Classical de Sitter Spacetime

- Maximally Symmetric Soln. to Einstein's Eqs. with a
  Positive Cosmological Constant (Vacuum Energy)

$$G_{ab} + \Lambda\, g_{ab} = 0$$

- Symmetry Group is *O(4,1)*: Hyperboloid of Revolution in D=5 flat spacetime

$$ds^2 = -dT^2 + dW^2 + dX^2 + dY^2 + dZ^2 \qquad \text{with fixed 'radius'}$$

$$-T^2 + W^2 + X^2 + Y^2 + Z^2 = H^{-2} \qquad H^2 = \Lambda/3$$

- Line Element in with closed S^3 spatial sections: globally complete

$$ds^2 = H^{-2} \sec^2\eta\, (-d\eta^2 + d\chi^2 + \sin^2\chi\, d\Omega^2)$$

- Line Element in FLRW form (flat spatial sections, proper time): inflation

$$ds^2 = -d\tau^2 + e^{2H\tau}(dx^2 + dy^2 + dz^2) \qquad \text{Scale Factor: } a(\tau) = e^{H\tau}$$

- Line Element in Static Coordinates: de Sitter's original form

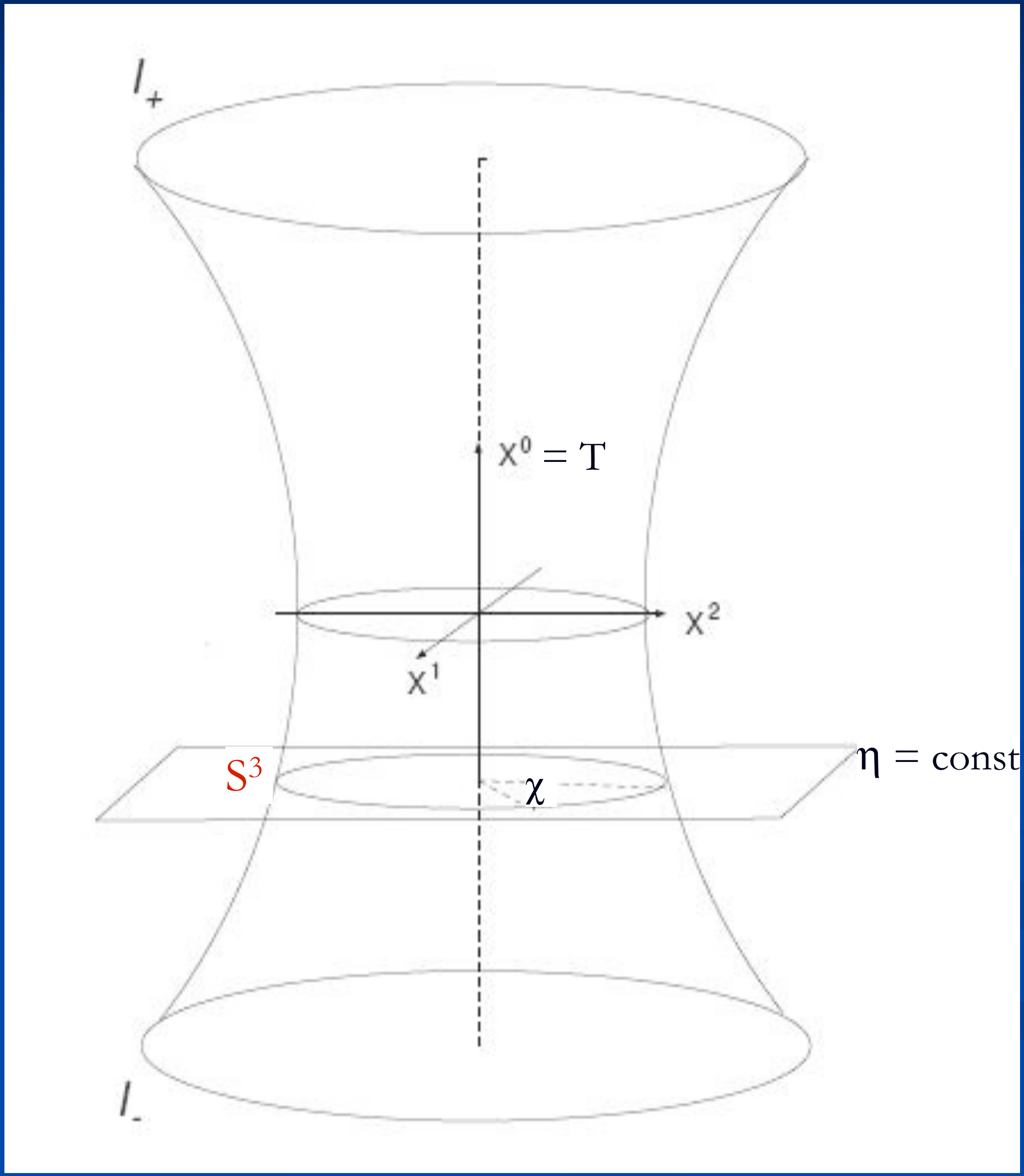
$$ds^2 = -(1 - H^2r^2)\, dt^2 + (1 - H^2r^2)^{-1} dr^2 + r^2\, d\Omega^2$$

$r_H = 1/H$ is the Hubble-de Sitter horizon scale

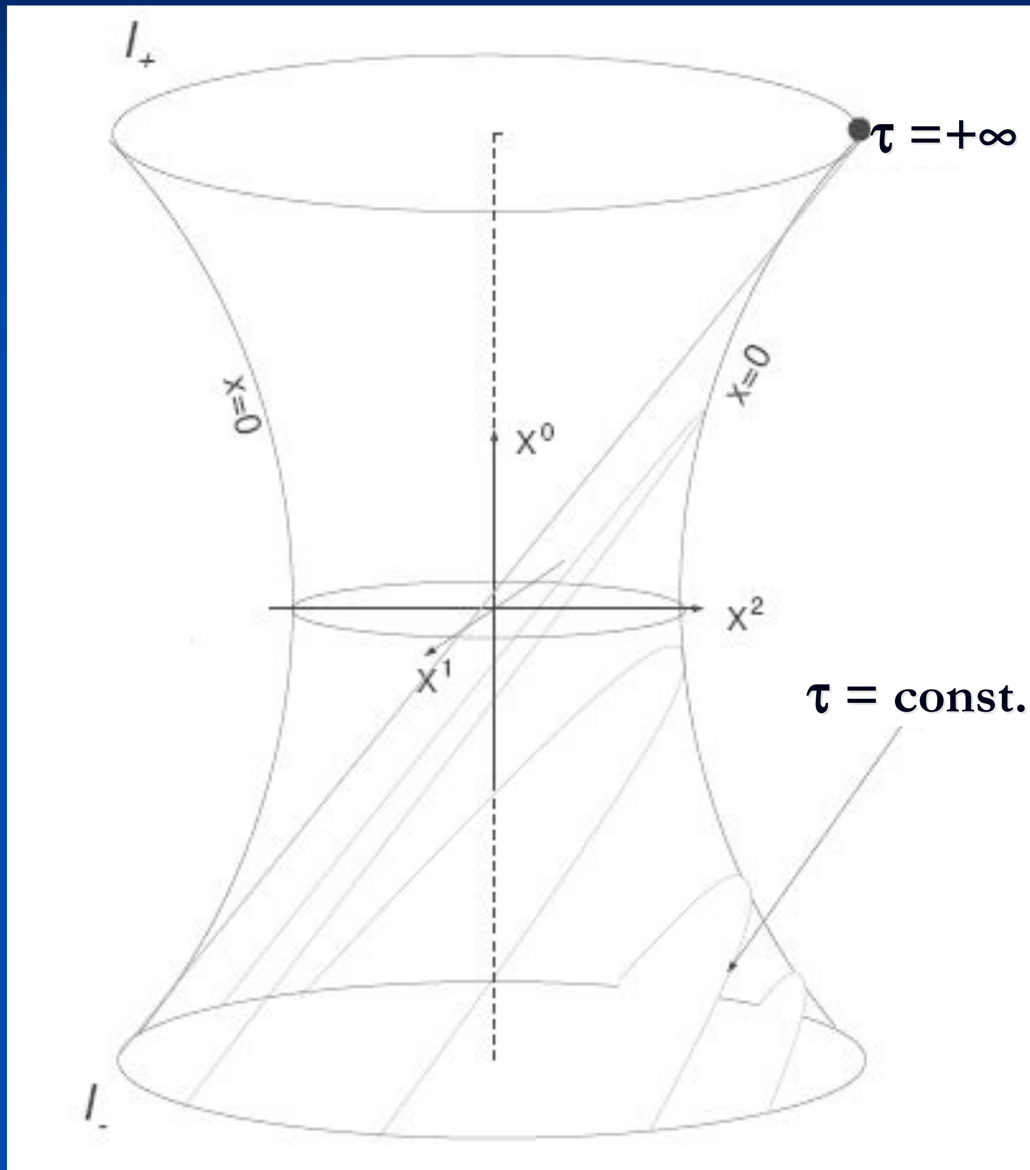
All the same de Sitter spacetime (or parts thereof) described in different coordinates

Classical de Sitter Hyperboloid

S^3 sections

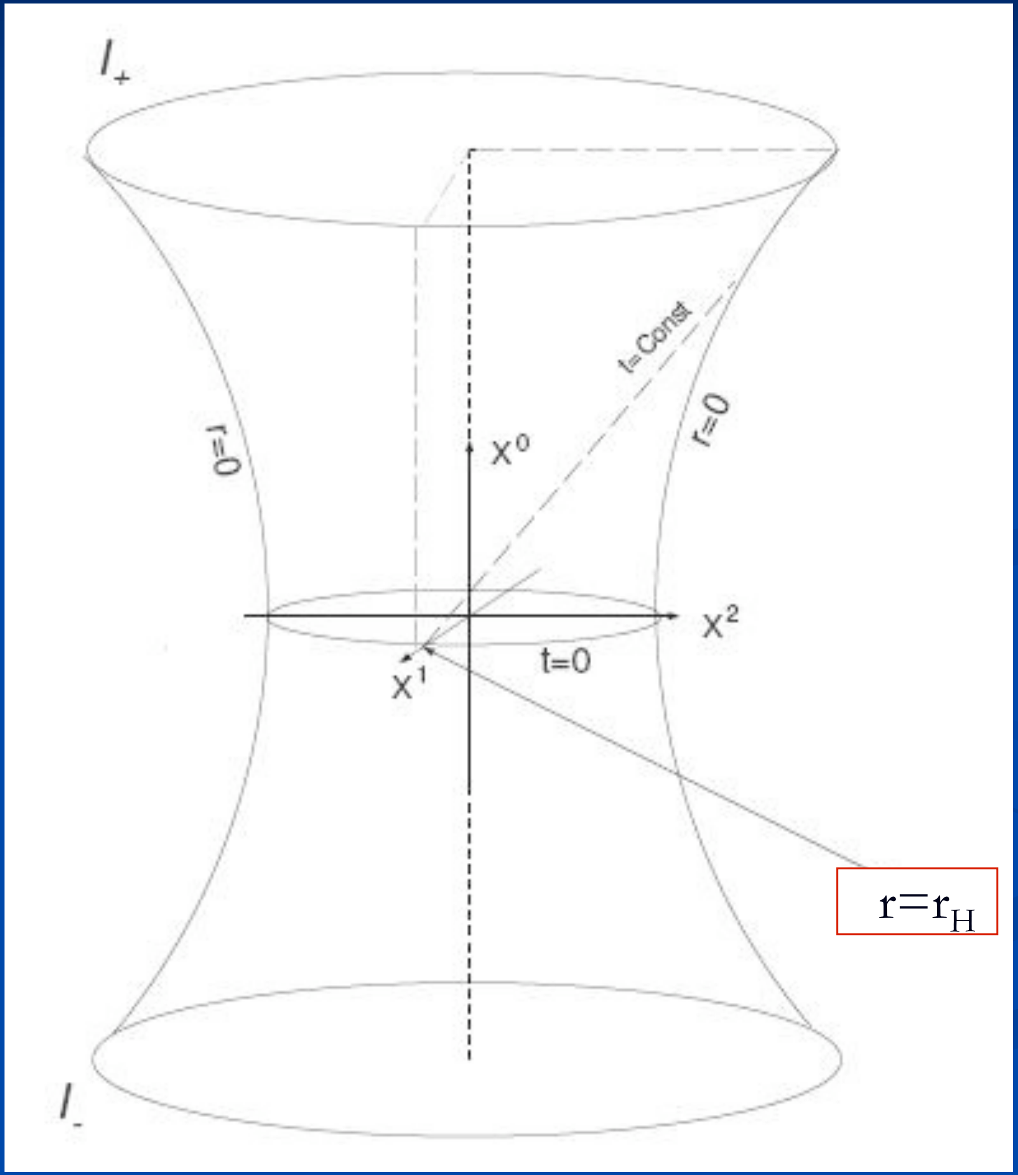


Classical de Sitter Hyperboloid



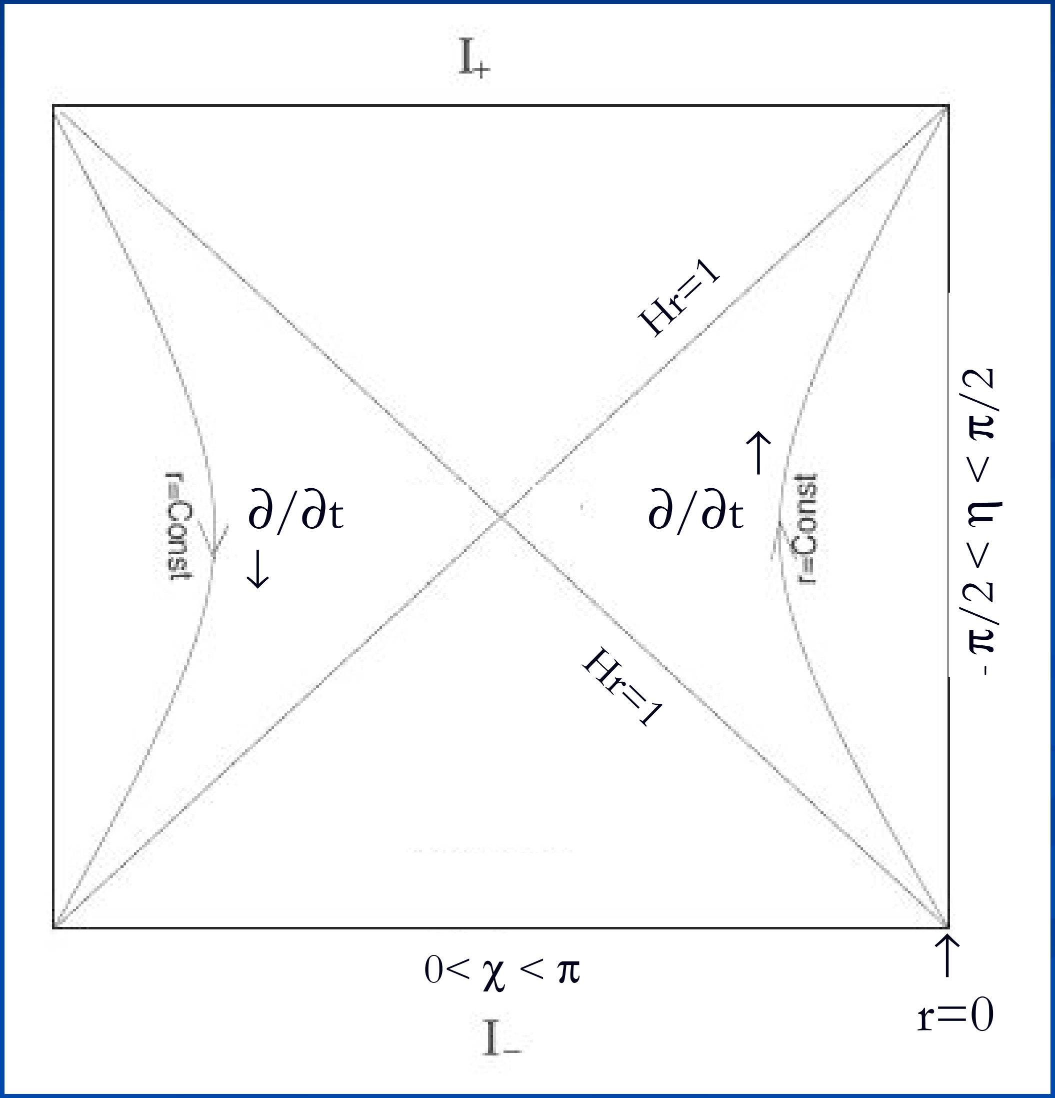
Flat sections

Classical de Sitter Hyperboloid



Static sections

Classical de Sitter Spacetime: Carter-Penrose Conformal Digram



Quantum Effects in de Sitter Space

- Quantum 'Vacuum' is non-trivial

  Spontaneous Particle Creation ('84)

  Decay Rate: $\Gamma \sim H^4 \exp(-m/T_H)$ $T_H = H/2\pi$

  for massive fields

- Compare to Schwinger Effect: 'Shorting' the vacuum

  $\Gamma \sim (eE)^2 \exp(-m^2/eE)$ $$\frac{dE}{dt} = -j$$

- Backreaction should decrease H $$\frac{dH}{dt} = -\frac{4\pi G}{c^2}(\rho + p)$$

- Maximally Symmetric *O(4,1)* Bunch-Davies State has

  exact time reversal symmetry -- thermodynamic equilibrium

  but negative heat capacity ('85)

  unstable to thermodynamic fluctuations

  (compare to black hole)

$$T_H = \frac{\hbar H}{2\pi k_B} \propto \left(\frac{c^5}{2GH}\right)^{-1} = E_H^{-1}$$

Quantum Effects in de Sitter Space

- Temperature Fluctuations lead to divergent stress tensor

  on the horizon: $\langle T^a{}_b \rangle \sim (T^4 - T_H{}^4)/(1-H^2r^2)^2$
- Infinite BlueShift (again)
- No *O(4,1)* Bunch-Davies Thermal State at all

  for massless, minimally coupled fields or gravitons
- Graviton Propagator grows logarithmically with distance

  No Cluster Decomposition, S-Matrix
- Global Symmetry Restoration: No Goldstone Bosons

  Similar to Massless Scalar Theory in D=2
- Non-trivial Infrared Properties
- Infrared Relevant Operator Missing in EFT of Gravity? (yes)

Effective Action of 4D Gravity

By inverting the eq. for σ the local W-Z action may be expressed as a difference of fully covariant but non-local actions,

$$\Gamma_{WZ}[\bar{g};\sigma] = S_{anom}[g] - S_{anom}[\bar{g}] \quad \text{and}$$

$$S_{eff}[g] = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,(R - 2\Lambda) + S_{anom}[g]$$

Consequences of Conformal Anomaly

- S_{anom} determined by general principles of covariance and QM, independently of any Planck scale physics.
- Additional term is relevant at large distances (scales as a marginally relevant operator under $\sigma \longrightarrow \sigma + \sigma_0$).
- Conformal factor, constrained in classical Einstein theory, contains new dynamical degrees of freedom.
- New conformally invariant phase of gravity in 4D
- Running of G^{-1} and Λ to zero in this new phase.
- Possible imprint on CMBR Spectrum and Statistics.

I. Antoniadis, P. O. Mazur, E. M., Phys. Rev. D 55 (1997) 4756, 4770; Phys. Rev. Lett. 79 (1997) 14

Implication: Vacuum Energy is Dynamical

Λ as Vacuum Energy of a Gravitational Bose-Einstein Condensate

- **The Conformal Factor of the metric $g_{ab} = e^{2\sigma}\bar{g}_{ab}$ is frozen by the classical Einstein's Eqs. $R = 4\Lambda$**
- **But the trace anomaly of massless quantum fields forces the scalar 'condensate' $\langle e^{2\sigma}\rangle$ to fluctuate** Auxiliary φ,ψ
- **This generates a well-defined additional term in the low energy action, and**
- **Describes a New Conformally Invariant Phase, Infrared Fixed Point of Gravity**
- **The quantum phase transition to this phase is characterized by 'melting' of the scalar condensate $\langle e^{2\sigma}\rangle$** Fluctuating φ,ψ

Λ_{eff} Dynamical, generated by SSB of Global Conformal Invariance $\sigma \rightarrow \sigma + \sigma_0$

Quantum Effects Near $r = R_S$

- Huge Vacuum Stresses for generic b. c. at horizon:

$$\langle T_t^t \rangle \sim \langle T_r^r \rangle \sim \left(1 - \frac{2GM}{r}\right)^{-n} \,,\; n = \begin{cases} 1 & \text{Unruh} \\ 2 & \text{Boulware} \end{cases}$$

- Gravitational effects of quantum matter become strong near $r = R_S$ and affect the geometry.
- Strong attractive self-interactions $\Rightarrow$ Condensation.
- If Quantum Correlations $\langle T_a^b(x)\, T_c^d(y) \ldots \rangle$ also grow when $x, y, \ldots$ approach the horizon $\Rightarrow$ Highly Entangled Quantum State.
- Possibility of Quantum Phase Transition to BEC-like phase near $r = R_S$.
- Critical region where Sound Speed = Light Speed:

$$c_S^2 = \frac{dp}{d\rho} = c^2$$

  Any Additional Increase in Pressure would violate Causality: Onset of Superluminal Modes is the Signature of a Relativistic Phase Transition.
- A Critical Surface Layer with $p = \rho$ is Necessary for Joining $p = -\rho$ Interior with Vacuum Exterior.

Bose-Einstein Condensation

- Bose-Einstein statistics imply any number of particles can occupy the same single particle state.
- At high enough densities and/or low enough temperatures a finite fraction of all the particles are in the lowest energy (ground) state.
- This tendency of bosons to condense takes place in the absence of interactions or even with (not too strong) repulsive interactions. Attractive interactions make it all the more favorable.
- Bose-Einstein Condensation is a generic macroscopic quantum phenomenon, observed in Superfluids, ^4He (even ^3He by fermion pairing), Superconductors, and Atomic Gases, ^{87}Rb.
- Relativistic Quantum Field Theory exhibits a similar phenomenon in Spontaneous Symmetry Breaking, in both the strong and electroweak interactions $\langle \bar{q}q \rangle \neq 0 \quad \langle \Phi \rangle \neq 0$.

A Macroscopic Quantum Effect

Gravitational Vacuum Condensates

- Gravity is a theory of spin-2 bosons
- Its interactions are attractive
- The interactions become strong near $r = R_s$
- Energy of any scalar order parameter must couple to gravity with the vacuum eq. of state,
$$p_V = -\rho_V = -V(\phi)$$
- Relativistic Entropy Density s is (for $\mu = 0$),
$$Ts = p + \rho = 0 \text{ if } p = -\rho$$
- Zero entropy density for a single macroscopic quantum state, $k_B \ln \Omega = 0$ for $\Omega = 1$
- This eq. of state violates the energy condition, $\rho + 3p \geq 0$ (if $\rho_V > 0$) needed to prove the classical singularity theorems
- Dark Energy acts as a repulsive core

A GBEC phase transition can stabilize a high density, compact cold stellar remnant to further gravitational collapse

Realizes idea of A. Sakharov, Ya. Zel'dovich, E. Gliner

A Solution to Einstein's Eqs.

$$R_a{}^b - \frac{1}{2} R\, \delta_a{}^b = 8\pi G\, T_a{}^b$$

- $1 - \frac{d(r\,h)}{dr} = 8\pi G\,\rho\, r^2$
- $\frac{rh}{f}\frac{df}{dr} + h - 1 = 8\pi G\, p\, r^2$
- $\frac{dp}{dr} + \frac{p+\rho}{2f}\frac{df}{dr} = 0 \qquad (\nabla_b T_r{}^b = 0)$

Other components follow by differentiating these

Define $h \equiv 1 - \frac{2Gm(r)}{r}$

Then $\frac{dm}{dr} = 4\pi\,\rho\, r^2$ and

$$\frac{dp}{dr} = -\frac{G(\rho+p)(m+4\pi p r^3)}{r\,(r-2Gm)} \qquad \text{(TOV eq.)}$$

Eqs. become closed when eq. of state is given:

$$p = \kappa\,\rho$$

with $\kappa = \begin{cases} -1, & r < r_1 \\ +1, & r_1 < r < r_2 \end{cases}$

$$p = \rho = 0\,, \quad r_2 < r$$

A Simple Model
2004

Planned: Use the EFT and Stress Tensor of the Trace Anomaly to solve the matching problem in the quantum phase boundary layer
(mean field approximation of auxiliary fields)

- I. Interior (Vacuum Condensate) de Sitter:
  $f(r) = Ch(r) = C\left(1 - H_0^2\, r^2\right)$,
  $\rho_V = -p_V = \frac{3H_0^2}{8\pi G}$
- III. Exterior (Vacuum) Schwarzschild:

  $f(r) = h(r) = 1 - \frac{2GM}{r}$

  C, H_0 and M are (so far) arbitrary parameters
- II. Only Non-Vacuum Region:
  Thin shell with $p = \rho \rightarrow pf = const.$
  Let $w \equiv 8\pi Gpr^2$ so the other two eqs. are

  - $\frac{dr}{r} = \frac{dh}{1-w-h} \simeq \frac{dh}{1-w}$
  - $\frac{dh}{h} = -\frac{1-w-h}{1+w-3h}\frac{dw}{w} \simeq -\frac{1-w}{1+w}\frac{dw}{w}$

  If region II shell is **thin**,$i.e.$ exists only near $r \simeq R_s \simeq H_0^{-1}$, then $h \ll 1$ in region II and h can be neglected on r.h.s. of •
  Elementary Integration gives then

  $h \simeq \epsilon\, \frac{(1+w)^2}{w} \ll 1 \rightarrow \epsilon \ll 1$, integ. const.

  $$r \simeq r_1 \left[1 - \epsilon \ln\left(\frac{w}{w_1}\right) + \epsilon\left(\frac{1}{w} - \frac{1}{w_1}\right)\right]$$

Integration of final (conservation) eq. gives

$$f(r) = \left(\frac{r}{r_1}\right)^2 \left(\frac{w_1}{w}\right) f(r_1) \simeq \left(\frac{w_1}{w}\right) f(r_1)$$

A consistent soln. matching at r_1 and r_2 is obtained if w is $\mathcal{O}(1)$ but $\Delta w \equiv w_2 - w_1 = \mathcal{O}(\epsilon)$. Then

$$r \simeq r_1 \simeq H_0^{-1} \simeq R_s \simeq r_2$$

barely changes in region II with

$$\Delta r \equiv r_2 - r_1 = \mathcal{O}(\epsilon^2)$$

and both f and h are of order ϵ in region II but nowhere vanishing. This means that the soln. has a globally defined time and **NO** event horizon. The physical meaning of $\epsilon \ll 1$ is that $\epsilon^{-\frac{1}{2}}$ is the order of the very large but finite redshift a photon emitted at the shell experiences in escaping to infinity.

The proper thickness of the shell is

$$\ell = \int_{r_1}^{r_2} dr\, h^{-\frac{1}{2}} \simeq R_s \epsilon^{\frac{1}{2}} \int_{w_1}^{w_2} dw\, w^{-\frac{3}{2}}$$

which is $\quad \mathcal{O}(\epsilon^{\frac{3}{2}} R_s) \ll R_s$

Likewise the energy in the thin shell of region II is

$$E_{II} = 4\pi \int_{r_1}^{r_2} \rho r^2 dr = \epsilon\, M \int_{w_1}^{w_2} dw$$

which is $\mathcal{O}(\epsilon M) \sim M_{pl} \ll M$

However, the entropy all resides in the shell since

$$p = \rho = \frac{a^2}{8\pi G}\left(\frac{k_B T}{\hbar}\right)^2$$

$$s = \frac{p+\rho}{T} = k_B \frac{a}{4\pi G\hbar}\frac{\sqrt{w}}{r}$$

$$S_{II} = 4\pi \int_{r_1}^{r_2} \frac{s r^2 dr}{\sqrt{h}} \simeq k_B \frac{a R_s^2}{\hbar G}\sqrt{\epsilon}\int_{w_1}^{w_2}\frac{dw}{w}$$

which is of order

$$a k_B \frac{M\ell}{\hbar} \sim S_{BH}\sqrt{\epsilon} \ll S_{BH} = 4\pi k_B \frac{GM^2}{\hbar}$$

and **very** much smaller than S_{BH}.

Eg. If $a \sim 1,\ \ell \sim \sqrt{L_{pl} R_s},\ M \sim M_\circ,$

$$S \simeq 10^{57} k_B \sim S_\circ \simeq 10^{58} k_B \ll S_{BH} \simeq 10^{77} k_B$$

Main Features of New Soln.

- Vacuum Schwarzschild Exterior
- de Sitter (GBEC) Interior, No Singularity
- $\Lambda > 0$ Casimir Energy due to b.c.
- GBEC similar to Gluon Condensate in Bag Model of Hadrons
- Thin Shell of $p = \rho$, No Event Horizon
- Global Time, Unitarity, No Hawking Radiation
- Modest Entropy, No Information Paradox
- Maximizes Entropy, Completely Stable
- No Planckian Pressures or Densities
- Hydrodynamic Einstein Eqs. Valid Everywhere except at r_1, r_2 Stationary Shock Fronts
- Interior de Sitter also a Cosmological Soln.

Analog to BEC quantum transition near the classical horizon

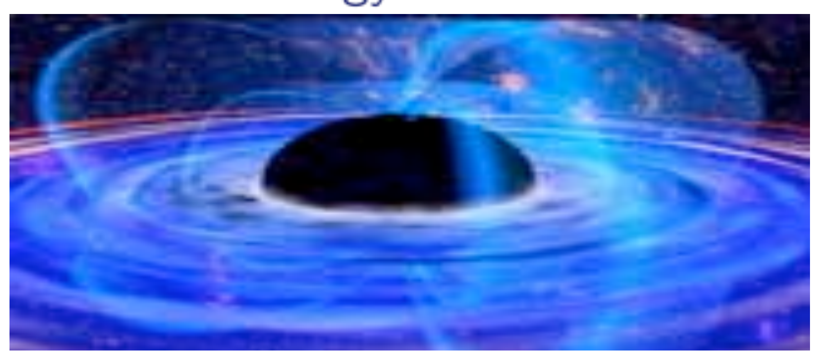
Proc. Natl. Acad. Sci., 101, 9545 (2004)

Gra(vitational) Va(cuum) Stars

Gravastars as Astrophysical Objects

- Cold, Dark, Compact, Arbitrary M
- Accrete Matter like a Black Hole
- But Matter does not Disappear down a Hole
- May be Re-emitted by Ultra-relativistic Shell
- Possible More Efficient Central Engine for Sporadic Gamma Ray Bursters, High-Energy Cosmic Rays, Other Sources?
- Formation could be Violent 'Bosenova'
- Should Support Angular Momentum, Magnetic Fields
- Gravitational Wave Signatures?
- Alternative to Black Holes for the Final State of Gravitational Collapse

- Cosmological Models of Dark Energy
- Much to be Done...



Cosmological Horizon Modes

(w. P. R. Anderson & C. Molina-Paris LA-UR-09-01895)

- Variation of $\langle T_{\mu\nu} \rangle$ in de Sitter space contains contributions from S_{anom} of scalar auxiliary fields φ, ψ
- Additional massless scalar degrees of freedom in cosmology
- The relevant scalar modes satisfy second order wave eqs. ("Inflaton without inflaton")
- Couple weakly to the metric with strength $G_N H^2 << 1$
- But grow significant close to the de Sitter horizon $r_H = H^{-1}$

$$G_N \delta \langle T^t_t \rangle \sim \frac{G_N H^4}{(1 - H^2 r^2)^2}$$

- Becomes of order of classical $R^t_t = 3H^2$ at a proper distance from r_H

$$\ell \sim \sqrt{r_H L_{Pl}}$$ "Healing Length"

- Same as proper distance outside the Schwarzschild horizon

New Horizons in Gravity

- Einstein's Theory receives Quantum Corrections relevant at *macroscopic* Distances & near Event Horizons
- These arise from ***new scalar degrees of freedom*** in the *extended EFT of Gravity* required by the **Conformal/Trace Anomaly**
- At horizons these massless scalar degrees of freedom have *macroscopically large effects*
- Their Fluctuations can induce a *Quantum Phase Transition* at the horizon of a 'black hole'
- Λ_{eff} is a ***dynamical condensate*** which can change in the phase transition & remove 'black hole' interior singularity
- **Gravitational Condensate Stars** resolve all 'black hole' paradoxes ⇒ New Astrophysics of 'gravastars' testable
- The observed dark energy of our Universe itself may be a ***macroscopic finite size effect*** whose value depends not on microphysics but on the cosmological horizon scale