Continuum reduction in large N gauge theory

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QCD:

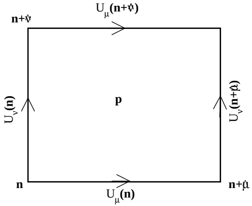
- Theory of strong interactions.
- Non-Abelian gauge theory with SU(3) as the gauge group.
- No free parameters.
  - Lattice gauge action has one bare parameter, namely, the coupling constant, g.
  - Quarks are massless.
  - The bare coupling constant, $g \to 0$, in the continuum limit and all dimensionless quantities (like ratios of masses) are uniquely determined.

Large N QCD:

- SU(3) $\rightarrow$ SU(N) with $\lambda = g^2 N$ fixed as $N \rightarrow \infty$ and $g \rightarrow 0$.
- $\lambda \rightarrow 0$ is the continuum limit.
- 't Hooft limit: Number of quark flavors, N_f, is fixed and finite.
- N^2 gauge degrees of freedom.
- NN_f quark degrees of freedom.
- Gauge fields dominate the dynamics.
- Quarks provide no back reaction.
- Quantum Yang-Mills theory with quark propagation in a gauge kackground.
- d=2: 't Hooft model – Serves as a good starting point to understand d=3 and d=4.

Wilson gauge action:

- $Z = \int [dU] e^{S(U)}$.
- $S(U) = 2bN \sum_p \mathrm{Re}\, U_p$; $b = \frac{1}{\lambda}$.
- $U_p = \mathrm{Tr}\, U_\mu(\mathbf{n}) U_\nu(\mathbf{n}+\hat{\mu}) U_\mu^\dagger(\mathbf{n}+\hat{\nu}) U_\nu^\dagger(\mathbf{n})$.
- $\mathbf{n}$ is a point on a d-dimensional periodic lattice.
- Gauge fields obey periodic boundary conditions on all directions.



Symmetries:

The action has two symmetries:

- Local gauge symmetry:

$$U_\mu(\mathbf{n}) \to g(\mathbf{n}) U_\mu(\mathbf{n}) g^\dagger(\mathbf{n}+\hat{\mu}).$$

- Global Z_N^d ($\to U^d(1)$ as $N \to \infty$) symmetry:

$$U_\mu(n_1, \cdots, n_{\mu-1}, L_\mu, n_{\mu+1}, \cdots, n_d)$$

$$\to$$

$$e^{i\frac{2\pi k}{N}} U_\mu(n_1, \cdots, n_{\mu-1}, L_\mu, n_{\mu+1}, \cdots, n_d);$$

$k = 0, \cdots, N-1;\ \mu = 1, \cdots, d.$

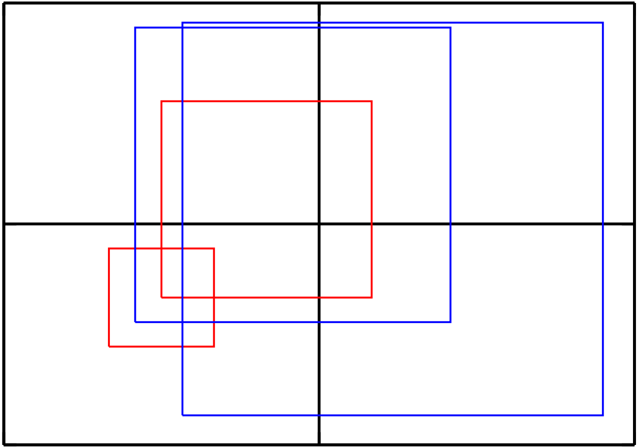
Eguchi-Kawai reduction:

If the global Z_N^d symmetries are not spontaneously broken on a L^d lattice for a given b, then physical observables do not depend on L.

- Write down the Schwinger-Dyson equations for Wilson loops on an infinite lattice – These are equations that relate expectation values of different Wilson loops.
- Use folded Wilson loops to write down the Schwinger-Dyson equations for Wilson loops on L^d lattice.
- Show that the equations are the same if the Z_N^d symmetries are not broken.

What is folding?

Wilson loops with (blue) and without (red) folding



The plaquette operator:

The eigenvalues of the plaquette operator U_p, $e^{i\theta_p^j}$, $j = 1, \cdots, N$ are gauge invariant with $-\pi \leq \theta_p^j < \pi$ for all j.

- We expect all eigenvalues, θ_p^j, to be close to zero at weak coupling (large b).
- We expect the eigenvalues, θ_p^j, to completely cover the range $[-\pi, \pi]$ at strong coupling (small b).
- The eigenvalue distribution, $\rho(\theta_p; b)$, for $-\pi < \theta_p \leq \pi$ exhibits behavior as a function of b in the large N limit.
- The eigenvalue distribution has a gap for $b > b_c$ (support is in $[-\theta(b), \theta(b)]$ with $\theta(b) < \pi$) and it does not have a gap for $b < b_c$.

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$d = 2, 3, 4$:

- The transition in the plaquette operator is a unphysical transition in the lattice theory – The continuum theory is always in the phase with a gap.
- It facilitates the lattice realization of gauge field topology – The eigenvalue distribution of the interpolating field between two lattice gauge fields with two different topological charge, will not satisfy the condition of the gap for some value of the interpolating parameter.
- It is known as the Gross-Witten transition in $d = 2$ and occurs at $b = 0.5$. The analytical calculation shows that tt is a third order transition.
- Numerical calculations in $d = 3$ suggest that it is possibly a third order transition and it occurs aroung $b \approx 0.43$.
- It is first order transition in $d = 4$ and it occurs around $b \approx 0.36$.

Large N QCD in two dimensions

- $\sqrt{b}$ can be used to set the scale on the lattice.
- $l_{1,2} = \frac{L_{1,2}}{\sqrt{b}}$ kept fixed as $L_{1,2}$ and b are taken to ∞.
- The two U(1) symmetries remain unbroken for all values of b and $L_{1,2}$.
- The problem reduces to a single site lattice with U_1 and U_2 being the two SU(N) degrees of freedom.
- Set $b > 0.5$ to be in the continuum side of the Gross-Witten transition.
- The continumm theory only exists in the confined phase – There is no deconfiment transition in two dimensional large N QCD.

Wilson loops in the continuum phase

- The rectangular folded Wilson loop operator of size $n \times m$ is given by $W(n,m) = U_1^n U_2^m \left(U_2^m U_1^n\right)^\dagger$.
- Let $t = \frac{4\pi nm}{b}$ be the parameter that characterizes the dimensional area. Consider a continuum Wilson loop of a fixed area by taking $b \to \infty$, $nm \to \infty$ while keeping t fixed.
- Area law is exact: $\mathrm{Tr}W(t) = e^{-\frac{t}{2}}$.
- Let $f(z,t) = \sum_{n=1}^{\infty} \frac{\langle \mathrm{Tr}W^n(t)\rangle}{z^n}$ be the generating function for $\langle \mathrm{Tr}W^n(t)\rangle$ where z is a complex variable.

$f(z,t)$:

- $f(z,t)$ satisfies

$$zf(z,t) = (1 + f(z,t))e^{-t\left(f(z,t)+\frac{1}{2}\right)}$$

- The expectation value of powers of Wilson loops are

$$\langle \mathrm{Tr} W^n(t) \rangle = \frac{1}{n} L_{n-1}^{(1)}(nt) e^{-\frac{nt}{2}}$$

- The expectation value of the distribution of the eigenvalues, $e^{i\theta}$, of W is given by

$$\rho(\theta, t) = -\frac{1}{\pi} \mathrm{Re} \left(f(e^{i\theta}, t) + \frac{1}{2} \right)$$

Critical behavior of Wilson loops as a function of area

- The expectation value of arbitrary powers of Wilson loops, $\langle \mathrm{Tr} W^n(t) \rangle$, are analytic functions of t.
- The eigenvalue distribution, $\rho(\theta, t)$, exhibits non-analytic behavior as a function of t:
- $t = 4$ is critical: Distribution has a gap for $t < 4$ and does not have a gap for $t > 4$.
- This transition in Wilson loops as a function of area from weak coupling ($t < 4$) to strong coupling ($t > 4$) is the **Durhuus-Olesen** transition.

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Double scaling limit of the Durhuus-Olesen transition

- Let
$$Q_N(e^y, a) = \langle \mathrm{det}(\mathrm{e}^{\mathrm{y}} + \mathrm{W}) \rangle \,; \quad a = \frac{4-t}{4t}$$
- The double scaling limit is defined by
$$y = \left[\frac{4}{3N^3}\right]^{\frac{1}{4}} \xi; \quad a = \frac{1}{4\sqrt{3N}}\alpha$$
$y, a \to 0$ and $N \to \infty$, such that ξ and α are fixed.
- $Q_N(e^y, a)$ is proportional to
$$f(\xi, \alpha) = \int_{-\infty}^{\infty} du e^{-u^4 - \alpha u^2 + \xi u}$$
in the double scaling limit.

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Fermions:

- Fermions are naturally quenched in the large N limit.
- The background gauge field is a constant gauge field on the infinite lattice, U_1 and U_2.
- Consider any lattice fermions operator in momentum space.
- The fermion operator on the single site lattice coupled to $U_1 e^{ip_1}$ and $U_2 e^{ip_2}$ with (p_1, p_2) being the fermion momentum.
- Strictly speaking, p_1 and p_2 should take only values of the form $\frac{2\pi k_1}{N}$ and $\frac{2\pi k_2}{N}$ with $0 \le k_1, k_2 \le N-1$ and becoming continuous as $N \to \infty$.

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Chiral condensate:

Let $G^{ij}_{\alpha\beta}(U_1, U_2; p_1, p_2; m_q)$ be the quark propagator computed on the lattice using some fermion discretization.

- i, j are the color indices.
- α, β are the spin indices.
- U_1, U_2 is the gauge field background on the single site lattice at the lattice coupling, b.
- (p_1, p_2) is the quark momentum and m_q is the bare quark mass.

$$\chi(b, m_q) = \langle \frac{1}{N} \sum_{i=1}^{N} \sum_{\alpha=1}^{2} G^{ii}_{\alpha\alpha}(U_1, U_2; p_1, p_2; m_q) \rangle$$

$$\langle O \rangle = \frac{\int [dU_1][dU_2] e^{2bN \mathrm{Re}\ \mathrm{Tr}\ \left[U_1 U_2 U_1^\dagger U_2^\dagger\right]} O}{\int [dU_1][dU_2] e^{2bN \mathrm{Re}\ \mathrm{Tr}\ \left[U_1 U_2 U_1^\dagger U_2^\dagger\right]}}$$

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$\chi(b, m_q)$:

$\chi(b, m_q)$ does not depend on p_1 or p_2 – This is because the Z_N^2 symmetries are not broken.
The chiral condensate at a fixed lattice coupling is

$$\Sigma(b) = \lim_{m_q \to 0} \lim_{N \to \infty} \chi(b, m_q).$$

Chiral symmetry breaking in the continuum implies

$$\sigma = \lim_{b \to \infty} \Sigma(b) b$$

is finite and non-zero.

Mesons:

Meson propagator:

$$\begin{aligned}
G_\Gamma(U_1, U_2; p_1, p_2, q_1, q_2; m_1, m_2) \\
= \sum_{i,j=1}^{N} \sum_{\alpha,\beta,\gamma,\delta=1}^{2} \quad & G^{ij}_{\alpha\beta}(U_1, U_2; q_1, q_2; m_1)\Gamma_{\beta\gamma} \\
& G^{ji}_{\gamma\delta}(U_1, U_2; p_1, p_2; m_2)\Gamma^\dagger_{\delta\alpha}
\end{aligned}$$

$$\langle G_\Gamma(U_1, U_2; p_1, p_2, q_1, q_2; m_1, m_2)\rangle = \bar{G}_\Gamma((\mathbf{p} - \mathbf{q})^2; m_1, m_2)$$

- No disconnected diagrams – Quarks have different flavors.
- Result only depends upon the difference of the quark momenta since the Z_N^2 symmetries are not broken.

Large N QCD in three dimensions

- Set $b > 0.43$ to be in the continuum side of the bulk transition.
- The tadpole improved coupling $b_I = b\langle \mathrm{Plaquette} \rangle$ can be used to set the scale.
- Consider a symmetric three torus with the physical size $l = \frac{L}{b_I}$ kept fixed as L and b_I are taken to ∞.
- The continuum theory exists in many phases:
  - 0c: $l_1 < l$ – None of the three U(1) symmetries are broken – Confined phase.
  - 1c: $l_2 < l < l_1$ – One of the three U(1) symmetries are broken – Deconfined phase.
  - 2c: $l_3 < l < l_2$ – Two of the three U(1) symmetries are broken – QCD in a small box at low temperatures.
  - 3c: $l < l_3$ – All three U(1) symmetries are broken – QCD in a small box at high temperatures.

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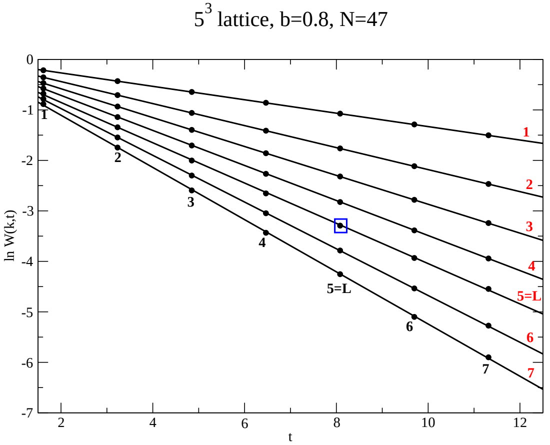
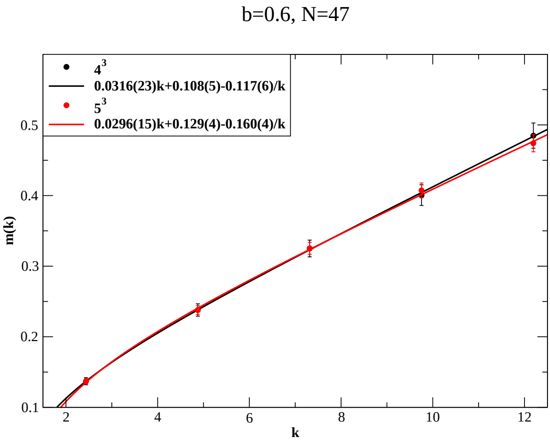
0c phase – Computation of the string tension:

- Consider a L^3 lattice at a coupling b such that $\frac{L}{b_l} > l_1$.
- Consider unfolded and folded rectangular Wilson loops, $W(K, T)$.
  - $k = \frac{K}{b_l}$ is the length of the string.
  - $t = \frac{T}{b_l}$ is the separation between strings.
  - Smear in the plane perpendicular to T to have good overlap with the ground state.
- Fit $\ln W(k, t) = -a - m(k)t$ to extract the energy of the string, $m(k)$, as a function of its length, k.
- Fit $m(k) = \alpha k + \beta + \frac{\gamma}{k}$.
  - γ: This should approach a constant in the continuum limit and describes the short distance behavior of the string.
  - $\alpha = \sigma b_l^2$: This should approach a constant in the continuum limit and σ is the string tension.
  - β: This is expected to logarithmically diverge in the continuum limit. Links in the T direction have not been smeared and the perimeter divergence has not been tamed.

Extraction of $m(k)$:



Extraction of the string tension:



1c phase – Spatial string tension

- The string tension does not depend on the temperature in the confined phase due to continuum reduction.
- Since there are two directions where the U(1) symmetry is not broken in the 1c phase, there will be a string tension associated with loops in the unbroken (spatial) directions.
- The spatial string tension will depend on the temperature in the deconfined phase.

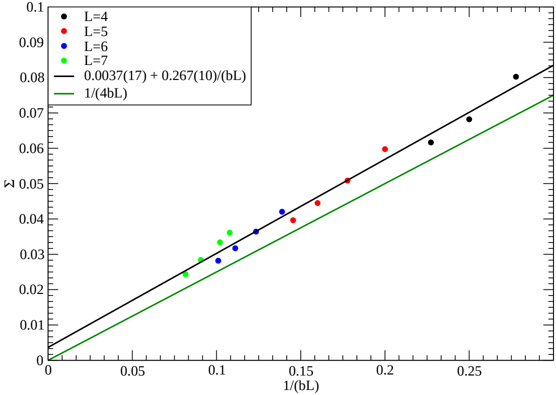
Question: How does the spatial string depend on the temperature in the deconfined phase?

Computation of the spatial string tension

- Pick the coupling b on a L^3 lattice such that one is in the 1c phase. $\frac{b_I}{L}$ sets the temperature in the deconfined phase.
- Let the "1" direction be broken.
- Consider the spatial Wilson loops, $W(L_2, L_3)$ and proceed in the same manner as in 0c to extract the string tension:
  - Fit $\ln W(L_2, L_3) = -A - V(L_2)L_3$ for a fixed L_2 to extract the energy, $V(L_2)$, for a string of length L_2.
  - Fit $V(L_2) = \Sigma L_2 + C_0 + \frac{C_1}{L_2}$ to extract the string tension, Σ.
- $b_2 = bL$ is the effective two dimensional coupling.
- If the spatial string tension, Σ in the deconfined phase is equal to the string tension in the two dimensional theory, then

$$\Sigma = \frac{1}{4b_2} = \frac{1}{4bL}$$

Spatial string tension:



Large N QCD in four dimensions

- The bulk transition at $b = 0.36$ is strongly first order.
- One can approach from the continuum side and stay in a metastable 0c phase as low as $b = 0.348$.
- The tadpole improved coupling, b_I, along with two-loop perturbation theory can be used to set the scale: Keep

$$l = L(b)\left[\frac{48\pi^2 b_I}{11}\right]^{\frac{51}{121}} e^{-\frac{24\pi^2 b_I}{11}}$$

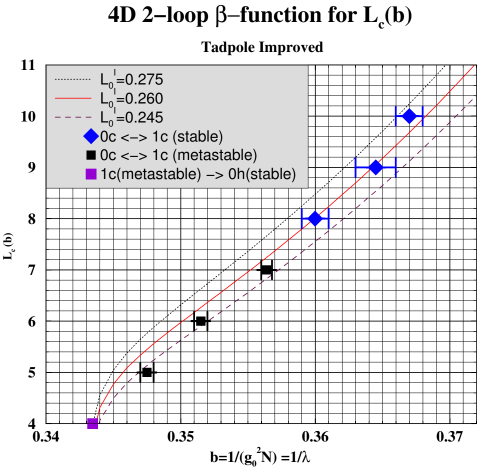
fixed with $L(b) \to \infty$ as $b \to \infty$.

Extraction of the deconfining temperature:

Fix the lattice size L and find the coupling $b_c(L)$ where one or more of the $U^d(1)$ symmetries break when $N_c \to \infty$.

$$L_c(b) = (0.260 \pm 0.015) \left[\frac{11}{48\pi^2 b_I} \right]^{\frac{51}{121}} e^{\frac{24\pi^2 b_I}{11}}$$

Scaling behavior:



Chiral condensate in finite volume:

- Compute the spectrum of the Dirac operator on a L^4 torus at a coupling b such that $L > L_c(b)$.
- The level spacing at the low end of the spectrum has to scale like $\frac{1}{N}$ if chiral symmetry is spontaneously broken.
- The size of the Dirac operator scales with N at a fixed volume and chiral symmetry is broken by level repulsion: We are essentially look at N eigenvalues of a $N \times N$ random matrix.

Strategy for computing the chiral condensate

- Pick some L and choose $b < b_c(L)$.
- No finite volume effects.
- Keep b close to $b_c(L)$ to minimize finite spacing effects.
- Use the overlap Dirac operator that respects exact chiral symmetry on the lattice. Let $A(\mu)$ denote the massive overlap Dirac operator with μ being the bare mass on the lattice.
-
$$\Sigma = \lim_{\mu \to 0} \lim_{N \to \infty} \frac{1}{L^4 N} \langle \mathrm{Tr} A^{-1}(\mu) \rangle_{N,L}$$
    - The first step is to the take the large N limit.
    - The second step is to take the massless limit.
- Absence of finite volume effects means that Σ does not depend on L at the given gauge coupling.
- Continuum limit is obtained by increasing b and suitably changing L such that one is always in the 0c phase.

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Quenched pathologies at finite N

- Quenched theory at finite N is an ill-defined field theory.
  - Anomalies are not taken into account.
  - Chiral condensate suffers from unphysical divergences: The chiral condensate $\bar{\Sigma}(L)$ defined in finite volume using chiral RMT diverges as L goes to infinity.
- Such pathologies are suppressed by $\frac{1}{N}$ – fermions are naturally quenched in the large N limit.
- It is important to take the large N limit before one takes the large volume limit if one wants to work with the quenched theory on the lattice.

Chiral Random Matrix Theory:

- Let $\pm i\lambda_i$ with $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_{2NL^4}$ be the eigenvalues of $A(0)$.
- Consider the scales variables $z_k = \lambda_k \Sigma N L^4$.
- Extensive work in the area of chiral RMT has shown that the probability distributions, $p(z_k)$, are universal functions as $L \to \infty$ at fixed N.
- Explicit formula are available for $p(z_k)$.
- Chiral RMT should work even better for fixed L and N goes to infinity since we have N^2 degrees of freedom in the underlying field theory and we are asking for the behavior of only N observables.
- Compute the two lowest eigenvalues λ_1 and λ_2.
- Check if the ratio $r = \frac{\lambda_1}{\lambda_2}$ obeys the universal function dictated by chiral RMT.
- Find the common Σ that converts λ_1 and λ_2 into z_1 and z_2.

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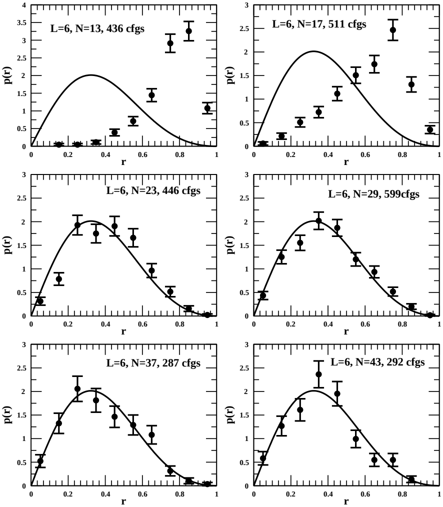
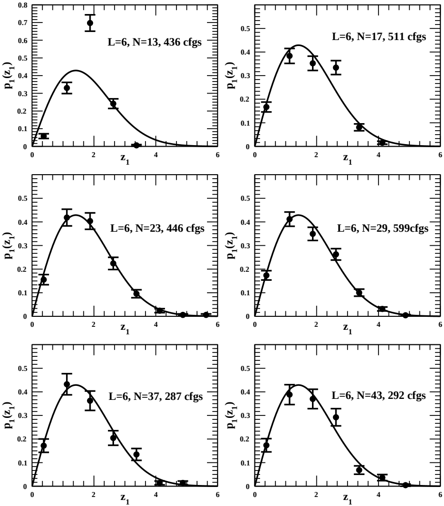
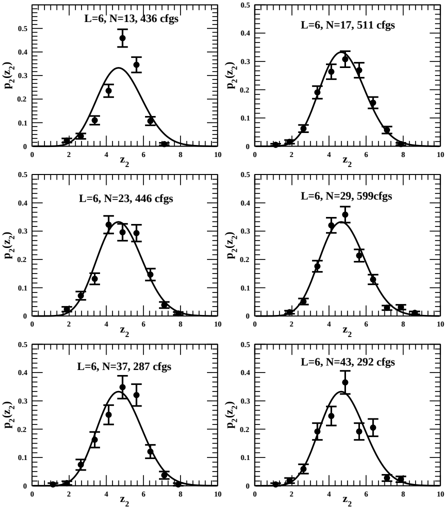
Behavior of the ratio:



Lowest eigenvalue:



Second lowest eigenvalue:

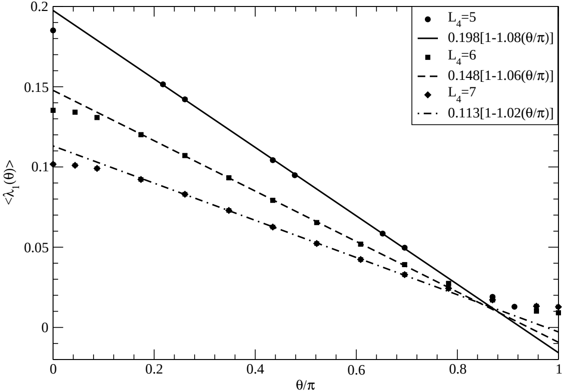


Main result for the chiral condensate

- The critical size l_c is close to $1/T_c$.
- If we use $T_c \approx 0.6\sqrt{\sigma} \approx 264\mathrm{MeV}$ we get $\Sigma_{\mathrm{R,cont}}^{1/3} \approx 155\mathrm{MeV}$.
- Using perturbative tadpole improved esimates for Z_S in the $\overline{MS}$ scheme, we get $\frac{1}{N}\langle\bar{\psi}\psi\rangle^{\overline{MS}}(2\mathrm{GeV}) \approx (174\mathrm{MeV})^3$
- Assuming $N = 3$ is large enough, we get $\langle\bar{\psi}\psi\rangle^{\overline{MS}}(2\mathrm{GeV}) \approx (251\mathrm{MeV})^3$for SU(3).

Restoration of chiral symmetry in the 1c phase

- Fermions do matter in the 1c phase even in the 't Hooft limit.
- Fermion determinant will depend on the momentum in the broken direction.
- In other words, boundary conditions in the temperature direction matters.
- Let θ be the phase associated with the $U(1)$ that defines the boundary condition with respect to the phase of the Polyakov loop in the broken direction. Let $\theta = 0$ define anti-periodic boundary conditions.
- The fermion determinant will depend on θ and one expects that fermions will pick $\theta = 0$.
- Consider the lowest eigenvalue of the overlap Dirac operator as a measure of the fermion determinant and look at this as a function of θ.
- The data shows a gap in the spectrum for all θ as long as $T > T_c$. This shows strong interaction in the color space.
- The gap is the biggest for $\theta = 0$.



L4=5
0.198[1-1.08(θ/π)]
L4=6
0.148[1-1.06(θ/π)]
L4=7
0.113[1-1.02(θ/π)]
<λ1(θ)>
θ/π
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1

Gap in the 1c phase

- Fermions are not quenched in the 1c phase but all we have to do is fix the boundary conditions to be anti-periodic with respect to the Polyakov loop.
- Fermions do not provide any other form of back reaction.
- Define the gap, G, to the average of the lowest eigenvalue of the overlap Dirac operator.
- Work on a $L^3 \times L_4$ lattice for several couplings b such that they are all in the 1c phase.
- Use $L_c(b)$ to define a dimensionless gap, $g = GL_c(b)$, and a dimensionless temperature, $t = L_4 L_c(b)$.
- A plot of g vs t shows that the data fall on a universal curve for small lattice spacing.
- The data fits $1.76\sqrt{t - 0.93}$ for $1 < t < 1.5$.
- There is clear numerical evidence for a first order phase transition in the fermionic sector.
- If we could supercool in the 1c phase below $t = 1$, we would find $T_c^{\mathrm{chiral}} < T_c^{\mathrm{deconfined}}$

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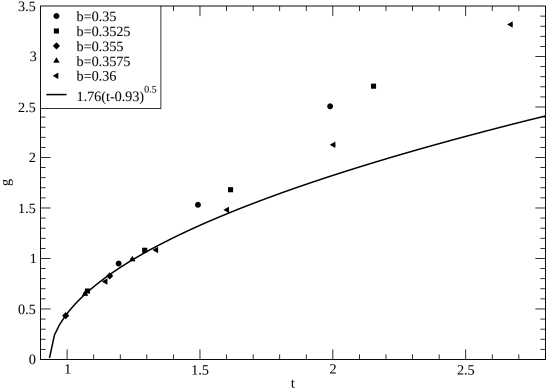
b=0.35
b=0.3525
b=0.355
b=0.3575
b=0.36
1.76(t-0.93)^0.5
g
t
3.5
3
2.5
2
1.5
1
0.5
0
1
1.5
2
2.5

Pion propagator

- Propagator is evaluated for several momenta and quark masses with the same gauge background.
- Both the scalar and the pseudoscalar meson propagator can be represented as a sum over infinite number of poles since all mesons are stable in the large N limit.
- The higher masses in the scalar and pseudoscalar meson sector are close to each other.
- We take the difference of the pseudoscalar and scalar mesons to isolate the contribution from the lowest pseudoscalar.
- We use smeared source and sink to further enhance the overlap on to the lowest pseudoscalar.
- The resulting propagator as a function of momentum at a fixed quark mass is fitted to $\frac{r_0}{p^2+m_\pi^2}$.

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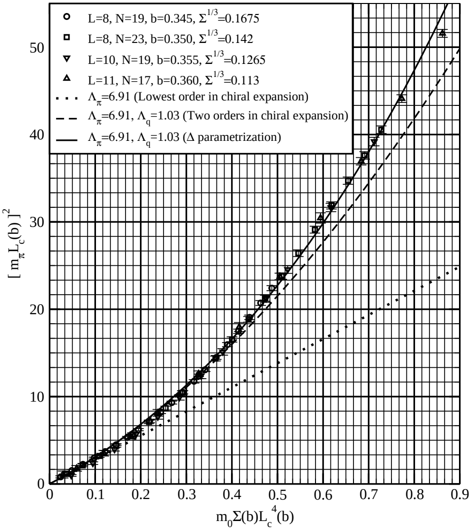
Extraction of pion decay constant

- The result of the numerical analysis is the pion mass, m_π as a function of the quark mass m_o.
- Quark mass is not physical and we convert it to a physical quantity by forming $m_o\Sigma$ where Σ is the chiral condensate on the lattice.
- In order to take the continuum limit, we form dimensionless quantities, $m_\pi L_c(b)$ and $m_0 \Sigma L_c^4(b)$. using the critical size l_c on the lattice.
- The parameter Λ_π gives $f_\pi = \frac{1}{\sqrt{2\Lambda_\pi l_c}}$. Using, $1/l_c = T_c = 264$ MeV, we get $f_\pi = 71$ MeV. This translates to $f_\pi = 123$ MeV for SU(3).



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SU(N) X SU(N) Principal Chiral Model

- Similar to four dimensional SU(N) gauge theory in many respects.
- $$S = \frac{N}{T} \int d^2x Tr \partial_\mu g(x) \partial_\mu g^\dagger(x)$$
  $g(x) \in$ SU(N).
- The global symmetry group $\text{SU(N)}_L \times \text{SU(N)}_R$ reduces down to a single SU(N) "diagonal subgroup" if we make a translation breaking "gauge choice", $g(0) = 1$.
- Model is asymptotically free and there are $N - 1$ particle states with masses
  $$M_R = M \frac{\sin(\frac{R\pi}{N})}{\sin(\frac{\pi}{N})}, \quad 1 \leq R \leq N - 1.$$
  The states corresponding to the R-th mass are a multiplet transforming as an R component antisymmetric tensor of the diagonal symmetry group.

Connection to multiplicative matrix model

- $W = g(0)g^\dagger(x)$ plays the role of Wilson loop with the separation x playing the role of area.
- One expects
$$G_R(x) = \langle \chi_R(g(0)g^\dagger(x))\rangle \sim C_R \binom{N}{R} e^{-M_R|x|}$$
where χ_R is the trace in the R-antisymmetric representation.
- Comparison with the multiplicative matrix model suggests that $M|x|$ plays the role of the dimensionless area.
- Numerical measurement of the correlation length using the lattice action
$$S_L = -2Nb \sum_{x,\mu} \Re Tr[g(x)g^\dagger(x+\mu)]$$
and
$$\xi_G^2 = \frac{1}{4} \frac{\sum_x x^2 G_1(x)}{\sum_x G_1(x)}$$
yields $M\xi_G = 0.991(1)$.

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Setting the scale

- ξ_G will be used to set the scale and it is well described by

$$\xi_G = 0.991 \left[\frac{e^{\frac{2-\pi}{4}}}{16\pi}\right] \sqrt{E}\ \exp\left(\frac{\pi}{E}\right)$$

in the range $11 \leq \xi_G \leq 20$ with

$$\begin{aligned} E &= 1 - \frac{1}{N}\Re\langle Tr[g(0)g^{\dagger}(\hat{1})]\rangle \\ &= \frac{1}{8b} + \frac{1}{256b^2} + \frac{0.000545}{b^3} - \frac{0.00095}{b^4} + \frac{0.00043}{b^5} \end{aligned}$$

The above equations will be used to find a b for a given ξ.

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Smeared SU(N) matrices

One needs to smear to defined well defined operators.

- Start with $g(x) \equiv g_0(x)$.
- One smearing step takes us from $g_t(x)$ to $g_{t+1}(x)$.
- Define $Z_{t+1}(x)$ by:

$$Z_{t+1}(x) = \sum_{\pm\mu} [g_t^\dagger(x) g_t(x+\mu) - 1]$$

- Construct antihermitian traceless $SU(N)$ matrices $A_{t+1}(x)$

$$A_{t+1}(x) = Z_{t+1}(x) - Z_{t+1}^\dagger(x) - \frac{1}{N}\mathrm{Tr}(Z_{t+1}(x) - Z_{t+1}^\dagger(x)) \equiv -A_{t+1}^\dagger(x)$$

- Set

$$L_{t+1}(x) = \exp[f A_{t+1}(x)]$$

- $g_{t+1}(x)$ is defined in terms of $L_{t+1}(x)$ by:

$$g_{t+1}(x) = g_t(x) L_{t+1}(x)$$

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Numerical details

- We need $L/\xi_G > 7$ to minimize finite volume effects.
- Since we want $11 \leq \xi_G \leq 20$, we chose $L = 150$.
- We used a combination of Metropolis and over-relaxation at east site x for our updates. The full SU(N) group was explored.
- 200-250 passes of the whole lattices was sufficient to thermalize starting from $g(x) \equiv 1$.
- 50 passes were enough to equilibriate if ξ_G was increased in steps of 1.

Test of the universality hypothesis

The test of the universality hypothesis proceeds in the same manner as for three D large N gauge theory.
Given an N and a ξ, we find the the d_c the makes the Binder cumulant $\Omega(d_c, N) = 0.364739936$.
We look at d_c as a function of ξ for a given N. This gives us the continuum value of d_c/ξ for that N.
We then take the large N limit and it gives us

$$\frac{d_c}{\xi_G}|_{N=\infty} = 0.885(3)$$

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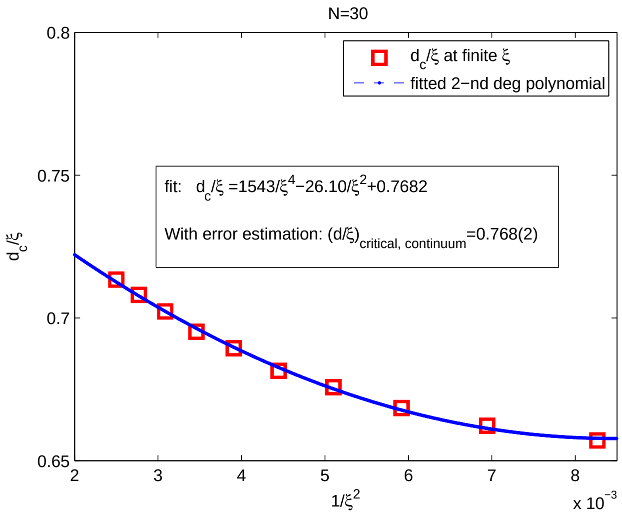
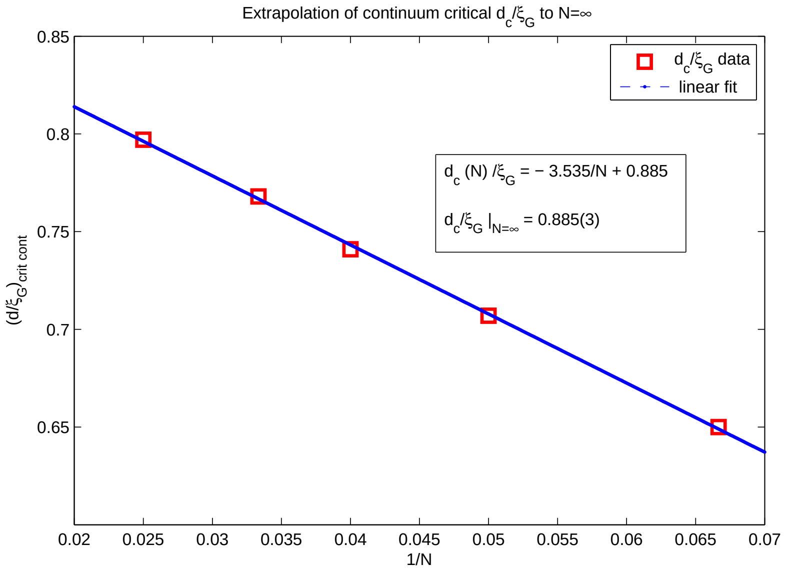


N=30
0.8
0.75
0.7
0.65
2
3
4
5
6
7
8
x 10^−3
1/ξ^2
d_c/ξ
d_c/ξ at finite ξ
fitted 2−nd deg polynomial
fit: d_c/ξ =1543/ξ^4−26.10/ξ^2+0.7682
With error estimation: (d/ξ)_critical, continuum=0.768(2)


Extrapolation of continuum critical d_c/ξ_G to N=∞
d_c/ξ_G data
linear fit
$d_c\,(N)\,/\xi_G = -\,3.535/N + 0.885$
$d_c/\xi_G\,|_{N=\infty} = 0.885(3)$
$(d/\xi_G)_{crit\ cont}$
1/N
0.85
0.8
0.75
0.7
0.65
0.02
0.025
0.03
0.035
0.04
0.045
0.05
0.055
0.06
0.065
0.07