Constructing the EFT of Gravity

- Assume *Equivalence Principle* (Symmetry)
- Metric Order Parameter Field g_{ab}
- Only two strictly *relevant* operators (R, Λ)
- Einstein's General Relativity is an EFT
- But EFT = General Relativity **+** <u>Quantum</u> <u>Corrections</u>
- Semi-classical Einstein Eqs. $(k << M_{pl})$:

$$G_{ab} + \Lambda\, g_{ab} = 8\pi\, G \langle T_{ab} \rangle$$

- But there is also a quantum (trace) anomaly:

$$\langle T_a^{\ a} \rangle = b\, C^2 + b' \left(E - \tfrac{2}{3} \Box R\right) + b'' \Box R$$

- *New* (marginally) relevant operator(s) *needed*

Effective Field Theory & Quantum Anomalies

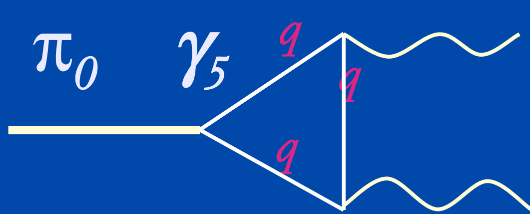
- Expansion of Effective Action in *Local* Invariants assumes

  Decoupling of Short Distance from Long Distance Modes
- But *Massless* Modes do not decouple
- Chiral, Conformal Symmetries are *Anomalous*
- Special Non-local Additions to Local EFT
- *IR* Sensitivity to *UV* degrees of freedom
- Macroscopic Effects of Short Distance (high energy) physics
- Conformal Symmetry & its Breaking controlled by the

  Conformal Trace Anomaly

Chiral Anomaly in QCD

- QCD with N_f massless quarks has an apparent
  $U(N_f) \otimes U_{ch}(N_f)$ Symmetry
- But $U_{ch}(1)$ Symmetry is Anomalous
- Effective Lagrangian in Chiral Limit has $N_f^2 - 1$
  (*not* N_f^2) massless pions at low energies
- Low Energy $\pi_0 \to 2\,\gamma$ dominated by the anomaly



$$\partial_\mu j^{\mu 5} = e^2 N_c F_{\mu\nu} \tilde{F}^{\mu\nu} / 16\pi^2$$

- ***No Local*** Action in chiral limit in terms of $F_{\mu\nu}$ but ***Non-local***
  IR Relevant Operator violates naïve decoupling of *UV*
- ***Measured*** decay rate verifies $N_c = 3$ in QCD
  Anomaly Matching *IR* ↔ *UV*

2D Gravity

$$S_{cl}[g] = \int d^2x \sqrt{g}(\gamma R - 2\lambda)$$

has no local degrees of freedom in 2D, since

$$g_{ab} = \exp(2\sigma)\bar{g}_{ab} \to \exp(2\sigma)\eta_{ab}$$

(all metrics conformally flat) and

$$\sqrt{g}R = \sqrt{\bar{g}}\bar{R} - 2\sqrt{\bar{g}}\,\bar{\Box}\,\sigma$$

gives a total derivative in S_{cl}.

Quantum Trace or Conformal Anomaly

$$\langle T_a{}^a \rangle = -\frac{c_m}{24\pi}R$$

$c_m = N_S + N_F$ for massless scalars or fermions.

Linearity in σ in the variational eq.

$$\frac{\delta \Gamma_{WZ}}{\delta \sigma} = \sqrt{g}\langle T_a{}^a \rangle$$

determines the Wess-Zumino Action by inspection:

2D Anomaly Action

- Integrating the linear anomaly gives

$$\Gamma_{WZ} = (c/24\pi) \int d^2x \sqrt{\bar{g}}\,(-\sigma\bar{\Box}\sigma + \bar{R}\sigma)$$

- This is local but non-covariant. Note kinetic term for σ
- By solving for σ the WZ action can be also written

$$\Gamma_{WZ} = S_{anom}[g] - S_{anom}[\bar{g}]$$

- Polyakov form of the action is covariant but non-local

$$S_{anom}[g] = (-c/96\pi) \int d^2x\sqrt{g_x} \int d^2y\sqrt{g_y}\;\; R_x(\Box^{-1})_{xy}R_y$$

- A covariant and local form requires an auxiliary dynamical field φ

$$S_{anom}[g;\varphi] = (-c/96\pi) \int d^2x \sqrt{g}\,\{(\nabla\varphi)^2 - 2R\varphi\}$$

$$-\Box\varphi = R$$

Effects of 2D Trace Anomaly

- Modification of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a{}_b \rangle$
- Metric conformal factor $e^{2\sigma}$ (was constrained) becomes dynamical & itself fluctuates freely
- Gravitational 'Dressing' of critical exponents: long distance macroscopic physics
- Non-perturbative/non-classical conformal fixed point of 2D gravity: Running of Λ
- Additional non-local Infrared Relevant Operator in S_{EFT}
- New Massless Scalar Degree of Freedom at low energies

Quantum Trace Anomaly in Flat Space

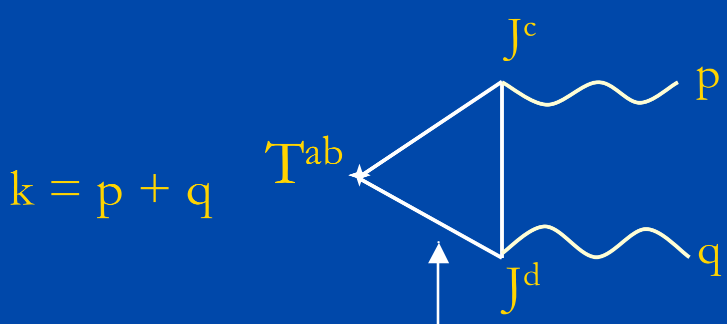
Eg. QED in an External EM Field A_μ

$$\langle T^{\mu}_{\mu} \rangle = \frac{e^2}{24\pi^2} F^{\mu\nu} F_{\mu\nu}$$

Triangle One-Loop Amplitude as in Chiral Case

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\cdot q - q^c p^d)\, F_1(k^2) + (\text{traceless terms})$$

In the limit of massless fermions, $F_1(k^2)$ must have a massless pole:



$$F_1(k^2) = \frac{e^2}{18\pi^2 k^2}$$

$$\rho_T(s) \to \frac{e^2}{6\pi^2}\delta(s)$$

Corresponding Imag. Part Spectral Fn. has a δ fn
This is a new massless scalar degree of freedom in
the two-particle correlated spin-0 state

<TJJ> Triangle Amplitude in QED

Determining the Amplitude by Symmetries and Its Finite Parts

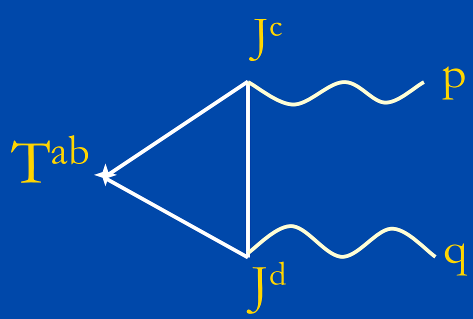
M. Giannotti & E. M. *Phys. Rev. D* **79, 045014 (2009)**

$$\Gamma^{abcd}(p,q) = \int d^4x \int d^4y \, e^{ip\cdot x + iq\cdot y} \, \frac{\delta^2 \langle T^{ab}(0)\rangle_A}{\delta A_c(x)\delta A_d(y)}\bigg|_{A=0}$$

Γ^{abcd}: Mass Dimension 2

Use low energy symmetries:

$k = p + q$



1. By Lorentz invariance, can be expanded in a complete set of 13 tensors $t_i^{abcd}(p,q)$, $i = 1, \ldots 13$:

$$\Gamma^{abcd}(p,q) = \Sigma_i \, F_i \, t_i^{abcd}(p,q)$$

2. By current conservation: $p_c t_i^{abcd}(p,q) = 0 = q_d t_i^{abcd}(p,q)$

All (but one) of these 13 tensors are dimension ≥ 4, so $\dim(F_i) \leq -2$ so these scalar $F_i(k^2; p^2, q^2)$ are completely UV Convergent

<TJJ> Triangle Amplitude in QED

Ward Identities

3. By stress tensor conservation Ward Identity: $\partial_b \langle T^{ab} \rangle_A = eF^{ab} \langle J_b \rangle \Rightarrow$

$$k_b \Gamma^{abcd}(p,q) = (g^{ac} p_b - \delta^c_b p^a)\Pi^{bd}(q) + (g^{ad} q_b - \delta^d_b q^a)\Pi^{bc}(p)$$

4. Bose exchange symmetry: $\Gamma^{abcd}(p,q) = \Gamma^{abdc}(q,p)$

Finally all 13 scalar functions $F_i(k^2; p^2, q^2)$ can be found in terms of

finite (IR) Feynman parameter integrals and the polarization,

$$\Pi^{ab}(p) = (p^2 g^{ab} - p^a p^b)\, \Pi(p^2)$$

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\cdot q - q^c p^d)\, F_1(k^2; p^2, q^2) + \ldots$$

(12 other terms, 11 traceless, and 1 with zero trace when m=0)

Result:
$$F_1(k^2; p^2, q^2) = \frac{e^2}{18\pi^2 k^2}\left\{1 - 3m^2 \int_0^1 dx \int_0^{1-x} dy\, \frac{(1-4xy)}{D}\right\}$$

with $D = (p^2 x + q^2 y)(1-x-y) + xy\, k^2 + m^2$

UV Regularization Independent

<TJJ> Triangle Amplitude in QED

Spectral Representation and Sum Rule

$$F_1(k^2;p^2,q^2) = \frac{1}{3k^2}\int_0^\infty \frac{ds}{k^2+s-i\epsilon}\left[(k^2+s)\rho_T - m^2\rho_m\right]$$

Numerator & Denominator cancel here

Im $F_1(k^2 = -s)$: Non-anomalous,vanishes when m=0

$$\rho_T(s;p^2,q^2) = \frac{e^2}{2\pi^2}\int_0^1 dx \int_0^{1-x} dy\ (1-4xy)\ \delta\left(s - \frac{(p^2x+q^2y)(1-x-y)+m^2}{xy}\right)$$

$$\int_0^\infty ds\,\rho_T(s;p^2,q^2) = \frac{e^2}{6\pi^2}$$

obeys a finite sum rule independent of p^2, q^2, m^2

and as p^2, q^2, $m^2 \to 0^+$

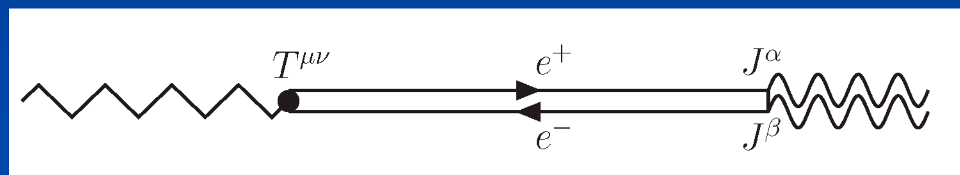
$$\rho_T(s) \to \frac{e^2}{6\pi^2}\delta(s)$$

$$F_1(k^2) \to \frac{e^2}{18\pi^2k^2}$$

Massless scalar intermediate two-particle state analogous to the pion in chiral limit of QCD

Massless Anomaly Pole

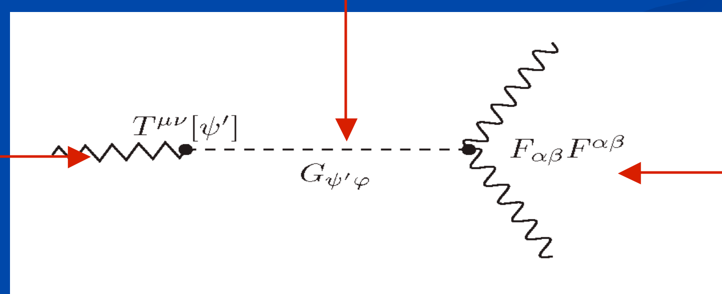
For $p^2 = q^2 = 0$ (both photons on shell) and $m_e = 0$ the pole at $k^2 = 0$ describes a massless $e^+ e^-$ pair moving at v=c collinearly, with opposite helicities in a total spin-0 state



⇒ a massless scalar 0^+ state which couples to gravity

Effective vertex

$h_{\mu\nu}\,(g^{\mu\nu}\,\Box - \partial^\mu\partial^\nu)\psi'$



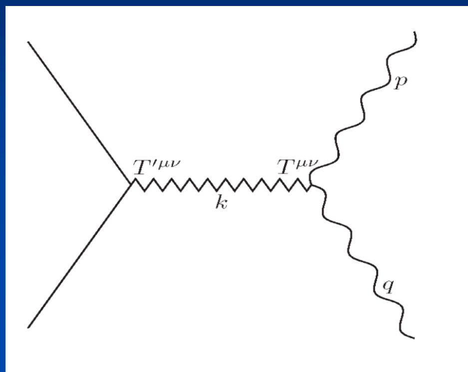
$\varphi\, F^{\alpha\beta}F_{\alpha\beta}$

Effective Action

$$\int d^4x\sqrt{-g}\left\{-\psi'\,\Box\,\varphi - \frac{R}{3}\psi' - \frac{e^2}{48\pi^2}\varphi F^{\alpha\beta}F_{\alpha\beta}\right\}$$

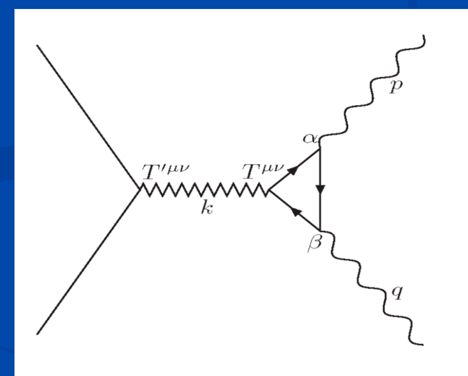
special case of general form

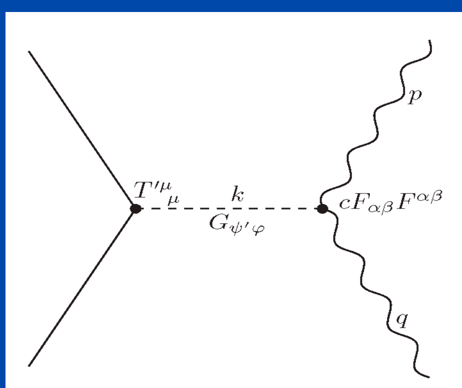
Scalar Pole in Gravitational Scattering



- In Einstein's Theory only transverse, tracefree polarized waves (<u>spin-2</u>) are emitted/absorbed and propagate between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$
- The scalar parts give only non-progagating constrained interaction (like Coulomb field in E&M)

- But for $m_e = 0$ there is a scalar pole in the $\langle TJJ \rangle$ triangle amplitude coupling to photons
- This scalar wave propagates in gravitational scattering between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$





- Couples to trace $T'^{\mu}{}_{\mu}$
- $\langle TTT \rangle$ triangle of massless photons has similar pole
- <u>Two</u> new scalar degrees of freedom in EFT

Trace Anomaly in Curved Space

$$\langle T_a^{\ a} \rangle = b\, C^2 + b' \left(E - \tfrac{2}{3} \Box R\right) + b'' \Box R + c F^2$$

(for $m_e = 0$)

$\langle T_{ab} \rangle$ is the Stress Tensor of Conformal Matter

- $\langle T_a^{\ a} \rangle$ is expressed in terms of Geometric Invariants E, C^2
- One-loop amplitudes similar to previous examples
- State-independent, independent of G_N
- No local effective action in terms of curvature tensor

But there exists a non-local effective action

which can be rendered local in terms of

two new scalar degrees of freedom

Macroscopic Quantum Modification of Classical Gravity

4D Anomalous Effective Action

Conformal Parametization

$$\rightarrow \qquad g_{ab} = \exp(2\sigma)\,\bar{g}_{ab}$$

Since $\sqrt{g}\,F_4 = \sqrt{\bar{g}}\,\bar{F}_4$

is independent of σ, and

$$\sqrt{g}\,(E_4 - \tfrac{2}{3}\Box R) = \sqrt{\bar{g}}\,(\bar{E}_4 - \tfrac{2}{3}\bar{\Box}\bar{R}) + 4\sqrt{\bar{g}}\bar{\Delta}_4\sigma$$

is linear in σ, the variational eq.,

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\langle T_a{}^a\rangle = b\,\sqrt{g}F_4 + b'\sqrt{g}\,(E_4 - \tfrac{2}{3}\Box R)$$

determines the Wess-Zumino Action by inspection:

$$\Gamma_{WZ} = 2b'\int d^4x\sqrt{\bar{g}}\,\sigma\bar{\Delta}_4\sigma + \int d^4x\sqrt{\bar{g}}\left[b\bar{F}_4 + b'\left(\bar{E}_4 - \frac{2}{3}\bar{\Box}\bar{R}\right)\right]\sigma\ ,$$

$$\Delta_4 \equiv \Box^2 + 2R^{ab}\nabla_a\nabla_b - \tfrac{2}{3}R\Box + \tfrac{1}{3}(\nabla^a R)\nabla_a$$

$F = C_{abcd}C^{abcd}$; $\qquad E = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$

Effective Action for the Trace Anomaly
Local Auxiliary Field Form

$$S_{anom} = \frac{b}{2}\int d^4x\sqrt{-g}\left[-2\varphi\triangle_4\psi + F_4\varphi + \left(E_4 - \frac{2}{3}\Box R\right)\psi\right]$$
$$+\frac{b'}{2}\int d^4x\sqrt{-g}\left[-\varphi\triangle_4\varphi + \left(E_4 - \frac{2}{3}\Box R\right)\varphi\right]$$

- Two New Scalar Auxiliary Degrees of Freedom
- Variation of the action with respect to φ, ψ -- the auxiliary fields -- leads to the equations of motion,

$$\triangle_4\varphi = \frac{1}{2}\left(E_4 - \frac{2}{3}\Box R\right) \qquad \triangle_4\psi = \frac{1}{2}F_4$$

$$\triangle_4 = \Box^2 + 2R^{ab}\nabla_a\nabla_b - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^a R)\nabla_a$$

Stress Tensor of the Anomaly

Variation of the Effective Action with respect to the metric gives stress-energy tensor

$$T_{\mu\nu}(g_{\mu\nu}, \varphi, \psi) = -\frac{2}{\sqrt{-g}}\frac{\delta S_{anom}}{\delta g_{\mu\nu}}$$

- Quantum Vacuum Polarization in Terms of (Semi-) Classical Auxiliary potentials
- φ, ψ are new scalar degrees of freedom in low energy gravity which depen upon the global topology of spacetimes and its boundaries, horizons

First Application: Schwarzschild Spacetime

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2$$

$$\varphi = \sigma = \ln\sqrt{f} = \frac{1}{2}\ln\left(1 - \frac{2M}{r}\right) \to \infty$$

solves homogeneous $\Delta_4\varphi = 0$

Timelike Killing field (Non-local Invariant)

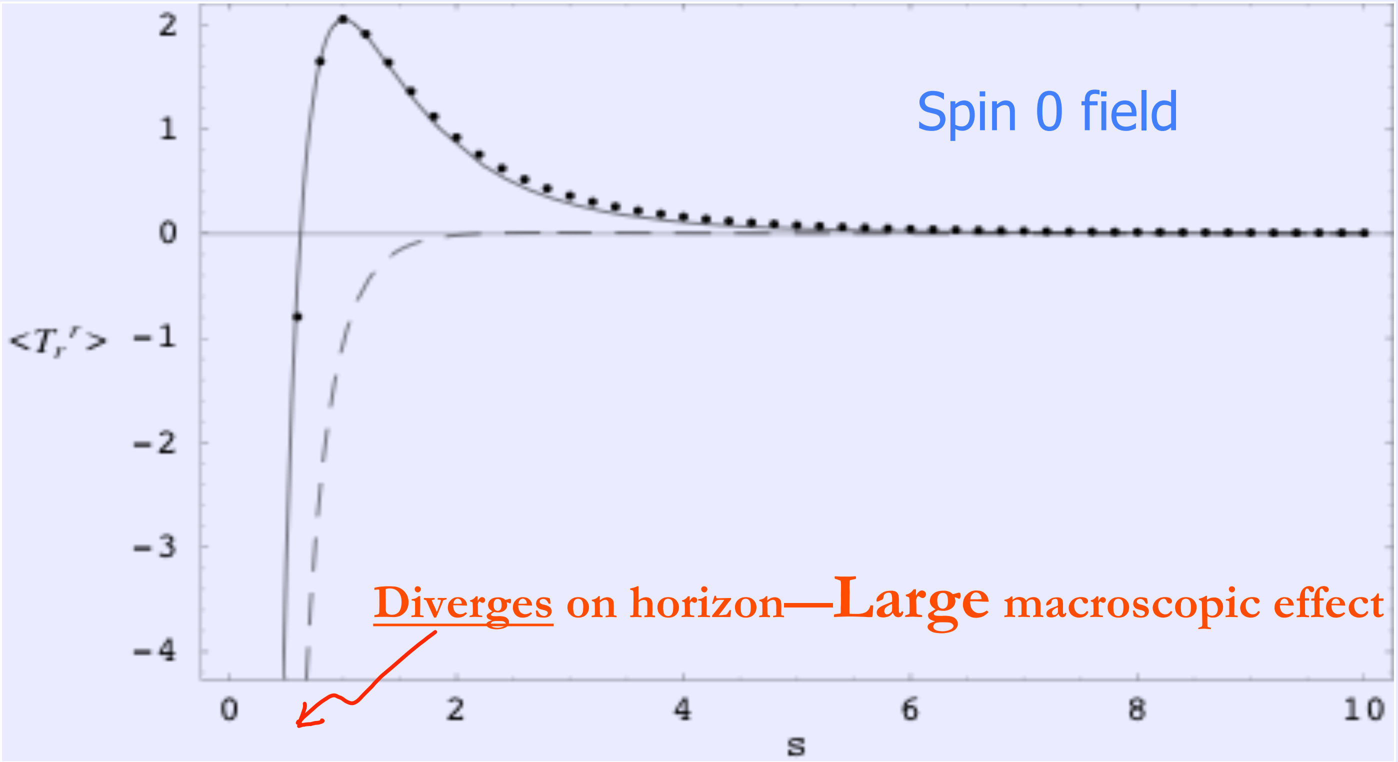
$$\xi^a = (1, 0, 0, 0) \qquad e^\sigma = (-\xi_a \xi^a)^{\frac{1}{2}} = f^{\frac{1}{2}}$$

Energy density scales like $e^{-4\sigma} = f^{-2}$

Auxiliary Scalar Potentials give Geometric (Coordinate Invariant) Meaning to Stress Tensor becoming Large on Horizon

Stress-Energy Tensor in Boulware Vacuum – Radial Component

Dots – Direct Numerical Evaluation of <T_a^b> (Jensen et. al. 1992)
Solid – Stress Tensor from the Auxiliary Fields of the Anomaly (E.M & R. Vaulin 2006)
Dashed – Page, Brown and Ottewill approximation (1982-1986)



IR Relevant Term in the Action

The effective action for the trace anomaly scales logarithmically with distance and therefore should be included in the low energy macroscopic EFT description of gravity—

Not given in terms of Local Curvature

This is a non-trivial modification of classical General Relativity required by quantum effects

$$S_{Gravity}[g,\varphi,\psi] = S_{H-E}[g] + S_{Anom}[g,\varphi,\psi]$$

Fluctuations of new scalar degrees of freedom allow Λ_{eff} to vary dynamically, and can generate a Quantum Conformal Phase of 4D Gravity where $\Lambda_{\text{eff}} = 0$