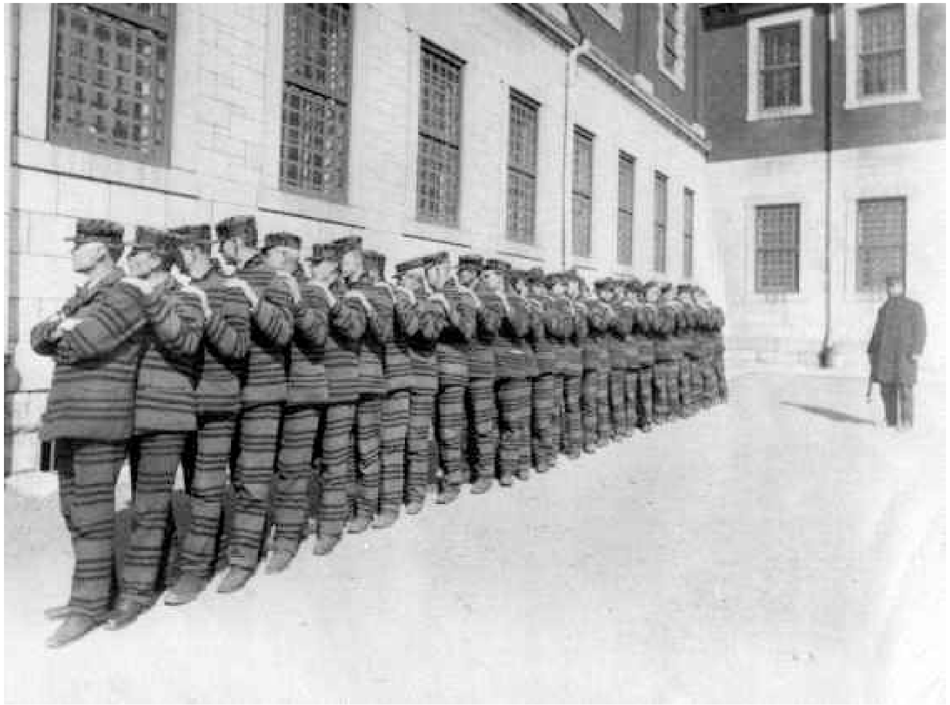
Some Current Approaches to the Confinement Problem



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We would like to also *understand* QCD, i.e. how it does what it does.

In particular we would like to understand confinement.

In this area progress has been slow, and there is still no general agreement about how confinement comes about.

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There is even disagreement about what we are trying to explain...

Outline

1. What *is* Confinement?

   Is it distinguished from "non-Confinement" by some symmetry?

2. Relevance of the gauge-Higgs model, and the ambiguity of spontaneous gauge symmetry breaking.

3. Order parameters, center symmetry, and some numerical facts.

4. Current approaches

   vortices, monopoles, calorons, Dyson-Schwinger eqns., vacuum wavefunctionals...

   (for AdS/CFT - other talks here?)

5. Confining Coulomb potential and the Gluon Chain Model

What is Confinement?

Juliet:
"What's in a name? That which we call a rose
By any other name would smell as sweet."

Romeo and Juliet (II, ii, 1-2)

What are people trying to prove, in order to “prove” confinement?
And what do they *mean* by that word?

1. linear static quark potential, rising to infinity ← most order parameters

2. colorless asymptotic particle states ← common terminology

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against #1 - in *real* QCD, with quarks, the static potential rises and then levels off, due to string breaking.

so is real QCD not confining?

against #2 - asymptotic particle states are also colorless in a Higgs theory, where there is no linear potential at all.

so are broken gauge theories confining?

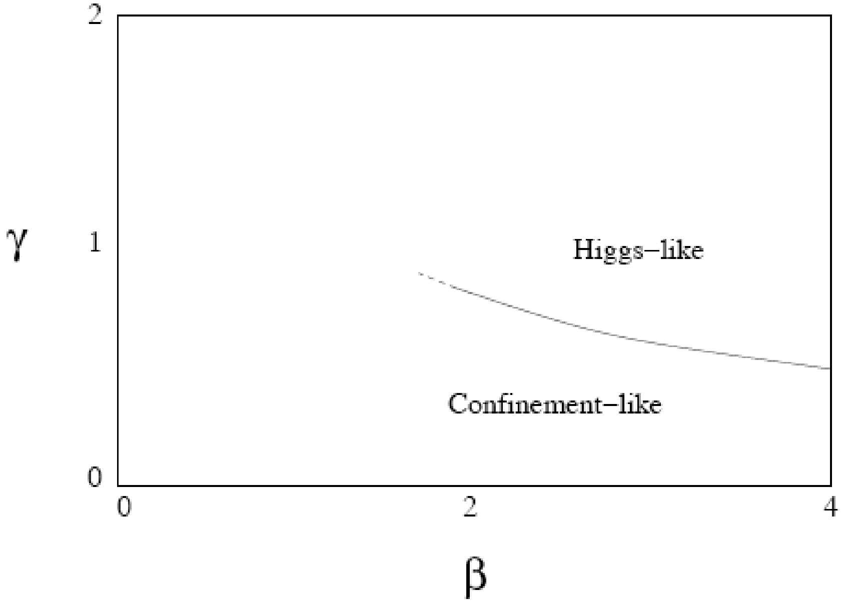
The Fradkin-Shenker-Osterwalder-Seiler Theorem

Consider an SU(2) gauge-Higgs theory with lattice action

$$S = \beta \sum_{plaq} \frac{1}{2}\mathrm{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{x,\mu} \frac{1}{2}\mathrm{Tr}[\phi^\dagger(x)U_\mu(x)\phi(x+\widehat{\mu})]$$

It has a phase diagram something like this:

The theorem says that there is no complete separation between the Higgs-like and the confinement-like regions.



More precisely: between a point

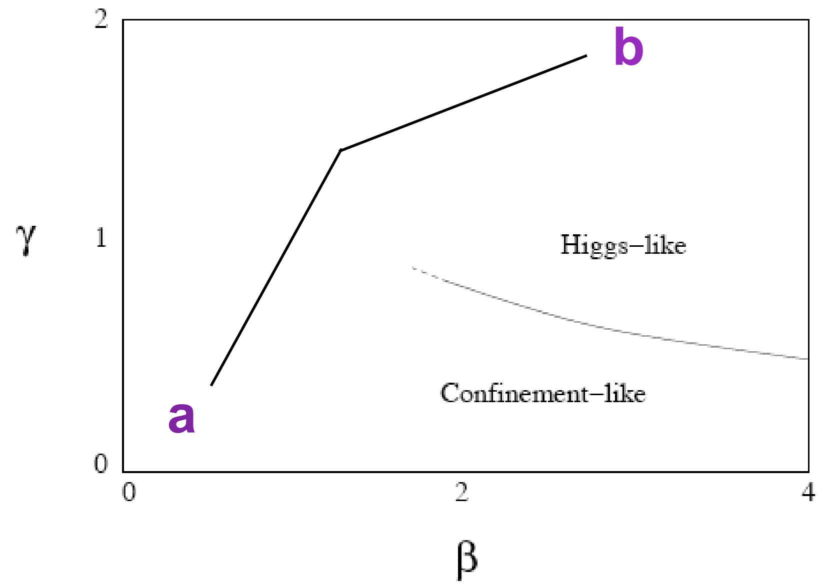
"**a**" deep in the confinement-like regime $(\beta, \gamma \ll 1)$, and a point

"**b**" deep in the Higgs regime $(\beta, \gamma \gg 1)$,

there is a path from **a** to **b** such that all Green's functions of all local, gauge-invariant operators

$$\langle A(x_1)B(x_2)C(x_3)...\rangle$$

vary analytically along the path.



This rules out an abrupt transition from a colorless to a color-charged spectrum.

What happened to "Spontaneous Symmetry Breaking"??

Elitzur's Theorem:

Local gauge symmetries do not break spontaneously. In the absence of gauge fixing, $\langle \varphi \rangle = 0$ regardless of the shape of the Higgs potential.

However, one can always fix to some gauge, e.g. Landau or Coulomb, having some residual *global* gauge symmetry. These residual symmetries *can* break spontaneously.

Gauge Condition	Residual symmetry	unbroken realization required by the
Landau gauge:	g(x,t) = g	Kugo-Ojima confinement criterion
Coulomb gauge:	g(x,t) = g(t)	Coulomb confinement scenario (global symmetry on a time-slice)

I. The Kugo-Ojima Criterion

Kugo and Ojima introduce a function $u^{ab}(p^2)$ defined by

$$u^{ab}(p^2)\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) = \int d^4x \; e^{ip(x-y)} \langle 0|T[D_\mu c^a(x) g(A_\nu \times \overline{c})^b(y)|0\rangle$$

where $c^a(x)$ is the ghost field in a covariant gauge. They then show that the expectation value of charge vanishes in any physical state

$$\langle \text{phys}\; |Q^a| \text{phys}\rangle = 0$$ *(confinement?)*

providing the following conditions are satisfied:

1. Remnant symmetry under $g(x) = g$ is unbroken

2. The criterion $u^{ab}(0) = -\delta^{ab}$ is satisfied.

It turns out that (2) implies that a spatially inhomogenous remnant symmetry in Landau gauge is also unbroken (Hata, Kugo).

Therefore, the Kugo-Ojima scenario requires that the entire remnant gauge symmetry in Landau gauge is unbroken, i.e. $\langle \varphi \rangle = 0$.

II. The Coulomb Criterion

Marinari, Parisi, Paciello, Taglienti (1993)
Olejnik, Zwanziger, JG (2004)

The idea is to show that

❶ The Coulomb energy of an isolated color charge is infinite;

❷ The color Coulomb potential is confining.

It turns out that both of these are implied by unbroken remnant gauge symmetry

$$g(\mathbf{x},t) = g(t)$$

which means that

$$\left\langle \mathrm{Tr}\left[L(\mathbf{x},T)\right]\right\rangle = 0$$

where

$$L(x,T) = P\exp\left[i\int_0^T dt\ A_0(x,t)\right]$$

Isolated Charge

$$\Psi_q^a = q^a(x)\Psi_0$$

propagation in time

$$\begin{aligned} G(T) &= \langle \Psi_q^a | e^{-(H-E_0)T} | \Psi_q^a \rangle \\ &\propto \left\langle \mathrm{Tr}\left[L(\mathbf{x},T)\right]\right\rangle \end{aligned}$$

infinite energy if ***G(T)=0*** , which implies $\langle \mathbf{Tr}[L] \rangle = 0$

Color-Coulomb Potential

$$V_{coul}(R) = -\lim_{T\to 0}\frac{d}{dT}\log\left[\mathrm{Tr}[L(\mathbf{x},T)L^\dagger(\mathbf{y},T)]\right]$$

$V_{coul}(R)$ goes flat at $G(R) \to \infty$ (no confinement) if $\langle \mathbf{Tr}[L] \rangle \neq 0$

So both conditions require unbroken remnant gauge symmetry.

Either criterion - Kugo-Ojima or Coulomb confinement - can work in real QCD, with matter fields.

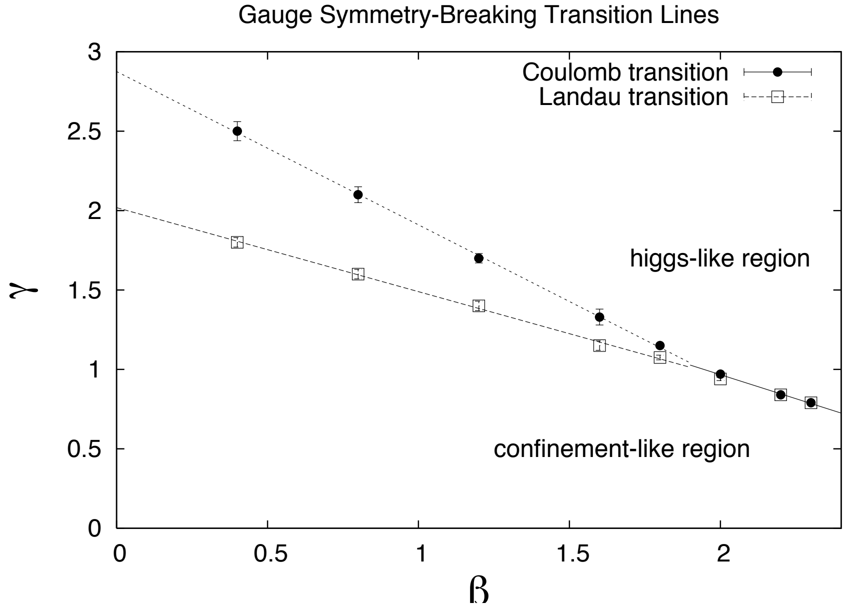
So is this what we mean by confinement?

The problem is:

1. these residual symmetries *break in different places*, and

2. they break in the absence of any other abrupt change in the physical state (Fradkin-Shenker)

Not a good criterion for confinement!



Caudy & JG (07)

Dual Superconductivity

Mandelstam and 't Hooft, mid-1970's

In compact U(1) gauge theories there is a conserved magnetic current

$$j^M_\mu = \partial^\nu \widetilde{F}_{\mu\nu}$$

associated with a dual U(1) gauge symmetry.

Spontaneous breaking of the dual gauge symmetry leads to confinement via a dual Meissner effect.

How to detect spontaneous breaking of a dual (global) gauge symmetry?

Pisa Proposal

Di Giacomo, Paffuti, D'Elia, Lucini, del Debbio...

The order parameter for dual symmetry breaking is a monopole creation operator, denoted μ, which doesn't commute with magnetic charge.

The monopole operator inserts a monopole field centered at a given point x

$$\mu(\mathbf{x})|A_i\rangle = |A_i + A_i^M\rangle$$

accomplished by

$$\mu(\mathbf{x}) = \exp\left[i \int d^3y \; A_i^M(y) E_i(y)\right]$$

(In a non-abelian theory, an abelian subgroup is picked out by abelian projection.)

In practice one computes

$$\rho = \frac{\partial}{\partial \beta} \log\langle\mu\rangle = \langle S\rangle_S - \langle S_M\rangle_{S_M}$$

A large negative peak in ρ at some $\beta=\beta_c$, growing with lattice volume, is the sign that $\langle\mu\rangle = 0$, and dual superconductivity disappears, for $\beta>\beta_c$.

In case after case, a symmetry restoration transition

$$\rho \rightarrow -\infty \ , \quad \langle\mu\rangle \rightarrow 0$$

occurs at the deconfinement temperature.

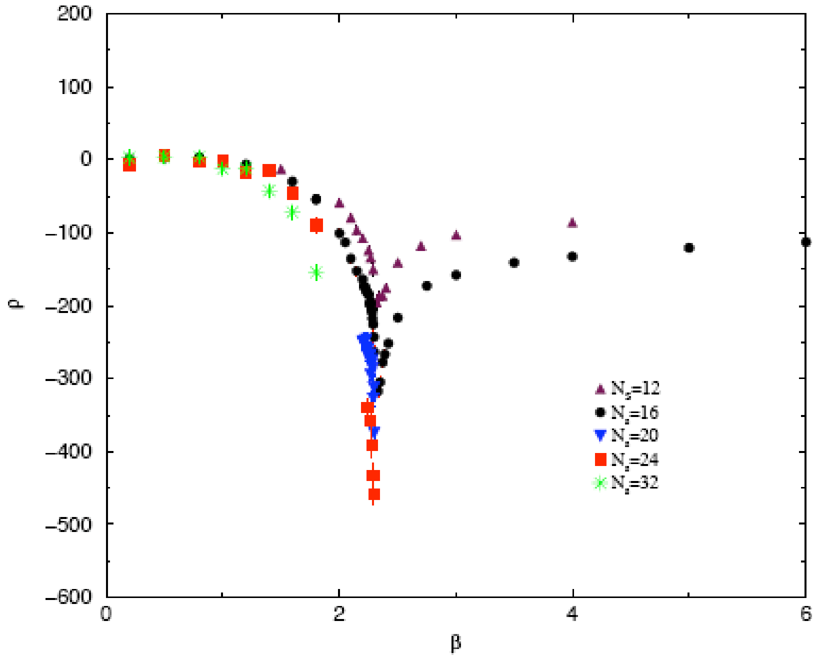


FIG. 3. ρ as a function of β for different spatial sizes at fixed $N_t = 4$. Plaquette projection.

Pure SU(2), N_T=4

Di Giacomo et al. (1999)

But what about the behavior of ρ near other types of transitions; e.g. in the gauge-Higgs model, at zero temperature?

There is strong evidence of $\mu \to 0$ (dual symmetry restoration) transitions in the *absence* of any transition from a confining to a non-confining phase, and *even in the absence of any change of phase whatever.*

We (Lucini & JG, 08) find such $\mu \to 0$ transitions, at zero temperature, in

1. SU(5) gauge theory

2. mixed fundamental-adjoint SU(2) gauge theory

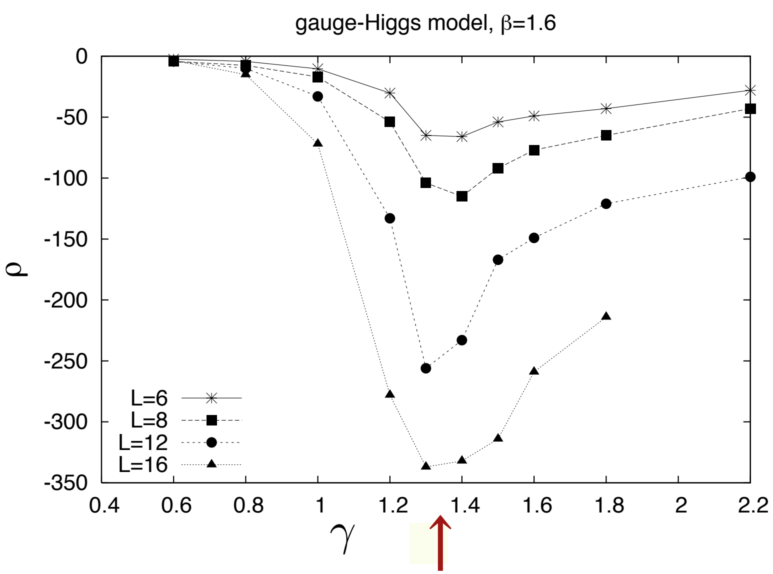
3. pure SU(2) (Wilson action)

4. gauge-Higgs theory

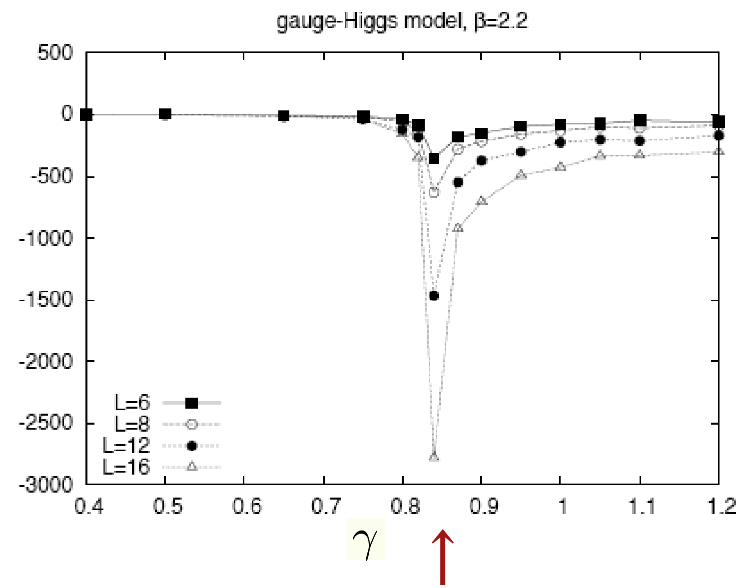
5. G(2) gauge theory (Cossu et al.)

Example - SU(2) gauge-Higgs action

We find μ → 0 transitions in the gauge-Higgs model, where the Fradkin-Shenker theorem tells us that the phase diagram is connected.



β=1.6, γ=1.3
off the crossover line



β=2.2, γ=0.84
on the crossover line

So, what's in a name?

If "confinement" means:

color-singlet spectrum

then there is probably no meaningful distinction between the confined and Higgs phases, at least in terms of symmetries

But there is a difference in physics! *Flux tube formation, linear potential, Regge trajectories*....as opposed to a Yukawa potential.

If we focus on these, rather than on color neutrality, then perhaps it is better to say that confinement is the phase of

magnetic disorder

Magnetic Disorder

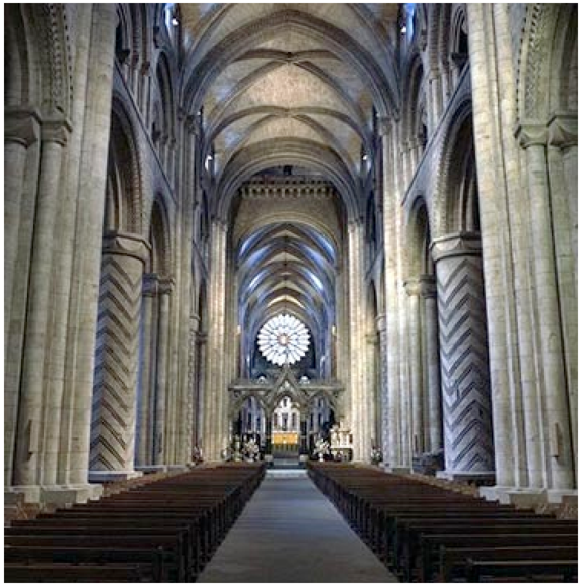
the existence of vacuum fluctuations strong enough to induce an area law falloff in Wilson loops at arbitrarily large scales.

The vacuum of the gauge-Higgs theory has this property in the $\gamma \rightarrow 0$ (Higgs decoupling) limit.

The QCD vacuum has this property in the quark mass $m_q \rightarrow \infty$ limit.

In these limits the static quark potential rises linearly forever, and the theory *acquires an unbroken global symmetry...*

Center Symmetry



When center symmetry is broken, either:

a) spontaneously

deconfinement at high T
adjoint rep matter fields

b) explicitly

fundamental rep matter fields

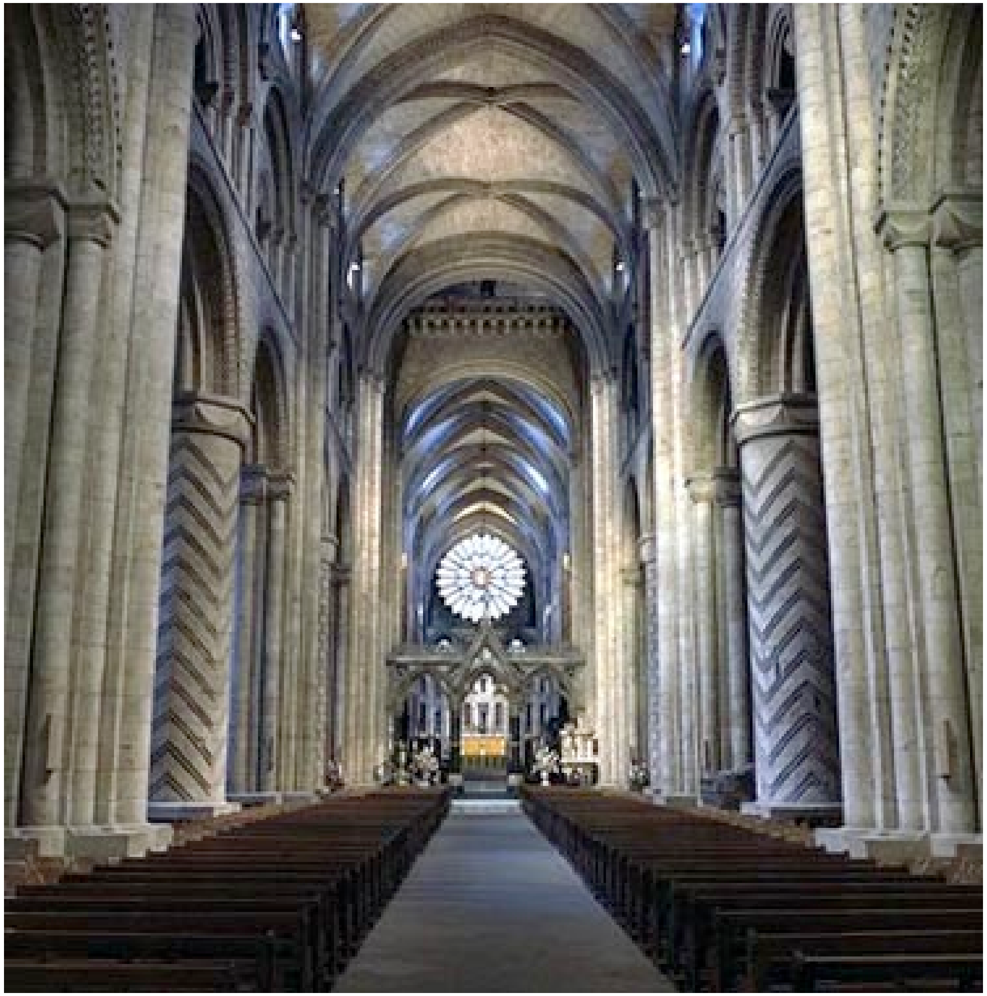
c) or doesn't exist in the first place

G(2) gauge group

magnetic disorder is lost.

Center Symmetry

and Order Parameters for Confinement



Traditional order parameters for confinement

A. finite asymptotic string tension $\sigma > 0$ (implies linear potential)

$$W(C) = \left\langle P\exp[i\oint_C dx^\mu A_\mu] \right\rangle \sim \exp[-\sigma \text{Area}(C)]$$

B. vanishing Polyakov lines (isolated charge has infinite energy)

$$P(\vec{x}) = \left\langle P\exp[i\int_0^T dt\ A_0(\vec{x},t)] \right\rangle = 0$$

C. 't Hooft loop (center vortex creation operator)

$$B(C) \sim \exp[-\mu \text{Perimeter(C)}]$$

D. center vortex free energy:

if $F_v = L_z L_t \exp[-\sigma' L_x L_y]$ then $\sigma \geq \sigma'$

None of these conditions are satisfied if global center symmetry is broken spontaneously (deconfinement) *or explicitly* (quarks).

A little group theory: the center subgroup is the set of all group elements that commute with the full group. For SU(N)

$$Z_N = \left\{ z_n = e^{2\pi i n/N} I_N, \ n = 0, 1, 2, ..., N-1 \right\}$$

Suppose *M[g]* is an irreducible representation of the group element *g*. Then there is a fixed power *k* - known as the N-ality - such that

$$M[z_n g] = \left(e^{2\pi i n/N} \right)^k M[g]$$

Gluons binding to a color charge can reduce the dimensionality of the representation, but not the N-ality.

Asymptotically, the string tension of a quark-antiquark pair can only depend on the N-ality of the quark color charge representation.

Center symmetry on the lattice is the global transformation

$$U_0(\vec{x}, t_0) \rightarrow z U_0(\vec{x}, t_0) \quad , \quad z \in Z_N \quad , \quad \text{all } \vec{x}$$

where $z = e^{2\pi i n/N}$ is an element of the center subgroup of SU(N)

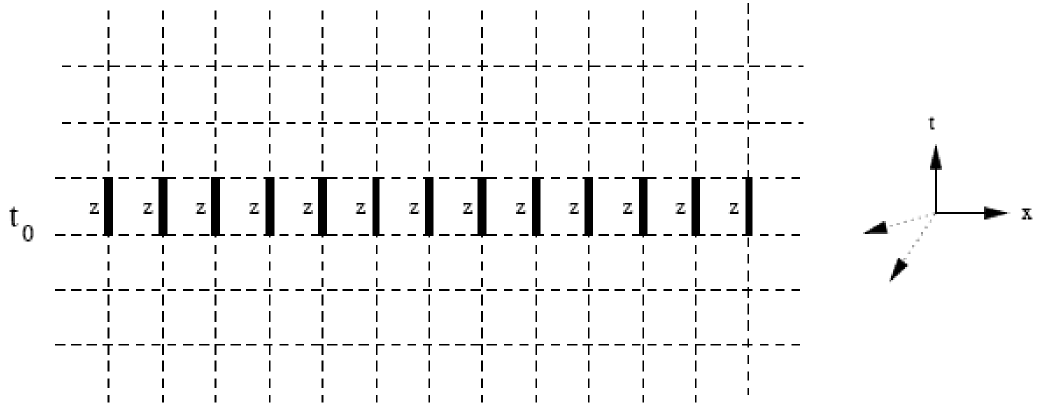


Figure 4: The global center transformation. Each of the indicated links in the t-direction, at $t = t_0$, is multiplied by a center element z. The lattice action is left unchanged by this operation.

This transformation does not change plaquettes or Wilson loops, but Polyakov lines are multiplied by the center element **z** .

$\langle P \rangle = 0$ **iff center symmetry is unbroken.**

If we take “Confinement” to mean “Magnetic Disorder”, then

Confinement is the phase of unbroken center symmetry.

Question:

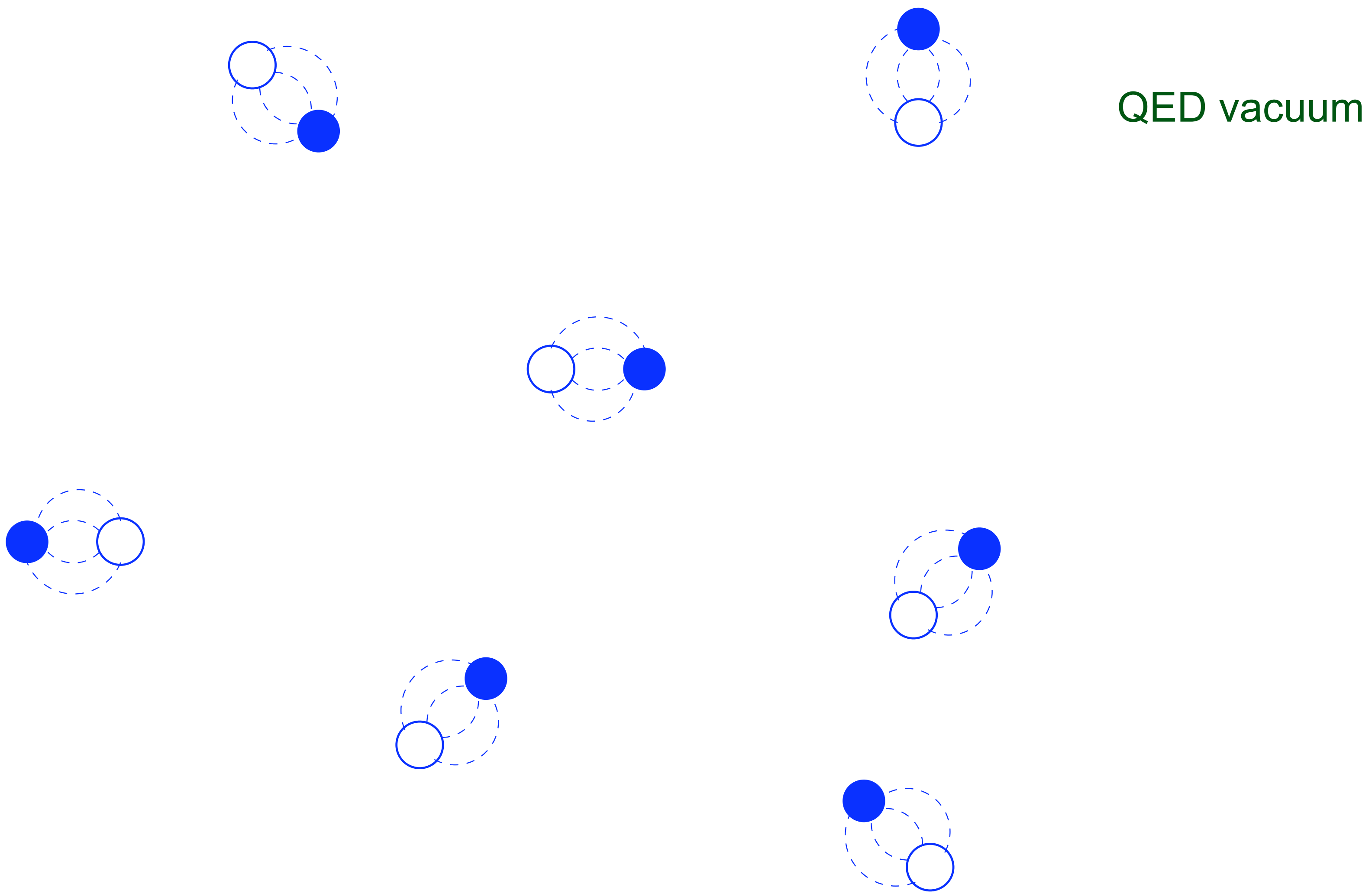
"If the center is so important, then what confines gluons?"

↑ spectrum sense!

Answer:

The same thing that "confines" large electric charge in QED.

In QED it is impossible to have an object of nuclear size having an electric charge much greater than $|Q_c| \approx 170$.



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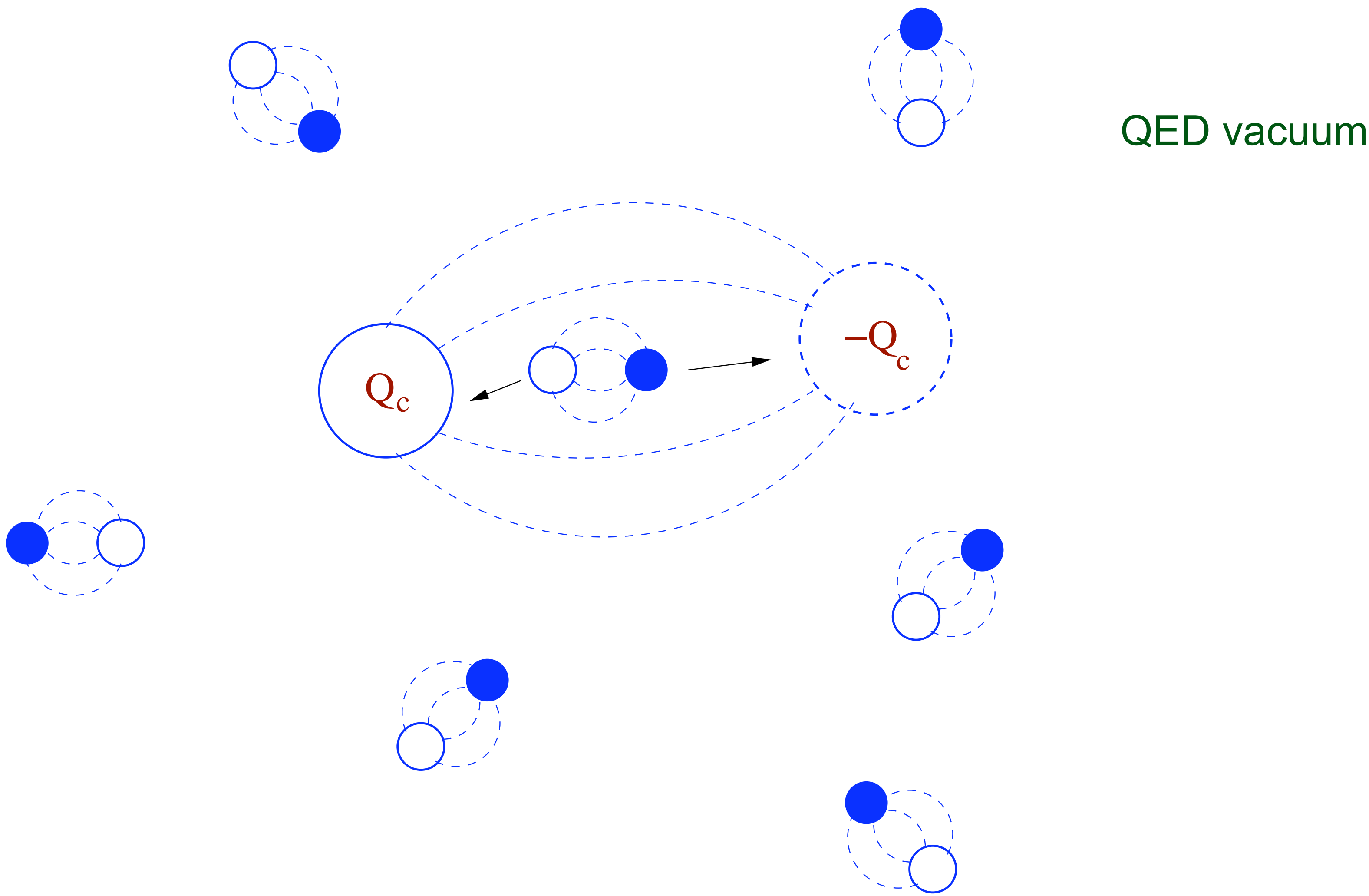

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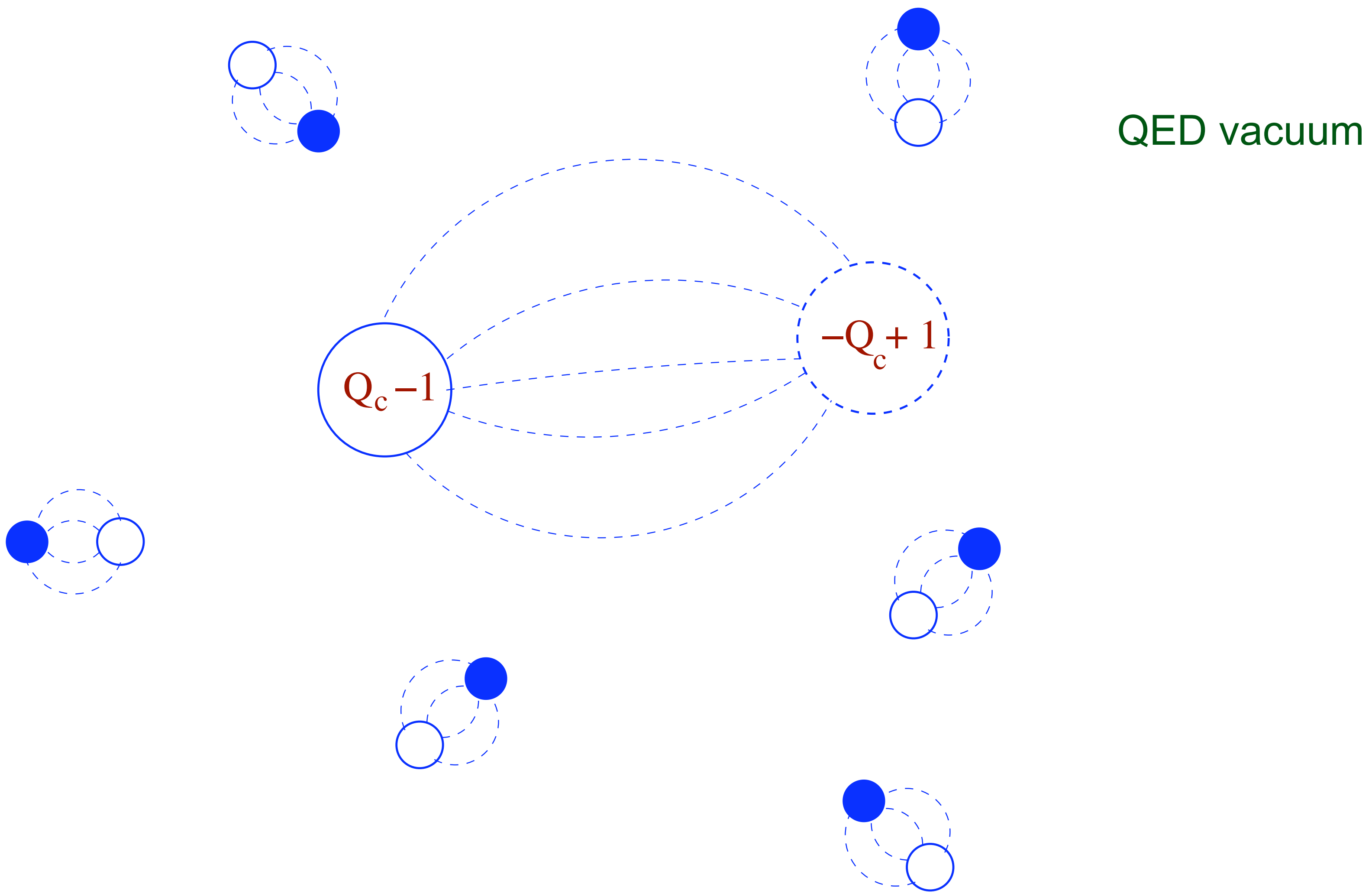
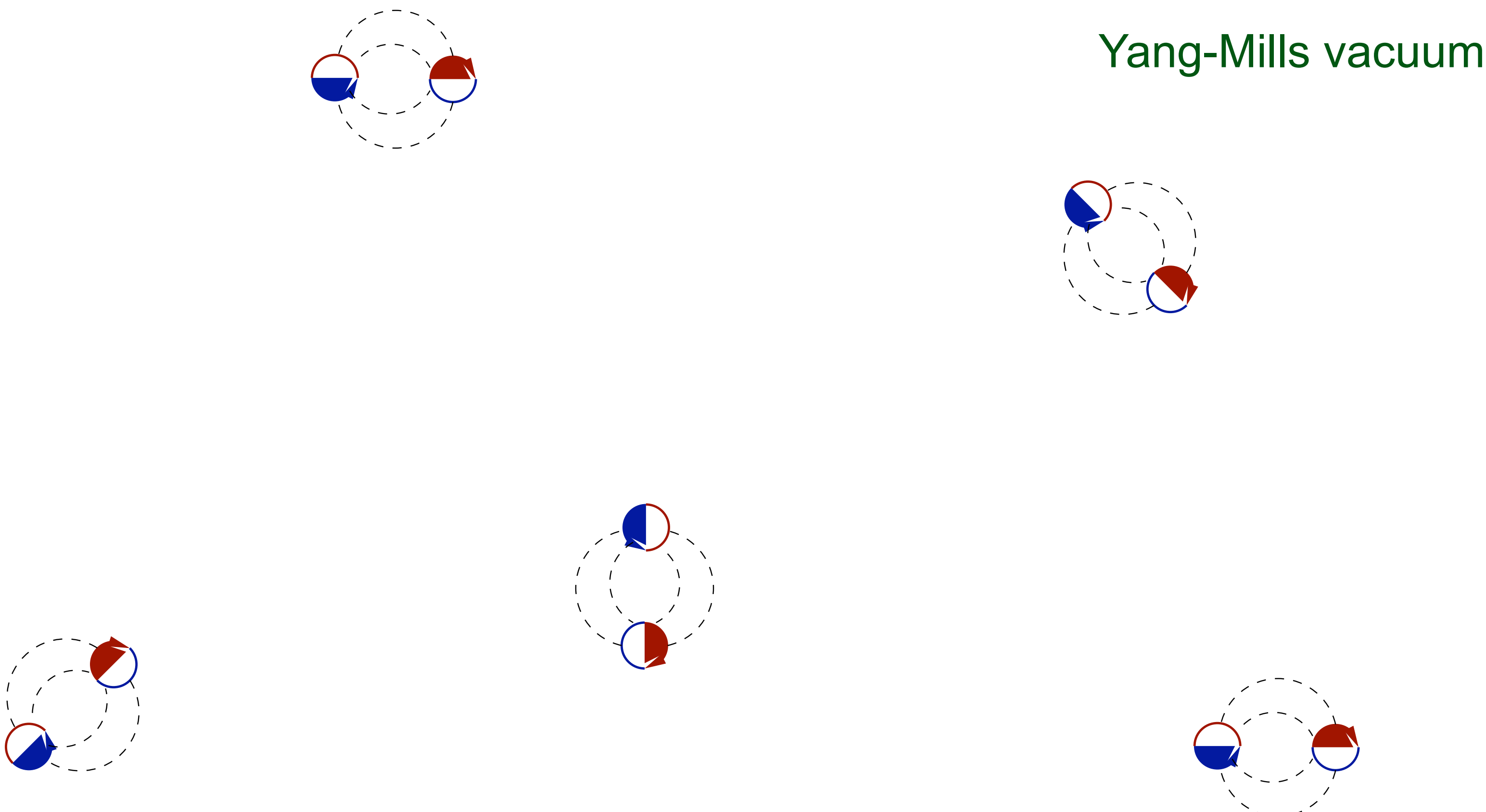


The same process goes on for adjoint charges in non-abelian theories, given sufficient charge separation



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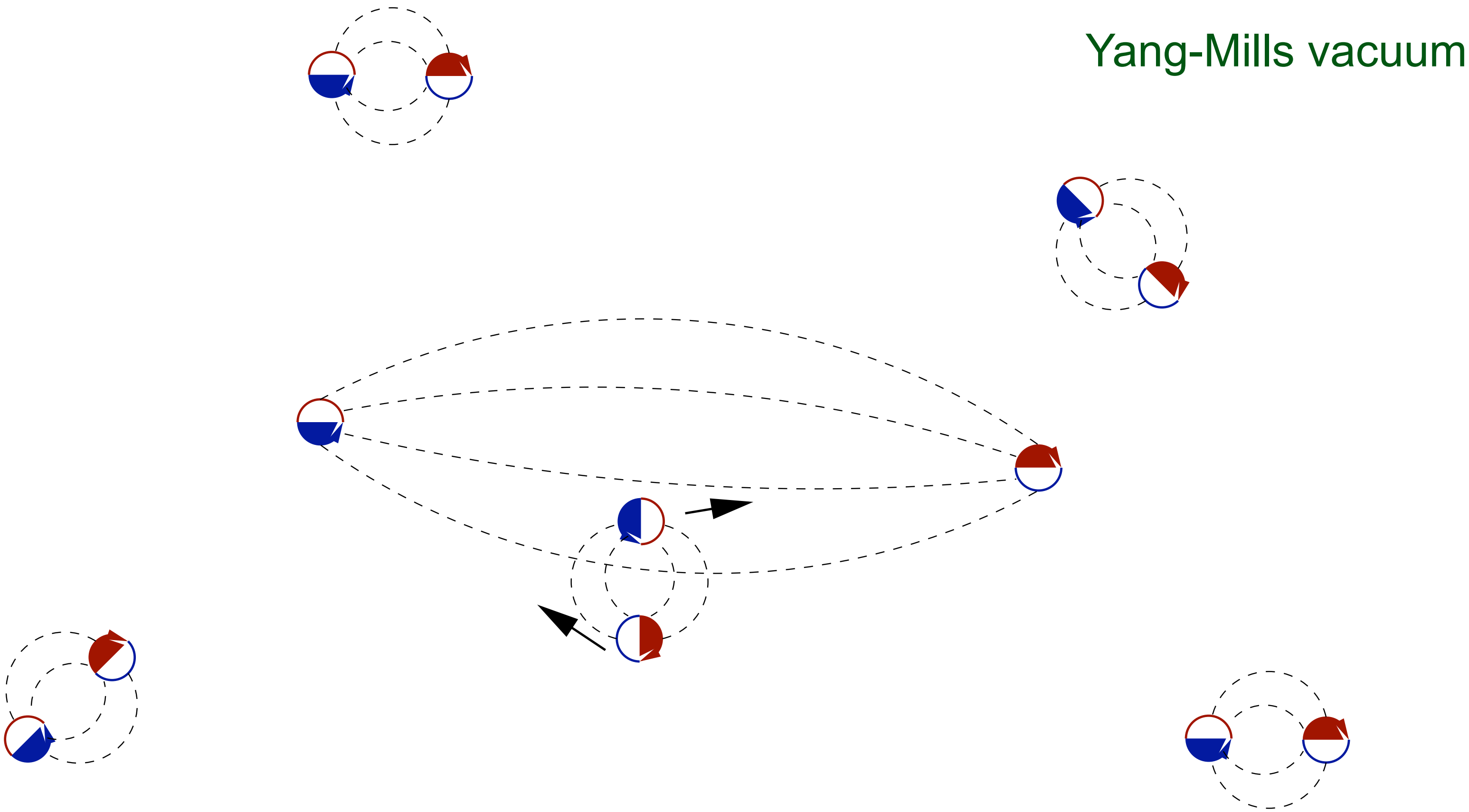

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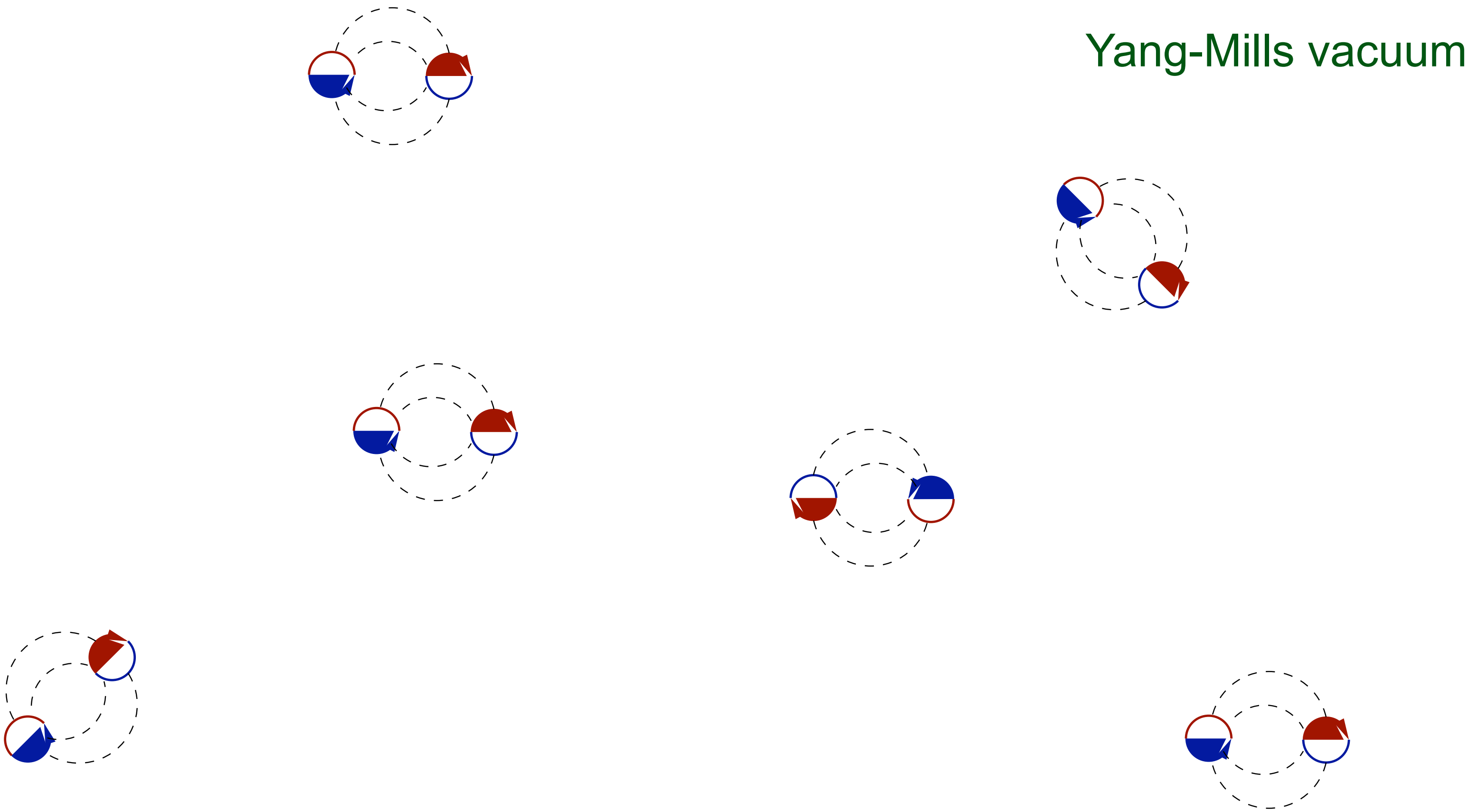

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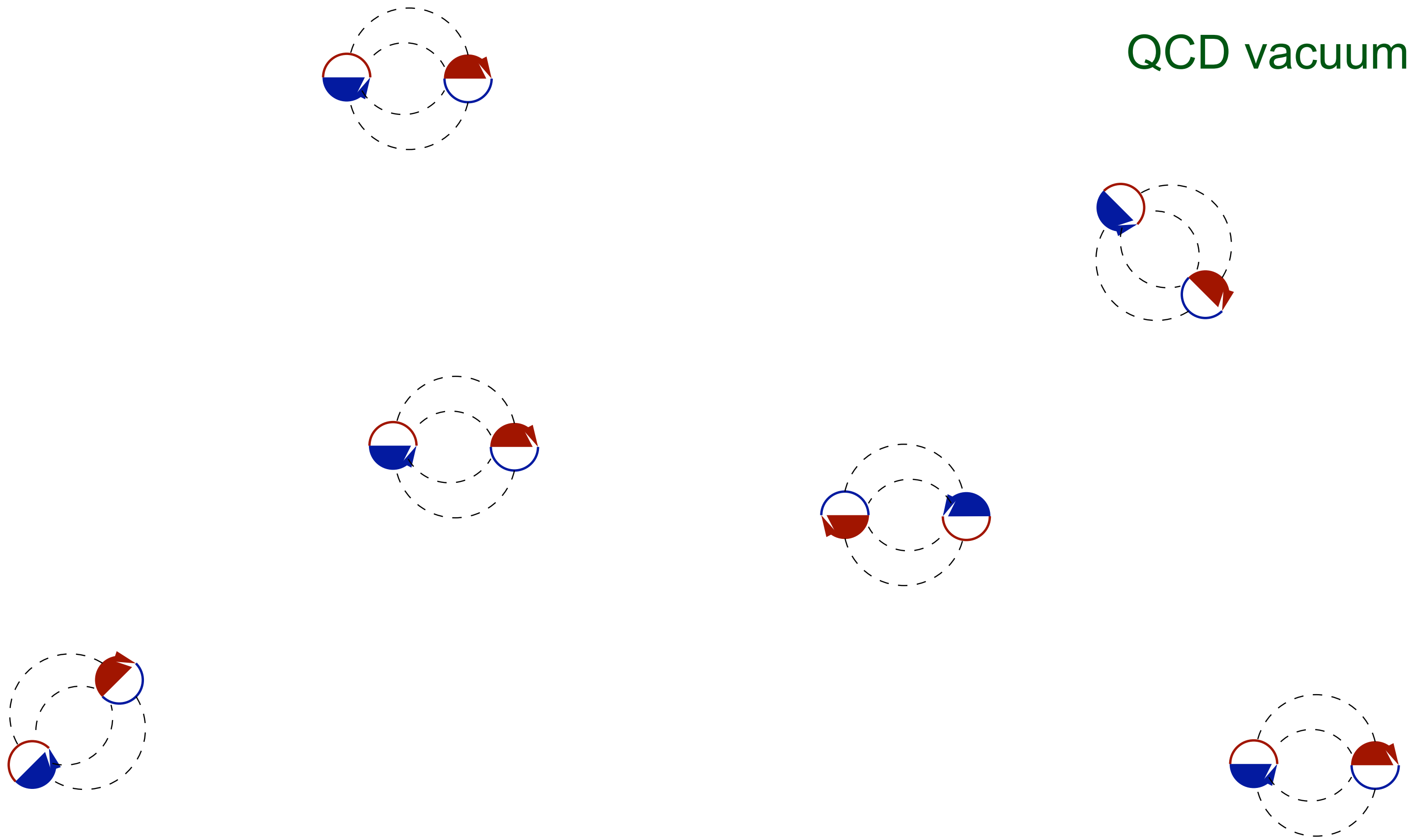

I prefer to call this “color screening”, rather than color “confinement”.

For theories with unbroken center symmetry, charges with non-zero N-ality cannot be *completely* screened by gluons.

These theories - mainly pure SU(2) and SU(3), but also higher N to some extent - have been studied extensively numerically.

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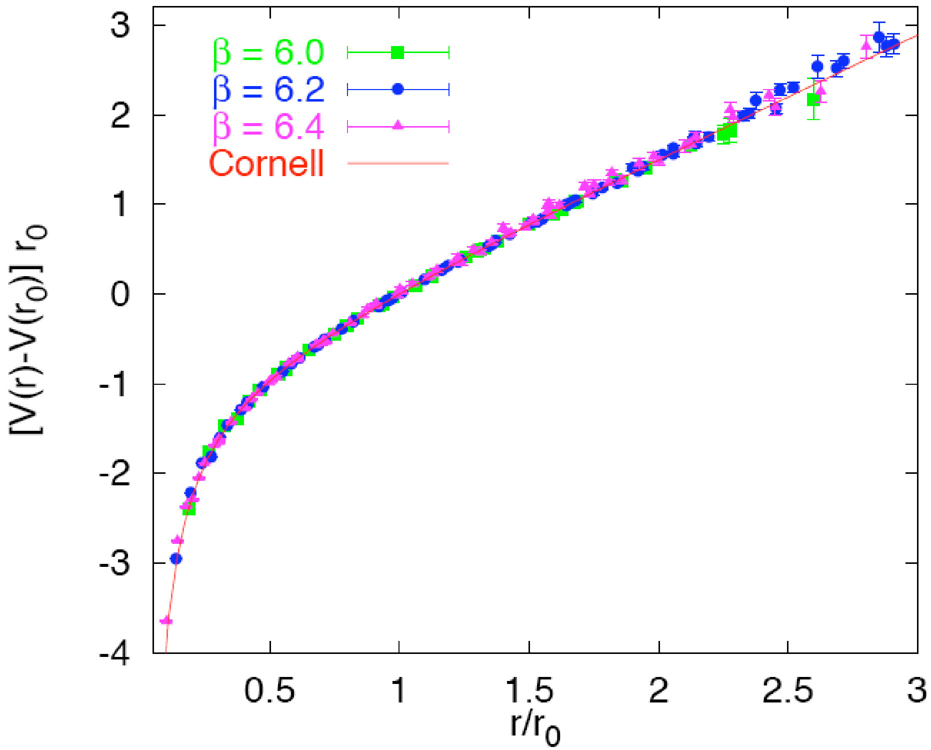
I. Linear potential



The static quark potential is asymptotically linear.

$$r_0 \approx 0.5 \text{ fm}$$

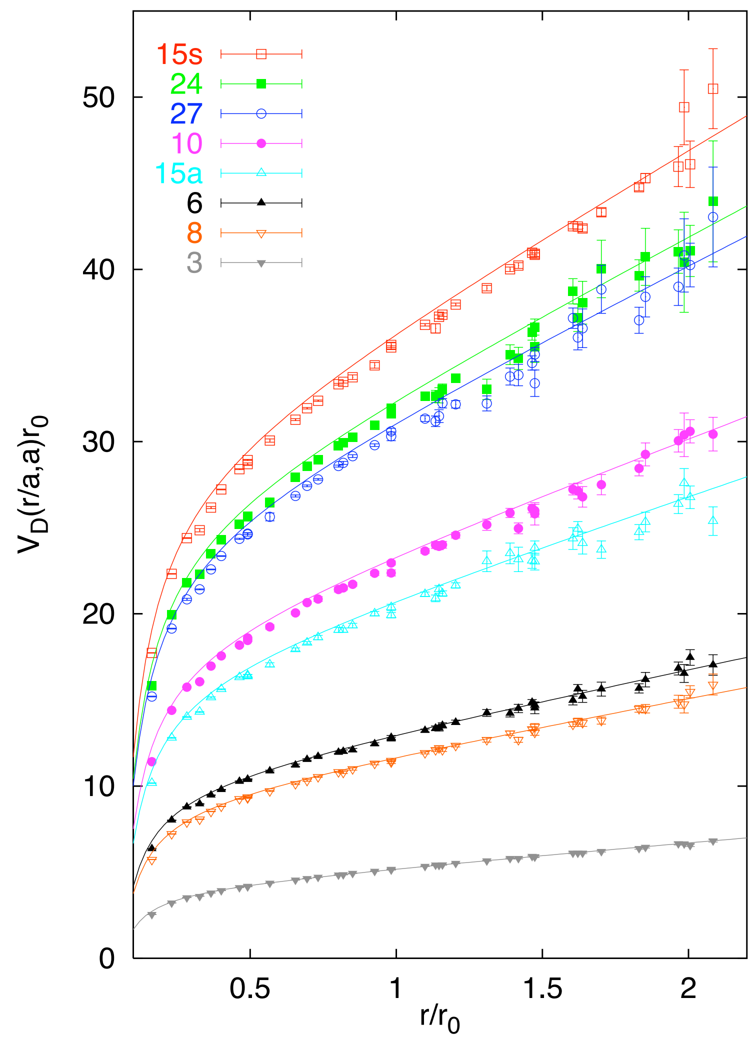
Bali (2000)

II. Casimir Scaling

For the static quark-antiquark pair in representation "r" of the color group:

At intermediate scales, string tensions are proportional to the quadratic Casimir of the color charge representation.

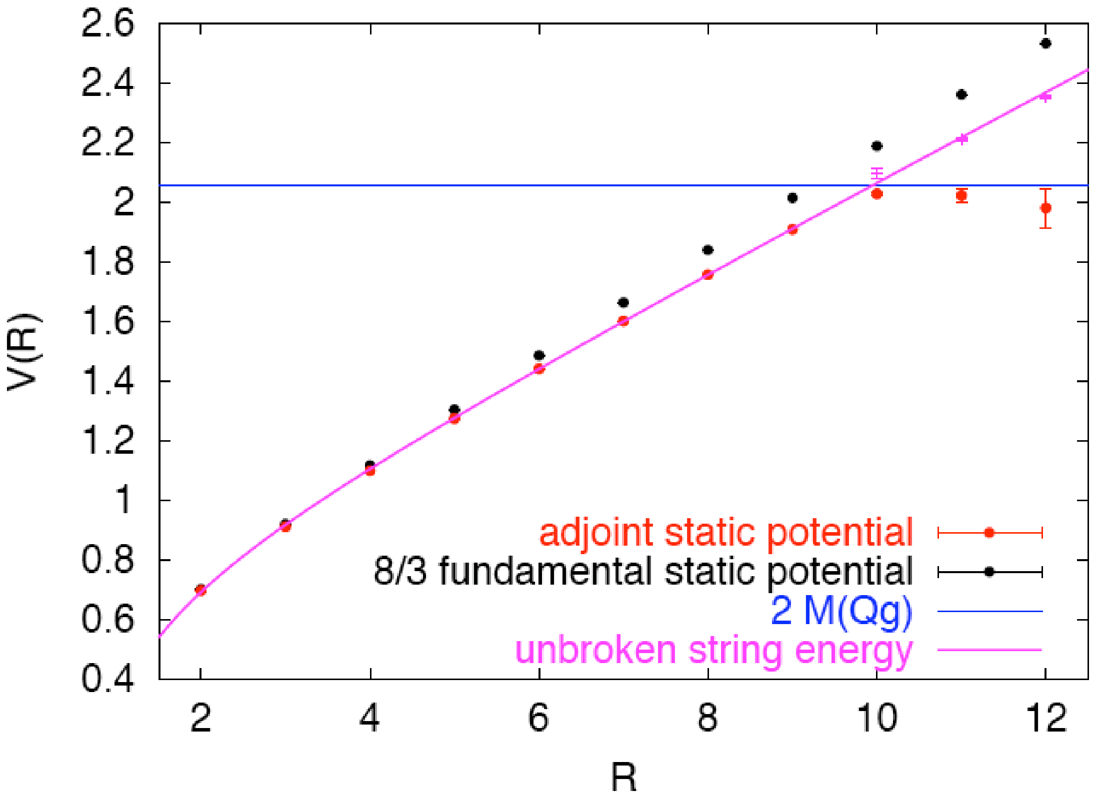
$$V_r(R) = \frac{C_r}{C_F} V_F(R)$$



(this numerical calculation is insensitive to string-breaking, and looks at metastable flux tubes)

III. Color Screening: N-ality

Asymptotically, the string tension depends only on the N-ality of the representation, not the quadratic Casimir.



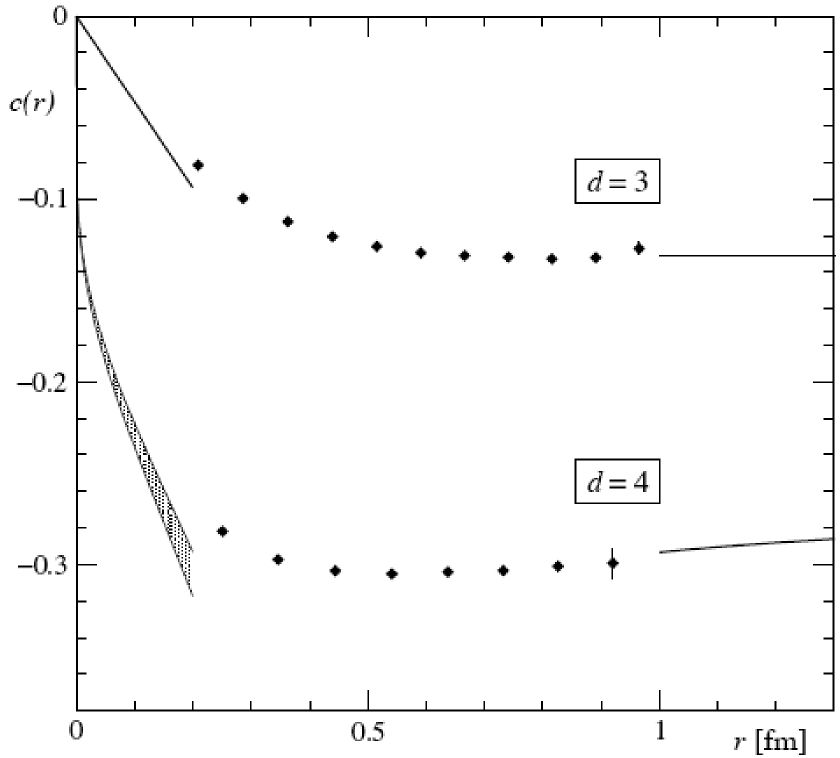
for the adjoint rep, N-ality = 0 , and the asymptotic string tension is also zero.

de Forcrand and Kratochvila (2002)

IV. String-like properties

the sign of these is the existence of a universal correction (the “Luscher term”) to the linear potential in D dimensions

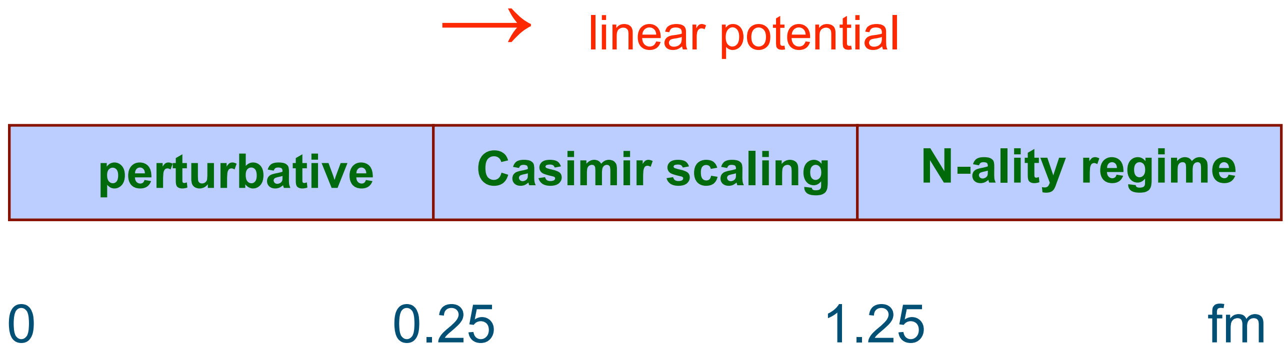
$$V_L(R) = -\frac{(d-2)}{24}\frac{1}{R}$$



Luscher and Weisz (2002)

$$V(R) = \sigma R + \frac{c(R)}{R}$$

So there is evidence, from numerical simulations, for qualitatively different behavior of V(R) at different distance scales:



For SU(N) gauge theories, the transition between Casimir scaling and N-ality runs off to infinity as $N \to \infty$.

In the N-ality regime, string tension can only depend on N-ality k, and there are two proposals on the table:

$$\frac{\sigma(k)}{\sigma_F} = \begin{cases} \dfrac{\sin(\pi k/N)}{\sin(\pi/N)} & \text{Sine Law scaling} \\[2ex] \dfrac{k(N-k)}{N-1} & \text{``Casimir'' k-string scaling} \end{cases}$$

asymptotic, not intermediate, string tensions

Neither seems to be exact.

Explanations of confinement generally fall into one of a few broad categories...

Current Approaches

I. “Topology” - special field configurations

a. Center Vortices
b. Monopoles
c. Calorons

II. “Propagators”

a. confining Coulomb potential
b. Dyson-Schwinger Equations

III. Vacuum Wavefunctionals

IV. AdS/CFT (other talks at this meeting?)

Center Vortices

't Hooft (1978)

Motivations:

1. The asymptotic string tension in pure gauge theories depends only on the N-ality of the static charges

   (i.e. how the charges transform under the center subgroup of the gauge group)

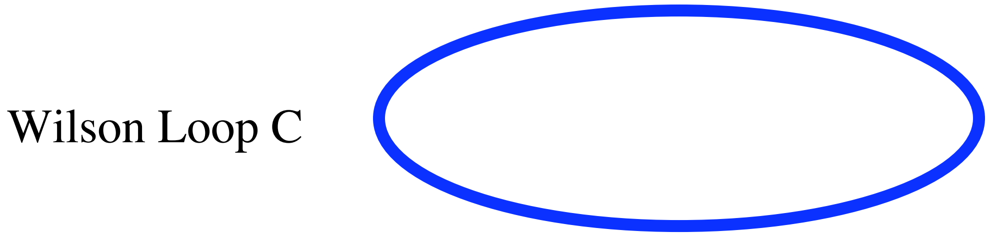
2. All of the unambiguous order parameters for confinement indicate that confinement is the phase of unbroken center symmetry.

The only scenario I know of, which explains point 1 in terms of vacuum field configurations, is the center vortex mechanism.

A center vortex is a loop of quantized magnetic flux which sweeps out a (thick) sheet as it propagates in time.

Creation of a center vortex, topologically linked to a Wilson loop, multiplies the Wilson loop by a center element.

$$U(C) = P\exp\left[i\oint_C dx^\mu \; A_\mu\right]$$



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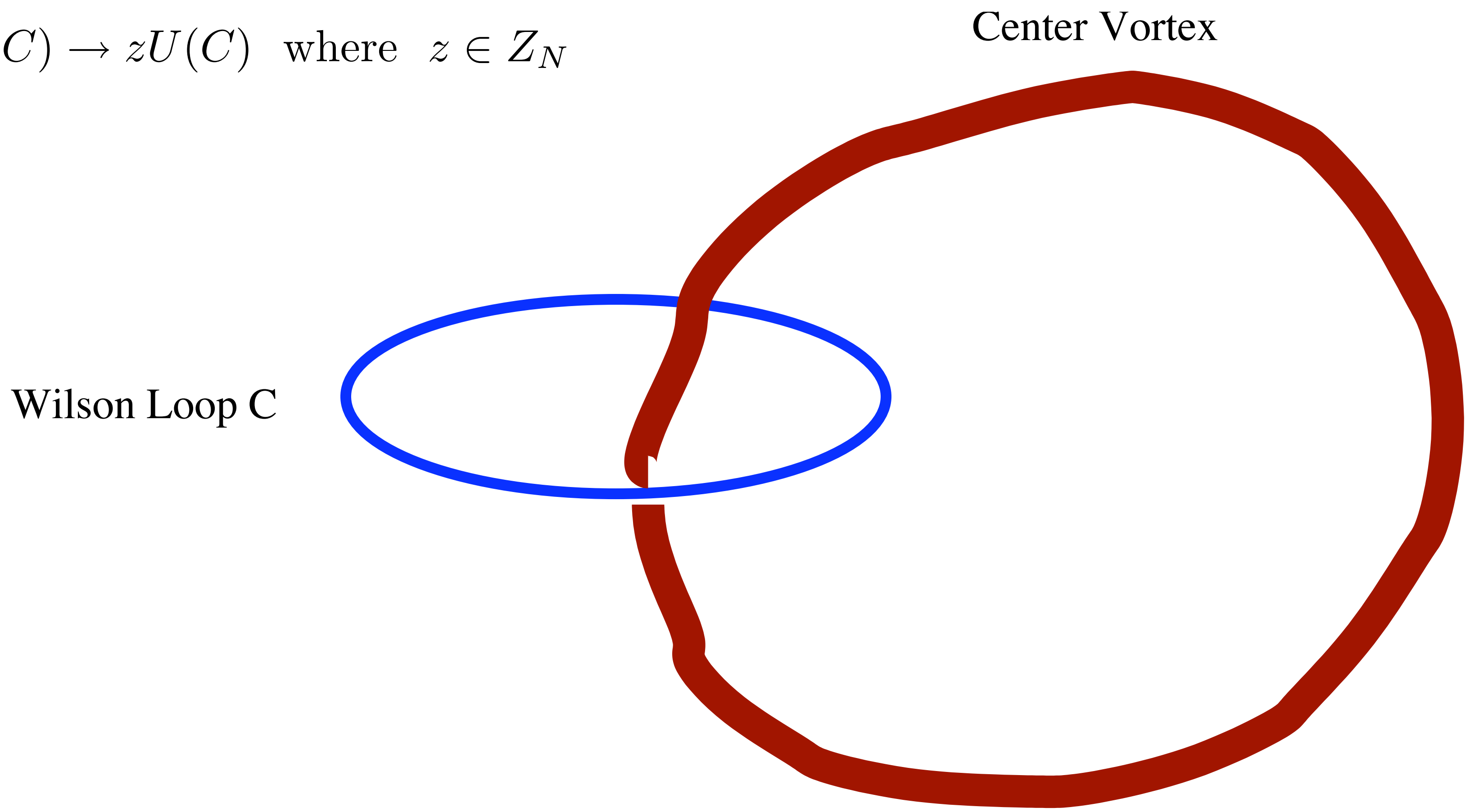
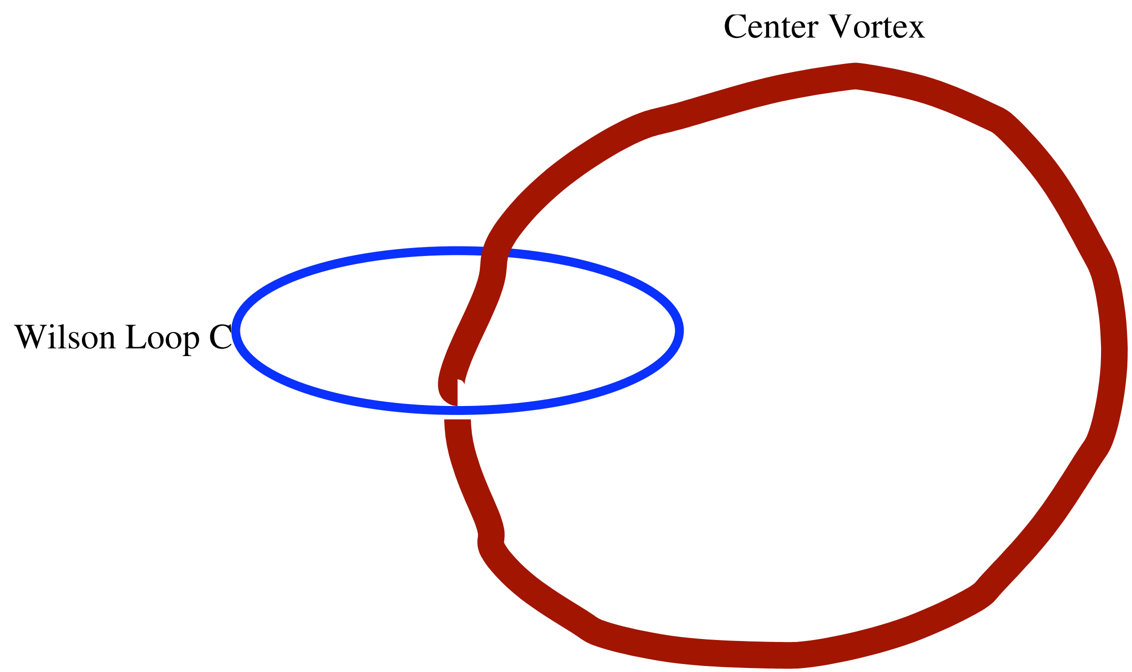
$$U(C) \to zU(C) \quad \text{where} \quad z \in Z_N$$





Area law   W(C) ~ exp[-Area(C)] is due to fluctuations in the number of vortices linked to loop C.

Asymptotic string tension depends only on N-ality.

There is a lot of numerical evidence in favor of this picture

based on methods, developed in 1997-98, for locating vortices in lattice configurations

1. Vortex linking number is correlated with the phase of the Wilson loop;

2. Vortices by themselves give about the right string tension;

3. Plaquette action is high on vortex surfaces;

4. Vortex density scales according to asymptotic freedom

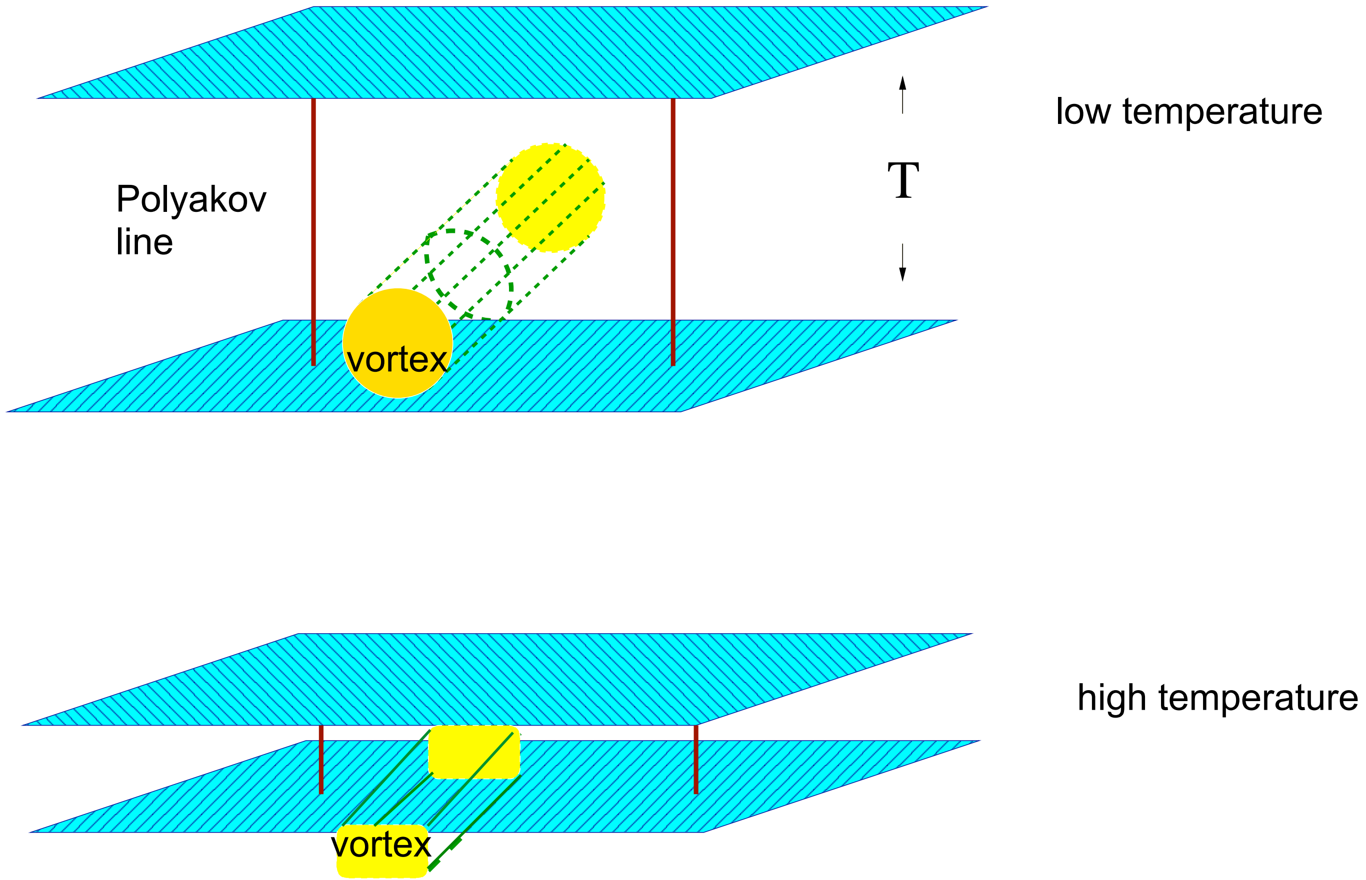
5. when vortices are removed from lattice configurations
   a. the string tension vanishes, and
   b. chiral symmetry breaking goes away

6. vortex thickness agrees with independent estimates (adjoint string breaking, vortex free energy measurements)

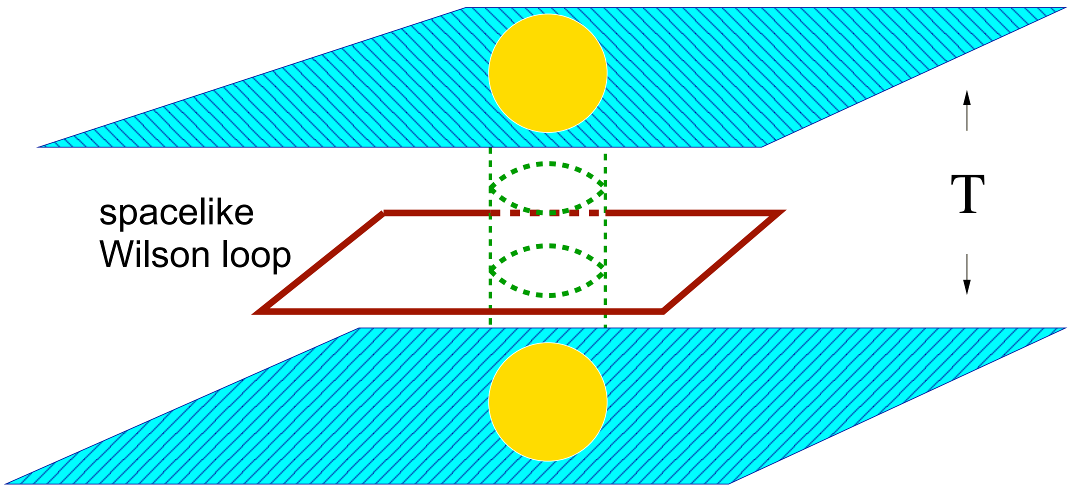
7. spacelike string tension at high T comes from vortices closed in the periodic time direction.

Faber, Olejnik, & JG,
Tubingen group: Reinhardt, Engelhardt,Langfeld...
ITEP group: Polikarpov, Gubarev, Zakharov,...
de Forcrand et al.

Deconfinement: As temperature increases, *spacelike* vortices get "squeezed", vortex free energy goes up, vortex percolation disappears, no confinement.



Timelike vortices (closed in the periodic time direction) do *not* get squeezed at high temperature, and remain to give an area law to spacelike Wilson loops.



$$V_r(R) = \frac{C_r}{C_F} V_F(R)$$

N-ality dependence is fine for the asymptotic string tension.

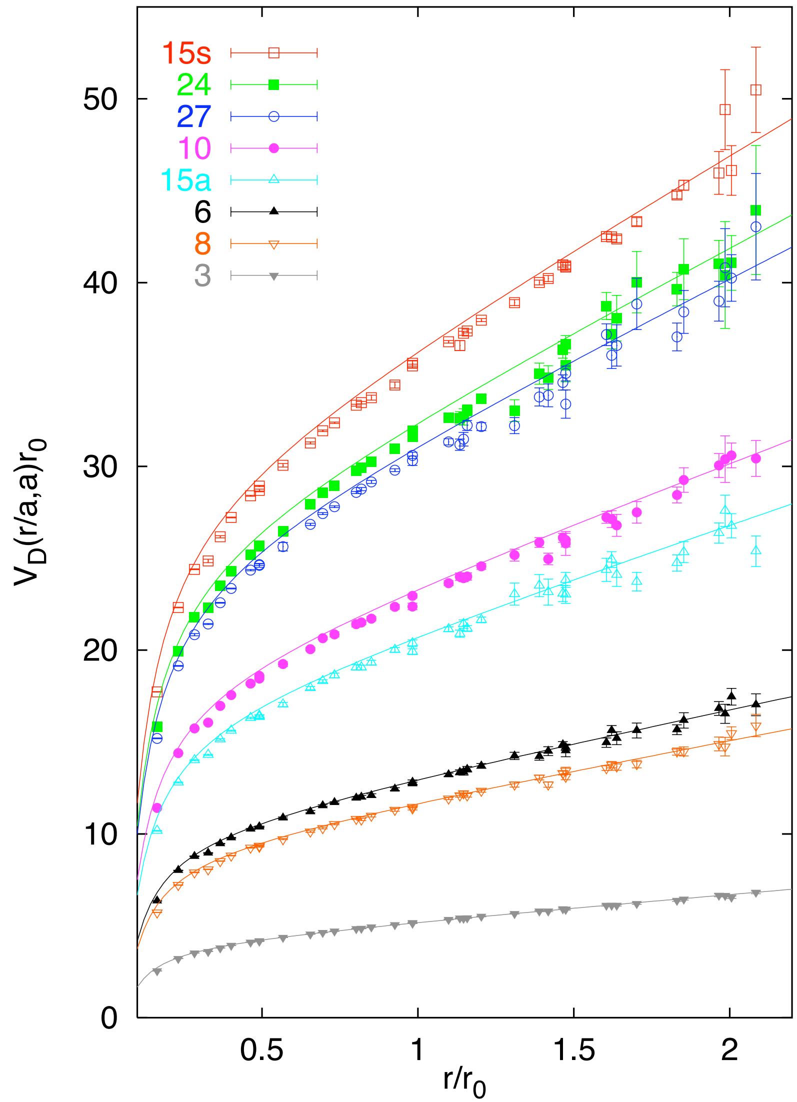
However, at intermediate scales, string tensions are proportional to the quadratic Casimir of the color charge representation, not the N-ality.

“Casimir Scaling”

In the vortex scenario, this can be explained in terms of

a. finite vortex thickness
b. fluctuations within vortices

Langfeld, Olejnik, Reinhardt, Tok & JG (2006)



Group Disorder and Center Disorder

K. Langfeld, S. Olejnik, H. Reinhardt, T. Tok, & J.G. (2006)

Representation dependence of the string tension:

- Casimir scaling at intermediate distances;
- N-ality dependence asymptotically.

The latter is due to string-breaking by gluons and/or matter fields. No big mystery.

But this is a “particle” explanation...

What is the “field” explanation, for both Casimir *and* N-ality behavior, in terms of vacuum fluctuations which dominate the relevant functional integral?

Basic idea: In a surface slice, the vacuum is dominated by overlapping center domains on some scale R. Fluctuations within each domain (beyond the confinement scale) are subject only to the weak constraint that the total magnetic flux adds up to a center element of the gauge group.

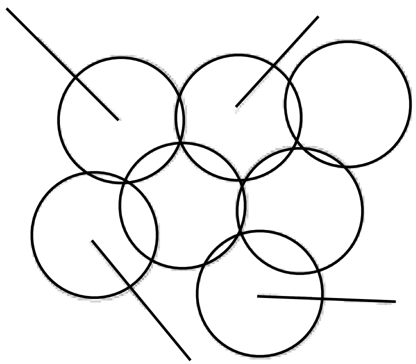


FIG. 1: A 2D slice of the D=4 Yang-Mills vacuum. Circular regions with (Dirac) lines correspond to $z = -1$ domains, circular regions without lines denote $z = +1$ domains.

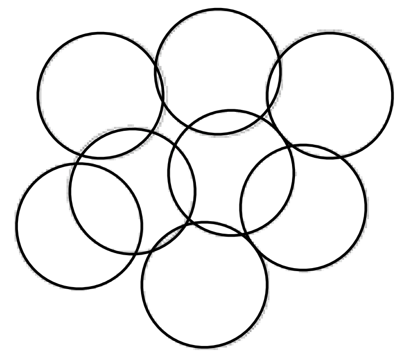
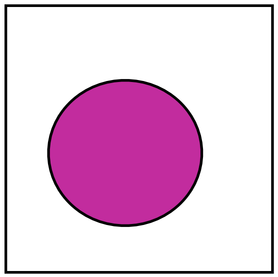


FIG. 2: A 2D slice of the D=4 vacuum of G(2) gauge theory. There is only one type of domain, corresponding to the single element of the center subgroup.

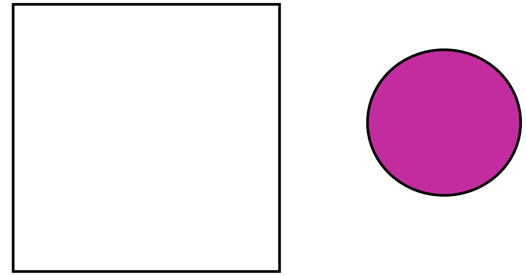
Fluctuations within a domain → Group disorder, Casimir scaling

Existence of domains → Center disorder, N-ality

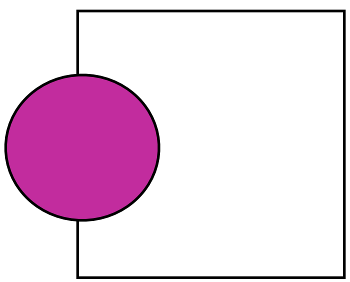
A simple model: center domain in the plane of a Wilson loop contributes a factor



$$z_n \in Z_N$$



$$z_0 = 1$$



$$z = S e^{i\vec{\alpha}^n \cdot \vec{H}} S^\dagger$$

(S a random group element)

$$\overline{z} = \frac{1}{d_r} \chi_r \left[\exp[i\vec{\alpha}^n \cdot \vec{H}] \right]$$

(average over S)

where χ_r is the group character, $\vec{H}$ the generators of the Cartan subalgebra, and the $\vec{\alpha}^n$ depend on the overlap of the domain with the interior of the loop.

$\overline{z}[\alpha]$ is proportional to the quadratic Casimir for small α, and goes to a center element (which may be $z_0 = 1$) for enclosed domains.

$\vec{\alpha} \cdot \vec{H}$ represents the average magnetic flux in the overlap region of loop and domain.

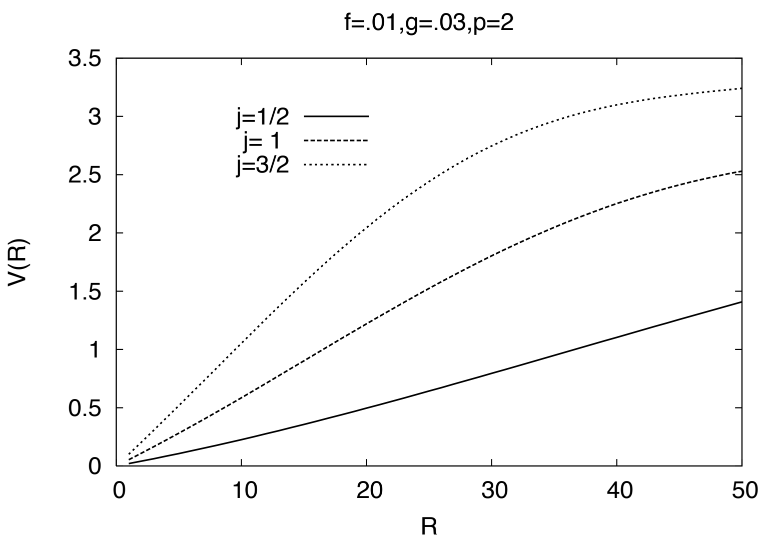
We suppose that fluctuations in different regions of each domain are correlated only by the constraint that they add up to center element.

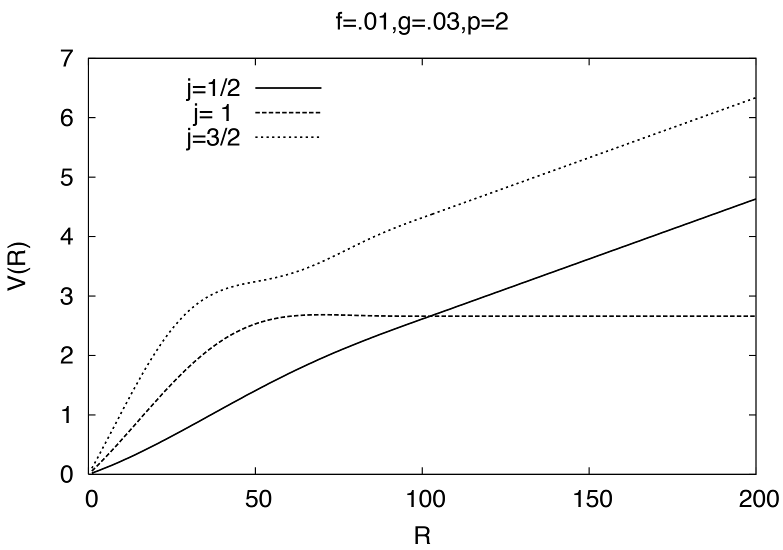
If A_D is the area of the domain, and A is the area contained in the loop, then for SU(2) we get

$$\begin{aligned}
\left(\alpha^1(x)\right)^2 &= \text{const.}\left[\frac{A}{A_D} - \frac{A^2}{A_D^2}\right] + \left(2\pi\frac{A}{A_D}\right)^2 \\
\left(\alpha^0(x)\right)^2 &= \text{const.}\left[\frac{A}{A_D} - \frac{A^2}{A_D^2}\right]
\end{aligned}$$

Difference between G(2) and SU(2): G(2) has only one type of center domain, only α^0 contributes, string tension is asymptotically zero.

For SU(2), the domain model gives results for the static potential like these:





Casimir scaling
(short distance)

Color screening
(asymptotic)

Monopoles

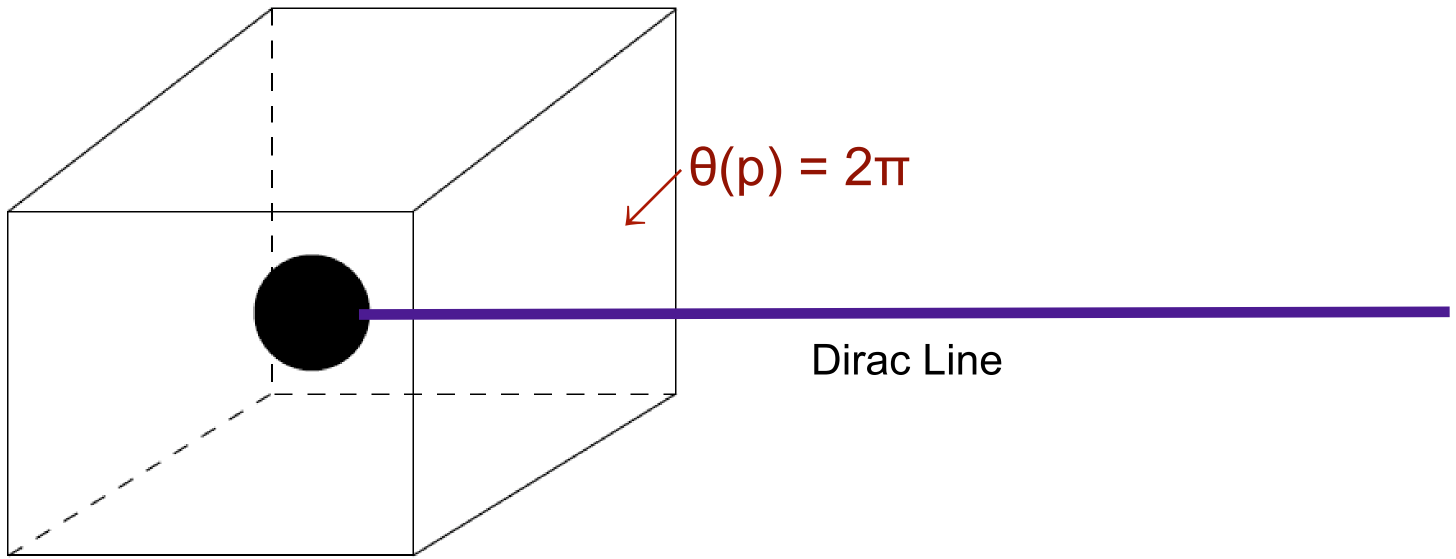
lattice investigations by the
Kanazawa group (Suzuki et al.)
Pisa group (di Giacomo et al.)
ITEP group (Polikarpov et al.)
among others...

Motivations:

1. "Dual Superconductivity" ('t Hooft & Mandelstam)
2. Compact U(1) in 2+1 dimensions (Polyakov)
3. Seiberg-Witten model

Confinement in Compact QED$_3$ (the monopole Coulumb gas)

Compact QED has monopole as well as photon excitations



$$U(p) = e^{i\theta(p)}$$

Polyakov showed that in D=3 dimensions, compact QED could be expressed as a monopole Coulomb plasma, with partition function

$$Z_{mon} = \sum_{m(r)=-\infty}^{\infty} \exp\left[-\frac{2\pi^2}{g^2 a}\sum_{r,r'} m(r')G(r-r')m(r)\right]$$

where the m(r) are integer-valued fields living on the dual sites of the lattice, and G(r) ~ 1/r .

Introduce a Wilson loop $\exp[i\oint_C dx^\mu A_\mu]$ into the partition function, where

$$\oint_C dr_\mu A_\mu(r) = \int_{S(C)} dS_\mu(r)\, H_\mu(r) = \int d^3r\, \eta_{S(C)}(r)\, m(r),$$

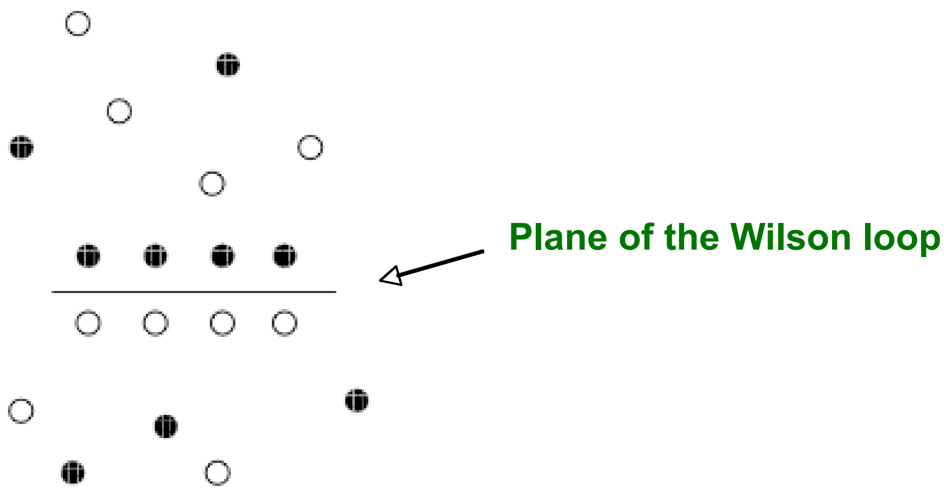
$$\eta_{S(C)}(r) = -\frac{1}{2}\frac{\partial}{\partial r_\mu}\int_{S(C)} dS_\mu(r')\, \frac{1}{|r-r'|}.$$

Everything can be calculated explicitly in D=3 dimensions, with the result, for a Wilson loop corresponding to abelian charge n ,

$$W_n(C) = \left\langle \exp\left[in \oint_C dx^\mu A_\mu \right] \right\rangle$$

$$= \exp[-n\sigma \mathrm{Area}(C)]$$

A very rough image of whats going on: monopoles and antimonopoles line up along the minimal area, and screen out the magnetic field that would be generated by the Wilson (current) loop source



Relativistic Superconductor

The abelian Higgs model

$$S = \int d^D x \left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left|(\partial_\mu + ieA_\mu)\phi\right|^2 + \frac{\lambda}{4}(\phi\phi^* - v^2)^2\right)$$

has a massive and a Coulomb phase. The Coulomb phase has Nielsen-Olesen vortices of magnetic flux, the analog of Abrikosov vortices in an ordinary type II superconductor, magnetic charge is confined.

Dual Superconductor Idea

The Higgs is magnetically, rather than electrically charged, it couples to a “dual” photon field. Electric, rather than magnetic fields are squeezed into flux tubes, and electrical charges are confined.

Extension to a non-abelian theory involves selection of an abelian subgroup of the gauge group, generated by the Cartan subalgebra. The subgroup is identified either via a Higgs field in the adjoint representation, or by imposition of an "abelian projection" gauge, which leaves the subgroup unfixed:

$$SU(N) \rightarrow U(1)^{N-1}$$

The most common such gauge is the **maximal abelian gauge**, which makes the gauge fields as abelian as possible. In lattice SU(2), the gauge maximizes the quantity

$$R = \sum_x \sum_\mu \mathrm{Tr}[U_\mu(x)\sigma_3 U_\mu^\dagger(x)\sigma_3]$$

Abelian projection means: project each link variable to the closest abelian group element.

Simplest versions of monopole confinement have difficulties with N-ality for Wilson loops in the abelian subgroup. E.g., for $SU(2) \rightarrow U(1)$, consider the double-charged abelian loop

$$W_2[C] = \left\langle P \exp \left[2i \oint dx^\mu A^3_\mu \frac{\sigma_3}{2} \right] \right\rangle$$

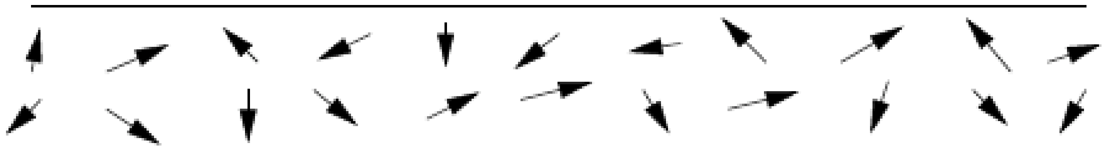
This loop has an area law in monopole Coulomb gas and dual superconductor pictures.

But in fact, the loop can be screened by abelian charged gluons, and the string tension is zero asymptotically.

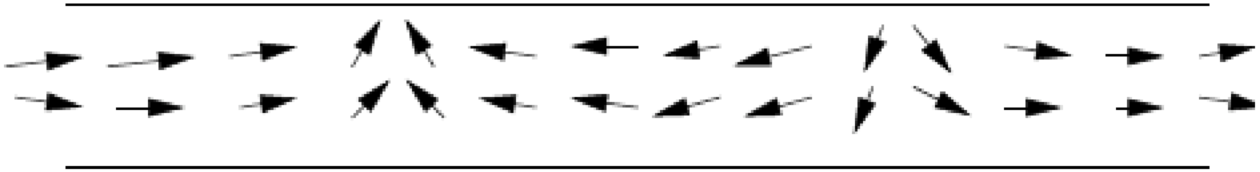
If the abelian charged degrees of freedom are integrated out, the resulting abelian theory must respect N-ality somehow.

In ‘t Hooft’s abelian projection picture, it turns out that the abelian monopole worldlines lie on vortex sheets.

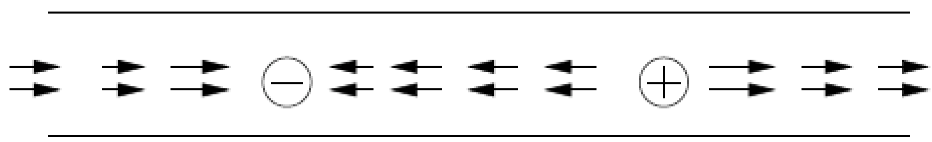
In the absence of gauge-fixing, the vortex field B^a points in random directions in the Lie algebra



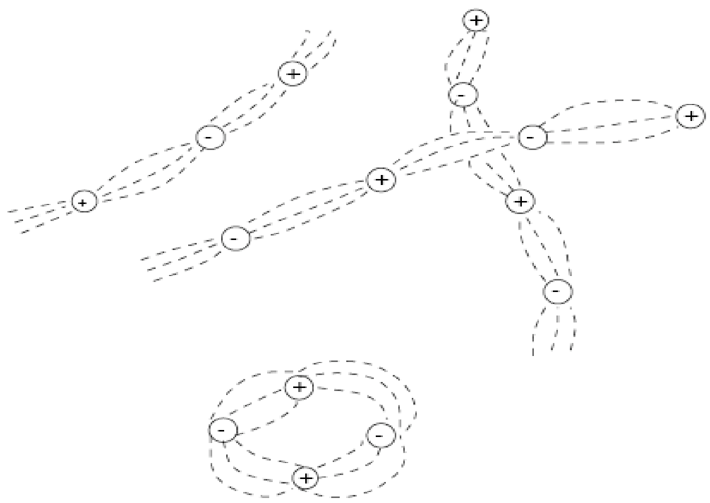
For the SU(2) gauge group, fixing to maximal abelian gauge, the field tends to line up in the $\pm\sigma_3$ direction. But there will still be regions where the B-field rotates in group space, from $+\sigma_3$ to $-\sigma_3$.



If we keep only the diagonal part of the link variables ("abelian projection"), a center vortex appears as a ***monopole-antimonopole chain***, with flux running between a monopole and neighboring antimonopole



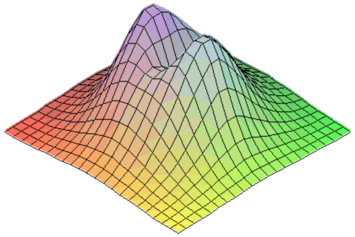
Then a typical vacuum configuration at a fixed time, after abelian projection, looks something like this:



This is the picture found in lattice simulations.

Ambjorn, Giedt, & JG, 2000

de Forcrand & Pepe, 2001



Calorons

van Baal, Bruckman, Diakonov, Ilgenfritz, Mueller-Preussker, Gattringer, Garcia-Perez...

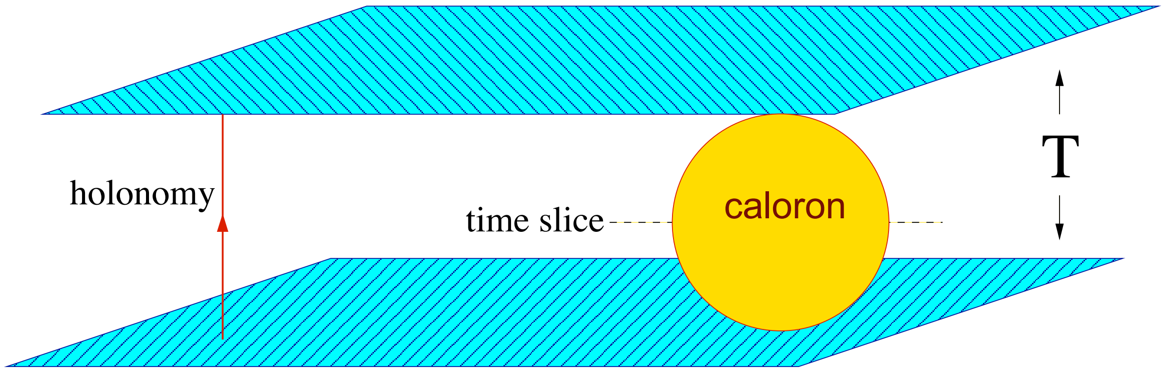
Calorons are instantons at finite temperature (volumes with finite time extent).

Kraan-van Baal-Lee-Lu calorons have non-trivial (i.e. non-center) Polyakov lines P_∞ asymptotically far from the caloron, and have monopole constituents.

There is evidence of calorons on cooled/smeared lattices at low temperatures.

A confinement mechanism at low temperature ??

In principle certain types of calorons (with $\mathrm{Tr}[P_\infty] = 0$) can give the correct N-ality dependence for Polyakov line correlators.



What is interesting about calorons is that they have monopole constituents, and these can be far apart.

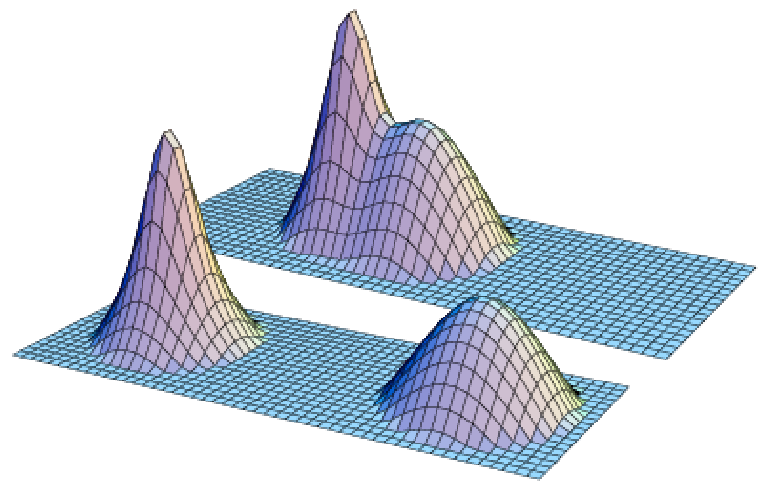


Figure 1. Two charge one SU(2) caloron profiles at $t = 0$ with $b = 1$ and $\mu_2 = -\mu_1 = 0.125$, for $\rho = 1.6$ (bottom) and 0.8 (top) on equal logarithmic scales (action density cutoff at $1/(2e^2)$).

caloron action density on a time-slice

The idea is that at low T, the monopole constituents of calorons are widely separated, and form a kind of *monopole Coulomb gas*, which leads to confinement.

A model calculation, by Diakonov and Petrov, gives just this result, with Sine-Law scaling of k-string tensions.

Their model calculation involves a guess for the appropriate measure of caloron collective coordinates.

However...

- Nobody really knows the right measure, and the Diakonov-Petrov guess has difficulties with non-positivity. (Ilgenfritz et al., 2009)
-
- Getting N-ality right for abelian Wilson loops seems problematic; I see no reason that the abelian string tensions should only depend only on N-ality.
-
- No explanation for the spacelike string tension above the deconfinement transition; a different mechanism is needed.
-
- Numerical evidence in favor of this mechanism is not very strong, at present.

The other main approach is

“Propagators”

Effects of the Gribov Horizon

In Landau gauge and Coulomb gauge there exist Gribov copies, all satisfying the given gauge-fixing condition.

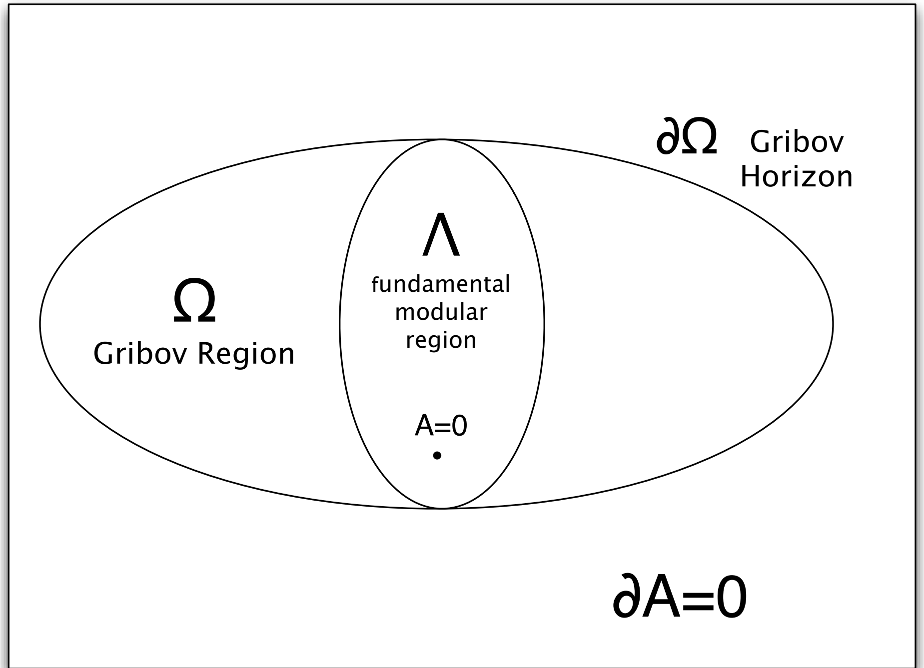
For BRST invariant actions, there is

Neuberger's Theorem

$$\langle Q \rangle = \frac{\int DU\, Dc D\overline{c}\; Q[U] e^{-(S+S_{gf})}}{\int DU\, Dc D\overline{c}\; e^{-(S+S_{gf})}} = \frac{0}{0}$$

Related to: In lattice regularization, there are even numbers of Gribov copies on a gauge orbit, half having a positive sign for the Faddeev-Popov determinant, half negative, so the sum over all copies vanishes.

Because of Gribov copies, the functional integration in Landau and Coulomb gauges should be restricted, e.g., to the Fundamental Modular Region Λ, where $\partial A = 0$ and ||A|| is minimized.



Gribov and Zwanziger argue that most of the volume within Λ (and Ω) is concentrated near the boundary $\partial\Omega$, *the Gribov Horizon*, where the Fadeev-Popov operator $\partial \cdot D$ has a zero eigenvalue.

Coulomb Confinement

In a non-abelian gauge theory, the Coulomb potential is given by

$$V_C(R) = -g^2 \left\langle \frac{1}{\nabla \cdot D}(-\nabla^2)\frac{1}{\nabla \cdot D} \right\rangle$$

where the VEV is taken in Coulomb gauge.

There are multiple gauge-equivalent configurations ("Gribov Copies") which satisfy the gauge-fixing condition

$$\nabla \cdot A = 0$$

At the Gribov Horizon, the Fadeev-Popov operator $\nabla \cdot D$ has a zero eigenvalue. If there is

i) a concentration of small eigenvalues for configurations near the horizon; and

ii) lattice configurations are typically close to the Gribov horizon;

then (it has been speculated) then this could lead to an enhancement of the Coulomb potential.

In fact, it *must* be so, because it can be shown rigorously that

$$V(R) \leq V_C(R)$$

(Zwanziger)

No confinement without Coulomb confinement.

eigenvalue equation of the F-P operator: $-\nabla \cdot D \ \phi_n(x) = \lambda_n \phi_n(x)$

Let $\rho(\lambda)$ be the eigenvalue density.

Coulomb self-energy of a static charge:

$$E_{self} = \frac{g^2 C_F}{N-1} \left\langle \left(\frac{1}{\nabla \cdot D} (-\nabla^2) \frac{1}{\nabla \cdot D} \right)^{aa}_{xx} \right\rangle$$

$$= \int_0^{\lambda_{max}} d\lambda \ \left\langle \rho(\lambda) \frac{(\phi_\lambda | (-\nabla^2) | \phi_\lambda)}{\lambda^2} \right\rangle$$

so the Coulomb confinement criterion is

$$\lim_{\lambda \rightarrow 0} \frac{\langle \rho(\lambda) (\phi_\lambda | (-\nabla^2) | \phi_\lambda) \rangle}{\lambda} > 0$$

which has been verified numerically (Olejnik, Zwanziger & JG, 2004) .

An easy way to compute the Coulomb potential: Define, in Coulomb gauge

$$\Psi_{q\overline{q}} = \overline{q}^a(0)q^a(R)\Psi_0$$

Then

$$G(R,T) = \langle \Psi_{q\overline{q}} | e^{-(H-E_0)T} | \Psi_{q\overline{q}} \rangle$$

and

$$\begin{aligned} V_C(R) &= \frac{\langle \Psi_{q\overline{q}} | H - E_0 | \Psi_{q\overline{q}} \rangle}{\langle \Psi_{q\overline{q}} | \Psi_{q\overline{q}} \rangle} \\ &= -\lim_{T\rightarrow 0} \frac{d}{dT} \log[G(R,T)] \end{aligned}$$

It's not hard to show that

$$\begin{aligned} G(R,T) &= \left\langle \Psi_{q\bar{q}} \left| e^{-(H-E_0)T} \right| \Psi_{q\bar{q}} \right\rangle \\ &= \left\langle L^\dagger(0,T) L(R,T) \right\rangle \end{aligned}$$

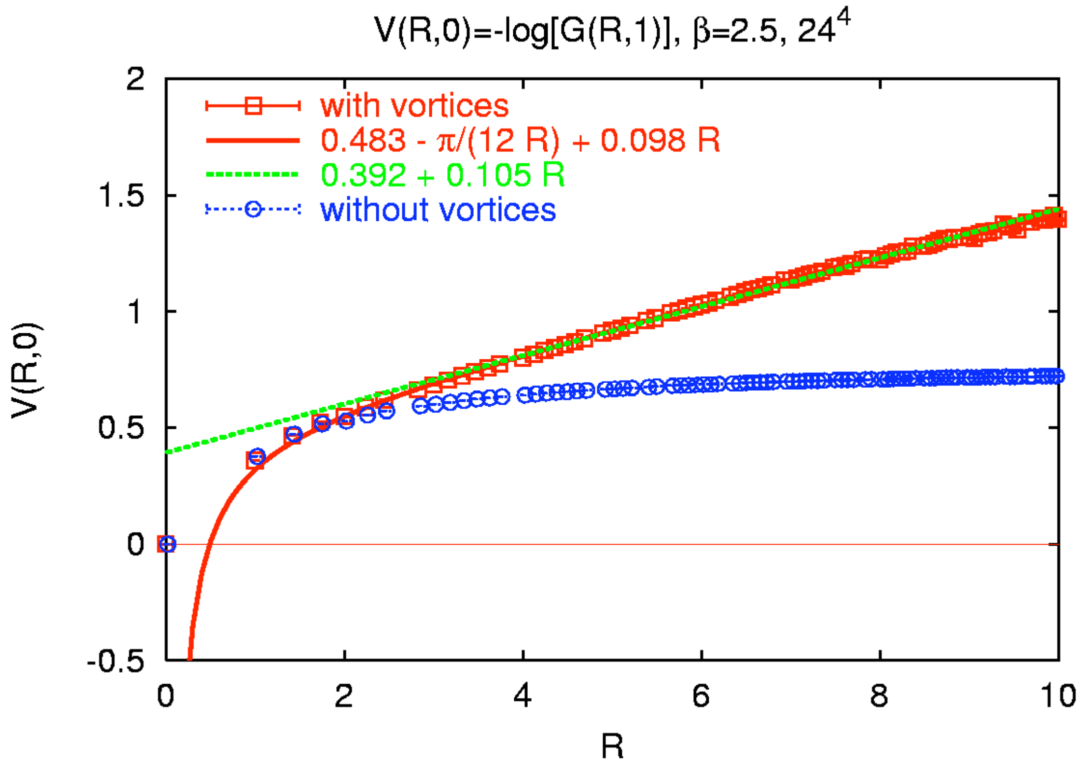
where

$$L(x,T) = P \exp\left[i \int_0^T dt \; A_0(x,t) \right]$$

is a timelike Wilson line. On the lattice, these are products of timelike link variables.

The upshot is, on the lattice, that the Coulomb potential can be extracted from the correlator of timelike link variables

$$V_C(R) = -\log\left[U_0^\dagger(\mathbf{x},t)U_0(\mathbf{x}+R,t)\right]$$

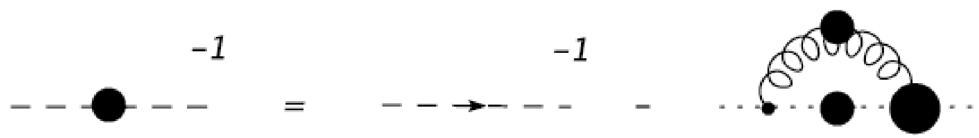


Olejnik & JG (2003)

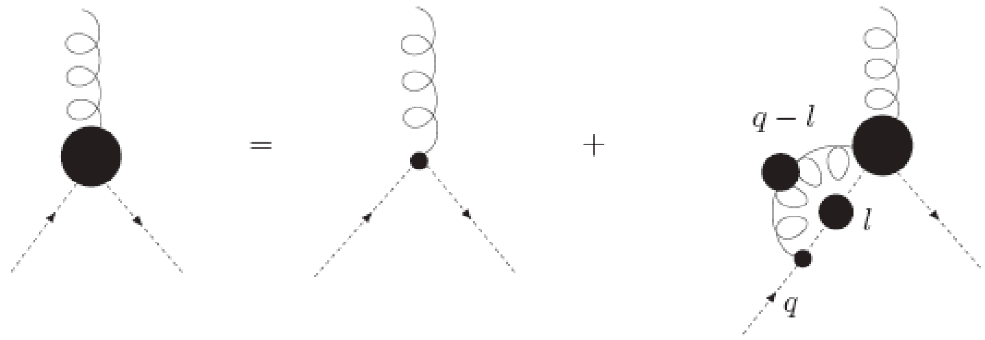
Dyson-Schwinger Equations

Alkofer, Fischer, Krassnig, Maris, Maas, Pawlowski, Roberts, von Smekal, Watson...

The idea is that DSE's for n-point functions may be soluble in the infrared. Look for power-law behavior. Diagramatically,



ghost propagator



ghost-gluon vertex

The claim is that these equations can be solved in a certain kinematical region (far infrared), and writing

$$D_{\mu\nu}(p) = \frac{Z(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \quad , \quad D_{ghost}(p) = -\frac{G(p^2)}{p^2}$$

Then

$$Z(p^2) \sim (p^2)^{2\kappa} \;\; \text{and} \;\; G(p^2) \sim (p^2)^{-\kappa}$$

with

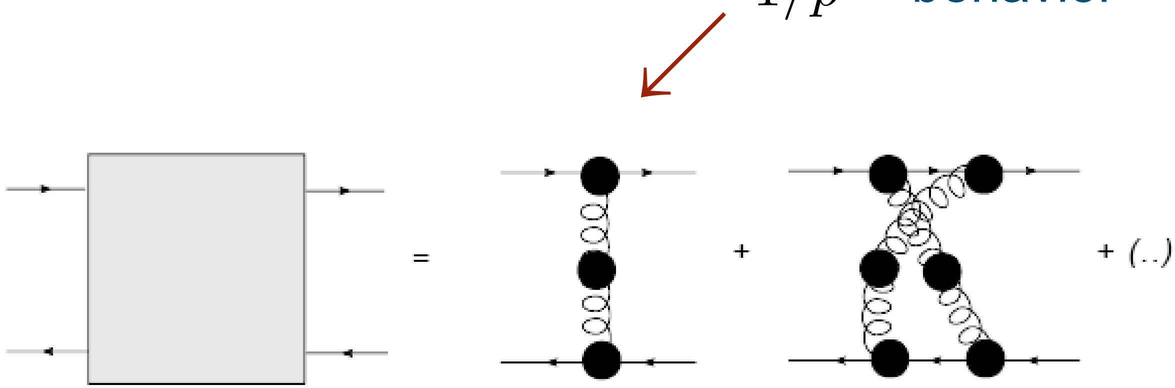
$$\kappa \approx 0.595353$$

So the gluon propagator is *less singular*, and the ghost propagator *more singular*, than the perturbative result.

This is called the **scaling solution**.

$1/p^4$ behavior

Alkofer, Fischer, Llanes-Estrada & Schwenzer, '08



The four-quark 1PI Green's function and the first terms of its skeleton expansion

Linear potential from "one-particle" exchange.

Unfortunately, the $\kappa > 0$ "scaling" solution appears to be **contradicted** by large volume lattice simulations, which support instead a different solution of the DSE's, the **decoupling solution**.

Lattice data on *huge* lattices (27 fm)^4 does not agree that the gluon propagator vanishes in the infrared...

Cuccieri & Mendes 07

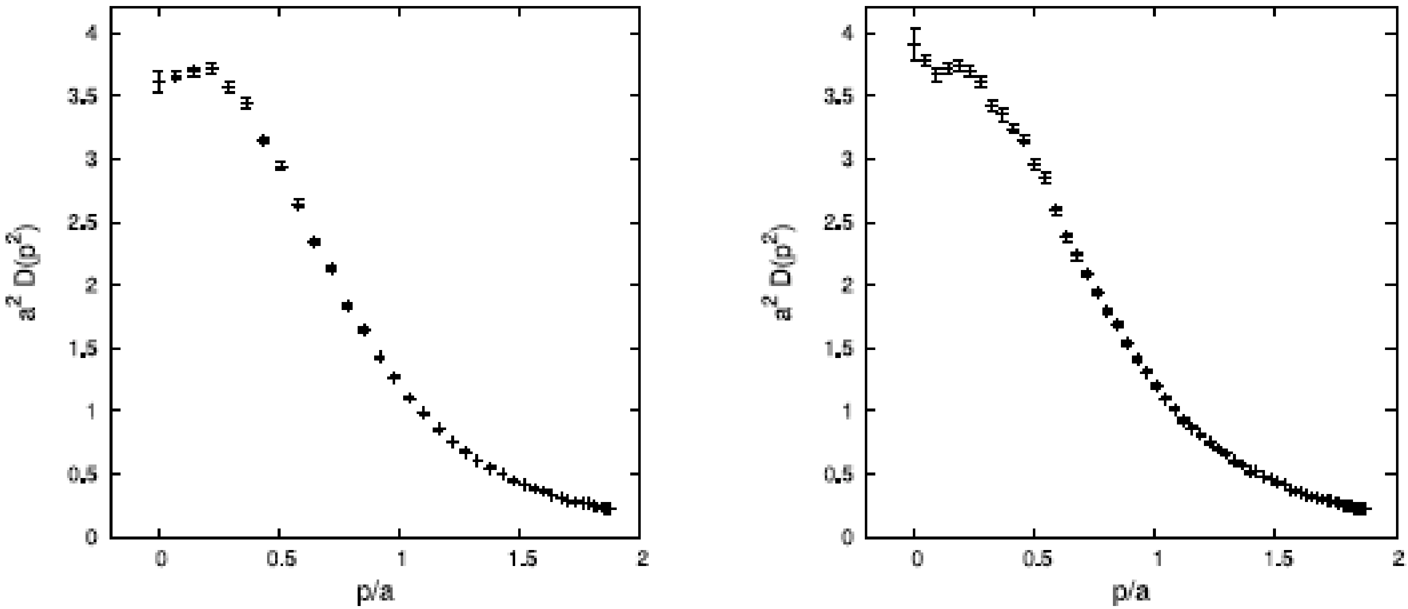


Figure 2: Unrenormalized gluon propagator $a^2D(p^2)$ (in GeV^{-2}) as a function of the momentum p/a (in GeV) for lattice volumes $V = 80^4$ (left) and $V = 128^4$ (right) at $\beta = 2.2$.

...nor is the ghost propagator more singular than a pole,
so the situation is a bit unclear, at present.

(more from Jan Pawlowki's lectures)

Yang-Mills Vacuum Wavefunctionals

The problem is to solve

$$H\Psi_0 = E_0\Psi_0$$

to see if anything can be learned about confinement and the mass gap. Currently there are several approaches.

1. **Coulomb Gauge** (Reinhardt et al., Szczepaniak et al.)

Use a gaussian ansatz and determine the kernel by minimizing <H>

$$\Psi_0[A] = \exp\left[-\int A_i(x) K^{ij}_{xy} A_j(y)\right]$$

I think this ansatz fails to give an area law for spacelike string tensions.

2. Temporal Gauge D=2+1 (Olejnik & JG)

$$\Psi_0[A] = \exp\left[-\frac{1}{2}\int d^2x d^2y\ B^a(x)\left(\frac{1}{\sqrt{-D^2-\lambda_0+m^2}}\right)^{ab}_{xy} B^b(y)\right]$$

adjust m^2 to get the string tension right, then we find that the mass gap (and other observables) comes out right.

3. New Variables D=2+1 (Karabali & Nair, Leigh et al.)

change variables from A^a_μ to gauge-invariant J^a, the tradeoff is local gauge invariance for local holomorphic invariance under

$$\overline{\partial} J \rightarrow h(z)\overline{\partial} J h^{-1}(z)$$

Start in temporal gauge,

then let $A = A_1 + iA_2$ and $z = x_1 - ix_2$

and define, for $M \in SL(2,C)$

$$A = -\frac{\partial M}{\partial z} M^{-1}$$

$$H = M^\dagger M$$

$$J = \frac{c_A}{\pi} \frac{\partial H}{\partial z} H^{-1}$$

H and J are gauge-invariant, the action in new variables is invariant under holomorphic transformations

$$J \to h(z) J h^{-1}(z) + \frac{c_A}{\pi} (\partial_z h) h^{-1}$$

The Hamiltonian has the form

$$H = m\left(\int_x J^a(x)\frac{\delta}{\delta J^a(x)} + \int_{z,w}\Omega^{ab}(z,w,J)\frac{\delta}{\delta J^a(z)}\frac{\delta}{\delta J^b(w)}\right) + \frac{\pi}{mc_A}\int_x \bar{\partial}J^a\bar{\partial}J^a$$

holomorphic-invariant ground-state wavefunctional:

$$\Psi_0 = \exp\left(-\frac{\pi}{2c_A m^2}\int \bar{\partial}J\ K\left(\frac{\Delta}{m^2}\right)\bar{\partial}J + \ldots\right)$$

Karabali and Nair, and also **Leigh et al** claim to have an (exact?) expression for the kernel, and calculate a string tension which is in impressive agreement with the lattice results.

I will return to this...

Temporal Gauge: Our claim is that the ground state solution in D=2+1 dimensions is approximated by

$$\Psi_0[A] = \exp\left[-\frac{1}{2}\int d^2x d^2y \; B^a(x) \left(\frac{1}{\sqrt{-D^2 - \lambda_0 + m^2}}\right)^{ab}_{xy} B^b(y)\right]$$

where

$B^a = F^a_{12}$	is the color magnetic field strength
$D^2 = D_k D_k$	is the covariant Laplacian in adjoint color representation,
λ_0	is the lowest eigenvalue of $-D^2$
m	is a constant proportional to g^2

Previous relevant work by: J.G. (1979)
Samuel (1996)
Diakonov (unpublished)

In support of this claim, we find that Ψ_0

- is a solution of the YM Schrodinger equation in the $g \to 0$ limit;
- solves the YM Schrodinger equation in the strong field, zero-mode limit;
- confines if $m > 0$, and that $m > 0$ seems energetically preferred;
- results in the numerically correct relationship between the mass gap and the string tension.

To begin at the beginning:

In Yang-Mills theory quantized in temporal gauge, all physical states must satisfy the Gauss Law constraint

$$\left(\delta^{ac}\partial_k + g\epsilon^{abc}A_k^b\right)\frac{\delta}{\delta A_k^c}\Psi = 0$$

which is equivalent to invariance of Ψ**[A]** under infinitesimal gauge transformations. The Hamiltonian is

$$H = \int d^d x \left\{-\frac{1}{2}\frac{\delta^2}{\delta A_k^a(x)^2} + \frac{1}{4}F_{ij}^a(x)^2\right\}$$

Free Field Limit

The proposed ground state

$$\Psi_0[A] = \exp\left[-\frac{1}{2}\int d^2x d^2y \; B^a(x)\left(\frac{1}{\sqrt{-D^2-\lambda_0+m^2}}\right)^{ab}_{xy} B^b(y)\right]$$

obviously satisfies the non-abelian physical state condition, and in the $g \rightarrow 0$ limit this becomes

$$\begin{aligned}\Psi_0[A] \quad &= \quad \exp\left[-\int d^2x d^2y \Big(\partial_1 A_2^a(x) - \partial_2 A_1^a(x)\Big)\right. \\ &\quad \left. \times \left(\frac{\delta^{ab}}{\sqrt{-\nabla^2}}\right)_{xy} \Big(\partial_1 A_2^b(y) - \partial_2 A_1^b(y)\Big)\right]\end{aligned}$$

which is the known ground state solution in the abelian, free-field case.

Zero Mode Limit

Consider gauge fields constant in space, variable in time, in D=2+1 dimensions. Lagrangian

$$
\begin{aligned}
L &= \frac{1}{2}\int d^2x \left[\partial_t A_k \cdot \partial_t A_k - g^2 (A_1 \times A_2)\cdot(A_1 \times A_2)\right] \\
&= \frac{1}{2} V \left[\partial_t A_k \cdot \partial_t A_k - g^2 (A_1 \times A_2)\cdot(A_1 \times A_2)\right]
\end{aligned}
$$

Hamiltonian operator

$$
H = -\frac{1}{2}\frac{1}{V}\frac{\partial^2}{\partial A_k^a \partial A_k^a} + \frac{1}{2} g^2 V (A_1 \times A_2)\cdot(A_1 \times A_2)
$$

Vacuum state

$$
\Psi_0 = \exp[-V R_0]
$$

With some algebra, one can verify that

$$\Psi_0 = \exp\left[-\frac{1}{2}gV\frac{(A_1 \times A_2)\cdot(A_1 \times A_2)}{\sqrt{|A_1|^2+|A_2|^2}}\right]$$

solves the zero-mode YM Schrodinger equation up to $1/V$ corrections.

Then we consider our proposal for the full vacuum state, for vacuum fluctuations in the ***strong A-field limit***, where the covariant Laplacian is dominated by the gauge-field zero-mode, i.e.

$$(-D^2)^{ab}_{xy} = g^2\delta(x-y)\Big[(A_1^2+A_2^2)\delta^{ab} - A_1^a A_1^b - A_2^a A_2^b\Big]$$

Then one finds (using also that $B \perp A_1, A_2$ in SU(2) color space)

$$\Psi_0[A]$$

$$= \quad \exp\left[-\int d^2x d^2y\ B^a(x) \left(\frac{1}{\sqrt{-D^2 - \lambda_0 + m^2}}\right)^{ab}_{xy} B^b(y)\right]$$

$$\Longrightarrow \quad \exp\left[-\frac{1}{2} g V \frac{(A_1 \times A_2) \cdot (A_1 \times A_2)}{\sqrt{A_1^2 + A_2^2}}\right]$$

So our wavefunctional

- satisfies the physical state constraint;
- has the proper perturbative $g \to 0$ limit.
- agrees with the calculable ground state of the zero-mode limit.

Supposing its right, what about confinement?

Dimensional Reduction

A long time ago it was suggested that at large distance scales, the pure Yang-Mills vacuum in a confining theory looks like

$$\Psi_0^{eff} \approx \exp\left[-\mu \int d^d x \; F_{ij}^a(x) F_{ij}^a(x)\right]$$

J.G. (1979)

This vacuum state has the property of ***dimensional reduction***: Computation of a spacelike loop in d+1 dimensions reduces to the calculation of a Wilson loop in Yang-Mills theory in d Euclidean dimensions.

Suppose $\Psi_0^{(3)}$ is the ground state of the 3+1 dimensional theory, and $\Psi_0^{(2)}$ is the ground state of the 2+1 dimensional theory. If these ground states both have the dimensional reduction form, and **W(C)** is a planar Wilson loop

$$\begin{aligned} W(C) &= \langle \mathrm{Tr}[U(C)] \rangle^{D=4} = \langle \Psi_0^{(3)} | \mathrm{Tr}[U(C)] | \Psi_0^{(3)} \rangle \\ &\sim \langle \mathrm{Tr}[U(C)] \rangle^{D=3} = \langle \Psi_0^{(2)} | \mathrm{Tr}[U(C)] | \Psi_0^{(2)} \rangle \\ &\sim \langle \mathrm{Tr}[U(C)] \rangle^{D=2} \end{aligned}$$

In D=2 dimensions the Wilson loop can be calculated analytically, and we know there is an area-law falloff, with *Casimir scaling* of the string tensions.

Mode number cutoff: Expand B(x) in eigenmodes of the covariant Laplacian:

$$
\begin{aligned}
(-D^2)^{ab}\phi^b_n(x) &= \lambda_n \phi^a(x) \\
B^a(x) &= \sum_{n=0}^{\infty} b_n \phi^a_n(x) \\
B^{a,\mathrm{slow}}(x) &= \sum_{n=0}^{n_{max}} b_n \phi^a_n(x)
\end{aligned}
$$

The cutoff mode sum defines the “slowly varying” B-field. Choosing n_{max} such that $\lambda_{n_{max}} - \lambda_0 \ll m^2$

$$
\begin{aligned}
&\int d^2x d^2y \; B^{a,\mathrm{slow}}(x) \left(\frac{1}{\sqrt{-D^2 - \lambda_0 + m^2}} \right)^{ab}_{xy} B^{b,\mathrm{slow}}(y) \\
&\approx \quad \frac{1}{m} \int d^2x \; B^{a,\mathrm{slow}}(x) B^{a,\mathrm{slow}}(x)
\end{aligned}
$$

So the part of the squared wavefunctional that involves B^{slow} is

$$|\Psi_0|^2 = \exp\left[-\frac{1}{m}\int d^2x \; B^{slow} B^{slow}\right]$$

which is the probability distribution of D=2 dimensional Yang-Mills (i.e. dimensional reduction). The string tension σ can be calculated analytically; in lattice units it is

$$\sigma = \frac{3}{4}\frac{m}{\beta}$$

Suppose we turn this around, and fix $m = \frac{4}{3}\beta\sigma$, with σ taken from the Monte Carlo data. Then the full vacuum wavefunctional

$$\Psi_0[A] = \exp\left[-\frac{1}{2}\int d^2x d^2y \; B^a(x)\left(\frac{1}{\sqrt{-D^2-\lambda_0+m^2}}\right)^{ab}_{xy} B^b(y)\right]$$

must imply a definite value for the mass gap. What is it?

Numerical Simulation of $|\Psi_0|^2$

To get the mass gap, we need to compute the connected correlator

$$G(x-y) = \langle (B^a B^a)_x (B^b B^b)_y \rangle - \langle (B^a B^a)_x \rangle^2$$

in the probability distribution

$$P[A] = |\Psi_0[A]|^2 = \exp\left[-\int d^2x d^2y \; B^a(x) K^{ab}_{xy}[A] B^b(y) \right]$$

where

$$K^{ab}_{xy}[A] = \left(\frac{1}{\sqrt{-D^2 - \lambda_0 + m^2}} \right)^{ab}_{xy}$$

Numerically, this looks hopeless! K^{ab}_{xy} is highly non-local, and is not even known explicitly for arbitrary gauge fields.

But suppose - after eliminating the variance along gauge orbits by a gauge choice - that K[A] has very little variation among thermalized configurations. Then things are more promising.

Define

$$P\Big[A; K[A']\Big] = \exp\left[-\int d^2x d^2y \; B^a(x) K^{ab}_{xy}[A'] B^b(y)\right]$$

where B is computed from A , not A' , and $P[A] = P\Big[A, K[A]\Big]$.
Then, assuming the variance of K is small,

$$\begin{aligned} P[A] &\approx P\Big[A, \langle K\rangle\Big] \\ &= P\left[A, \int DA' \; K[A'] P[A']\right] \\ &\approx \int DA' \; P\Big[A, K[A']\Big] P[A'] \end{aligned}$$

solve this equation iteratively...

$$P^{(1)}[A] \quad = \quad P\Big[A; K[0]\Big]$$

$$P^{(n+1)}[A] \quad = \quad \int DA' \; P\Big[A; K[A']\Big] P^{(n)}[A']$$

General idea of the simulation: work in an axial A_1=0 gauge, and change integration variables from A_2 to B. Then:

1. given A_2 , set $A'_2 = A_2$

2. $P[A; K[A']]$ is gaussian in B. Diagonalize $K^{ab}_{xy}[A']$ and generate a new B-field (or set of B-fields) stochastically.

3. from B, calculate A_2 in axial gauge, and compute observables

4. go to step 1, repeat as necessary.

(all in a lattice regularization)

Observables of interest include

- The eigenvalue spectrum $\{\lambda_n\}$ of the adjoint covariant Laplacian $(-D^2)$

- The connected field-strength correlator

$$\langle B^2(x) B^2(y) \rangle_{conn} \propto G(x-y)$$

where

$$G(x-y) = \left\langle (K^{-1})^{ab}_{xy} (K^{-1})^{ba}_{yx} \right\rangle$$

$$K^{-1} = \sqrt{-D^2 - \lambda_0 + m^2}$$

with the parameter ***m*** chosen to reproduce the known string tension σ

$$m = \frac{4}{3}\beta\sigma$$

From G(R), we can extract the mass gap.

For Comparison

We can also compute $\{\lambda_n\}$, K_{xy}^{ab}, and

$$G(x-y) = \left\langle (K^{-1})^{ab}_{xy} (K^{-1})^{ba}_{yx} \right\rangle$$

$$K^{-1} = \sqrt{-D^2 - \lambda_0 + m^2}$$

on 2D slices of lattices generated by 3D lattice Monte Carlo.

This is like simulating the ground state of the transfer matrix in the Euclidean theory.

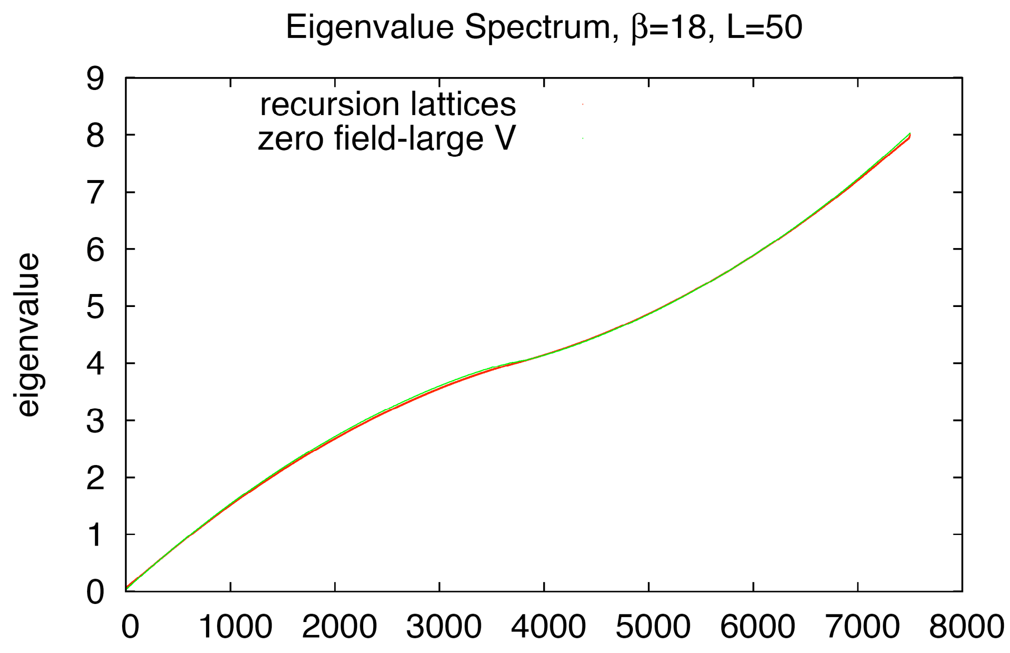
Results obtained from "MC" lattices, generated by ordinary lattice Monte Carlo, can be compared with results obtained by simulating $|\Psi_0|^2$ ("recursion" lattices).

Eigenvalue Spectrum β=18, 50x50 lattice

This is a plot of eigenvalue vs mode number of

- the zero-field operator $(-\nabla^2 + m^2)$
- the covariant operator $(-D^2 - \lambda_0 + m^2)$, computed on 10 lattices.

These are ***not*** averaged; the values for each lattice are plotted, and (almost) fall on top of one another.

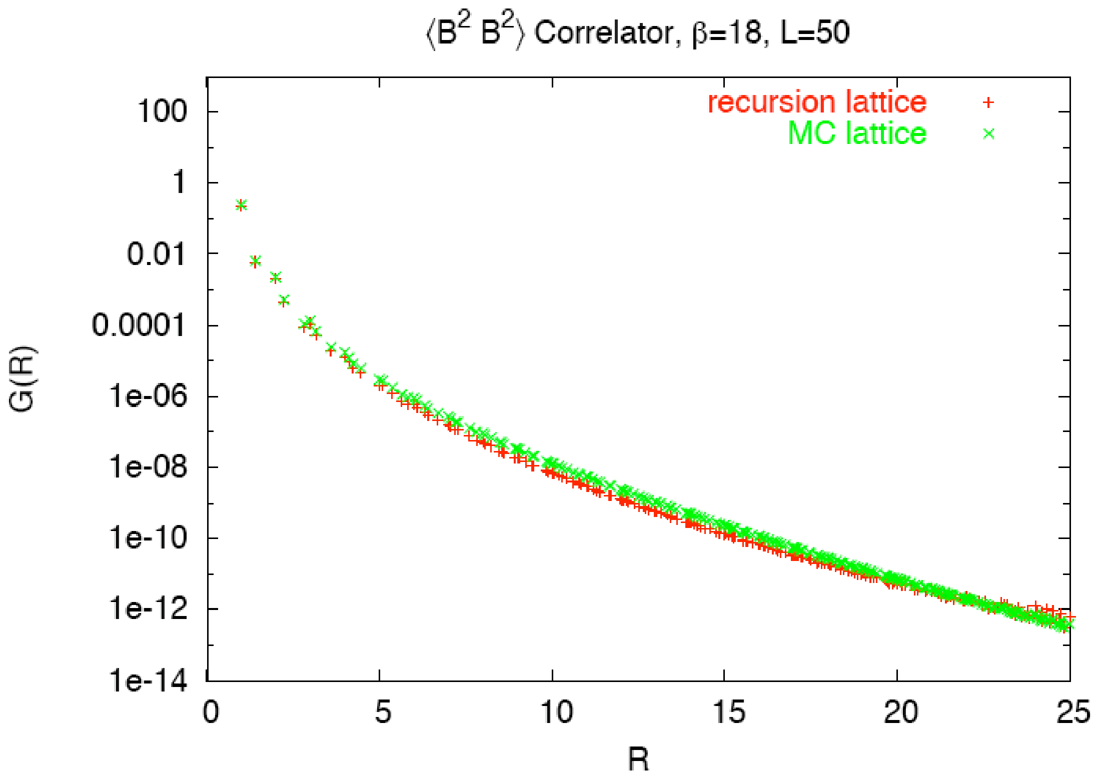


There is <u>very little variance</u> in the spectrum of $-D^2-\lambda_0$ from one lattice to the next..

Mass Gap

Here is the data for

$$G(x-y) = \left\langle (K^{-1})^{ab}_{xy} (K^{-1})^{ba}_{yx} \right\rangle$$

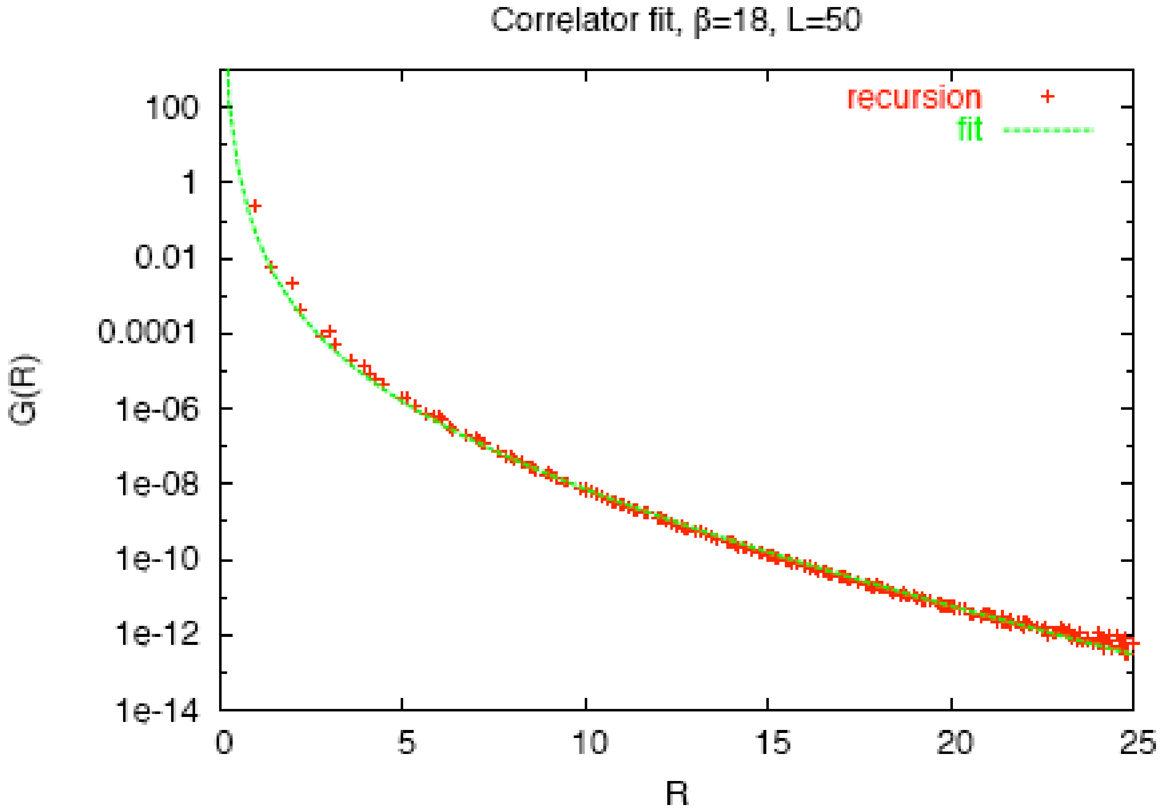


The data is obtained from ten recursion lattices, and ten MC lattices. Note the tiny values of G(R) obtained at larger R. This requires a near-absence of fluctuation in K^{-1} from one lattice to the next.

The mass gap is obtained by fitting the data for G(R) to extract the exponential falloff. Define

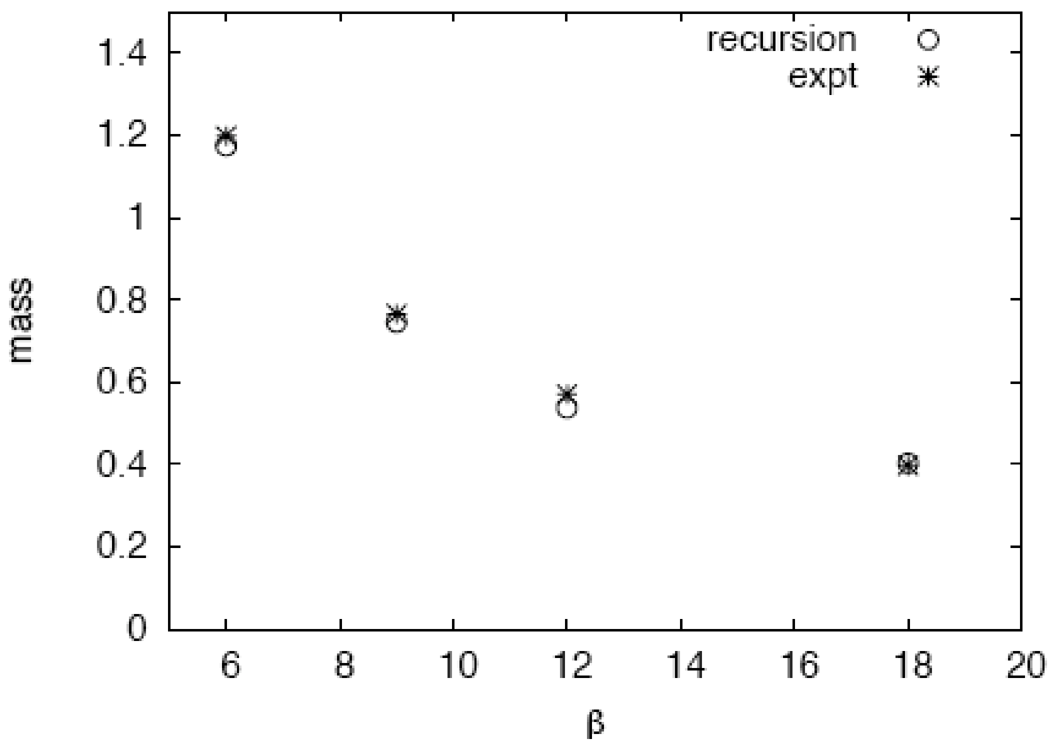
$$
\begin{aligned}
G_0(R) &= \delta^{ab}\delta^{ba}\left[\left(\sqrt{-\nabla^2+\mu^2}\right)_{xy}\right]^2 \\
&= \frac{3}{4\pi^2}(1+\frac{1}{2}MR)^2\frac{e^{-MR}}{R^6}
\end{aligned}
$$

where
R=|x-y| and
M=2μ.



Results for the mass gap

- "recursion" is our result.
- "expt" is the Monte Carlo result for the 0^+ glueball, obtained by Meyer and Teper.

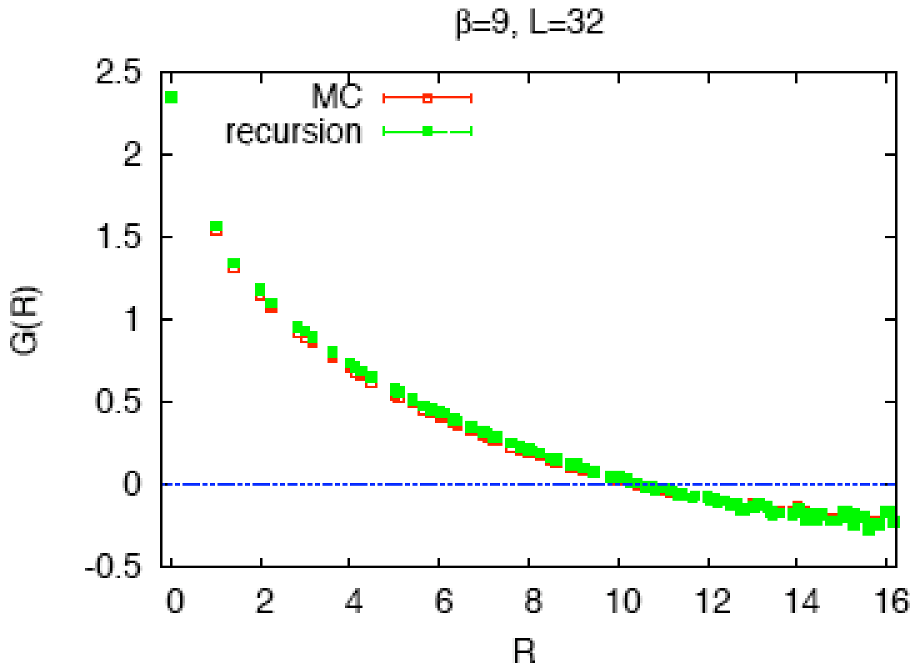


Given string tension σ, we have determined fairly accurately the 0^+ glueball mass.

Another observable we have looked at is the Coulomb gauge ghost propagator. This is evaluated by transforming each (MC or recursion) lattice to Coulomb gauge, and evaluating

$$G_{ghost}(\mathbf{x}-\mathbf{y}) = \left\langle \left(\frac{1}{-\nabla \cdot D} \right)^{aa}_{\mathbf{xy}} \right\rangle$$

with the (preliminary!) result



The Coulomb potential is very sensitive to “exceptional” configurations with very small λ_0 ; these lead to huge errorbars. To compare recursion and MC results, we impose cuts on the data, throwing away these rare configurations.

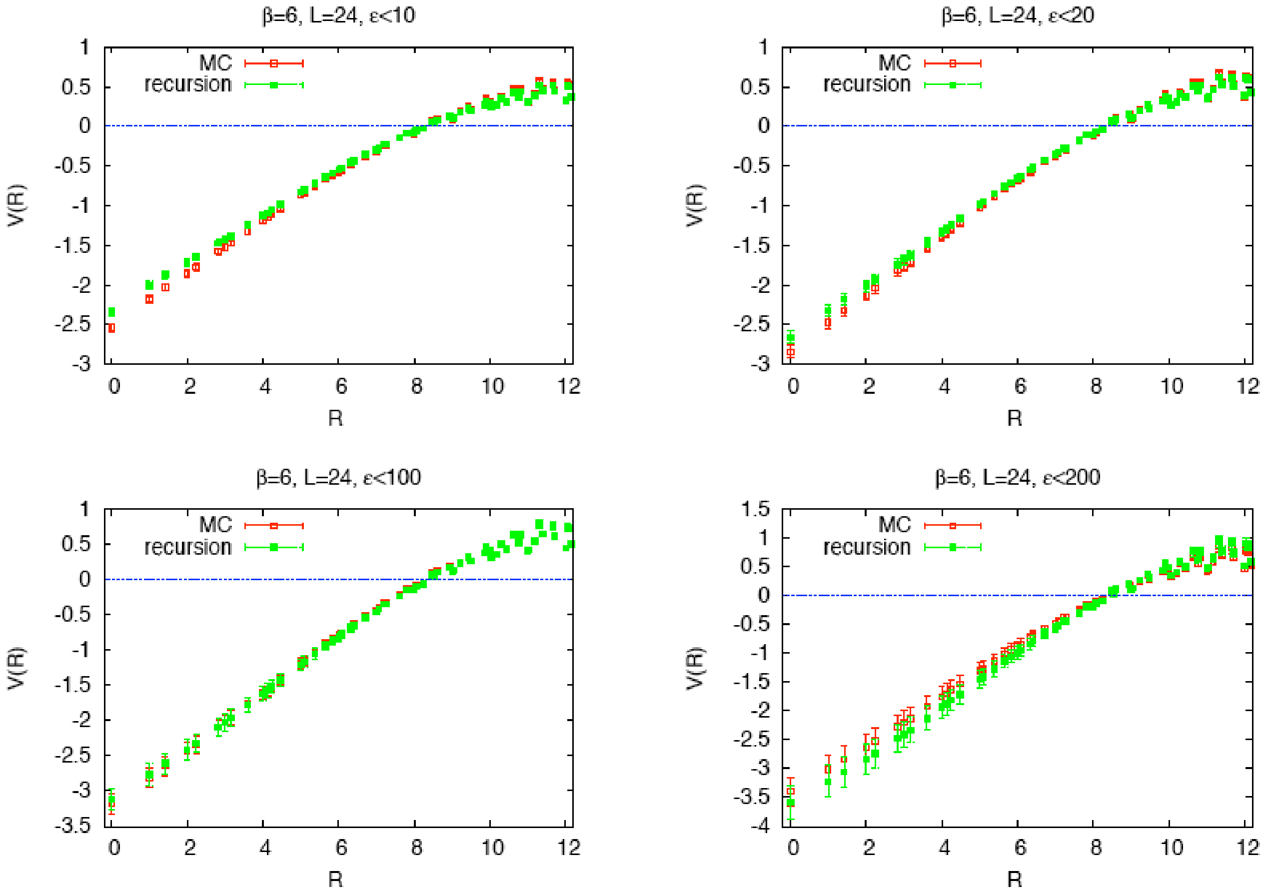


Figure 3: Color-Coulomb potential, $\beta = 6, L = 24$, computed from configurations with $\epsilon = |V(0)| < 10, 20, 100$ and 200.

Karabali-Kim-Nair wavefunctional

When converted from "new variables" to old variables, it has the bilinear form

$$\Psi_0 \approx \exp\left[-\frac{1}{2g^2}\int d^2x d^2y \; B^a(x)\left(\frac{1}{\sqrt{-D^2+m^2}+m}\right)^{ab}_{xy} B^b(y)\right]$$

where $m = \dfrac{g^2 C_A}{2\pi}$

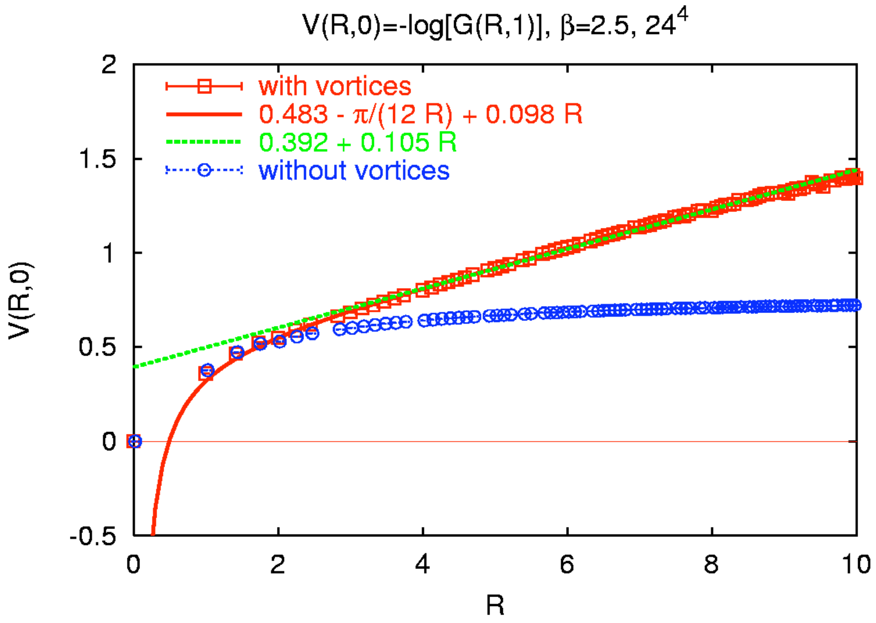
and KKN quote a string tension prediction (from Dimensional Reduction) derived by setting $-D^2=0$. The result is accurate to a few percent.

I think this success is a coincidence. The analysis ignores the fact that the lowest eigenvalue λ_0 of $-D^2$ is positive definite. When the KKN wavefunctional is simulated numerically, the error in the string tension is around 50%, and may even be infinite in the continuum limit.

Constituent Gluons, and the Gluon Chain Model

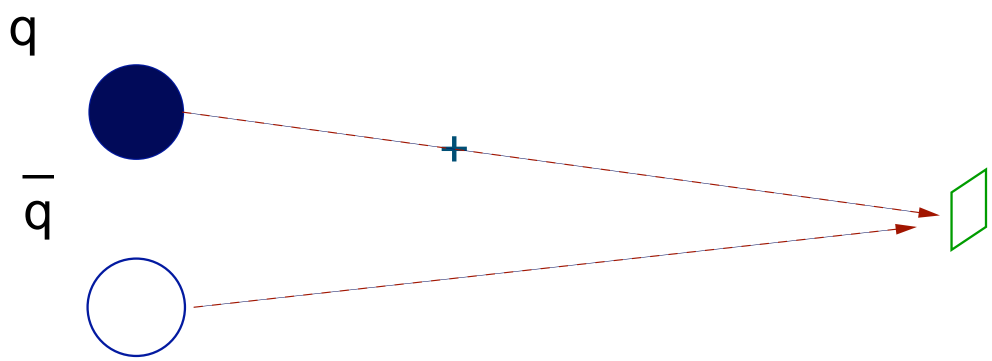
We have seen that the Coulomb potential is linear:

but there are (at least) two serious difficulties, in claiming that the Coulomb potential "explains" confinement...



i) The Coulomb string tension σ_c is about three times larger than the asymptotic string tension σ .

ii) Long-range Coulombic dipole fields.



$$\vec{E}^L = g\vec{\nabla}\frac{1}{\nabla \cdot D}\rho$$

Problem (ii) is generic to “one-gluon exchange” or ladder-diagram models of confinement. In hadrons, there would be long range van der Waals forces.

Basic Problem: No flux tube!

So how does a flux tube form in Coulomb gauge?

The Coulomb potential *$V_c(R)$* is the interaction energy of the physical state

$$\Psi_{q\overline{q}} = \overline{q}^a(0) q^a(R) \Psi_0$$

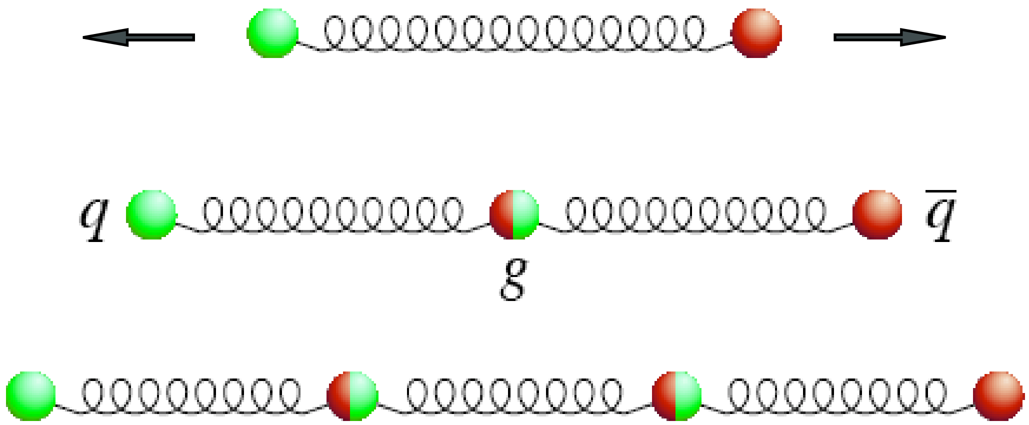
and the Coulomb string tension σ_c comes out too high.

Can we bring the string tension down to σ by adding constituent gluons?

Schematically,

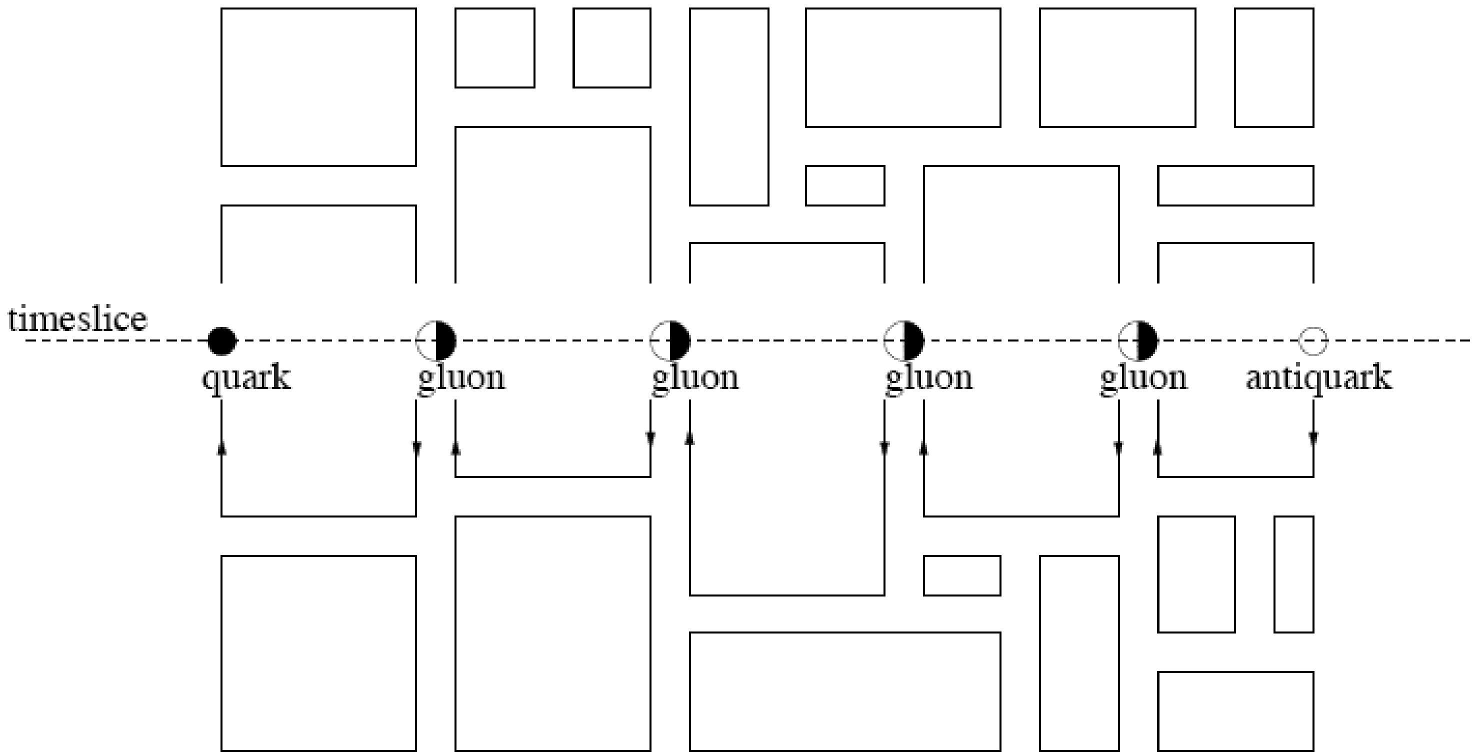
$$\Psi_{q\overline{q}} = \overline{q}^a(0) \Big\{ c_0 + c_1 A + c_2 AA + ... \Big\} q^a(R) \Psi_0$$

The Gluon Chain Model *(Thorn & JG)*

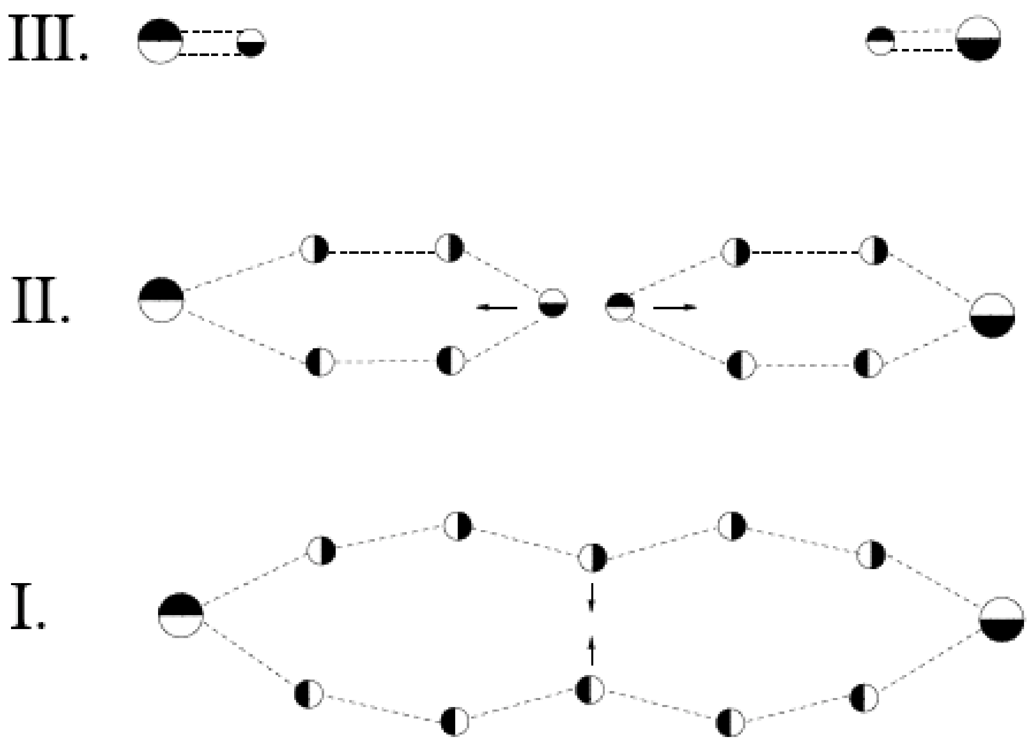


As a quark-antiquark pair move apart, they pull out a chain of constituent gluons between them.

One of the motivations of this model is that a gluon chain can be regarded as a time-slice of a high-order planar Feynman diagram



A gluon chain has string-like properties (e.g. a Luscher term), Casimir scaling is natural at large N, and it is also consistent with N-ality dependence



String breaking for adjoint representation sources

On the lattice, define the rescaled transfer matrix

$$T = \exp[-(H - E_0)a]$$

(E_0 is the vacuum energy)

Ideally, we would like to diagonalize this in the subspace of states containing two static charges. In practice, diagonalize in a finite M-dimensional subspace. Let

$$|k\rangle = \overline{q}^a(\mathbf{x}) Q_k^{ab} q^b(\mathbf{y}) |\Psi_0\rangle \quad , \quad k = 1, 2, ..., M$$

where the Q_k are functionals of the link variables.

Use lattice Monte Carlo to compute the quantities...

$$\begin{aligned} O_{mn} &= \langle m|n\rangle \\ &= \langle \tfrac{1}{2}\mathrm{Tr}[Q_m^\dagger(t)Q_n(t)]\rangle \end{aligned}$$

$$\begin{aligned} t_{mn} &= \langle m|T|n\rangle \\ &= \langle \tfrac{1}{2}\mathrm{Tr}[Q_m^\dagger(t+1)U_0^\dagger(\mathbf{x}_0,t)Q_n(t)U_0(\mathbf{x}_L,t)]\rangle \end{aligned}$$

From these quantities we construct (Gram-Schmidt procedure) an orthonormal set of states $\{\varphi_k, k=1,2,...,M\}$, and also derive the matrix elements

$$T_{ij} = \langle\varphi_i|T|\varphi_j\rangle$$

Diagonalize the M x M matrix T. Then

$$V(R) = -\log(\lambda_{max})$$

is an estimate of the static quark potential.

Choice of Q's and variational parameter: Define

$$A_k(\mathbf{x},t) = \frac{1}{2i}\Big(U_k(\mathbf{x},t) - U_k^\dagger(\mathbf{x},t)\Big)$$

Fourier transform, and suppress high-momentum components in directions transverse to direction "j" (of line joining $\overline{q}q$)

$$\begin{aligned} A_i(\mathbf{k},t) &\rightarrow \exp\Big[-\rho(\mathbf{k}^2 - k_j^2)\Big] A_i(\mathbf{k},t) \\ &\rightarrow \exp\Big[-\rho\mathbf{k}_\perp^2\Big] A_i(\mathbf{k},t) \end{aligned}$$

where ρ is a variational parameter. Transform back to position space, and denote the resulting "transverse-smoothed" operator

$$A_i(\mathbf{x},t,j)$$

which is the A-field smeared in directions transverse to direction $\widehat{e}_j$.

We also define

$$B_i(\mathbf{x}, t) = 1 - \tfrac{1}{2}\mathrm{Tr}[U_i(\mathbf{x}, t)]$$

and smear in the same way to obtain the “transverse-smeared” operator $B_i(x,t,j)$.

The Q operators are then defined in terms of the $A_i(x,t,j)$ and $B_i(x,t,j)$, for an antiquark at site $\mathbf{x}_0$ and a quark at site $\mathbf{x}_0 + R\mathbf{e}_j$:

0-gluon state

$$Q_1(t) = \mathbb{1}_2$$

1-gluon state

$$Q_2(t) = \sum_{n=0}^{R-1} A_j(\mathbf{x}_0 + n\mathbf{e}_j, t, j)$$

2-gluon states

$$Q_3(t) = \sum_{n=-2}^{R+1} \sum_{n'=n}^{R+1} A_j(\mathbf{x}_0 + n\mathbf{e}_j, t, j) A_j(\mathbf{x}_0 + n'\mathbf{e}_j, t, j)$$

↓

$$Q_4(t) = \sum_{n=-2}^{R+2} \sum_{n'=n}^{R+2} \sum_{i \neq j} \overline{A}_i(\mathbf{x}_0 + n\mathbf{e}_j, t, j) \overline{A}_i(\mathbf{x}_0 + n'\mathbf{e}_j, t, j)$$

$$Q_5(t) = \sum_{n=0}^{R-1} B_j(\mathbf{x}_0 + n\mathbf{e}_j, t, 1) \mathbb{1}_2$$

$$Q_6(t) = \sum_{n=0}^{R-1} \sum_{i \neq j} \overline{B}_i(\mathbf{x}_0 + n\mathbf{e}_j, t, j) \mathbb{1}_2$$

We use this to define

$$|k\rangle = \overline{q}^a(\mathbf{x})Q_k^{ab}q^b(\mathbf{y})|\Psi_0\rangle \quad , \quad k = 1, 2, ..., M$$

(with *M=6*) and use lattice Monte Carlo to obtain an orthonormal set of states, and the elements of the transfer matrix $T_{ij} = \langle\phi_i|T|\phi_j\rangle$ among those states.

Choose a variational parameter ρ which maximizes the largest eigenvalue λ_{max} of T_{ij} . Denote the corresponding eigenmode

$$|\psi(R)\rangle = \sum_{k=1}^{6} a_k(R)|\phi_k\rangle$$

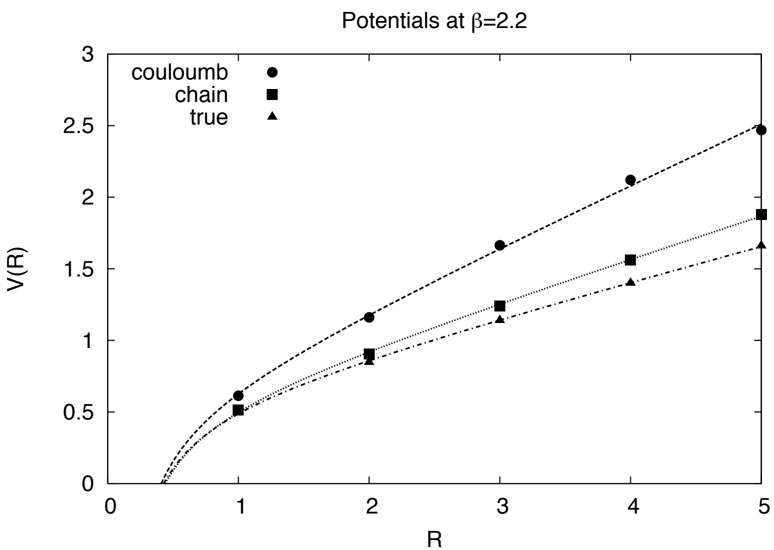
so

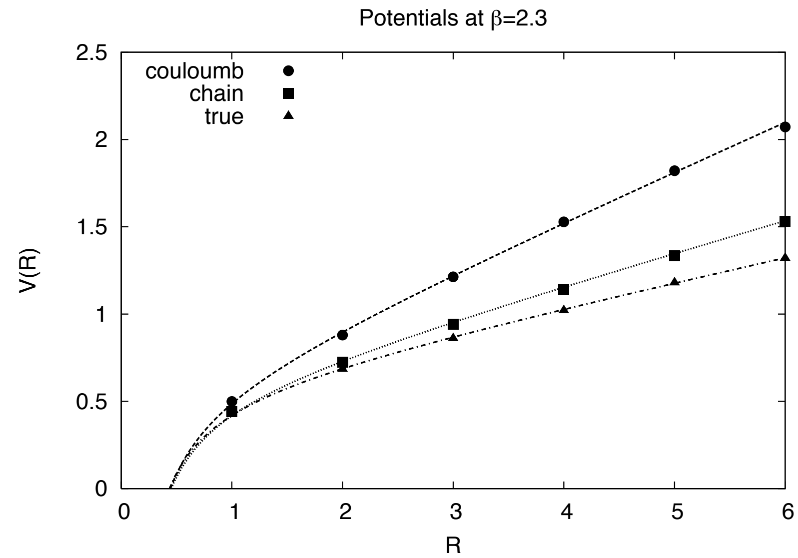
$(a_1)^2$ is the fraction of the norm from the 0-gluon state

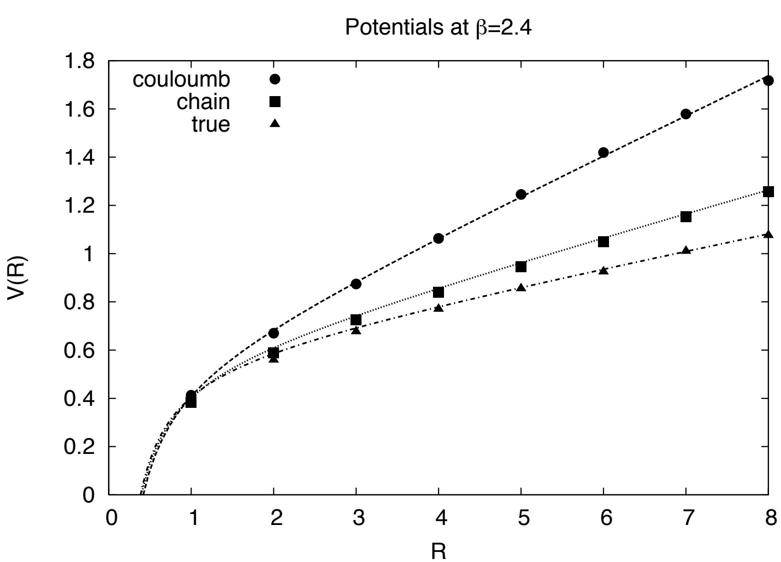
$(a_2)^2$ is the fraction of the norm from the 1-gluon state

$1-(a_1)^2-(a_2)^2$ is the fraction of the norm from the 2-gluon states

and $V_{chain}(R) = -\log(\lambda_{max}) \quad , \quad V_C(R) = -\log(T_{11})$



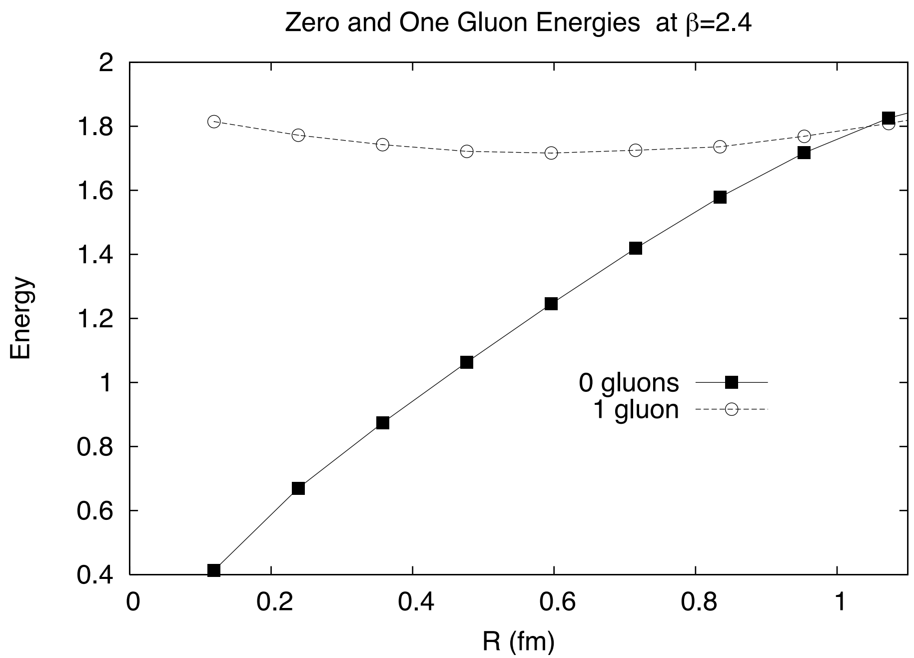




Constituent gluons bring the potential much closer to the true static quark potential.

The “chain” potential remains linear.

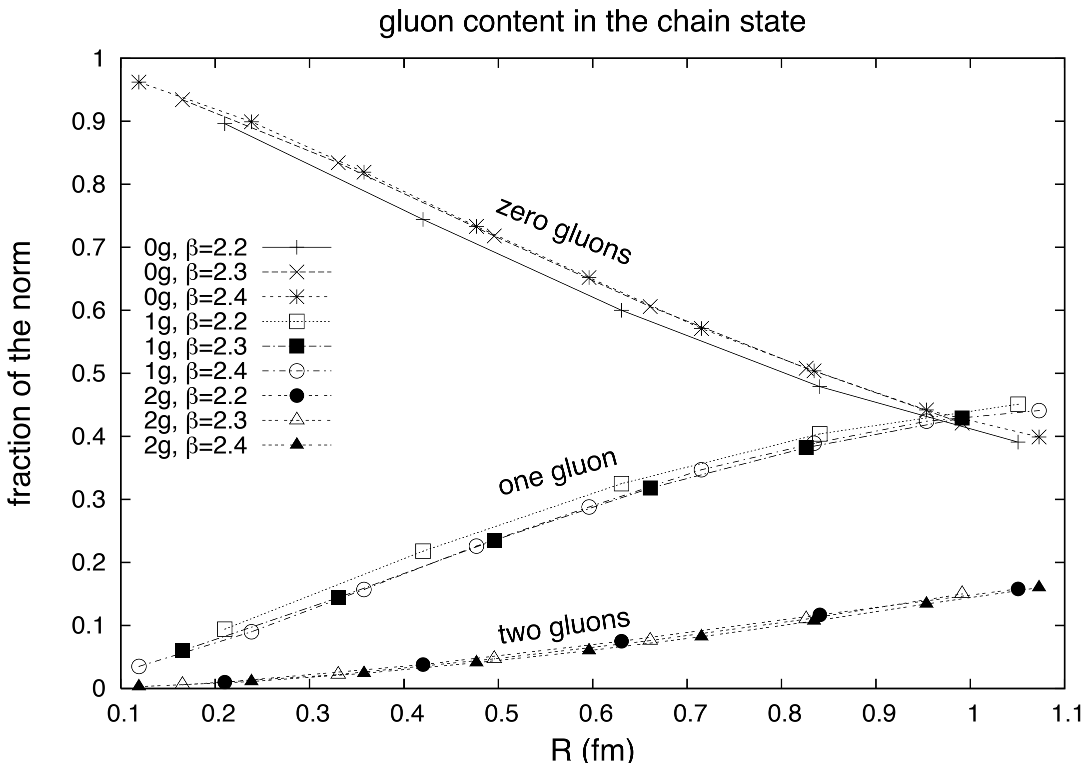
Energy expectation values of the zero-gluon and 1-gluon states



$$V_C(R) = -\log(T_{11}) \qquad \text{0-gluon}$$

$$V_1(R) = -\log(T_{22}) \qquad \text{1-gluon}$$

Gluon content of the lowest-energy state



This is also a test of scaling.

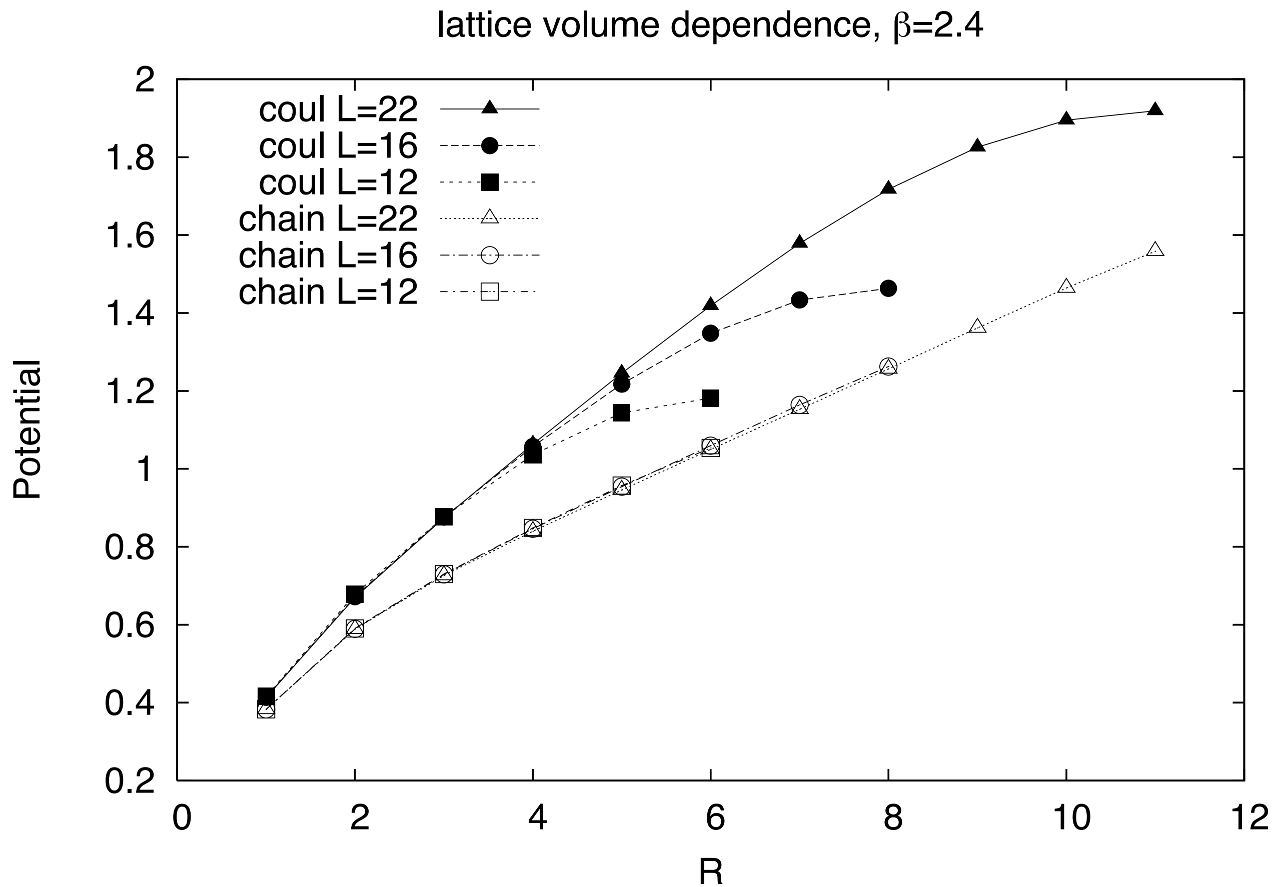
Note the 0-gluon/1-gluon crossover, again around one fermi.

Help for the dipole problem?

The color Coulomb field is not expected to be collimated into a flux tube. This means that there should be strong sensitivity to lattice volume, on a lattice of spatial extension L, for quark-antiquark separations close to $R=L/2$.

The reason is that for separations of that size, the finite volume cuts off a region where the field energy is still significant.

If the field energy were collimated into a flux tube of diameter d, and if $L>>d$, then there would not be a similar sensitivity to the finite volume.



The gluon-chain states seem to be insensitive to lattice size, in contrast to the Coulomb potential. Perhaps a hint that the dipole problem is much less severe for the multi-gluon states.

Conclusions

Until asymptotically-free pure gauge theories are solved analytically in the infrared, there is likely to be disagreement about the structure of the vacuum, the origin of confinement and the origin of the mass gap.

Several approaches - not necessarily compatible with each other - seem promising. So far, however:

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Until asymptotically-free pure gauge theories are solved analytically in the infrared, there is likely to be disagreement about the structure of the vacuum, the origin of confinement and the origin of the mass gap.

Several approaches - not necessarily compatible with each other - seem promising. So far, however:

The confinement problem remains open...

it remains a challenge to our understanding of non-abelian gauge theories.

Is there now a proof?

Tomboulis, arXiv: 0707.2179

We can insert a center vortex into a finite volume using twisted, rather than periodic boundary conditions.

Let Z_- denote the SU(2) partition function with t.b.c, and
F_v denote the center vortex free energy

then

$$e^{-F_v} = \frac{Z_-}{Z}$$

Confinement is proven if $F_v = cL_zL_t \exp[-\sigma' L_x L_y]$

for a vortex sheet in the z-t plane.

Migdal-Kadanoff blocking

This is an RG decimation scheme, involving an uncontrolled approximation, which takes a lattice action with spacing ***a*** to a lattice action with spacing ***2a*** .

The idea is to take a 2^4 hypercube, move the interior plaquettes to the exterior faces and integrate out some of the link variables.

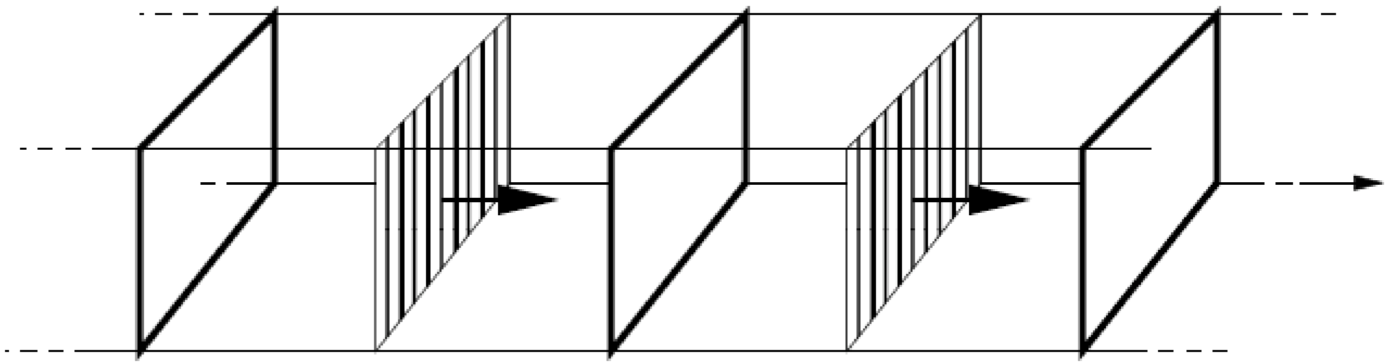

Figure 1: Basic plaquette moving operation, $b = 2$

Tomboulis's idea is to use to the MK procedure to prove an inequality, after ***n*** blocking steps,

$$\frac{Z_-}{Z} \geq \frac{Z_-^{MK}(n)}{Z^{MK}(n)}$$

If ***n*** is large enough, the rhs can be evaluated by strong-coupling methods, and confinement is proved.

This comes the closest to a proof that I've seen...

...but I think there is one crucial step in the argument which has not yet been shown to be true.

The point has been made in a very recent article by Ito & Seiler, arXiv:0711.4930.

Dressed ghost and gluon propagators in Landau gauge

$$D_{\mu\nu}(p) = \frac{Z(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \quad , \quad D_{ghost}(p) = -\frac{G(p^2)}{p^2}$$

the Zwanziger Horizon Conditions are that at $p^2 \to 0$

$$D_{\mu\nu}(p) \;\to\; 0 \qquad \text{“Gluon Confinement”}$$

$$G(p^2) \;\to\; \infty \qquad \text{Kugo-Ojima condition}$$

Ties in nicely with both Kugo-Ojima and the Dyson-Schwinger Equation (DSE) approach.

In Coulomb gauge, where the color Coulomb potential is related to the operator

$$\frac{1}{\nabla \cdot D}(-\nabla^2)\frac{1}{\nabla \cdot D}$$

the proximity to the Gribov horizon can, in principle, enhance the potential in the infrared.

In fact, Monte Carlo measurements of the color Coulomb potential find that it *does* rise linearly

Olejnik & JG, 2003

$$V_{coul}(R) \sim \sigma_{coul} R \quad \text{with} \quad \sigma_{coul} \approx 3\sigma$$

albeit with a slope which is three times larger than the asymtotic string tension.

Monopoles

lattice investigations by the
Kanazawa group (Suzuki et al.)
Pisa group (di Giacomo et al.)
ITEP group (Polikarpov et al.)
among others...

Motivations:

1. "Dual Superconductivity" ('t Hooft & Mandelstam)
2. Compact U(1) in 2+1 dimensions (Polyakov)
3. Witten-Seiberg model

In the absence of a Higgs field in the adjoint representation, it is necessary to single out an abelian subgroup using an "abelian projection" gauge. ('t Hooft, '80)

It turns out that in abelian projection gauges, the abelian monopole worldlines lie on vortex sheets...

Kugo-Ojima criterion (covariant gauges)

Says that $\langle \text{phys}|Q^a|\text{phys}\rangle = 0$ if a certain operator condition is satisfied, *and* if the remnant gauge symmetry in the covariant gauge is unbroken. In the gauge-Higgs model it requires

$$\langle \phi \rangle = 0$$

Coulomb confinement (Coulomb gauge)

Confining color Coulomb potential. Gribov and Zwanziger
(measured from the correlator of timelike links)

The scenario implies unbroken remnant gauge symmetry in Coulomb gauge

$$\left\langle \frac{1}{L^3} \sum_{\mathbf{x}} \mathrm{Tr}[U_0(\mathbf{x},t)] \right\rangle \to 0$$

Marinari, Paciello, Parisi, Taglienti

Either criterion can work in real QCD, so is this what we mean by confinement?