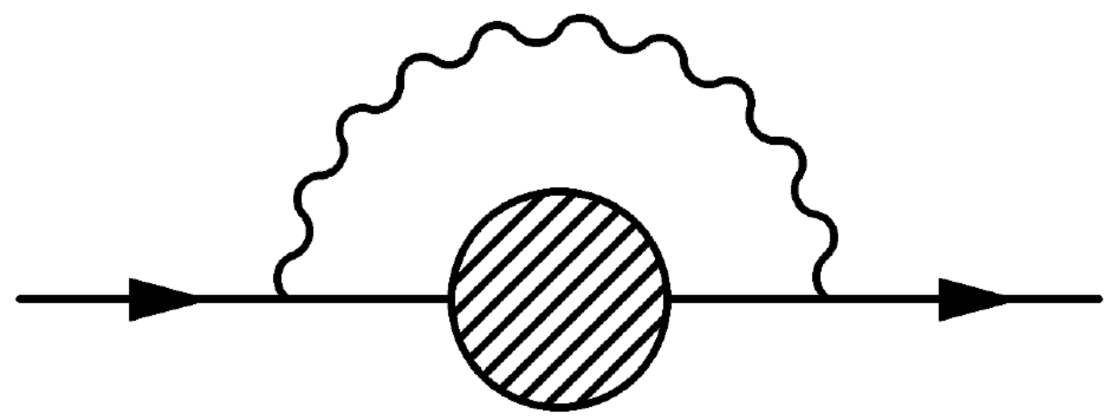
't Hooft Anomaly Matching Conditions

Thermal degrees of freedom count

Exact Renormalization Methods

* Appelquist, Bowick, Chivukula, Cohen, Dobrescu, Eichten, Gies, Hill, Holdom, Karabali, Jaeckel, Lane, Mahanta, Miransky, Sannino, Shrock, Simmons, Terning, Wijewardhana....

Rainbow - Schwinger-Dyson



The full nonperturbative fermion propagator reads:

$$iS^{-1}(p) = Z(p)\left(\not{p} - \Sigma(p)\right)$$

The Euclidianized gap equation in Landau gauge is:

$$\Sigma(p) = 3C_2(R) \int \frac{d^4k}{(2\pi)^4} \frac{\alpha\left((k-p)^2\right)}{(k-p)^2} \frac{\Sigma(k^2)}{Z(k^2)k^2 + \cancel{\Sigma^2}(k^2)}$$

$$Z(k^2) = 1$$

$$\beta(\alpha) \simeq 0 \qquad \alpha(\mu) \approx \alpha_c \qquad \alpha_c = \frac{\pi}{3C_2(r)}$$

In practice

$$\beta(g) = -\frac{\beta_0}{(4\pi)^2} g^3 - \frac{\beta_1}{(4\pi)^4} g^5$$

$$\beta = 0 \longrightarrow \frac{\alpha^*}{4\pi} = -\frac{\beta_0}{\beta_1}$$

$$\alpha_c = \frac{\pi}{3C_2(r)} \longleftrightarrow \gamma(2-\gamma) = 1$$

$$\alpha^* \leq \alpha_c$$

SU(N)   Dirac Fermions in representation “r”

$$N^c_{f\,\mathrm{Ladder}} = \frac{17C_2(G) + 66C_2(r)}{10C_2(G) + 30C_2(r)} \frac{C_2(G)}{T(r)}$$

$$C_2(G) = C_2(Adj) = N$$

Back to the Example

SU(N) Adjoint Dirac Matter

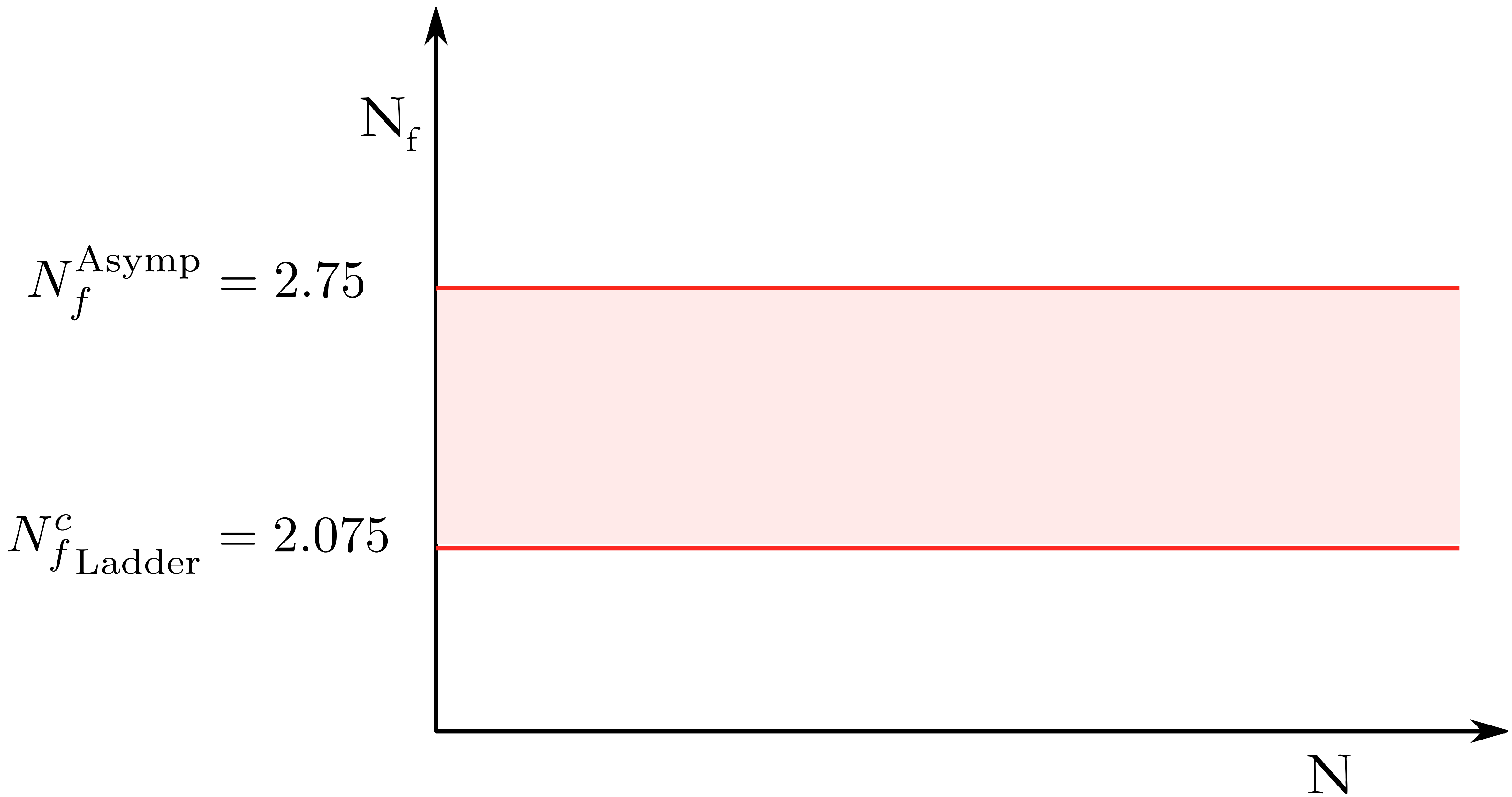
$$C_2(r) = C_2(G) = T(G) = N$$

$$\beta_0 = \frac{11}{3}N - \frac{4}{3}N N_f = 0 \ , \quad \rightarrow \quad N_f^{\text{Asymp}} = \frac{11}{4} = 2.75$$

$$N^c_{f\,\text{Ladder}} = \frac{17+66}{10+30} = 2.075$$

Back to the Example

SU(N) Adjoint Dirac Matter



Historical Example

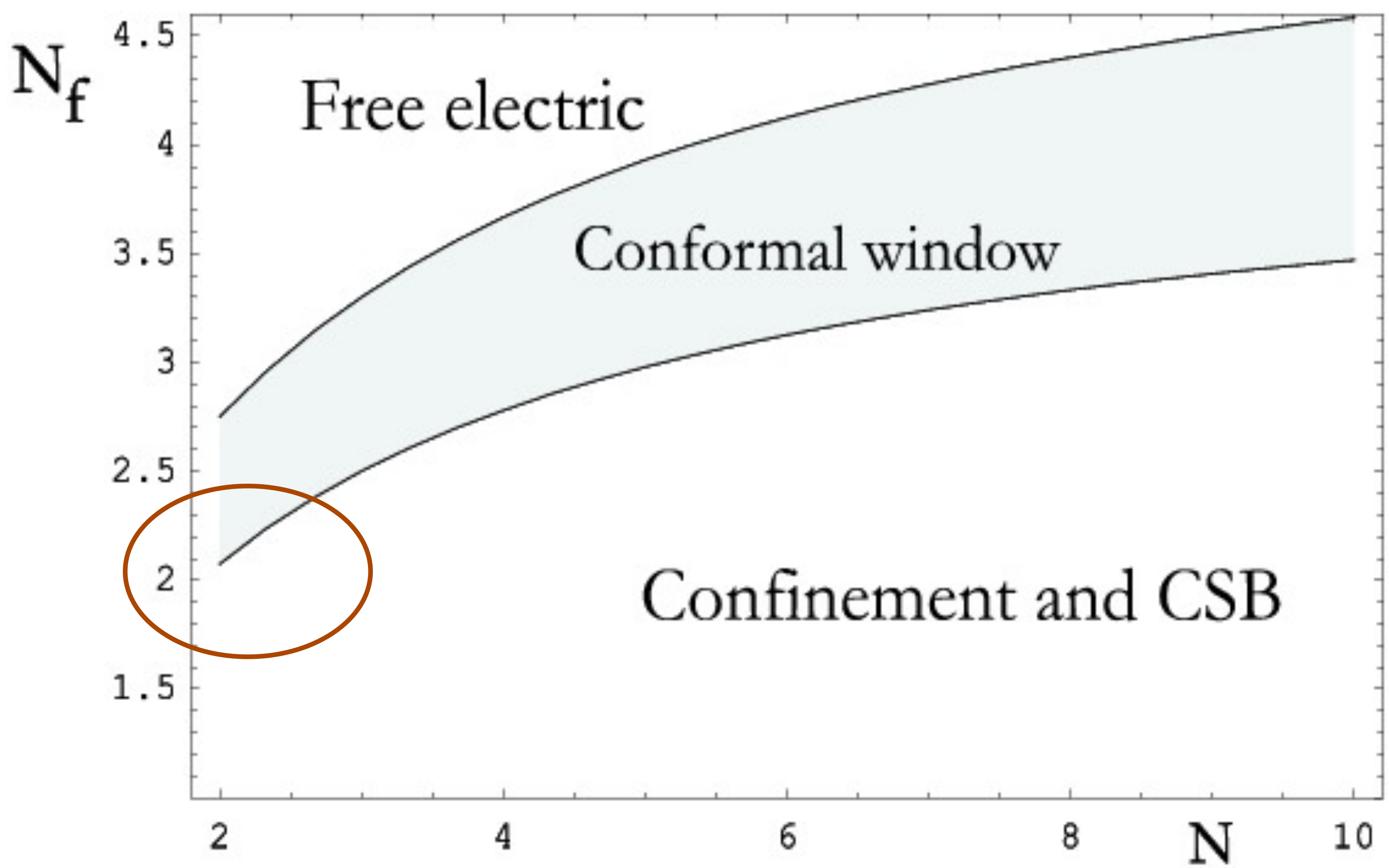
SU(N)   2-index Symmetric Rep

	$\color{magenta}{SU(N)}$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$	$U_A(1)$
$Q_{\{ij\}}$	$\square\square$	$\square$	1	1	1
$\widetilde{Q}^{\{ij\}}$	$\overline{\square\square}$	1	$\overline{\square}$	-1	1
G_μ	Adj	0	0	0	0

Here Q and $\widetilde{Q}$ are Weyl fermions.

The A-type is obtained by substituting $\square\square$ with $\begin{smallmatrix}\square\\\square\end{smallmatrix}$.

Phase Diagram for Symmetric Rep.

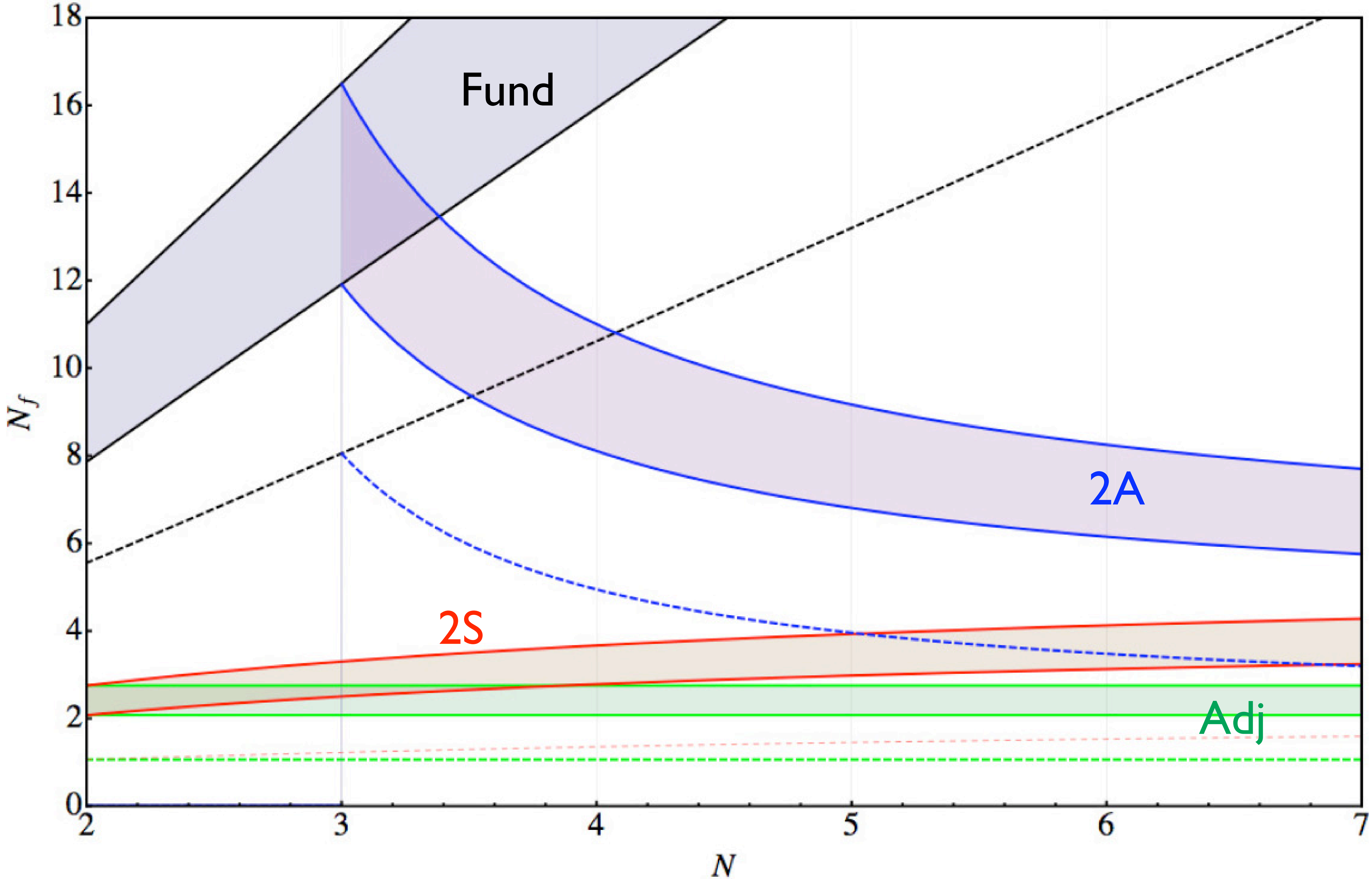


F.S. - Tuominen 04

Using Ladder Approximation

Is this the minimal walking theory?

Non-SUSY Phase Diagram for HDRs



Ladder approximation

Dietrich and F.S. 06

SUSY - Diagram

Exact Susy Beta Function

$$\beta(g) \ = \ -\frac{g^3}{16\pi^2}\frac{\beta_0 + 2T(r)N_f\gamma(g^2)}{1-\frac{g^2}{8\pi^2}C_2(G)}$$

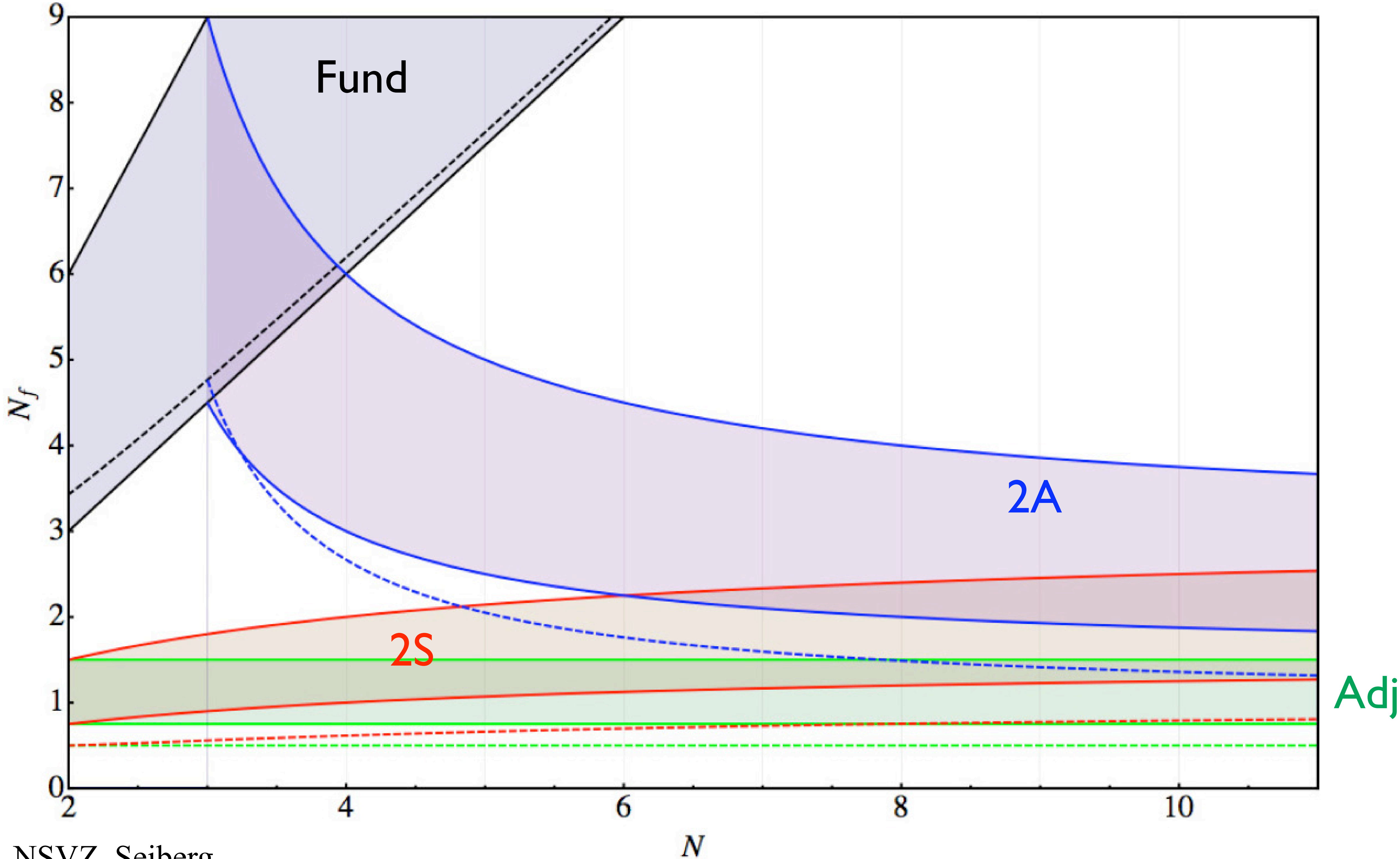
$$\gamma(g^2) \ = \ -\frac{g^2}{4\pi^2}C_2(r) + O(g^4)$$

$$\gamma(g^2) = -d\ln Z(\mu)/d\ln\mu$$

$$\beta_0 = 3C_2(G) - 2T(r)N_f$$

NSVZ

SUSY Phase Diagram for HDRs



NSVZ, Seiberg
Intriligator-Seiberg

Ryttov and F.S. 07

Beta Function

Ryttov and F.S. 07

All orders beta function conjecture

$$\beta(g) = -\frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3}T(r)N_f\gamma(g^2)}{1 - \frac{g^2}{8\pi^2}C_2(G)(1 + \frac{2\beta_0'}{\beta_0})}$$

$$\gamma = -d\ln m/d\ln\mu$$

$$\gamma(g^2) = \frac{3}{2}C_2(r)\frac{g^2}{4\pi^2} + O(g^4)$$

$$\beta_0 = \frac{11}{3}C_2(G) - \frac{4}{3}T(r)N_f$$

$$\beta_0' = C_2(G) - T(r)N_f$$

Ryttov and F.S. 07

Recovering SYM

$SU(N)$ $\qquad N_f = \frac{1}{2}$ $\qquad$ Adjoint Matter

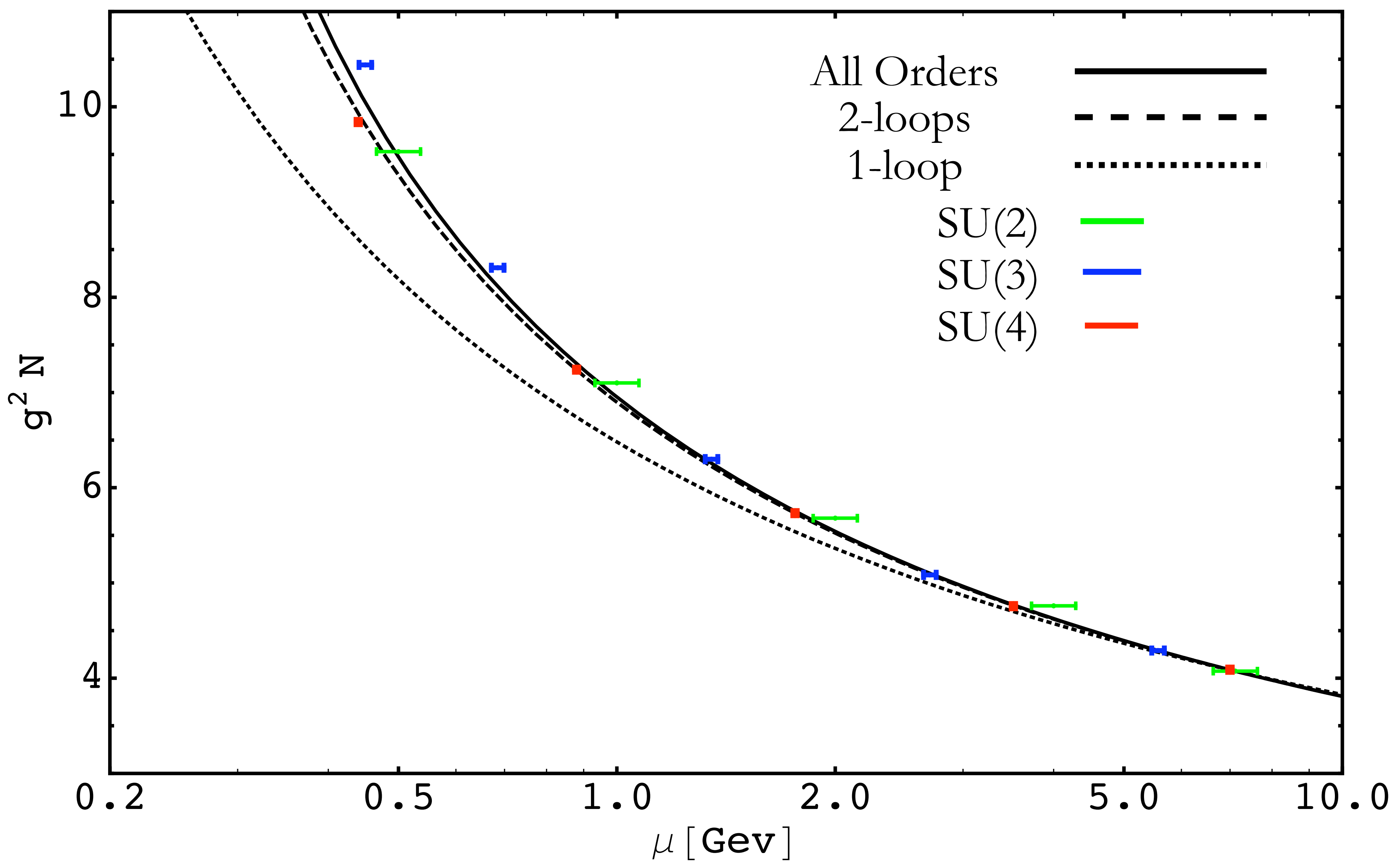
$$\beta(g) = -\frac{g^3}{(4\pi)^2} 3N \frac{1 - \frac{\gamma_{\text{Adj}}}{9}}{1 - \frac{g^2}{8\pi^2}\frac{4N}{3}}$$

$$\beta_{SYM}(g) = -\frac{g^3}{(4\pi)^2} \frac{3N}{1 - \frac{g^2}{8\pi^2}N}$$

$$\gamma_{\text{Adj}} = \frac{g^2}{8\pi^2} \frac{3N}{1 - \frac{g^2}{8\pi^2}N}$$

Ryttov and F.S. 07

Running in Yang - Mills for different N



Luscher, Sommer, Wolff, Weisz, 92 SU(2)

Luscher, Sommer, Weisz, Wolff 94 SU(3)

Lucini and Moraitis 07 SU(4)

Bounds on the Conformal Window

$$\beta = 0 \quad \longrightarrow \quad \gamma = \frac{11C_2(G) - 4T(r)N_f}{2T(r)N_f}$$

Unitarity of the Conformal Operators demands:

$$\gamma \leq 2$$

Ryttov and F.S. 07

Back to the Example

SU(N)   Adjoint Dirac Matter

$$\gamma \leq 2 \,, \quad \Longrightarrow \quad N_f^c \geq \frac{11}{8}$$

$$\gamma = 1 \,, \quad \Longrightarrow \quad N_f^c = \frac{11}{6} = 1.8\bar{3}$$

Pleasing Discovery!

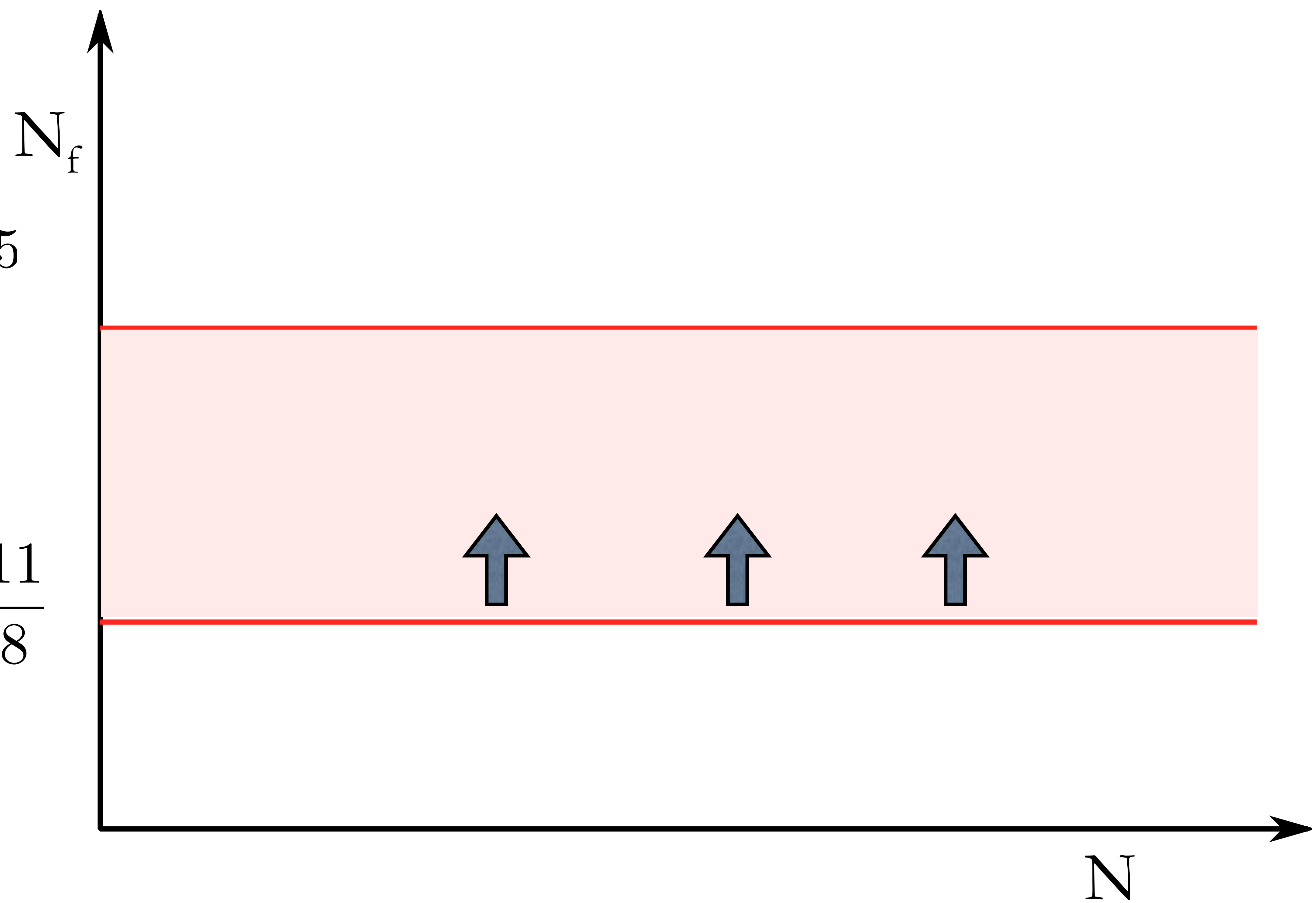
$${N_f^c}_{\mathrm{Ladder}} = 2.075$$

Back to the Example

SU(N) Adjoint Dirac Matter

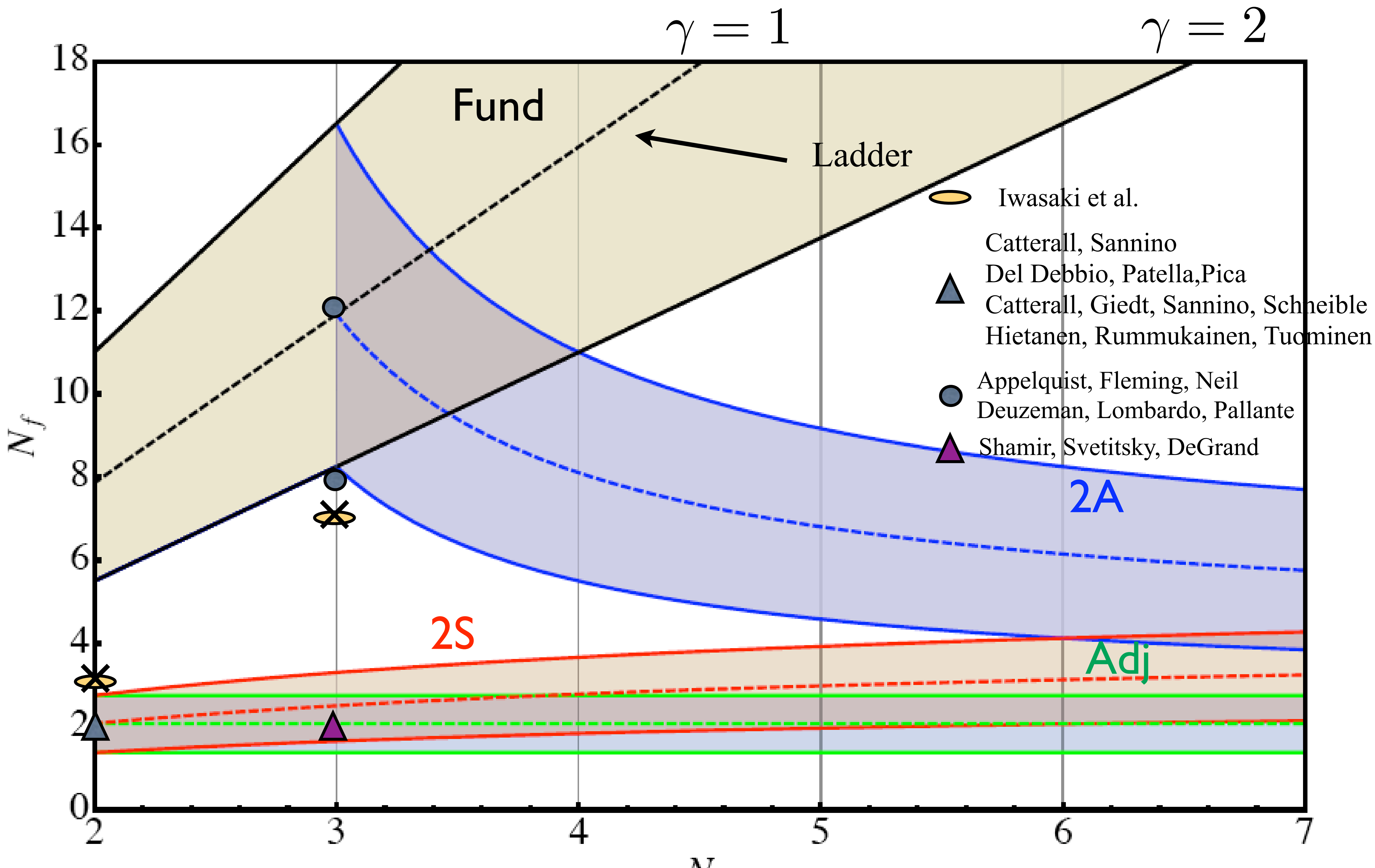
$$N_f^{\text{Asymp}} = 2.75$$

$$\gamma \leq 2 \rightarrow N_f^c \geq \frac{11}{8}$$



Universal Picture

Non-SUSY Phase Diagram Bound



All Orders Beta Function

Ryttov and F.S. 07

Large Lattice Technicolor Activity

Lattice

Any Rep.

Catterall, F.S. 07

Catterall, Giedt, F.S., Schneible 08

Del Debbio, Frandsen, Panagopoulos, F.S. 08

Del Debbio, Patella, Pica. 08

Hietanen, Rantaharju, Rummukainen, Tuominen 08

DeGrand, Shamir, Svetitsky, 08

Del Debbio, Patella, Pica, 08

Hietanen, Rummukainen, Tuominen 09

Fund. Rep

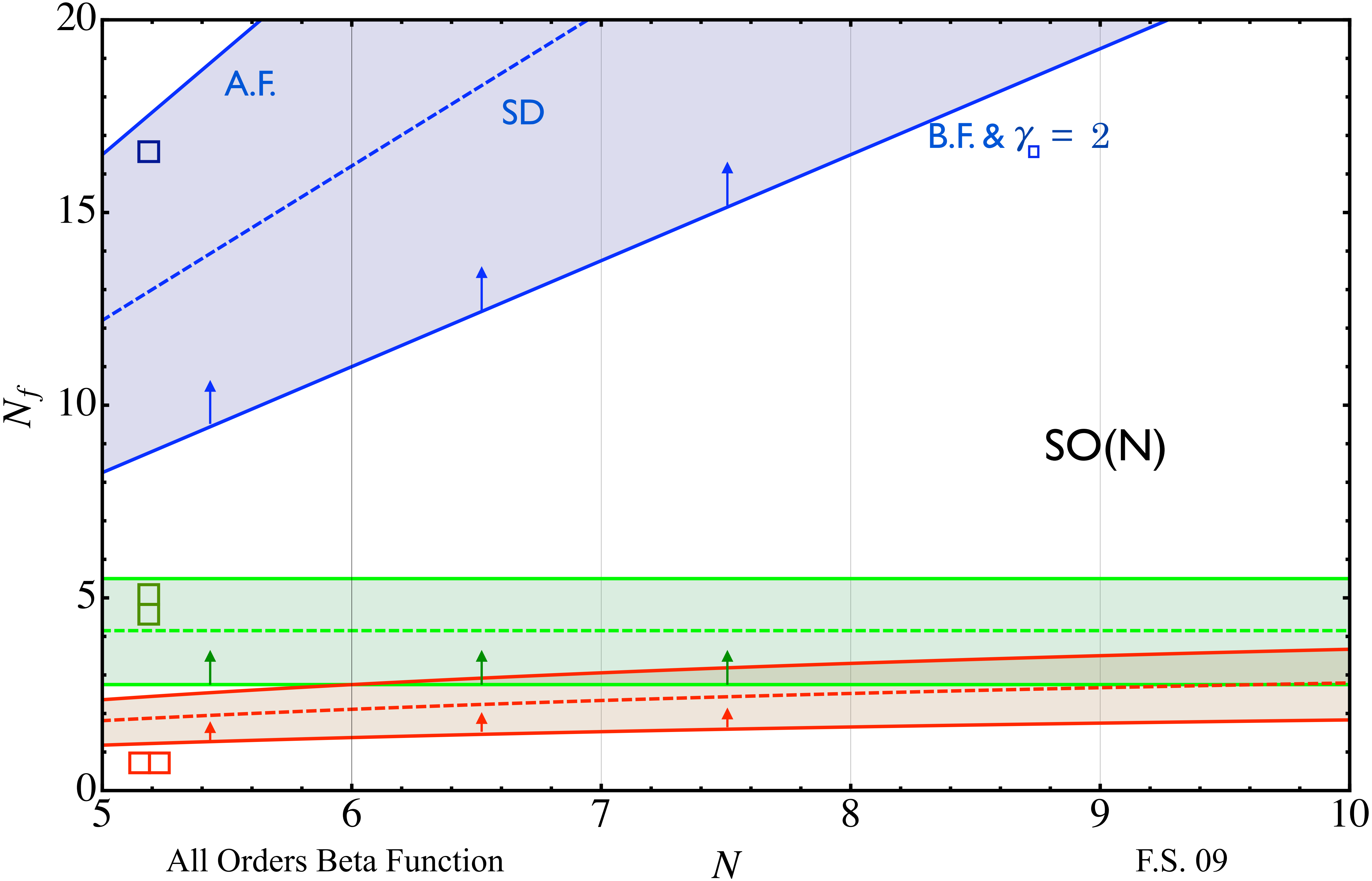
Appelquist, Fleming, Neil 07

Deuzman, Lombardo, Pallante 08

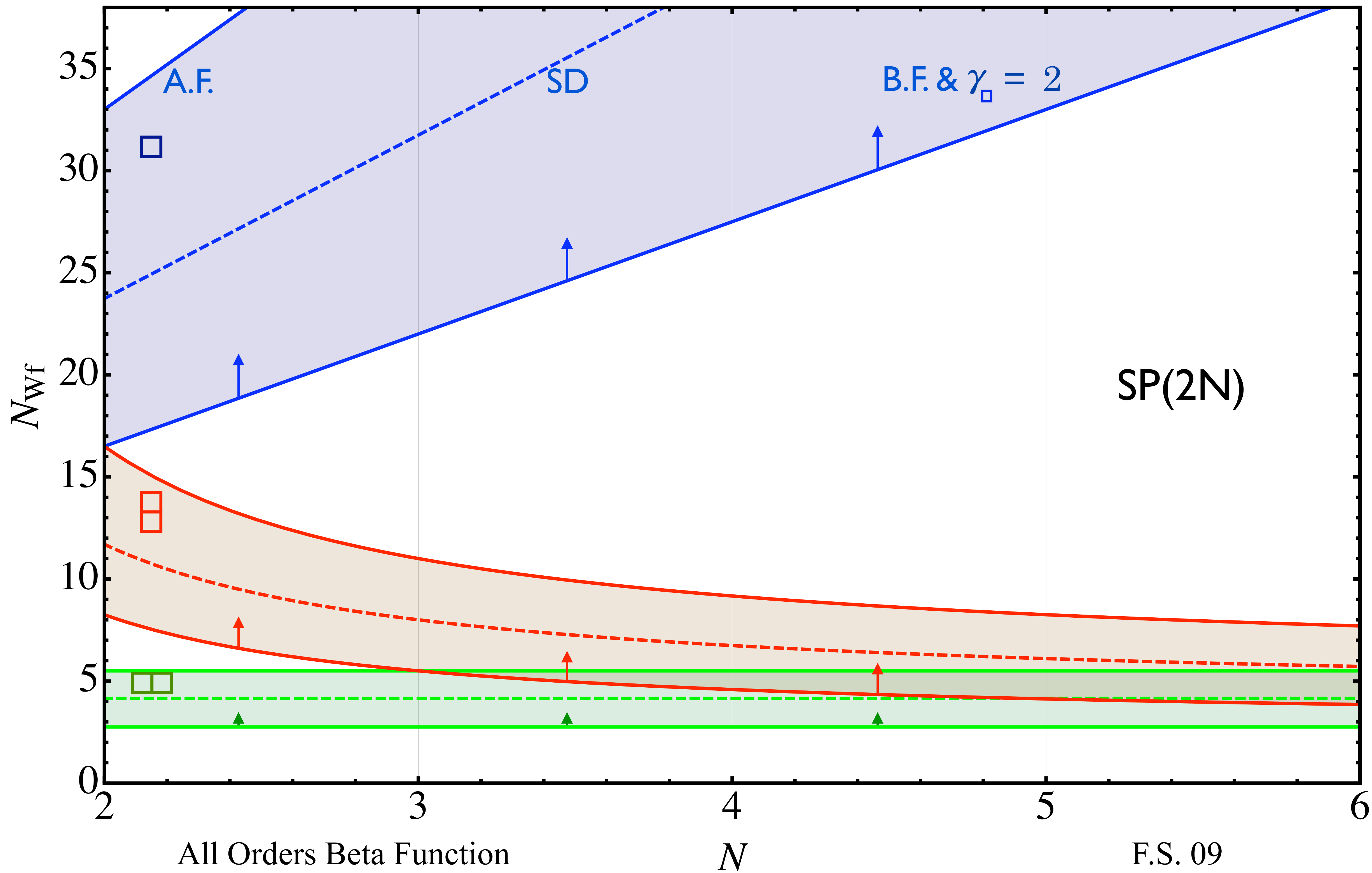
Fodor, Holland, Kuti, Nogradi, Schroeder, 08

Be Imaginative!

SO(N) Phase Diagram



Sp(2N) Phase Diagram



Conformal House

Ryttov and F.S. 09

today ;-)

Generalized Beta function

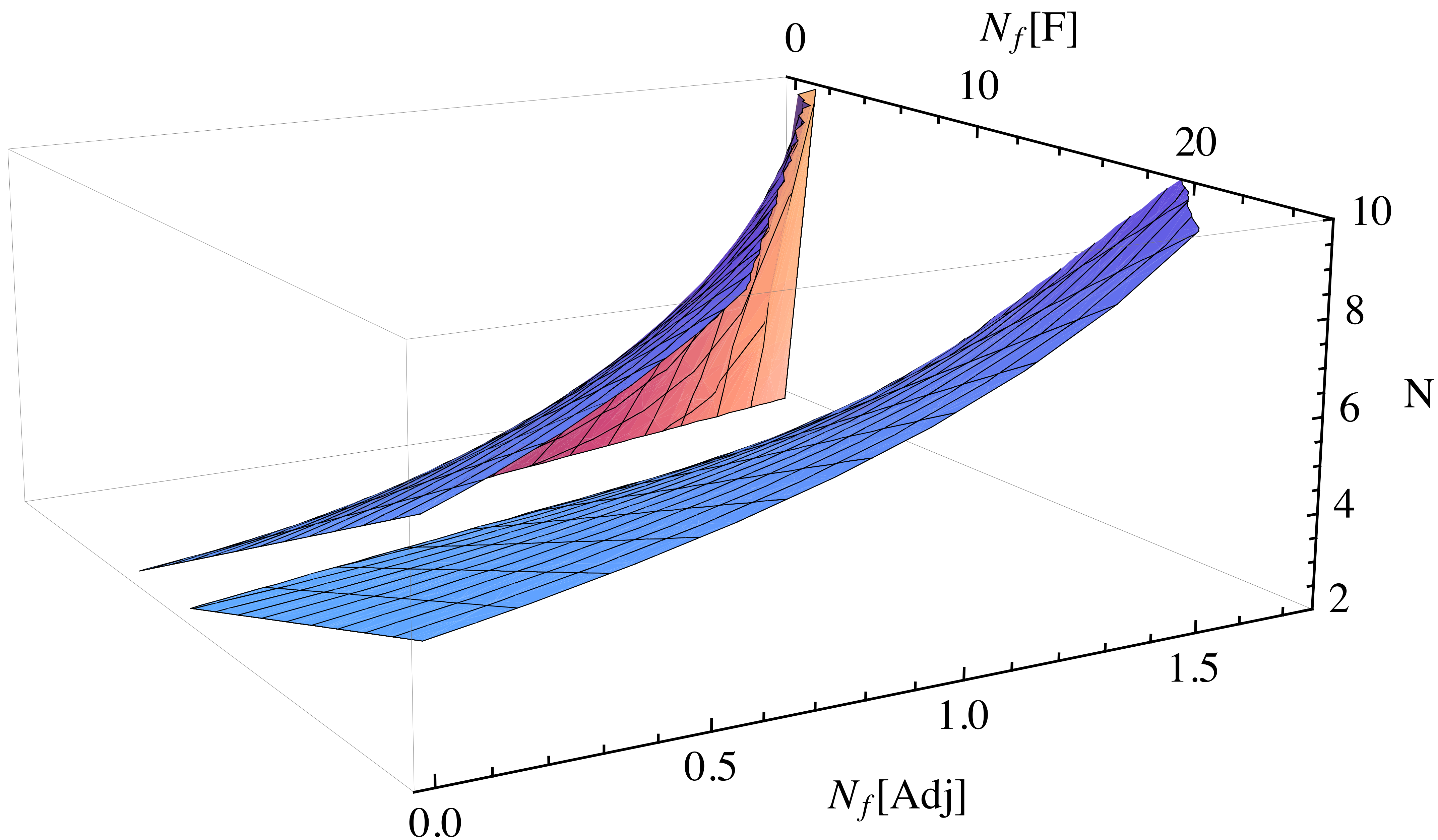
$$\beta(g) = -\frac{g^3}{(4\pi)^2}\frac{\beta_0 - \frac{2}{3}\sum_{i=1}^{k} T(r_i)N_f(r_i)\gamma_i}{1 - \frac{g^2}{8\pi^2}C_2(G)(1 + \frac{2\beta_0'}{\beta_0})}$$

$$\beta_0 = \frac{11}{3}C_2(G) - \frac{4}{3}\sum_{i=1}^{k} T(r_i)N_f(r_i)$$

$$\beta_0' = C_2(G) - \sum_{i=1}^{k} T(r_i)N_f(r_i)$$

2 Rep. Example

SU(N) Adjoint & Fundamental Dirac Matter



Method	Fund-Rep	Higher Rep.	House	γ	Susy
BF	+	+	+	+	+
SD	+	+	-	+	-
Thermal	+	-	-	-	+

Physical Applications

Sensible Models of DEWSB

Viable Models

(Ultra) Minimal Walking Technicolor (MWT)

Higher Dimensional Representations

Beyond Minimal Walking Technicolor

Partially EW Gauged Technicolor

Split Technicolor

Additional Fermions in SM

Custodial TC

Minimal Walking Technicolor

F.S. and Tuominen 04

Dietrich, F.S. Tuominen 06

Gudnason, Kouvaris, F.S. 06

Gudnason, Ryttov, F.S. 06

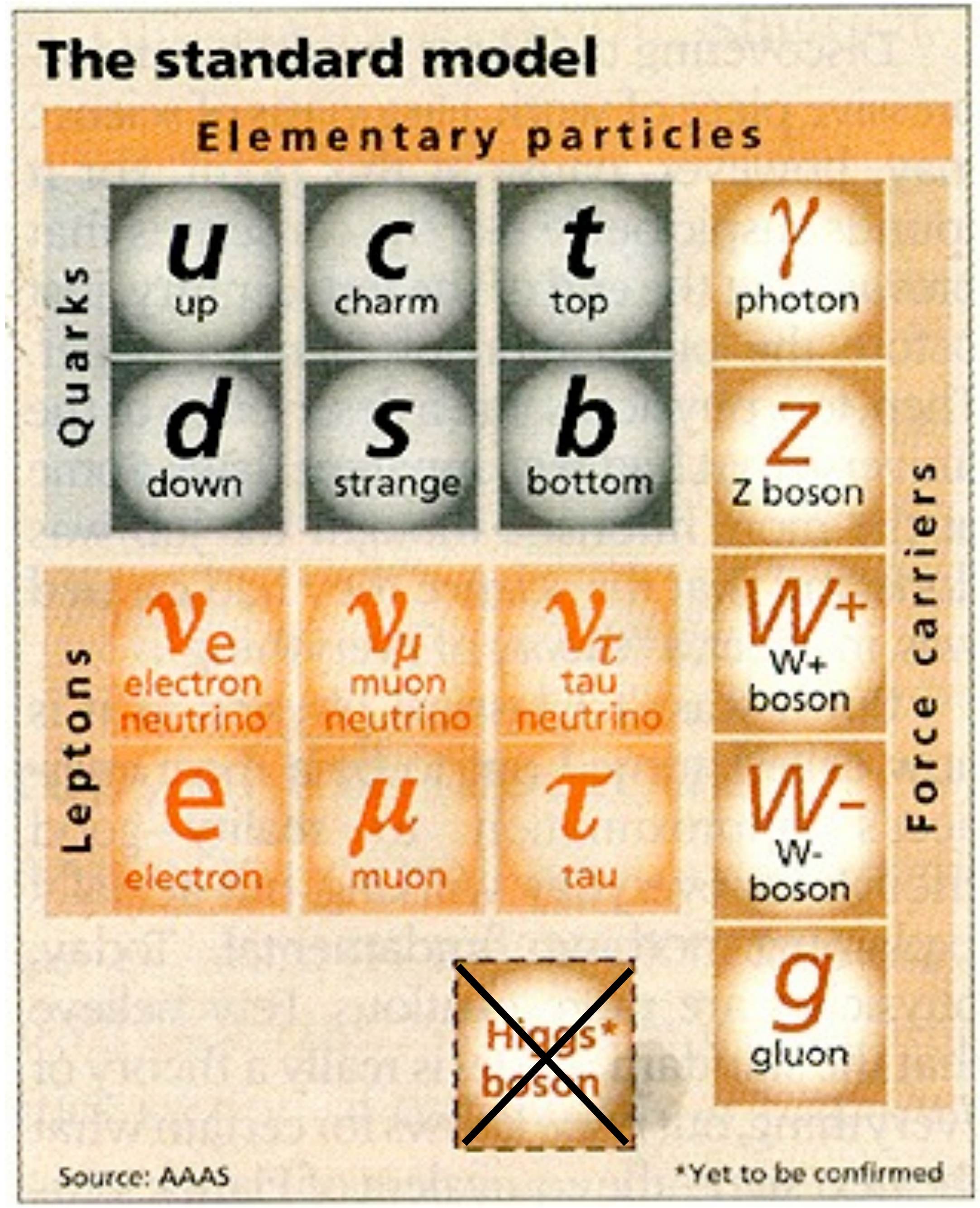
Foadi, Frandsen, Ryttov, F.S. 07

Cline, Jarvinen, F.S. 08

Belyaev, Foadi, Frandsen, Jarvinen, Pukhov, F.S. 08

N
Extra Neutrino

E
Extra Electron



U(1)

SU(2)

SU(3)

U
t-up

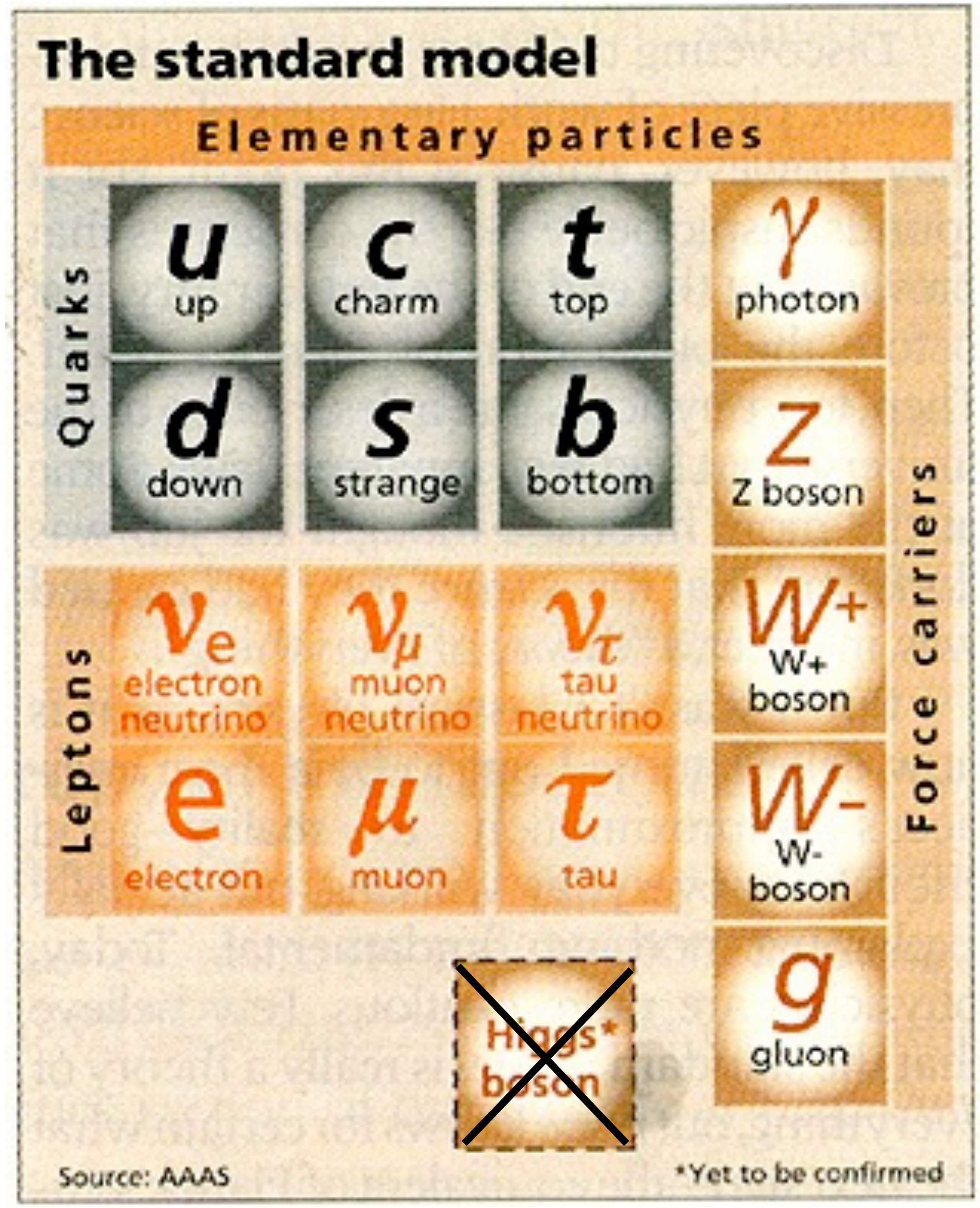
D
t-down

G
t-glue

SU(2)

Ultra Minimal Technicolor

Ryttov and F.S. 08



U(1)

SU(2)

SU(3)

U t-up

G t-glue

D t-down

λ^1 λ^2 t-elle

SU(2)

Progress in Strong Dynamics I:

Analytically unveiled a universal picture

Phase Diagrams for SO(N) and Sp(2N) — F.S. 09

Phase Diagrams for SU(N) — Ryttov, F.S. 07; F.S. and Tuominen 04; Dietrich, F.S. 06

Useful methods to understand Near-Conformal Dynamics

Conjectured all-orders beta function — Ryttov, F.S. 07

Conformal Chiral Dynamics — F.S. 08; F.S. and R. Zwicky 08; Stephanov 07

Summary 2

- Phases of Gauge Theories
- Non Perturbative Methods to explore PGT
- All order beta function conjecture
- Multiple Representations
- Physical Applications