New Horizons in Gravity: Dark Energy & Condensate Stars

The Trace Anomaly & Macroscopic Quantum Effects in Gravity

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Proc. Natl. Acad. Sci., 101, 9545 (2004)

w. R. Vaulin, *Phys. Rev. D* 74, 064004 (2006)
w. P. Anderson & R. Vaulin, *Phys. Rev. D*, 76, 024018 (2007)
Review Article: w. I. Antoniadis & P. O. Mazur, *N. Jour. Phys.* 9, 11 (2007)
w. M. Giannotti, *Phys. Rev. D*, 79, 045014 (2009)
w. P. Anderson & C. Molina-Paris, LA-UR-09-01895 (2009)

Outline

- Classical Black Holes in General Relativity
- Quantum Effects -- Microscopic & Macroscopic
  - Entropy & the Second Law of Thermodynamics
  - Temperature & the 'Trans-Planckian Problem'
  - Negative Heat Capacity & the 'Information Paradox'
  - Vacuum Polarization & Backreaction Effects near the Horizon
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  - Quantum Effects in de Sitter space
  - Fluctuations in Hawking-de Sitter Temperature
- Effective Theory of Low Energy Gravity: Role of the Trace Anomaly
  - Massless Scalar Poles in Flat Space Amplitudes
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- The Trace Anomaly and the Dynamical Cosmological 'Constant'
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Classical Black Holes

Schwarzschild Metric (1916)

$$ds^2 \;=\; -dt^2\, f(r) + \frac{dr^2}{h(r)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$f(r) = 1 - \frac{2GM}{r} = h(r)$$

Classical Singularities:

- $r = 0$: Infinite Tidal Forces, Breakdown of Gen. Rel.
- $r \equiv R_S = 2GM \quad (c = 1)$: Event Horizon, Infinite Blueshift, Change of sign of f, h

Trapping of light inside the horizon is what makes a black hole

BLACK

The $r = R_S$ singularity is purely kinematic, removable by a coordinate transformation

iff $\hbar = 0$

The Event Horizon & Analytic Extension

- Classical Matter reaches the Horizon in Finite Proper Time
- The Local Riemann Tensor & its Contractions remain Finite
- Kruskal-Szekeres Coordinates (1960) ($G/c^2 = 1$)

$$ds^2 = (32M^3/r)\, e^{-r/2M} (-dT^2 + dX^2) + r^2\, d\Omega^2$$

$$(r/2M - 1)\, e^{r/2M} = X^2 - T^2$$

$$t = 4M\, \mathit{tanh}^{-1} (T/X)$$

For $r > 2M$:

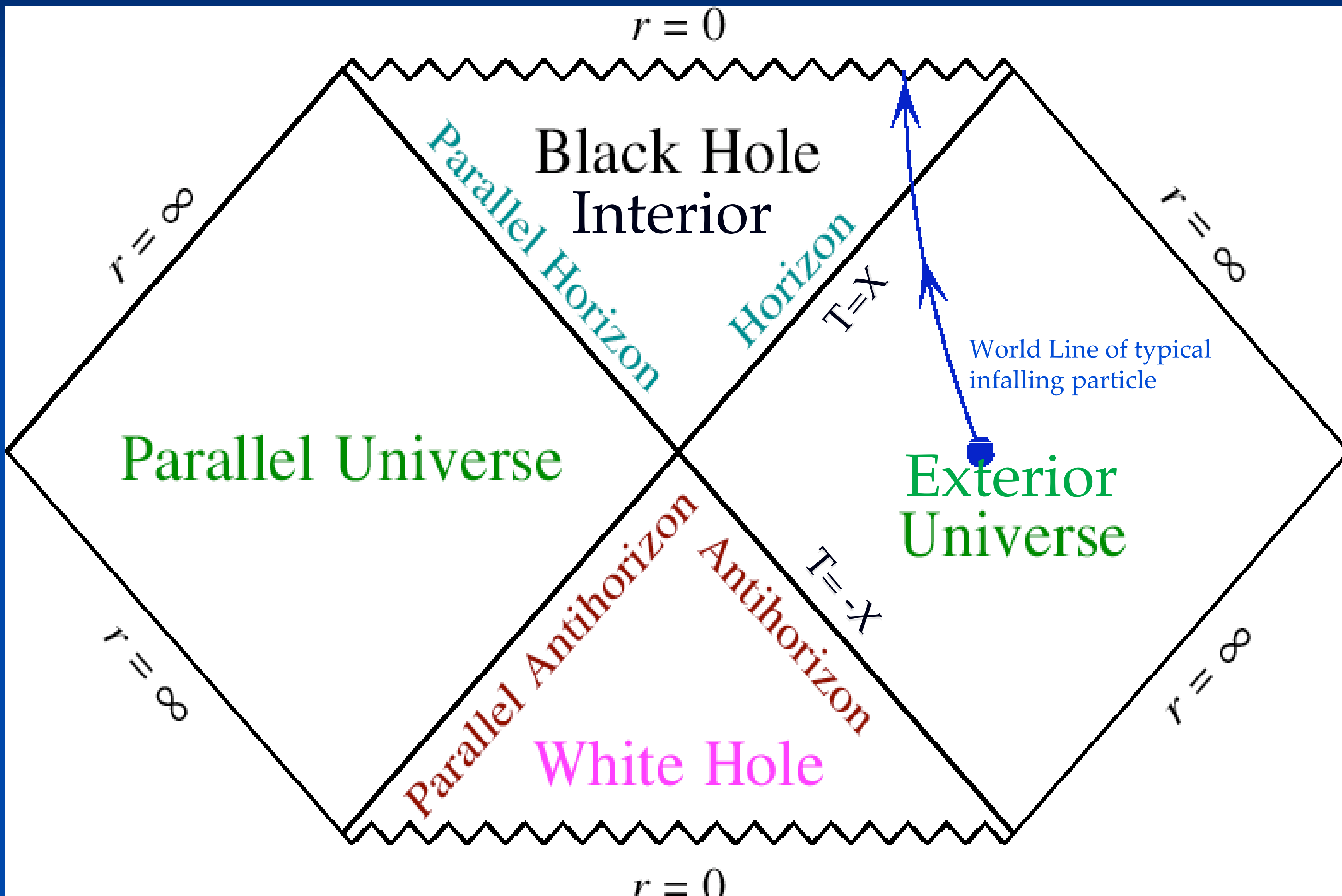
$$X = (r/2M - 1)^{1/2}\, e^{r/4M}\ \mathit{cosh}\, (t/4M)$$

$$T = (r/2M - 1)^{1/2}\, e^{r/4M}\ \mathit{sinh}\, (t/4M)$$

- Future/Past Horizon at $r = 2GM/c^2$ is $T = \pm X$ Regular
- It is possible to use Kruskal coordinates to analytically continue inside $r < 2GM/c^2$ all the way to $r = 0$ singularity
- Necessarily involves complexifying the metric of spacetime

Schwarzschild Maximal Analytic Extension

Carter-Penrose Conformal Diagram



Black Holes & Classical Irreversibility

- Schwarzschild soln. is time *reversible*
  (white hole as well as black hole)
  But Classical Matter falls into the black hole *irreversibly*
- Rotating Black Hole Soln. found by Kerr (1963)
  More complicated analytic extension (and more unphysical: multiple asymptotic regions, closed timelike curves)
- All Gen. Rel. Black Holes specified by M, J, Q (no "hair")
- Irreducible Mass M_{irr} *increases monotonically* (Christodoulou, 1972)

$$M^2 = (M_{irr} + Q/4M_{irr})^2 + J^2/4M_{irr}^2$$

$$M_{irr}^2 = (\text{Area})/16\pi G$$

$$\Delta M_{irr}^2 \geq 0$$

Black Holes and Entropy

- A fixed classical solution usually has no entropy :
  (What is the "entropy" of the Coulomb potential $\Phi = Q/r$?)
  … But if matter/radiation disappears into the black hole,
  what happens to its entropy? (Only M, J, Q remain)
- Maybe M_{irr}^2 (which always increases) is a kind of "entropy"?
  To get units of entropy need to divide Area, A by $(\text{length})^2$
  … But there is no fixed length scale in classical Gen. Rel.
- Planck length $\ell_{Pl}^2 = \hbar G / c^3$ involves $\hbar$
- Bekenstein suggested $S_{BH} = \gamma\, k_B\, A/\ell_{Pl}^2$ with $\gamma \sim O(1)$
- Hawking (1974) argued black holes emit thermal radiation at

$$T_H = \frac{\hbar c^3}{8\pi G k_B M}$$

Apparently then the first law, $dE = T_H\, dS_{BH}$ fixes $\gamma = 1/4$

Great ! But …

A few problems remained …

- Hawking Temperature requires trans-Planckian frequencies
- $S_{BH} \propto A$ is *non-extensive* and ***HUGE !***
- In the classical limit $T_H \to 0$ (cold) but $S_{BH} \to \infty$ (? !)
- $E \propto T^{-1}$ implies negative heat capacity

$$\frac{dE}{dT} << 0 \quad \Rightarrow \text{highly unstable}$$

Equilibrium Thermodynamics cannot be applied

- Information Paradox: Where does the information go?

  (Pure states → Mixed States? *Unitarity* ?)

- What is the statistical interpretation of S_{BH} ?

  Boltzman asks: $S = k_B \ln W$??

Statistical Entropy of a Relativistic Star

- $S = k_B \ln W(E)$ (microcanonical) is equivalent to

$$S = - k_B \, Tr\,(\rho \ln \rho)$$

- Maximized by canonical thermal distribution

Eg. Blackbody Radiation $E \sim V\,T^4$, $S \sim V\,T^3$

$$S \sim V^{1/4}\,E^{3/4} \sim R^{3/4}\,E^{3/4}$$

For a fully collapsed relativistic star $E = M$, $R \sim 2GM$,

so $$\boxed{S \sim k_B\,(M/M_{Pl})^{3/2}}$$ ← *note 3/2 power*

$S_{BH} \sim M^2$ is a factor $(M/M_{Pl})^{1/2}$ larger or 10^{19} for $M = M_\odot$

- There is *no way* to get $S_{BH} \sim M^2$ by any standard statistical thermodynamic counting of states

Horizon in Quantum Theory

- Infinite Blueshift Surface

$$\omega_{local} = \omega_\infty (1 - 2GM/r)^{-1/2}$$

No problem classically, but in quantum theory,

$$E_{local} = \hbar\omega_{local} = \hbar\omega_\infty (1 - 2GM/r)^{-1/2} \rightarrow \infty$$

$\hbar \rightarrow 0$ and $r \rightarrow 2GM$ limits do not commute ! ($\Rightarrow$ non-analyticity)

Singular coordinate transformations need not be harmless (e.g. Vortices)

- Energies becoming trans-Planckian should call into doubt the semi-classical fixed metric approximation
- Large local energies must be felt by the gravitational field
- Large local energy densities/stresses are generic near the horizon

$$\langle T_a^{\ b} \rangle \sim \hbar\omega_{local}^{\ 4} \sim \hbar M^{-4} (1 - 2GM/r)^{-2}$$

The geometry does not remain unchanged down to $r = 2GM$

Quantum Backreaction is important

Wave Eq. in Schwarzschild Geometry

$$-\Box\Phi = 0$$

$$\Phi = e^{-i\omega t} Y_{lm}(\theta,\phi)\frac{\psi_{\omega l}(r^*)}{r}$$

$$dr^* = \frac{dr}{1-\frac{2GM}{r}}$$

$$r^* = r + 2GM\ln\left(\frac{r}{2GM}-1\right)$$

$$\left[-\frac{d^2}{dr^{*2}} + V_l(r^*)\right]\psi_{\omega l} = \omega^2\psi_{\omega l}$$

'Potential' $V_l = \left(1-\frac{2GM}{r}\right)\left(\frac{l(l+1)}{r^2}+\frac{2GM}{r^3}\right)$

vanishes as $r \to 2GM$, $r^* \to -\infty$

- Free Waves at the horizon
- Horizon is null Cauchy Surface of the Wave Eq.
- Boundary Conditions on horizon determine the behavior of the stress energy tensor expectation value $\langle T_a^b \rangle$ as $r \to 2GM$.

Boulware Vacuum (Free b. c. on horizon)

$$\overrightarrow{a}_{\omega lm}\,|B\rangle = \overleftarrow{a}_{\omega lm}\,|B\rangle = 0$$

Renormalized Stress Tensor Expectation Value

$$\langle B|T_a^b|B\rangle \to -\frac{\pi^2}{90}\frac{(k_B T_{loc})^4}{(\hbar c)^3}\,\mathrm{diag}\,(-3,1,1,1)$$

near the horizon is the negative of the stress tensor of radiation at the local blueshifted temperature,

$$T_{loc} = T_H\left(1-\frac{2GM}{r}\right)^{-\frac{1}{2}} \qquad \text{(Tolman)}$$

Coherent modes with wavelength $\lambda \sim R_S$ and frequency $\omega \sim \frac{1}{GM}$ dominate the mode sum.

The local frequency of these modes,

$$\omega_{loc} = \omega\left(1-\frac{2GM}{r}\right)^{-\frac{1}{2}} \sim \frac{1}{GM}\left(1-\frac{2GM}{r}\right)^{-\frac{1}{2}}$$

becomes trans-Planckian as $r \to R_S$.

The same trans-Planckian kinematics appears in Hawking's derivation of black hole radiance, since the same near horizon modes are the important ones at late retarded times, $u = t - r^* \gg R_S$.

Backreaction

• Because of the unbounded blueshifting of waves near the horizon, these Planckian effects can appear at finite values of ω in Schwarzschild spacetime.

• The fluctuations give the perturbed stress tensor Boulware components, hence unbounded growth near $r = R_S$, even if not present in the unperturbed $\langle T_a^b \rangle$.

• This implies that the backreaction of the quantum stress tensor on the geometry is large near $r = R_S$.

• In fact, the Riemann tensor component,

$$R^{tr}{}_{tr} = \frac{h}{4}\left(\frac{f'^2}{f^2} - \frac{2f''}{f} - \frac{h'}{h}\frac{f'}{f}\right) \to -\frac{f''}{2}$$

is of order M^{-2} and small only if $h = f$.

• But by the Einstein eqs.,

$$\frac{d}{dr}\left(\frac{h}{f}\right) = -8\pi G(\rho + p_r)\frac{r}{f}$$

so $h \neq f$ if $\rho + p_r \neq 0$, $R^{tr}{}_{tr} \sim (M^2 f)^{-1}$ and even the Riemann tensor can become large near R_S.

- In fact, the quantum backreaction already significantly alters the classical geometry when $\langle T_a^b \rangle \sim \frac{1}{M^4 f^2} \sim \frac{1}{M^2}$ or at $\left(1 - \frac{2GM}{r}\right)^{-1} \sim M$ well before the Planckian regime is reached.

- Coherent quantum wave amplitudes over the scale of R_S (non-local on this scale!) can alter the local stress tensor and local geometry near $r = R_S$, which is a 'breakdown' of the Strong Equivalence Principle embodied in classical GR, but only in the mildest sense required by Quantum Mechanics.

In a self-consistent solution of the semi-classical Einstein eqs. the horizon singularity and trans-Planckian divergences are removed.

Strong Equivalence Principle
(Strict Locality of All Physics)

vs.

Quantum Correlations
(Non-locality of Entanglement, EPR, Macroscopic Coherence)

- QM is about matter waves not point particles.
- Waves satisfy wave eqs. whose solns. depend upon boundary conditions.
- Macroscopic Quantum Cohererence, BEC, Cooper pairing in Superconductors, Bohm-Aharonov Effect, Entanglement are not strictly local because of this.
- Local Casimir 'vacuum' stresses depend on boundary conditions (but $G = 0$ causes no problems).
- Strict locality (SEP) cannot be maintained when both $\hbar \neq 0$ and $G \neq 0$.

A Second Problem of Some Gravity

The 'Cosmological Constant' Problem

The Classical Einstein's Equations,

$$R_{ab} - \frac{1}{2}R g_{ab} + \Lambda\, g_{ab} = 8\pi G\, T_{ab}$$

relate

- **Curvature of Spacetime,** R_{ab} **to**
- **Energy-Momentum of Matter,** T_{ab}

Metric: g_{ab} **fixes lengths,** $ds^2 = g_{ab} dx^a dx^b$

Flat spacetime: $g_{ab} = \mathrm{diag}(-1, +1, +1, +1)$

Λ **is equivalent to**

Constant Vacuum Energy Everywhere:

$$T^{(vac)}_{ab} = -\frac{\Lambda}{8\pi G} g_{ab} \qquad \text{or}$$

$$p_\Lambda = -\rho_\Lambda$$

Negative Pressure

Classically, Λ **may be set to zero but ...**

Gravity weighs Everything even Quantum Vacuum Fluctuations:

$$\rho_\Lambda = N \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{\hbar\omega_k}{2} \rightarrow \frac{N\hbar c}{16\pi^2} L_{min}^{-4} = -p_\Lambda$$

Quartic Dependence on $L_{min} \rightarrow 0$

With any "reasonable" L_{min}, ρ_Λ is HUGE:

The "natural" scale would seem to be

$$\Lambda \simeq \frac{c^3}{\hbar G} = L_{Pl}^{-2} \simeq \left(\frac{1}{10^{-33}\mathrm{cm}}\right)^2$$

The Universe would be curled up then to a radius of 10^{-33} cm. (!)

Since the observable Universe is of order 10^{28} cm (Hubble scale),

$$\Lambda < 10^{-121} \frac{c^3}{\hbar G}$$

our estimate is wrong by some 121 orders of magnitude (!)

Requires <u>both</u> $\hbar$ and G different from zero

<u>Macroscopic</u> Quantum Gravity

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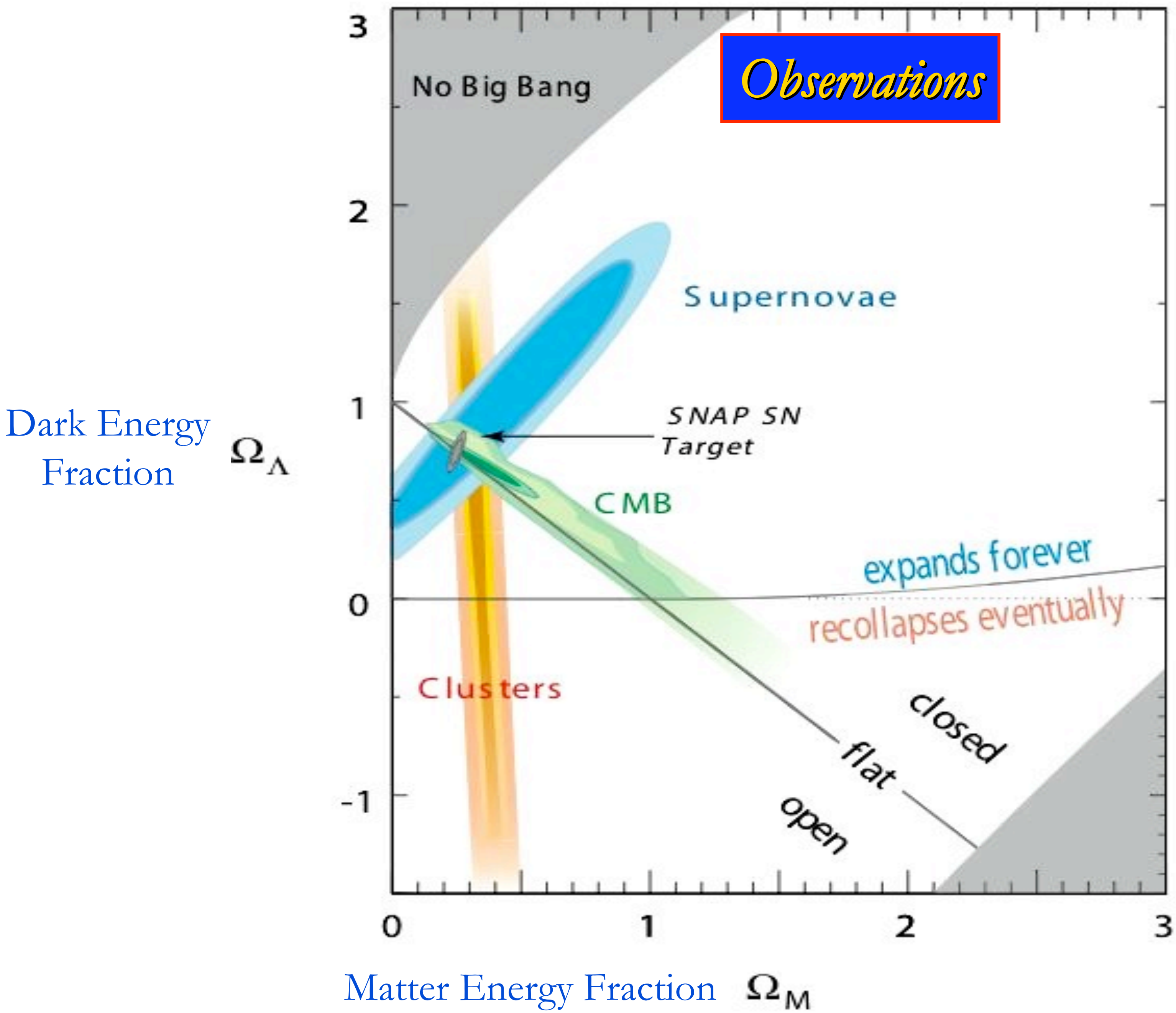
Quantum Effects:
Macroscopic or Microscopic ?

- We deal with UV divergences by **Renormalization**, and now understand most (all?) QFT's as *Effective* Theories
- Λ is a free parameter of the **Low Energy Effective Theory**

  "Just because something is infinite does not necessarily mean that it is zero." –W. Pauli

- The Standard Model has **Spontaneous Symmetry Breaking**
  When the ground state changes, so does its energy – so we should expect generically $\Lambda \neq 0$ now.
- More Symmetries at Very High Energy (UV) Cannot Help
- This is a problem of fixing the *infrared*

  Quantum Vacuum State of Macroscopic Gravity


Observations
No Big Bang
Supernovae
SNAP SN
Target
CMB
expands forever
recollapses eventually
Clusters
closed
flat
open
3
2
1
0
-1
0
1
2
3
Dark Energy
Fraction
Ω_Λ
Matter Energy Fraction Ω_M

The New Cosmology

- Non-Luminous (Dark) Matter, presumed
  Non-Baryonic is **25-30%** of the Universe
- Relativistic Dark Energy with ***negative*** pressure,
  $$p \approx -\rho < 0$$
  is **70-75%** of all the energy in the observable universe
- Ordinary Baryonic Matter is only a few percent
- Since $\rho + 3p < 0$, the expansion is ***accelerating***
- **Tiniest** pure number in Nature: (note involves ħ and **G**)

$$\hbar G \Lambda_{obs} / c^3 \simeq 3.6 \times 10^{-122}$$

We live in a de Sitter-like Universe **dominated**
by Vacuum Dark Energy ➔

Something is clearly missing in the Standard Model