Recent developments for higher-dimensional black holes

I. Phase structure of black holes, strings and rings

Cracow School, Zakopane, June 3, 2009

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0902.0427 (**PRL)** (with R. Emparan, T. Harmark, V. Niarchos)

090y.xxxx:: To appear (with R. Emparan, T. Harmark, V. Niarchos)

0708.2181 (JHEP) (with R. Emparan, T. Harmark, V. Niarchos, M.J. Rodriguez)

0802.0519 (Springer Lectures Notes)

0701022 review CQQ (with V. Niarchos and T. Harmark)

Motivations to study higher-dimensional gravity

- **Applications**:
  - String/M theory
    - BH entropy, new brane solutions
  - AdS/CFT
    - new phases of thermal gauge theories, phase transitions
    - plasma balls/rings in AdS (fluid/gravity correspondence)
  - Large extra dimensions + TeV gravity
    - possible objects in universe/accelators
  - math: Lorentzian geometry

- **Intrinsically interesting**:

  Can regard D as tunable parameter for gravity + black holes

  which BH properties are:
  - intrinsic $\rightarrow$ Laws of BH mechanics
  - D-dependent $\rightarrow$ uniqueness, topology, shape, stability

For various reviews see:
- Kol
- Harmark,Niarchos,NO
- Kleihaus,Kunz,Navarro-Larida
- Emparan,Reall
- NO

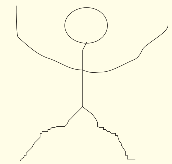
Progress in the last years

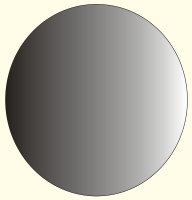
What do we know about black objects (i.e. with event horizon) in higher dimensional Einstein gravity

⇒ some, but still lot to discover

stationary black holes: ⟵ $\mathcal{M}^D$

KK black holes: ⟵ $\mathcal{M}^{D-1} \times S^1$





▶ Two cases studied: most progress in recent years - less explored

- asymptotically flat spaces: (stationary solutions)
  - five dimensions
    - MP black holes, black rings, black Saturns, black di-rings,
  - six and beyond
    - but: recent progress !

- Kaluza-Klein spaces: (static solutions)
  - d-dim Minkowski x circle (tori)
    - non-uniform strings, localized black holes, bubble-black hole sequences, merger point, evolution of GL instability
  - other Ricci flat..
    - e.g. CY

Plan

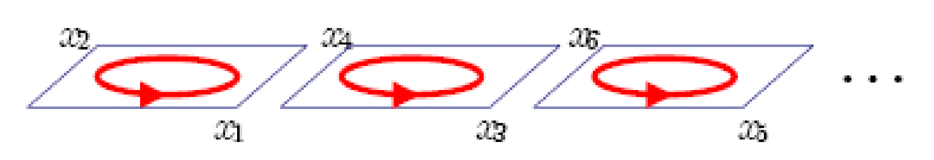
- Introduction: going beyond four dimensions
- Static Kaluza-Klein black holes
- Stationary asymptotically flat black holes in five and more dimensions
- Pinched rotating black holes in six and more dimensions
- Thin rotating black rings in six and more dimensions

Going beyond four spacetime dimensions

► why is $D > 4$ richer ?

- more degrees of freedom
- rotation:

more rotation planes



gravitational attraction $\Leftrightarrow$ centrifugal repulsion

- $\exists$ extended black objects: black p-branes
- compact directions: extra scale(s)

► why is $D > 4$ harder ?

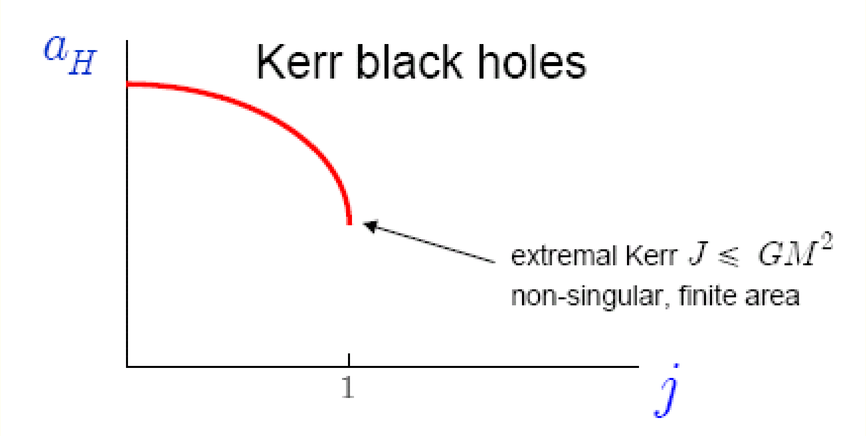
- more degrees of freedom
- less symmetries
  - not enough axial symmetries to reduce to 2D σ -model for $D > 5$
  - broken symmetries along compact directions

Uniqueness in four spacetime dimensions

▶ in four dimensional pure gravity:
given mass and angular momentum: unique black hole solution

- static: Schwarzschild BH
- stationary: Kerr BH

phase plots: to compare solutions one needs to fix common scale
• classical GR does not have intrinsic scale $\rightarrow$ fix mass M
• in presence of compact directions: use length scale L of compact circle(s)



$$a_H \equiv \frac{\mathcal{A}_H}{(GM)^2}$$

$$j \equiv \frac{J}{GM^2}$$

Black holes in $D > 4$

- for D–dimensional asymptotically flat space times:
  - ▶ generalization of Schwarzschild: Schwarzschild-Tangherlini black hole

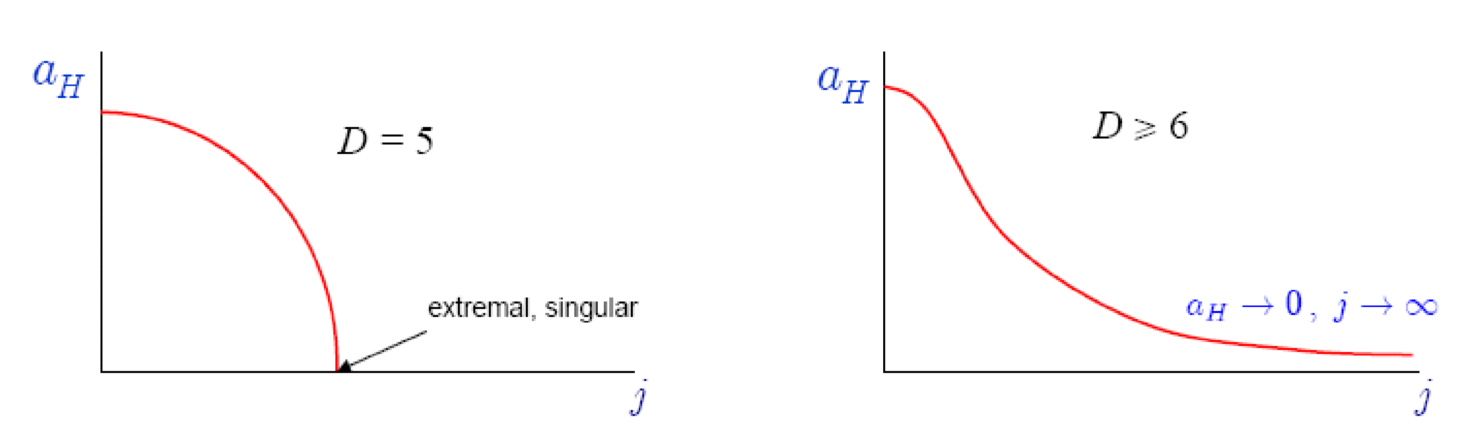
Tangherlini (1963)

$$ds^2 = -\left(1 - \frac{r_0^{D-3}}{r^{D-3}}\right) dt^2 + \left(1 - \frac{r_0^{D-3}}{r^{D-3}}\right)^{-1} dr^2 + r^2 d\Omega_{D-2}^2$$

- uniqueness: only static and neutral black hole in pure gravity — Gibbons,Ida,Shiromizu
- dynamically linearly stable — Ishibashi,Kodama

▶ generalization of Kerr: Myers-Perry black holes (1986)

- rotating BHs with angular momentum in arbitrary number of planes
- spherical topology $S^{D\text{-}2}$



More BHs in $D > 4$?

► for stationary solutions in asymptotically flat space:

- combine black branes & rotation

⇨ • black rings + other blackfolds in $D \geq 5$
• pinched black holes in $D \geq 6$

$D = 5$: end may be in sight

► for static solutions in spaces with compact directions (Kaluza-Klein):

⇨ • uniform black string = Schwarschild-Tangherlini BH x circle
(circle $\rightarrow$ tori: black branes)
• new solutions that are non translationally invariant in circle direction
- localized black holes
- non-uniform black string (Gregory-Laflamme instability)
• bubble-black hole sequences (not discussed in this talk)

→ recent insight:
for $D \geq 6$: phase structure of these two cases is intimately linked !

reason: ultraspinning black holes become (pancaked) rotating branes

Kaluza-Klein black holes

► black holes asymptoting to d-dimensional Minkowvski space times a circle
(Kaluza-Klein spaces) = $\mathcal{M}^d \times S^1$
= time x d -dimensional cylinder $\mathbb{R}^{d-1} \times S^1$ circumference of S^1: L

circle direction breaks symmetry $\rightarrow$ gives rise to new possibilities of BH solutions

- what are the static & neutral BH solutions on the cylinder ?

two (gauge-invariant) asymptotic quantities:

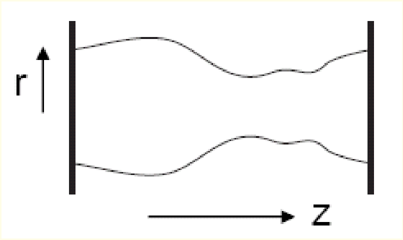
mass M tension $\mathcal{T}$

Harmark,NO/Kol,Sorkin,Piran/
Traschen,Fox/Towsend,Zamaklar

dimensionelss quantites $\mu = \frac{M}{GL^{d-2}}$, $n = \frac{L\mathcal{T}}{M}$

► restrict to case with spherical symmetry for $\mathbb{R}^{d-1}$ part of cylinder:

at ∞ can think of any BH solution as coming from Newtonian source located at origin of $\mathbb{R}^{d-1}$ but with mass distribution in circle direction: source $\rho(z)$

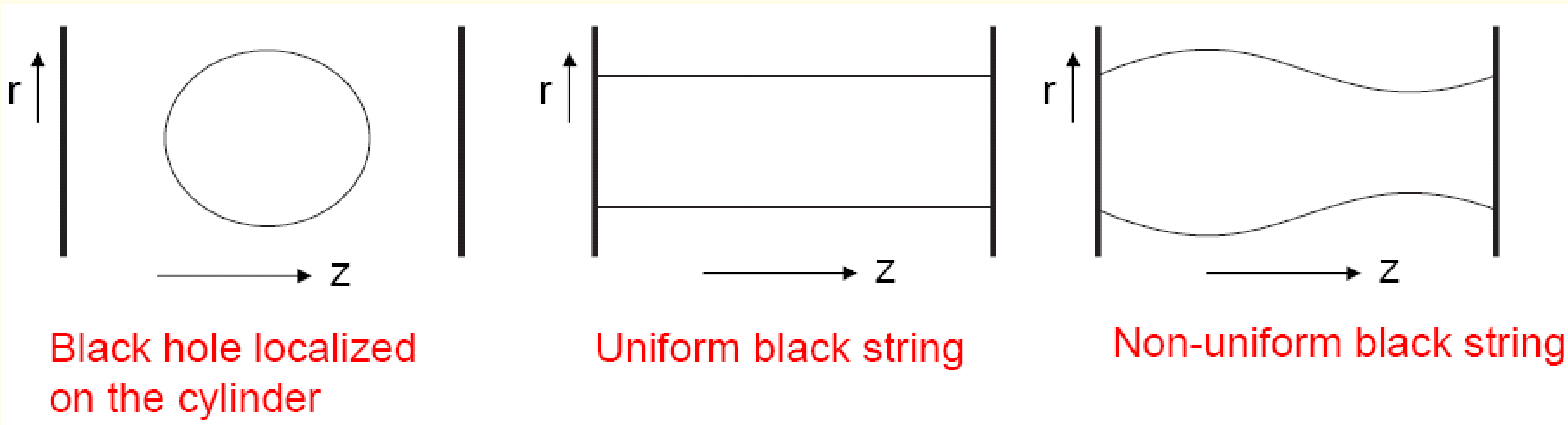


Possible BH solutions on circle

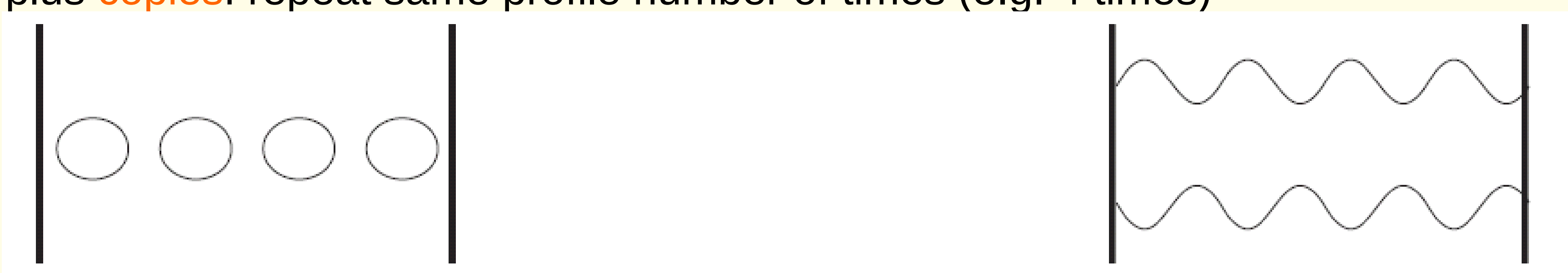
► do all profile/mass distributions correspond to BH solutions ?

- clearly No – BH solution in GR automatically takes into account self-gravitation of the mass distribution, so not even for Newtonian matter would we expect that

what are possible BH solutions ?



plus copies: repeat same profile number of times (e.g. 4 times)

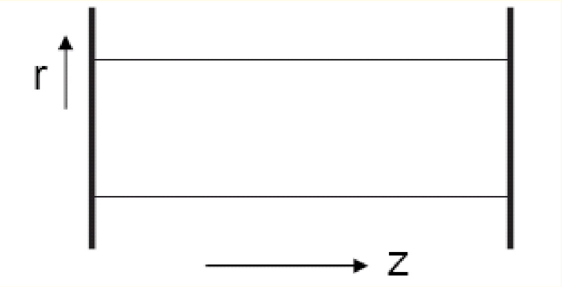


these + unequal mass multi BHs are presently known solutions on cylinder (assuming spherical symmetry)

Uniform and non-uniform black string, GL instability

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_{d-2}^2 + dz^2$$

$$f = 1 - \frac{r_0^{d-3}}{r^{d-3}}$$

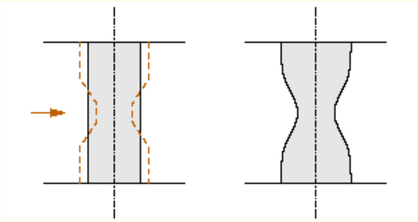


d-dim Schw-Tang. BH x flat compactified direction

GL found: classically unstable $\mu < \mu_{\rm GL}$ classically stable μ

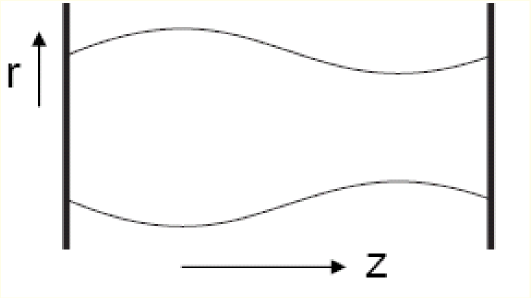
Gregory,Laflamme

threshold mode $\Omega = 0$



$$\delta g_{\mu\nu} \sim e^{ikz} e^{\Omega t}$$

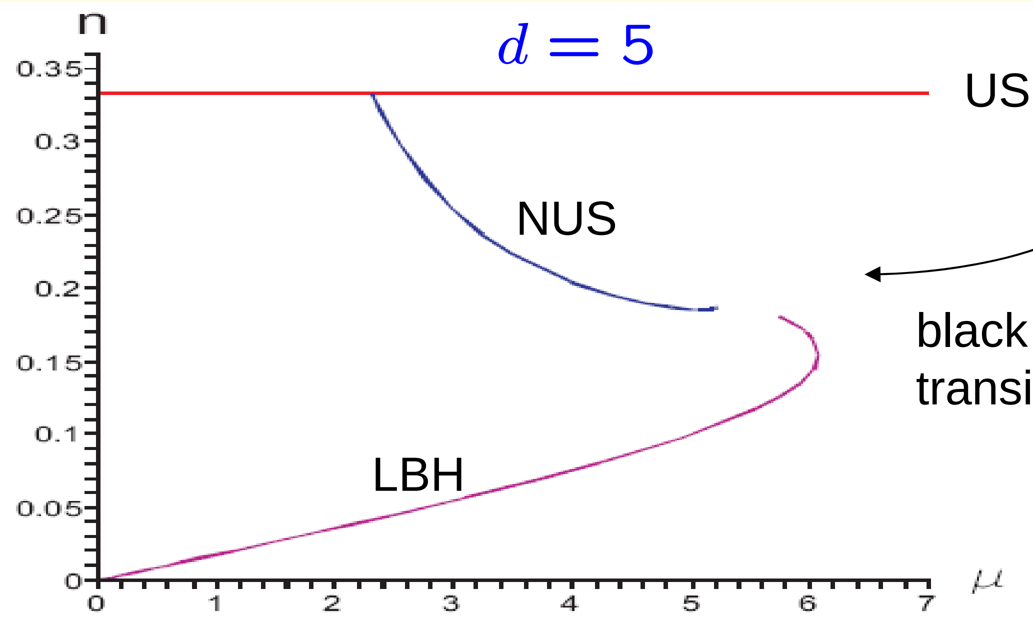
non-uniform static solution emerges:



long wave-length instability
– present when critical GL wavelength can fit in the compact direction

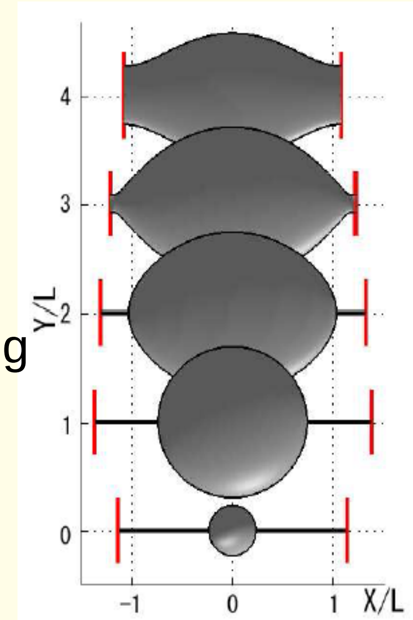
KK BHs: Phase Diagram (example 6D)

relative tension



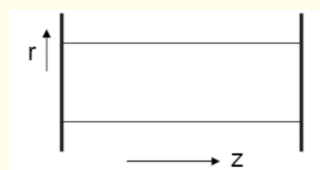
rescaled mass

black hole/black string transition



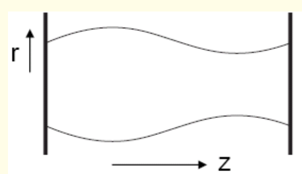
Koduh,Wiseman

- uniform black string (UBS) $S^{d-2} \times S^1$



Schwarzschild (d) $\times$ S^1

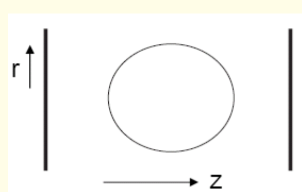
- non-uniform black string (NUBS) $S^{d-2} \times S^1$



emanates from uniform string at Gregory-Laflamme point

Gregory,Laflamme/Gubser/Wiseman/Sorkin /Kleihaus,Kunz,Radu
motivated in part by: Horowitz,Maeda

- localized black hole (LBH) S^{d-1}

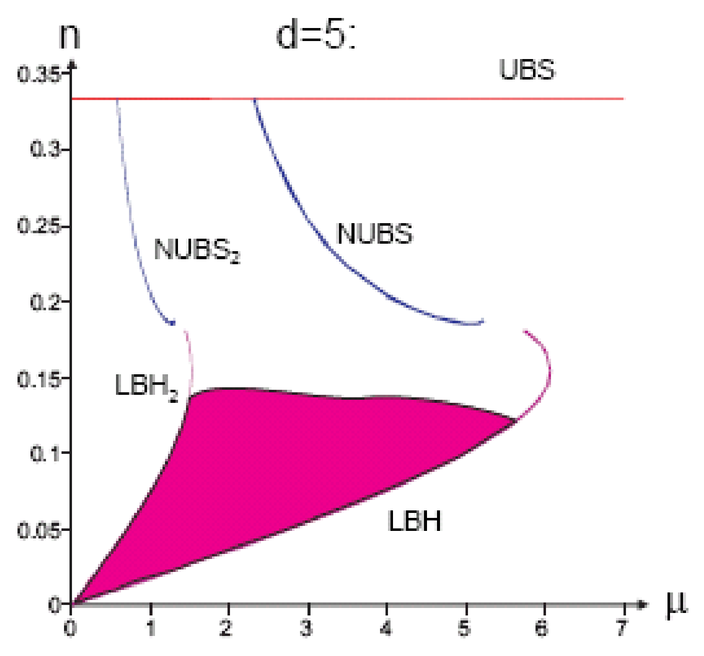


Schwarzschild (d +1) + $\mathcal{O}$ (μ)

Harmark,NO/Harmark/Kol,Gorbonos/ Sorkin,Kol,Piran/Koduh,Wiseman

Multi-black hole configurations

► thermal equilibrium phases also include copies of localized black hole phase and non-uniform black string phase

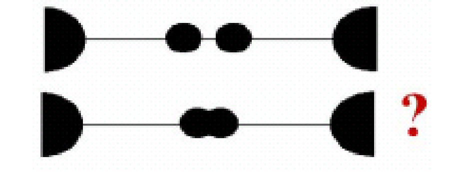


there exist also unequal mass multi-black hole configurations

Dias,Myers,Harmark,NO

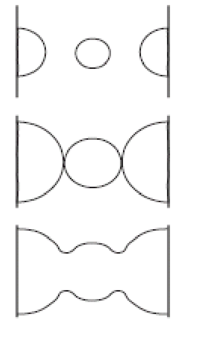
- continuous non-uniqueness

• speculation: existence of (static) lumpy black holes (one big BH + two small BHs: small ones can merge into lumpy object before all horizons merge)

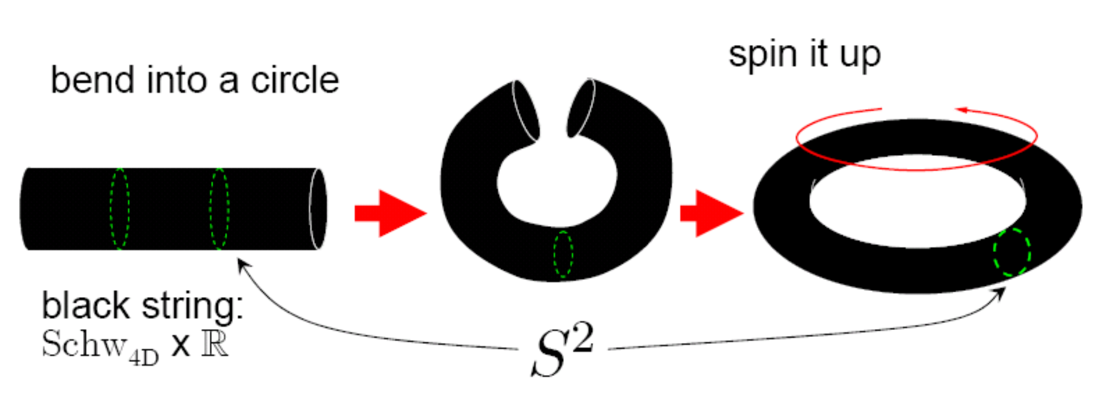


• what happens when we crank up mass of multi-BH config existence of new non-uniform black strings ? (bumpy black strings)

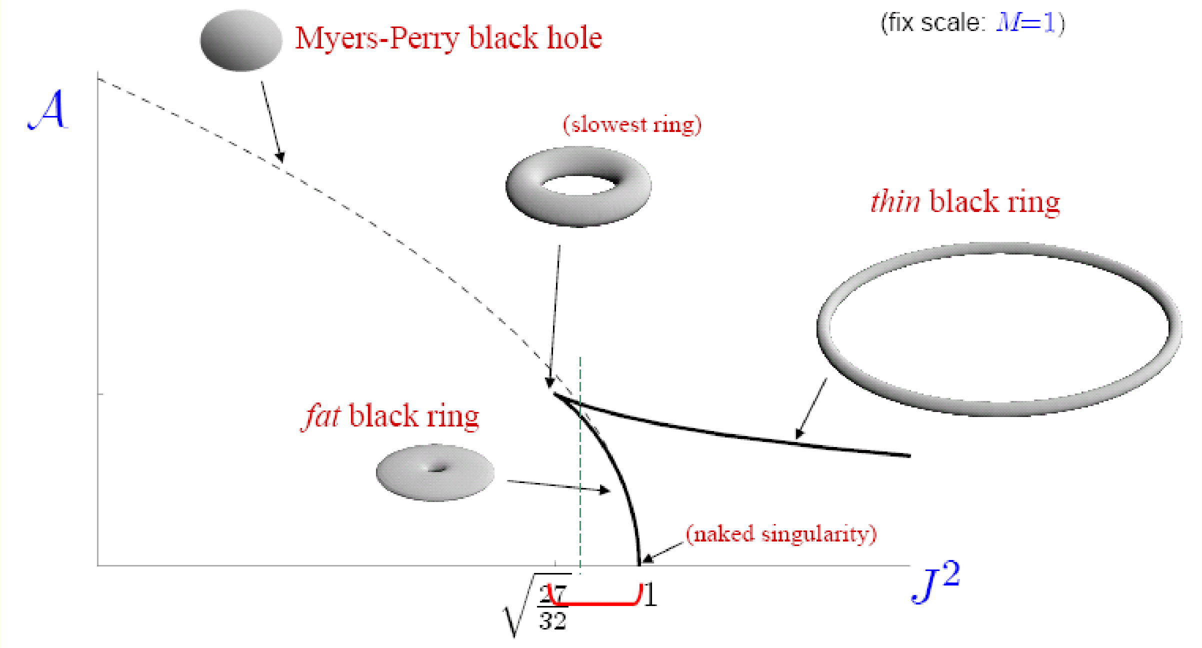
- open question: how connected to GL point



The Black Ring in five dimensions



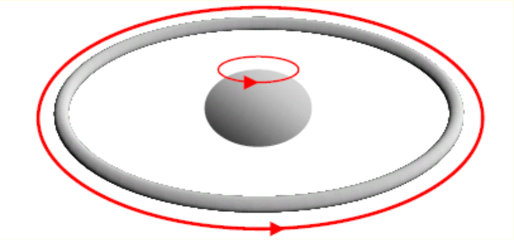
- horizon topology: $S^1 \times S^2$
- exact solution known (generalized Weyl ansatz): Emparan, Reall (2001)



non-uniqueness:
3 different BHs for
same value of M, J

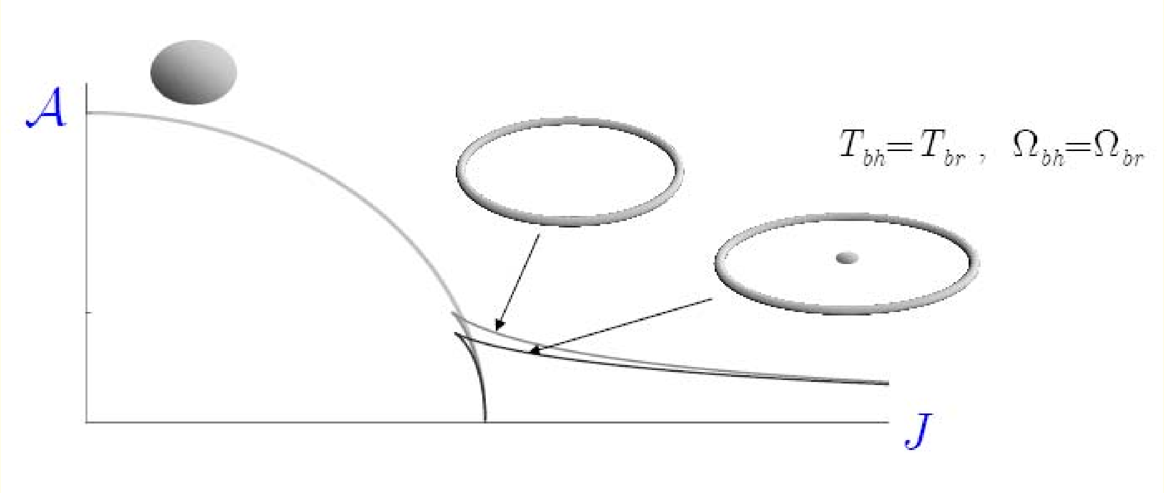
Multi-black holes

black Saturn



Elvang,Figueras

► 5D phases in thermal equilibrium



also: two-spin solutions

Pomeransky,Sen'kov/Elvang,Rodriguez

- infinite non-uniqueness for configurations not in thermal equilibrium
  - di-rings
  - Saturns with multi rings
  - bicycling rings

Iguchi,Mishima/Evslin,Krishnan/
Elvang,Emparan,Figueras
Izumi/Elvang,Rodriguez

Beyond five dimensions: difficulties

- difficulties:

  ► explicit construction techniques

  -not enough isometries to reduce to 2D σ–model — Emparan,Reall; Harmark/Harmark,Olesen

  D = 4: $R \times U(1)$, D = 5: $R \times U(1) \times U(1)$

  but $D \geq 6$: $R \times U(1)^{[(D-1)/2]}$ i.e. less than D -2 isometries

  ► horizon topology

  D =4: S^2 — Hawking

  D =5: S^3 , $S^1 \times S^2$ — Galloway,Schoen

  D = 6: S^4 , $S^1 \times S^3$, $S^2 \times S^2$, $M_g \times S^2$ — Helfgott,Oz,Yanay

  exact soln. approximate soln. — Emparan,Harmark,Niarchos,NO,Rodrigues

  $D \geq 6$: S^{D-2} , $S^1 \times S^{D-3}$, $T^p \times S^{D-p-2}$, ….

  from blackfold approach (2nd lecture)

  Emparan,Harmark,Niarchos,NO

Beyond five dimensions: ideas

► more qualitative (& less rigorous) methods

- physical insights
- analogies
- "probe" approximation

► perturbative constructions (around known solution)

- identify parameter + construct in a perturbative regime
- method of matched asymptotic expansion

  → Blackfold approach

⇨ one can already infer remarkable many new results/conjectures

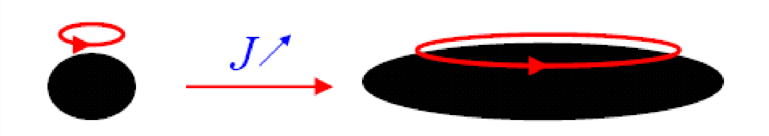
can form basis for:

- numerical attacks
- inspiration for possible exact methods

Ultraspinning MP BHs and pinched (lumpy) BHs in $D \geq 6$

MP BH approaches black membrane geometry $\mathbb{R}^2 \times S^{D-4}$ for large J

Emparan,Myers

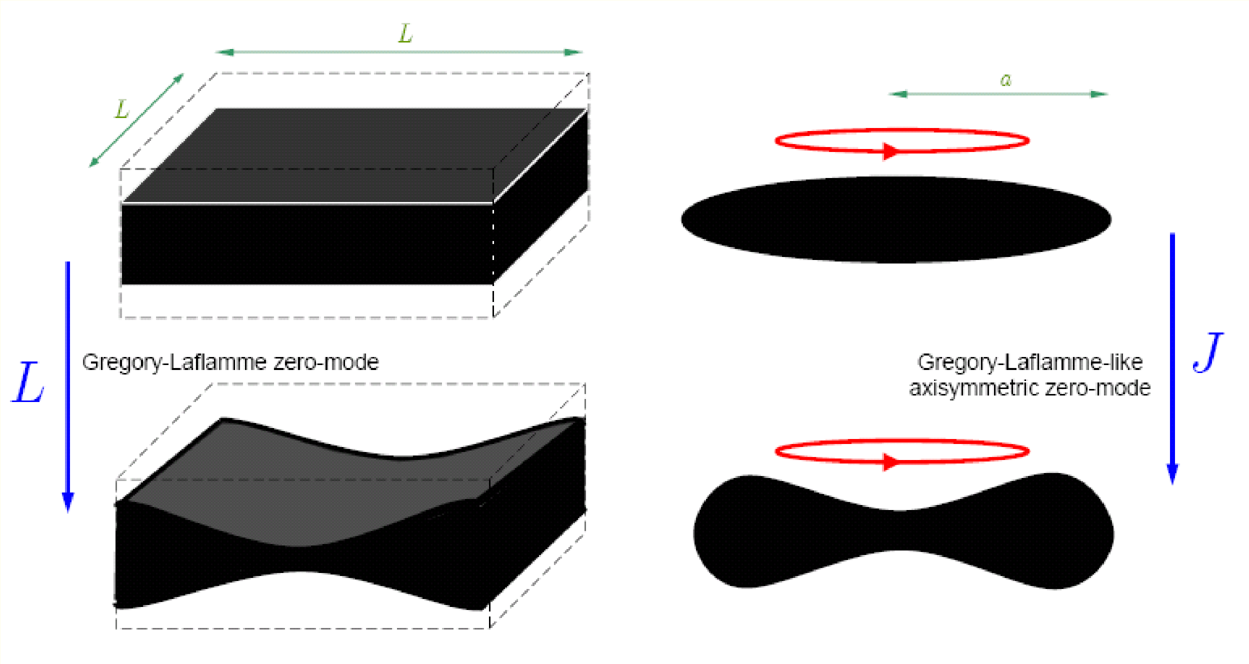


black membrane along rotation plane

⇨ black strings exhibit GL instablity: Gregory,Laflamme/Gubser/Wiseman

in particular: new non-uniform black string at threshold (zero-) mode

- same holds for black branes, in particular black membrane

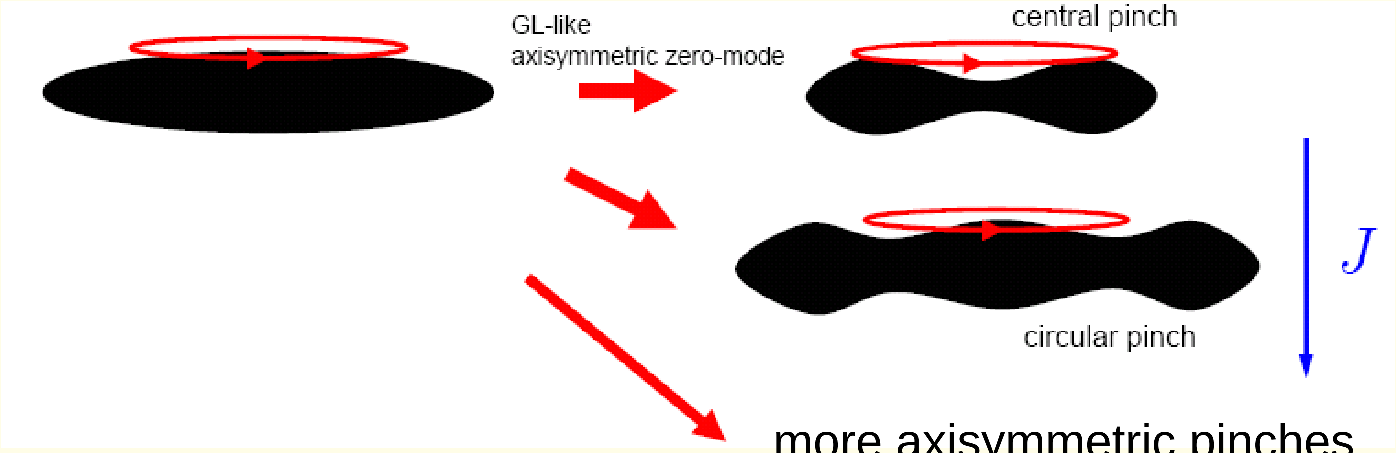


← pinched rotating BH

Multi-pinched BHs from axisymmetric zero-modes

for black string: integer multiples of the GL zero-mode also give rise to new non-uniform solutions (with repeated pattern of wiggles)

⇒ apply to pancaked (membrane-like) rotating BH



more axisymmetric pinches

- not yet found explicitly (need presumably numerics, cf. non-uniform string)
- necessary to complete phase diagram
  - will become black rings/black Saturns when pinched through
- pinched plasma balls in N=4 SYM recently found
  - dual to large pinched black holes in AdS

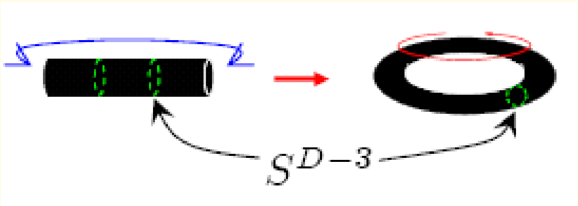
Lahiri,Minwalla

Thin black rings in $D \geq 6$

▶ heuristic construction: in analogy with 5D black string:

think of black ring as black string bent around in a circle

- horizon topology $S^1 \times S^{D-3}$



- thin ring limit means the string is only a little bent
- zeroth order approximation: straight black string
- rotation of the ring $\rightarrow$ boosted straight black string
- first order perturbation $\rightarrow$ bending of the string

Emparan,Harmark,Niarchos,NO,Rodrigues

small mass limit $M \rightarrow 0$ ⟹ thin ring limit

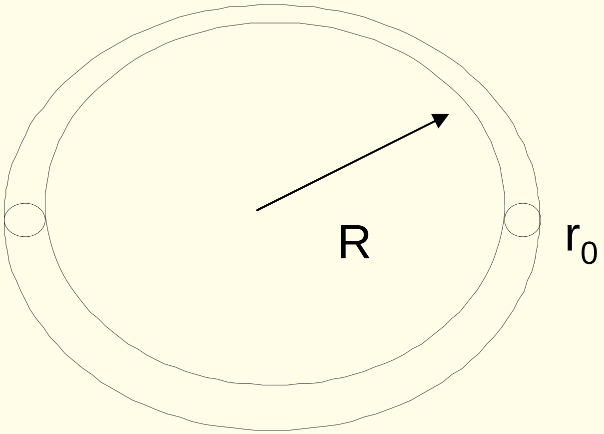
corresponds to $\frac{GM^{D-2}}{J^{D-3}} \ll 1$

ultraspinning limit

same as $R \gg r_0$

R = radius of S^1

r_0 = radius of S^{D-3}



Equilibrium condition

► boosted black string limit of black ring is described by three parameters

$r_0\ ,\quad R\ ,\quad \alpha$ (boost parameter)

expect physically: two parameters (e.g. given radius and mass, spin is fixed)

dynamical balance condition relates the three parameters

$$K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0 \longrightarrow \frac{T_{zz}}{R} = 0 \longrightarrow \sinh^2\alpha = \frac{1}{D-4}$$

EOM for probe brane-like objects

Carter

critical boost:

enables computation of all leading order thermodynamic quantities !

$$R = \frac{n+2}{\sqrt{n+1}} \frac{J}{M}$$

valid in large J limit of black ring

crucial assumption: horizon remains regular when boosted black string is curved

■ important check: rederive equilibrium condition from regularity condition

→ shows how GR encodes EOM of BHs as regularity conditions on geometry

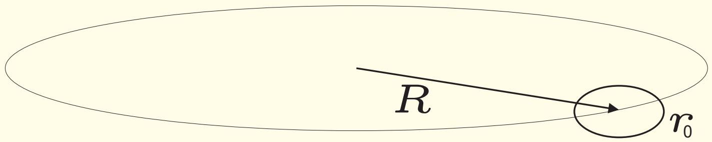
Matched asymptotic expansion

- MAE = systematic approach to iteratively construct solution given known solution in some limit + then correcting it in perturbative expansion

  - applied e.g. to construct metric of small black holes on circle Harmark/Gorbonos,Kol

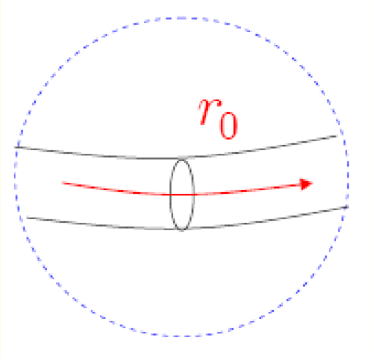
1. linearized solution around flat space

   - asymptotic zone: $r \gg r_0$



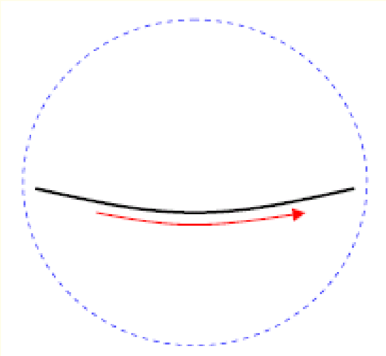
r_0: S^{n+1} radius

r is distance from ring



2. perturbations of boosted black string

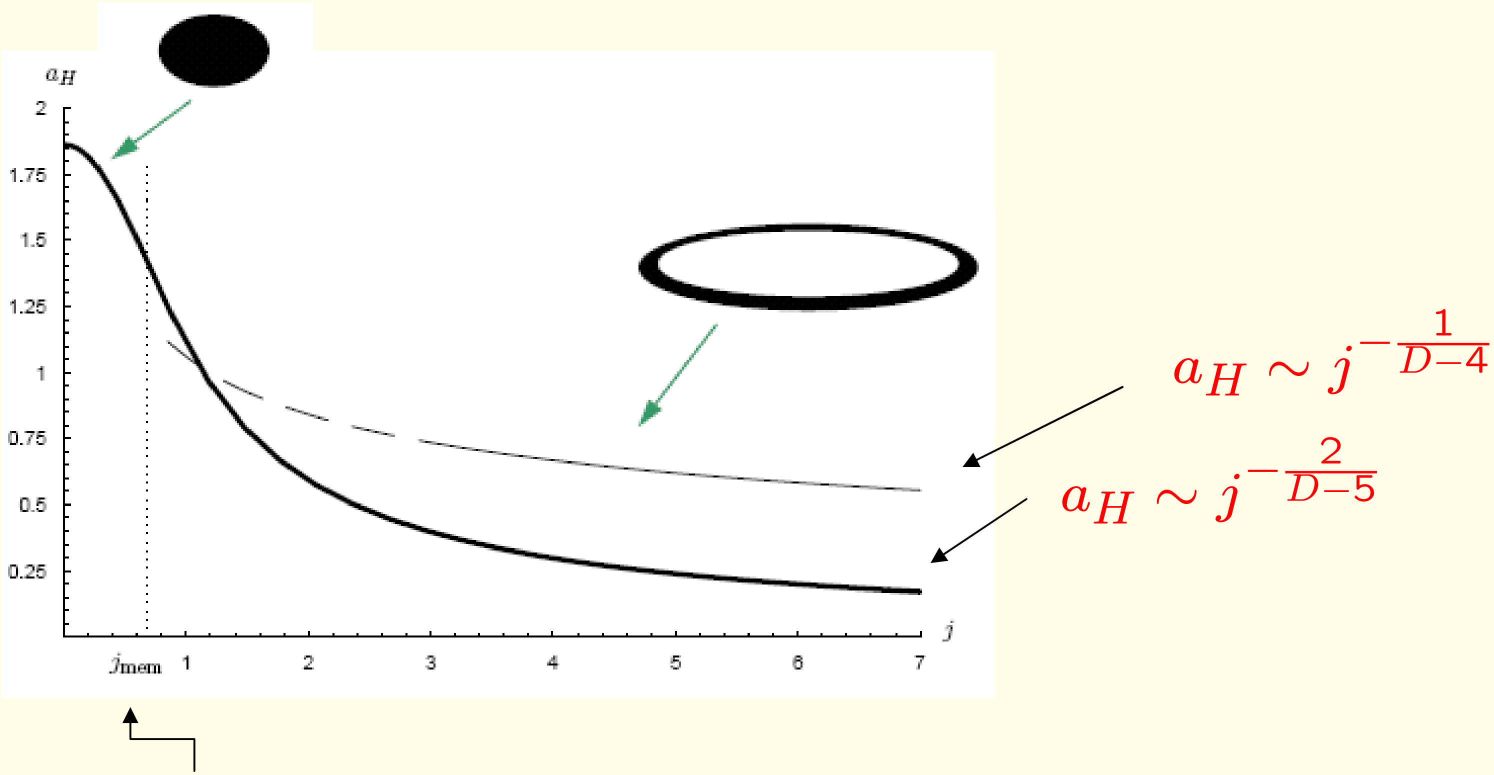
   - near-horizon zone: $r \ll R$



3. match in overlap zone

$$r_0 \ll r \ll R$$

Higher-dimensional black rings vs. MP black holes: $D \geq 6$

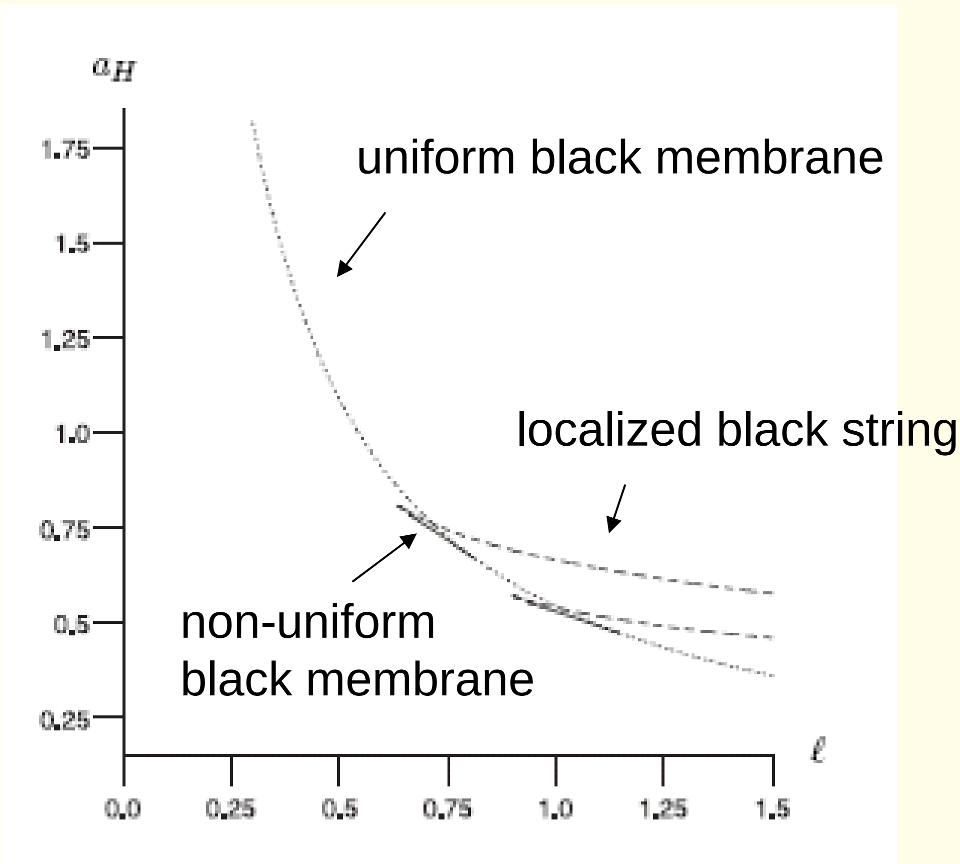


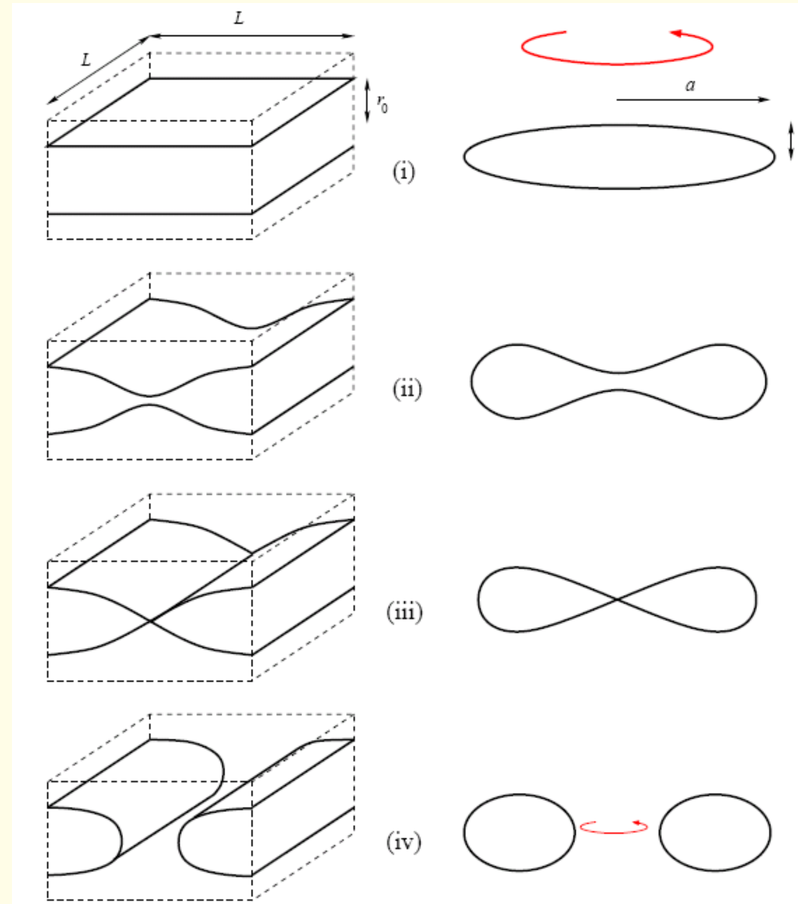
onset of membrane-like behavior of MP BH

⇒ black rings dominate entropically in ultraspinning regime

Black membrane/rotating black hole correspondence

- import knowledge of phase of
  KK black holes on $\mathbb{T}^2$

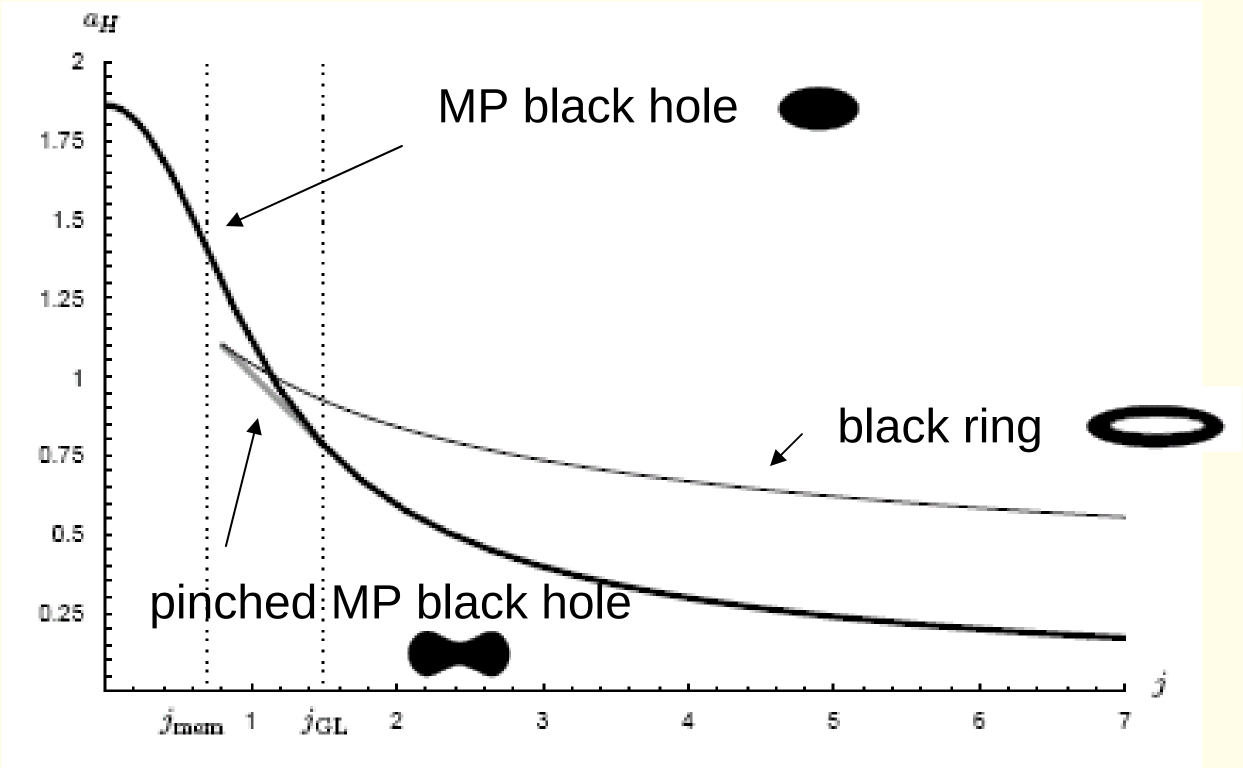




Towards completing the phase diagram

◀ based on analogy with phase diagram for KK BHs on torus: extrapolate to $j = \mathcal{O}(1)$ regime

- proposal for phase diagram of stationary BHs (one angular momentum) in asymptotically flat space: main sequence = MP BH, pinched MP BH, black ring (uniform, non-uniform, localized)

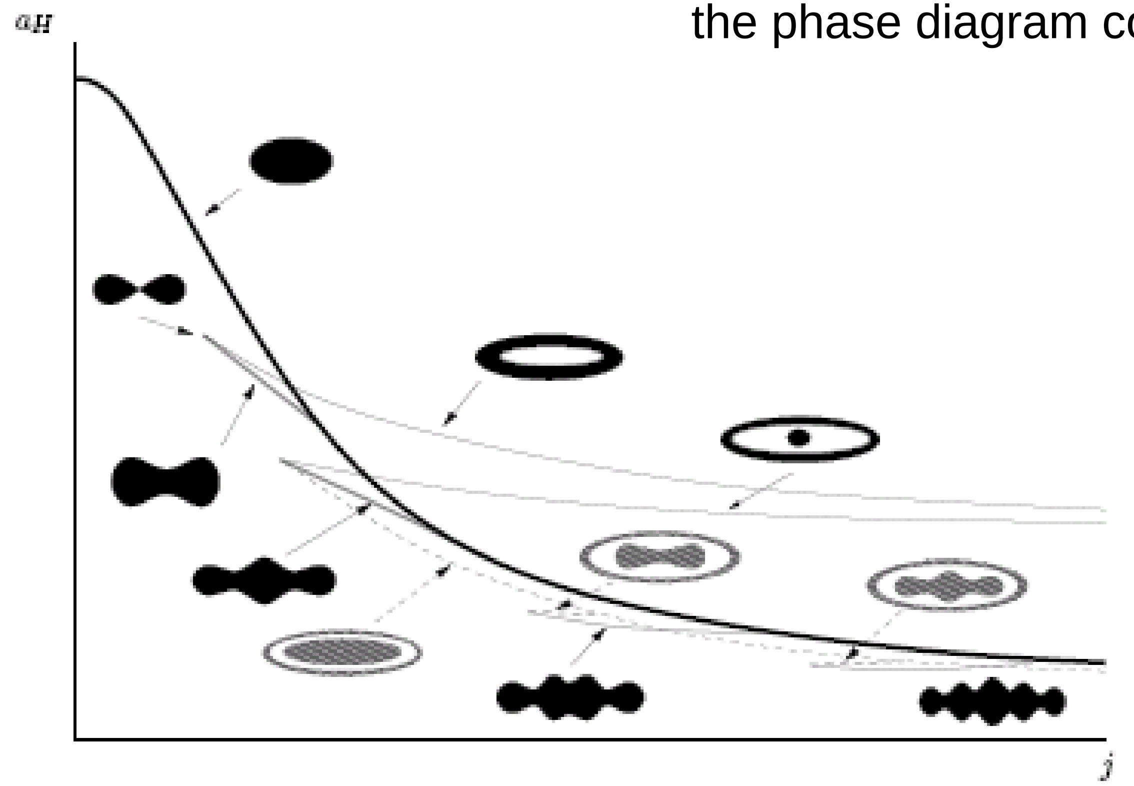


Black saturns and multi-pinches

most likely features

- main sequence: BH with pinch at rotation axis meets black ring phase
- infinite sequence of pinched BHs emanating from BH curve (from copies of the GL zero mode)
- upper black Saturn curve + merger to circular pinch

less compelling arguments for: pancaked + pinched black Saturns (but admit a simple and natural way for completing the phase diagram consistent with available info)



Blackfolds: a new approach to higher-dimensional BHs

Crucial feature of BHs in more than four dimensions:
- BH horizons can possess two (or more) different length scales

→ Origin of rich landscape of higher-dimensional black holes

Recently: effective theory has been contructed that captures the long-distance physics when scales are widely separated

Blackfold approach (2nd lecture)

► based on curving thin black branes on compact submanifolds of spacetime

⇒ dynamical constraints on possible horizon topologies

(global properties important: embedding of different topologies)