

# Recent developments for higher-dimensional black holes

## II. Blackfolds: a new approach to higher-dimensional BHs

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0902.0427 (**PRL**) (with R. Emparan, T. Harmark, V. Niarchos)

090y.xxxx:: To appear (with R. Emparan, T. Harmark, V. Niarchos)

0708.2181 (JHEP) (with R. Emparan, T. Harmark, V. Niarchos, M.J. Rodriguez)

0802.0519 (Springer Lectures Notes)

0701022 review CQQ (with V. Niarchos and T. Harmark)

# Plan

- Introduction
- Separation of scales in higher-dimensional black holes
- Blackfold approach
- Examples of novel black hole families
- Lessons and outlook

# Progress in the last years



What do we know about black objects (i.e. with event horizon) in **higher dimensional Einstein gravity** ?

→ Dynamics of BHs in  $D \geq 5$  much richer than four dimensions

In this lecture: restrict (mostly) to **asymptotically flat solutions of pure gravity**

$$R_{\mu\nu} = 0 \quad \mathcal{M}^D$$

but:- interesting parallels with BHs in KK spaces

- techniques are readily generalized to AdS/dS space + adding charge

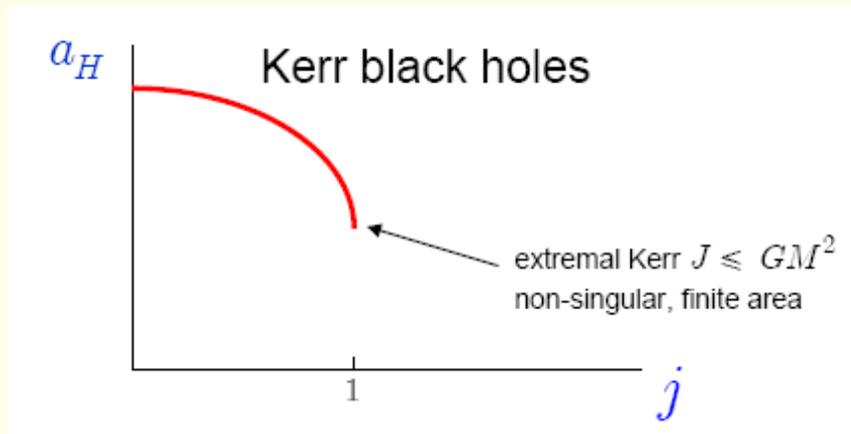
- D=4: **black hole uniqueness**
- D=5: MP black hole ( $S^3$ ), ER black ring ( $S^2 \times S^1$ ), black Saturn, ...
  - **4D inspired techniques** successful
  - (assuming 2 axial Killing vector fields → integrability
  - full classification of BHs in terms of “rod-structure” + asympt. charges )
- D  $\geq$  6: MP black holes ( $S^{D-2}$ ) are only known exact solutions
  - **full dynamics too complex** to be captured by conventional approaches
  - ➡ but recent progress: thin black rings ( $S^1 \times S^{D-3}$ ) in any dimension

# Novel feature of higher D neutral BHs

- ▶ in some regimes horizons are characterized by (at least) **two separate scales**

$$r_0 \ll R$$

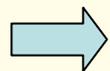
Cf.  $D=4$



shape of Kerr BH is  
always approx. round  
with radius

$$r_0 \sim GM$$

$D \geq 5$ : no Kerr bound anymore



two classical length scales can be **widely separated**

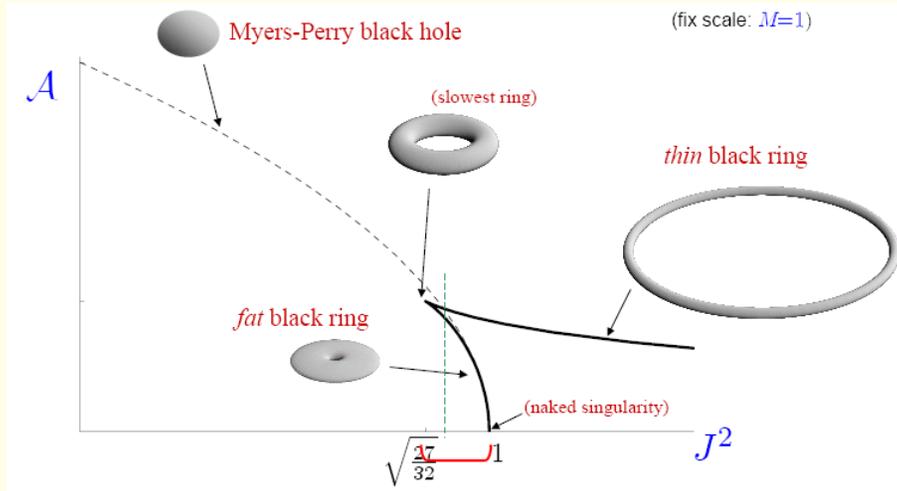
$$\frac{J}{M} \quad \text{vs.} \quad (GM)^{1/(D-3)}$$

Analogue for **KK black holes**: **size of compact manifold vs. horizon radius**

# Separation of scales

- observe separation of scales in **explicitly known solutions**

D=5

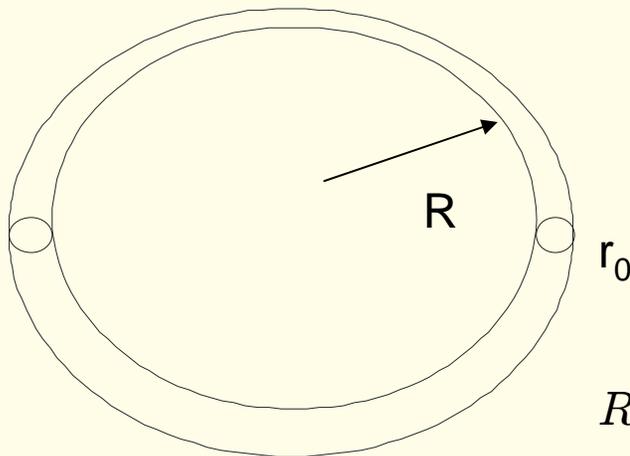


- Kerr bound for MP

but: rotating black ring can have arbitrarily large angular momentum for given mass

Emparan, Reall

ultraspinning (small mass) limit



$$\frac{J^2}{GM^3} \rightarrow \infty$$

corresponds to:  $R \gg r_0$

radius of ring  $\gg$  thickness of ring

$R$  = radius of  $S^1$

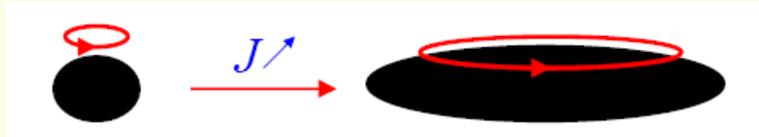
$r_0$  = radius of  $S^{D-3}$

# Separation of scales (cont'd)

$D \geq 6$ : no Kerr bound for MP BHs:

→ ultraspinning regimes with **pancaked horizons**

Emparan, Myers



$$\frac{J^{D-3}}{GM^{D-2}} \rightarrow \infty$$

(approaches **black membrane** geometry  $\mathbb{R}^2 \times S^{D-4}$  for large  $J$ )

radius of disc  $\gg$  thickness of disc

Note: **GL instability** is also property of horizons in higher  $D$  depending on separation of two length scales along horizon

**length vs. thickness** (of black brane)

→ inhomogeneous black branes arise when the two begin to differ

Gregory, Laflamme

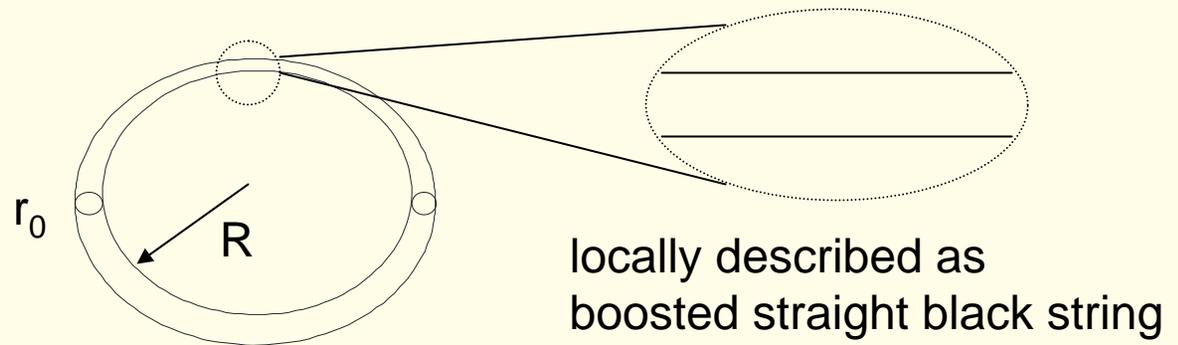
# Long distance effective theory

▶ two widely separated scales  $\rightarrow$  integrate out short-distance dynamics

$\rightarrow$  long-distance effective theory

• use to construct BHs perturbatively

e.g. 5D rotating black ring



◀ by employing method of **matched asymptotic expansion (MAE)**

thin black ring solution for  $D \geq 6$  has been constructed Emparan, Harmark, Niarchos, NO, Rodriguez

• MAE was first developed for localized BHs in KK space in limit:  $L \gg r_0$

Harmark/Kol, Gorbonos/Karsik et.al  
Dias, Harmark, Myers, NO

• other technique has been developed as well: **classical effective field theory (CIEFT)**

Chu, Goldberger, Rothstein/Kol

Goal: use these methods to develop a **leading order theory for the long-distance dynamics** of higher-dimensional black holes

# General idea

- ▶ start with general theory of gravity  $S[\Psi]$   $\Psi = \{\text{graviton, p-forms, scalars}\}$ 
  - look for BH solutions that have two characteristic scales
    - + **integrate out short-distance physics**  $\Psi = \Psi_{\text{short}} + \Psi_{\text{long}}$
- ◀ to leading order: blackfold = black-brane **probe in asymptotic background**

$$S_{\text{full}} \rightarrow S[\Psi_{\text{long}}] + S_{\text{wv}}[X^\mu]$$

Aim: give general prescription for  $S_{\text{wv}}$  to leading order in  $r_0/R$

- solve EOM of  $S[\Psi_{\text{long}}]$ : defines **asymptotic background**
- solve  $S[\Psi_{\text{long}}]$  to find black brane soln with flat worldvolume:
  - asymptotic charges define the blackfold locally
    - provides **short-distance input** for  $S_{\text{wv}}$
- $S_{\text{wv}}$  describes **embedding of blackfold** in background

$$S_{\text{wv}}[X^\mu] = \int \sqrt{-\gamma} \mathcal{L}[X^\mu(\sigma^a)]$$

determined by:

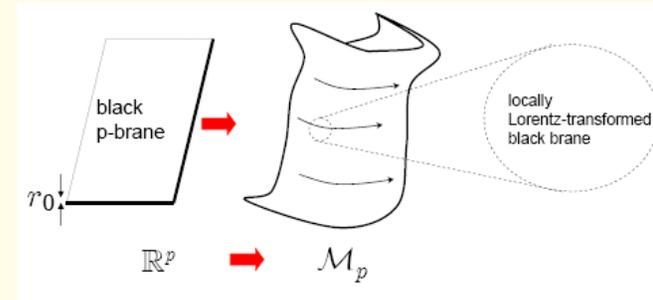
- **consistent coupling** to bulk fields  $\Psi$  (EM/charge conservation)
- use locally stress-energy/currents of the flat black brane solution
- + enforce **horizon regularity**

# Blackfold approach

Empanan, Harmark, Niarchos, NO

**Blackfold** = **Black** p-brane whose worldvolume extends along a curved submanifold (of embedding space)

- ▶ start with **flat p-brane**: horizon  $\mathbb{R}^p \times S^{n+1}$   
 $\downarrow$   
 bend spatial world-volume into submanifold  $\mathcal{B}_p$   
 characterized by length scale:  $R$   
 size:  $r_0$



- for asymptotically flat blackfolds in D dims:  
 start with a compact embedding  $X^\mu(\sigma^a)$  of submanifold  $\mathcal{B}_p \subset \mathbb{R}^{D-1}$

- consider regime of **widely separated scales**:

curvature radius of submanifold  $\gg$  brane thickness

$$R \gg r_0$$

→ can approximate the blackfold locally with flat black brane

Question: which  $\mathcal{B}_p$  are possible ?

Geometric Censorship

# Classical brane dynamics

embedding  $X^\mu(\sigma)$  of  $\mathcal{B}_p$  determines induced metric:  $\gamma_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$

what governs **dynamics of a blackfold** ?

- For point particles this is Newton's 2<sup>nd</sup> law or geodesic eqn. in GR

► For infinitely thin branes we have **Carter equation** (brane probe approximation)

$$T^{\mu\nu} K_{\mu\nu}{}^\rho = 0 \quad (\Leftrightarrow \quad \nabla_\mu T^{\mu\nu} = 0)$$

extrinsic curvature tensor (2<sup>nd</sup> fund. form)

energy momentum tensor on brane

$$T_{\mu\nu}(\sigma^\alpha) = \tau_{\mu\nu}(\sigma^\alpha) \delta^{(D-p-1)}(x - X(\sigma^\alpha))$$

• Blackfold equations are equivalent to **generalized geodesic equation**

$$\tau^{\alpha\beta} \left( \nabla_\alpha^{(\gamma)} \partial_\beta X^\rho + \Gamma_{\mu\nu}^\rho \partial_\alpha X^\mu \partial_\beta X^\nu \right) = 0$$

follow from the **world-volume action**  $I_{\text{wv}}[X^\mu(\sigma^\alpha)] = \int_{\text{wv}} \sqrt{-\gamma} \tau^{\alpha\beta} \gamma_{\alpha\beta}$

# Brane stress tensor

- ▶ effective stress tensor of blackfold det'd by **matching to short-distance physics**
  - demand that locally ( $r \ll R$ ) blackfold is equivalent to black p-brane up to position dependent Lorentz transformation

We know gravitational field of black p-brane + weak field at  $r \gg r_0$

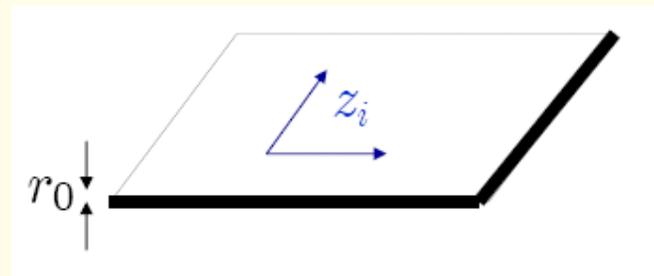
- determines **equivalent distributional stress tensor**  
(sources same field in matching region:  $r_0 \ll r \ll R$ )

- ◀ static (flat) black p-brane **metric and stress tensor** (define  $n = D - p - 3$ )

$$ds^2 = -f dt^2 + \sum_{i=1}^p dz_i^2 + f^{-1} dr^2 + r^2 d\Omega_{n+1}^2, \quad f(r) = 1 - \frac{r_0^n}{r^n}$$

$$\tau_{tt} = r_0^n (n + 1)$$

$$\tau_{ii} = -r_0^n, \quad i = 1 \dots p$$



setup of embedding:  $m$  rotation planes that we want blackfold to rotate in:

$$(r_l, \phi_l), \quad l = 1 \dots m \quad + D-1 - 2m \text{ other spatial coordinates}$$

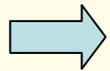
- angular directions correspond locally to **boost of flat black brane**  $z_l \sim z_l + 2\pi r_l (\sigma^\alpha)$
- align  $t$  with worldvolume time

## Boosting the p-brane

- We now act with **Lorentz transformation**:  $z \rightarrow \Lambda z$   
 $\Lambda \in SO(1, m) \subset SO(1, p)$

brane is invariant under spatial rotations: parameterize  $m$  boosts as:

$$\Lambda_0^0 = \cosh \alpha, \quad \Lambda_i^0 = \nu_i \sinh \alpha, \quad \sum_{i=1}^m \nu_i^2 = 1$$



**boosted EM tensor**  $\tau_{ij} \rightarrow (\Lambda \tau \Lambda^T)_{ij}(\sigma)$  is

$$\tau_{tt} = r_0^n [n \cosh^2 \alpha + 1]$$

$$\tau_{ii} = r_0^n [n \nu_i^2 \sinh^2 \alpha - 1], \quad i = 1 \dots m$$

$$\tau_{i \neq j} = r_0^n n \nu_i \nu_j \sinh^2 \alpha, \quad i, j = 1 \dots m$$

$$\tau_{ti} = r_0^n n \nu_i \cosh \alpha \sinh \alpha, \quad i = 1 \dots m$$

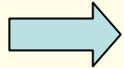
$$\tau_{ii} = -r_0^n, \quad i = m + 1 \dots p$$

$m$  boost parameters  $\alpha(\sigma)$ ,  $\nu_i(\sigma)$  and brane thickness  $r_0(\sigma)$  may depend on worldvolume coordinate !

- determined here EM tensor with flat indices since we are using local Lorentz frame to map with flat black brane

# Blackness condition

- ▶ blackfold is now locally a **boosted black brane**  
but still need to impose that it is **overall black** (regular horizon)



Blackness condition:  
surface gravity and angular velocities constant on the blackfold

can find these locally in terms of the embedding

$$\kappa = \frac{n}{2r_0(\sigma^\alpha) \cosh \alpha(\sigma^\alpha)}, \quad \Omega_{Hi} = \frac{\nu_i(\sigma^\alpha)}{r_i(\sigma^\alpha)} \tanh \alpha(\sigma^\alpha)$$

- ▶ blackness determines the **thickness and the boosts** in terms of local velocity components

$$r_0(\sigma^\alpha) = \frac{n}{2\kappa} \sqrt{1 - \Xi(\sigma^\alpha)^2}, \quad \tanh \alpha(\sigma^\alpha) = \Xi(\sigma^\alpha), \quad \nu_i(\sigma^\alpha) = \frac{r_i(\sigma^\alpha) \Omega_{Hi}}{\Xi(\sigma^\alpha)}$$

with **local velocity field** defined by:  $\Xi(\sigma^\alpha) = \left( \sum_{i=1}^m (r_i(\sigma^\alpha) \Omega_{Hi})^2 \right)^{1/2}$

insert these in EM tensor  $\rightarrow$  completely determined in terms of

$$\kappa, \Omega_i, r_i(\sigma)$$

## Final form of boosted stress tensor

$$\tau_{00} = \left(\frac{n}{2\kappa}\right)^n (1 - \Xi^2)^{\frac{n-2}{2}} (n + 1 - \Xi^2)$$

$$\tau_{0i} = \left(\frac{n}{2\kappa}\right)^n (1 - \Xi^2)^{\frac{n-2}{2}} n r_i \Omega_i, \quad i = 1, \dots, m$$

$$\tau_{ii} = \left(\frac{n}{2\kappa}\right)^n (1 - \Xi^2)^{\frac{n}{2}} \left( \frac{n(r_i \Omega_i)^2}{1 - \Xi^2} - 1 \right), \quad i = 1, \dots, m$$

$$\tau_{ii} = - \left(\frac{n}{2\kappa}\right)^n (1 - \Xi^2)^{\frac{n}{2}}, \quad i = m + 1, \dots, p$$

$$\tau_{i \neq j} = \left(\frac{n}{2\kappa}\right)^n (1 - \Xi^2)^{\frac{n-2}{2}} r_i r_j \Omega_i \Omega_j \quad i, j = 1, \dots, m$$

local velocity field:  $\Xi(\sigma^\alpha) = \left( \sum_{i=1}^m (r_i(\sigma^\alpha) \Omega_{Hi})^2 \right)^{1/2}$

EM tensor determined in terms of  $\kappa, \Omega_i, r_i(\sigma)$

# Worldvolume action and Carter equation

- Easy way to find brane EOMs is now by first computing the **worldvolume action**

$$I_{\text{wv}} = \int \sqrt{-\gamma} \tau^{ab} \eta_{ab} \propto - \int \sqrt{\gamma} [1 - \Xi(\sigma)^2]^{\frac{n}{2}}$$

+ varying with respect to the embedding coordinates  $r_i(\sigma)$

→ Carter equation becomes a set of **purely geometric equations** for embedding of  $\mathcal{B}$  and given temperature + angular velocities

⇒ **Geometric censorship for blackfolds**

(much stronger than topological restrictions)

- e.g. round  $S^1$  satisfies Carter +blackness, but a wiggly  $S^1$  does not

◀ Technical aside: how many EOMs are there ?

embedding is defined by functions  $F_i(X^\mu) = 0$  ,  $i = 1 \dots n + 2$

number of eqs. = codimension

(number of non-trivial EOMs typically reduced using symmetries)

# Thermodynamic quantities and horizon topology

◀ can compute **mass and angular momentum** by integrating appropriate EM tensor components over brane worldvolume

$$M = \int_{\mathcal{B}_p} \sqrt{-\gamma} \tau_{tt}, \quad J_i = \int_{\mathcal{B}_p} \sqrt{-\gamma} r_i \tau_{ti}$$

◀ to compute **total area**: use that locally we have area of a boosted black brane

$$a_H(\sigma^\alpha) = \Omega_{n+1} r_0^{n+1}(\sigma^\alpha) \cosh \alpha(\sigma^\alpha)$$

- small  $s^{n+1}$ -sphere at each point of blackfold

→ **horizon is fibration** of  $s^{n+1}$  over  $\mathcal{B}_p$

- if fiber is regular, horizon topology:  $(\text{topology of } \mathcal{B}_p) \times S^{n+1}$

- but  $r_0(\sigma)$  can go to zero at codimension-1 locus on  $\mathcal{B}$   
(where local boost is light-like)

- e.g. if  $\mathcal{B}_p$  is  $p$ -ball with  $s^{n+1}$  shrinking at boundary:  $S^{p+n+1} = S^{D-2}$

total area of horizon: 
$$A_H = \int_{\mathcal{B}_p} \sqrt{-\gamma} a_H(\sigma^\alpha)$$

# 1st law of thermodynamics

► consider **Gibbs free energy functional**:  $I_G[X^\mu(\sigma)] = M - \Omega_i J_i - 4\pi\kappa A_H$

by explicit computation one finds that this is proportional to the worldvolume brane action:  $(D - 2)I_G = -I_{\text{wv}}$

varying  $I_G \Rightarrow$  1<sup>st</sup> law of thermodynamics



1<sup>st</sup> law of thermo  $\Leftrightarrow$  geometric blackfold equations

also get from this:

$$(D - 3)M = (D - 2) \left( \Omega_i J_i + TS \right) + \mathcal{T}$$

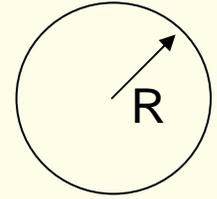
total **integrated tension of blackfold**  $\mathcal{T} = - \int \sqrt{-\gamma} \sum_{i=1}^p \tau_{ii}$

• same as **integrated version of local Smarr** for black  $p$ -brane! Harmark,NO/Kastor,Traschen

but asymptotically flat solutions should obey Smarr above with zero tension:  $\mathcal{T} = 0$

→ **total tension vanishes for blackfold** (explicitly checked in examples)

## Example: Black ring



► **wrap black string** on a compact 1D space (topologically  $S^1$ )

specify embedding:  $S^1$  in  $\mathbb{R}^2$  (times point in  $\mathbb{R}^{D-3}$ )

$$\mathbb{R}^2 : (r, \phi) \quad r = R(\sigma) , \quad \phi = \sigma$$

action  $I_{\text{WBV}} \propto \int \sqrt{-\gamma} (1 - \Xi^2)^{\frac{n}{2}} = \int d\sigma \sqrt{(R')^2 + R^2 (1 - \Omega^2 R^2)^{\frac{n}{2}}}$

→ full EOM is:

$$(1 - \Omega^2 R^2) R R'' + ((n+2)\Omega^2 R^2 - 2) R'^2 + ((n+1)\Omega^2 R^2 - 1) R^2 = 0$$

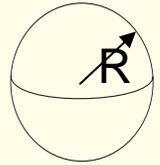
- highly non-linear DE; **simple solution** with constant  $R$ .  $R = \frac{1}{\sqrt{n+1}} \frac{1}{\Omega}$

or directly from Carter equation:  $\frac{\tau_{11}}{R} = 0$  (total tension vanishes)

◀ **zero tension condition** is equivalent to balancing forces on ring

- centrifugal repulsion balances gravitational tension
- solution with horizon topology  $S^1 \times S^{D-3}$

## New solutions: odd-spheres



► **black brane** wrapped on  $\mathcal{B}_{2k+1} = S^{2k+1}$

embed in  $\mathbb{R}^{2k+2}$ :  $d\rho^2 + \rho^2 \left( \sum_{i=1}^{k+1} d\mu_i^2 + \mu_i^2 d\phi_i^2 \right)$ ,  $\sum_{i=1}^{k+1} \mu_i^2 = 1$

sphere embedded as  $\rho = R$  worldvolume coordinates:  $\mu_i$ ,  $\phi_i$

• assume  $R = \text{const.}$  and take all  $\Omega_i$  equal (simple solution ansatz)

► action is:  $I_{\text{WB}} \propto \int R^p (1 - \Omega^2 R^2)^{\frac{n}{2}}$

EOM solved by  $R = \sqrt{\frac{p}{n + p\Omega}} \frac{1}{\Omega}$

(equivalent to  $\sum_{i=1}^p \tau_{ii} = 0$  so total tension vanishes)

Novel family of blackfolds with horizon topology:  $S^{2k+1} \times S^{n+1}$

- includes black rings for  $k = 0$
- for  $k \geq 1$ : **boosts depend on location** on the  $S^{2k+1}$
- uniform thickness

# products of odd-spheres

► black brane wrapped on

$$\mathcal{B}_p = \prod_a S^{p_a} , \quad p_a = \text{odd} , \quad \sum_a p_a = p$$

- number of spheres cannot be larger than  $n+2$

• assume  $R = \text{const.}$  for each sphere

+ take all  $\Omega_i$  equal for each sphere (simple solution ansatz)

► action is:  $I_{\text{WV}} \propto \prod_a \int R_a^{p_a} (1 - (\Omega^{(a)})^2 R_a^2)^{\frac{n}{2}}$

$$\text{EOM solved by } R_a = \sqrt{\frac{p_a}{n + p} \frac{1}{\Omega^{(a)}}}$$

many new blackfolds with non-trivial horizon topology:

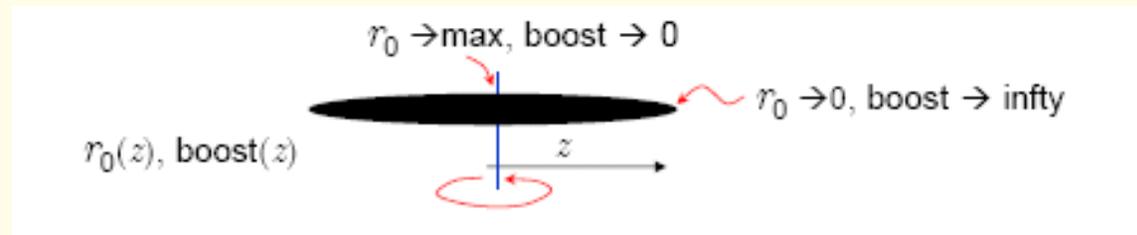
$$\begin{aligned} & \mathbb{T}^p \times S^{n+1} , \quad (\mathbb{T}^{p-3} \times S^3) \times S^{n+1} , \\ & (S^3 \times S^3) \times S^{n+1} , \quad \dots \end{aligned}$$

# Ultraspinning MP BHs as even-ball blackfolds

- ▶ blackfold eqs. do **not admit even-sphere** solutions for  $\mathcal{B}_p$ 
  - tension at fixed points of rotation group cannot be counterbalanced by centrifugal forces
  - instead solutions with  $\mathcal{B}_p = \text{ellipsoidal even-ball}$ 
    - thickness  $r_0$  shrinks to zero at boundary of ball so including the  $S^{n+1}$  fibers, horizon topology is  $S^{D-2}$
- **reproduce precisely all physical quantities of MP BH** with  $p/2$  ultra-spins
  - highly non-trivial check on approach (rotation has fixed points at center of ball,  $r_0(\sigma)$  varying)

◀ simplest example: black disc:  $D_2 \subset \mathbb{R}^2$

boost depends  
on radius:



- corresponds to MP BH with one angular momentum in **ultraspinning limit**

# Blackfold Bestiary

- ▶ blackfold construction shows existence of **new types** of asymptotically flat stationary black holes in higher dimensions

$D = 4$	$D = 5$	$D = 6$	$D = 7$	$D = 8$	$D = 9$
$S^2$	$S^3$	$S^4$	$S^5$	$S^6$	$S^7$
		$B_2 \otimes s^2$	$B_2 \otimes s^3$	$B_2 \otimes s^4$ $B_4 \otimes s^2$	$B_2 \otimes s^5$ $B_4 \otimes s^3$
	$S^1 \times s^2$	$S^1 \times s^3$	$S^1 \times s^4$	$S^1 \times s^5$	$S^1 \times s^6$
		$T^2 \times s^2$	$T^2 \times s^3$	$T^2 \times s^4$	$T^2 \times s^5$
			$S^3 \times s^2$ $T^3 \times s^2$	$S^3 \times s^3$ $T^3 \times s^3$	$S^3 \times s^4$ $T^3 \times s^4$
				$S^1 \times S^3 \times s^2$	$S^1 \times S^3 \times s^3$ $T^4 \times s^3$

Kerr, MP BH

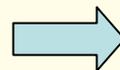
ultraspinning  
MP BH

black ring

black torus

- ◀ for **product odd-sphere and even-ball blackfolds** with equal sizes and angular momenta (at fixed mass):

$$A(J) \sim J^{-p/n}$$



tori dominate entropically

# Caveats

- **regularity of black brane horizon** after bending ?
  - shown for **black 1-folds** (i.e. black strings)
  - extension to  **$p$ -folds** (to appear)  
(use matched asymptotic expansion)
- **backreaction of blackfold** on background geometry is neglected (to leading order in  $r_0/R$ )
  - could make it impossible for leading-order solution to remain stationary  
(must be analyzed case-by-case)
- blackfolds may be (classically) **unstable**
  - can use blackfold equations to analyze stability under long wavelength perturbations ( $\lambda \gg r_0$ )
  - there are short wavelength ( $\lambda \sim r_0$ ) instabilities (GL-type) outside approach

# Lessons from blackfold approach

- ▶ dynamics of higher-dimensional black holes naturally organized in **relative value of scales**

$$0 \leq J \lesssim M(GM)^{\frac{1}{D-3}}$$

- single length scale: **Kerr BH behavior**

$$J \gtrsim M(GM)^{\frac{1}{D-3}}$$

- regime of **mergers and connections** between phases when two horizon scales meet  $r_0 \sim R$ 
  - not accessible to effective methods; requires extrapolation or numerics

$$J \gg M(GM)^{\frac{1}{D-3}}$$

- **blackfolds**
  - extreme rich physics in this regime; study dynamics rather than exact solutions for all possible BHs

◀ blackfold **horizon topologies**  $\mathcal{B}_p \times S^{n+1}$

supported by  
mechanical equilibrium

supported by internal  
structure of the BH

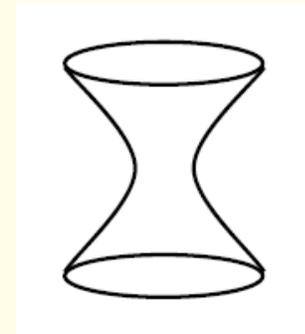
- purely topological analysis cannot distinguish between these two factors

## Other cases

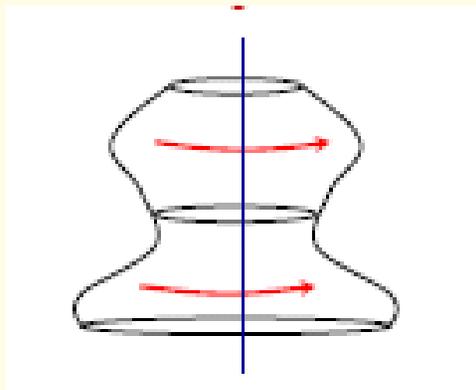
- ▶ **static minimal blackfolds**  $\tau_{ij} = -P\eta_{ij}$   
(no boost)  $\rightarrow K^\rho = 0$  (mean curvature vector)

minimal submanifold

e.g. hyperboloid (static non-compact blackfold)



- ▶ **axisymmetric blackfolds**



use numerics or further perturbative approach ?

# New blackfolds in 5D: helical rings and strings

Emparan, Harmark, Niarchos, NO (in progress)

for black 1-folds we can take curves with tangent vector equal to a linear combination of isometries

$$\zeta = \sum_i c_i \xi^{(i)} \Big|_{x=X(\sigma)}$$

→ for critical boost this satisfies Carter + blackness

- helical black string:  $\zeta \sim (k\partial_x + \partial_\phi) \Big|_{r=R}$

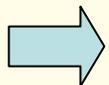
- helix with pitch  $k$

- boost along string gives momentum along  $x$  and angular momentum along  $\phi$

- helical black ring:  $\zeta \sim (n\partial_\phi + m\partial_\psi) \Big|_{r_1=R_1, r_2=R_2}$

- helix of radius  $R_2$  around circular trajectory of radius  $R_1$  that closes on itself after  $m$  turns

- boost is linear combo of two angular momenta



has only single spatial U(1) isometry:

first evidence of such a solution in 5D ! (as admitted by rigidity theorem)

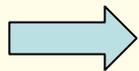
# Charged blackfolds

Empanan, Harmark, Niarchos, NO (in progress)

Carter equation:  $K_{\alpha\beta}^{\rho} \tau^{\alpha\beta} = F^{\rho} , \nabla_{\mu_1} J^{\mu_1 \dots \mu_{p+1}} = 0$

worldvolume action  $I_{\text{wv}} = \int \sqrt{\gamma} \tau^{ij} \gamma_{ij} + A \cdot J$

- use **branes in EMD-gravity** (includes supergravities relevant for string theory)
  - must now also add **charge conservation** to blackness conditions



seems to generate highly non-trivial blackfolds

- odd-sphere solutions
  - S<sup>1</sup>: **dipole rings** in any dimension (includes known dipole ring in 5D)
    - from boosting and bending a charged string Empanan
  - higher spheres (in progress)
- even-ball solutions
  - **charged rotating discs** ? .....

could potentially be stable ! (under investigation)

# Further Outlook

- **charged blackfolds**
  - in progress [Empanan, Harmark, Niarchos, NO](#)
- method can also be applied to **blackfolds in other backgrounds** (AdS, dS)
  - black rings in (A)dS [Caldarelli, Empran, Rodriguez](#)
- SUSY blackfolds ?
  - extremal black holes and black rings [Figueras, Kunduri, Lucetti, Rangamani](#)  
cf. 5D supersymmetric black ring [Elvang, Empanan, Mateos, Reall](#)
- stability analysis
- relation with DBI
- higher-order analysis (via MAE/CIEFT) (in progress: horizons stay regular)
- **blackfold motion** + relation to **fluid/gravity correspondence**
- duality of higher D black holes to **plasma balls + rings** in AdS  
(cf. [Lahiri, Minwalla](#)) – many similar features

The end