Gravity as an effective theory

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Chiral effective theory

The chiral lagrangian is a non-renormalizable theory describing accurately pion physics at low energies.

It contains a (infinite) number of operators organized according to the number of derivatives

$$\mathcal{L} = f_\pi^2 \mathrm{Tr}\ \partial_\mu U \partial^\mu U^\dagger + \alpha_1 \mathrm{Tr}\ \partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger + \alpha_2 \mathrm{Tr}\ \partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger + \ldots$$

$$U = \exp i\tilde{\pi}/f_\pi$$

$$\mathcal{L} = \mathcal{O}(p^2) + \mathcal{O}(p^4) + \mathcal{O}(p^6) + ...$$

Pions are the Goldstone bosons associated to the (global) symmetry breaking pattern of QCD

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

Locality, symmetry and relevance (in the RG sense) are the only guiding principles to construct $\mathcal{L}$.

The effective lagrangian still has the *full* symmetry

$$U \to L U R^\dagger$$

Loops

$$A_{N^\pi}(p_i) = \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{f_\pi}\right)^{N^\pi} (k^2)^{N_V} \left(\frac{1}{k^2}\right)^{N_P}$$

Consider e.g. $\pi\pi \to \pi\pi$ scattering. $N_\pi = 4$, $N_V = 2$ and $N_P = 2$

$$A_{N^\pi} \sim \frac{1}{16\pi^2 f_\pi^2} p^4$$

This counting works to all orders and IR divergences are absent (Weinberg)

At each order in perturbation theory the divergences that arise can be eliminated by redefining the coefficients in the higher order operators, e.g.

$$\alpha_i \to \alpha_i + \frac{c_i}{\epsilon}$$

Also logarithmic non-local terms necessarily appear. For instance (in a two-point function) they appear in the combination

$$\frac{1}{\epsilon} + \log \frac{-p^2}{\mu^2}$$

Unitarity

The cut provided by the log is absolutely required by unitarity. Let us separate

$$S = I + iT.$$

The identity corresponds to having no interaction at all.

Unitarity implies

$$S^{\dagger}S = I = I + i(T - T^{\dagger}) + T^{\dagger}T.$$

$$i(T - T^{\dagger}) = -T^{\dagger}T.$$

Loops are essential, even for effective theories. There is no such thing as a ‘classical effective theory’.

Chiral counting

The lowest-order, tree level contribution is $\sim \frac{p^2}{f_\pi^2}$

The one-loop chiral corrections is $\sim \frac{p^4}{16\pi^2 f_\pi^4}$

$\Rightarrow$ The counting parameter in the loop (chiral) expansion is

$$\frac{p^2}{16\pi^2 f_\pi^2}$$

Each chiral loop gives an additional power of p^2

$\mathcal{O}(p^{2n})$ counts as p^{2n}

Soft breaking terms: $\mathrm{Tr}\ \mu m(U + U^\dagger)$

$\Rightarrow\ m$ counts as p^2.

All coefficients in the chiral lagrangian are nominally of $\mathcal{O}(N_c)$.

Loops are automatically suppressed by powers of N_c, but enhanced by logs.

Relevance of chiral corrections

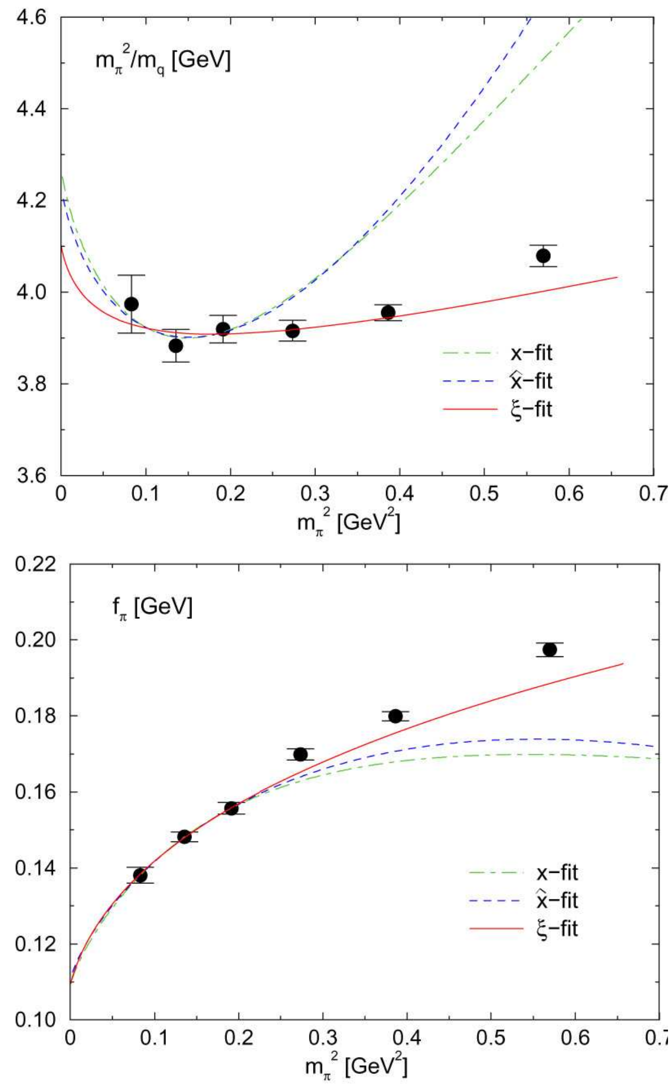

Figure 1: Recent fits using chiral perturbation theory at the NLO

The gravity analogy

Einstein-Hilbert action shares several aspects with the chiral lagrangian (non-renormalizable, dimension two, massless quantum,...)

$$\mathcal{L} = M_P^2 \sqrt{-g}\mathcal{R} + \mathcal{L}_{matter}$$

$$\kappa^2 \equiv \frac{2}{M_P^2} = 32\pi G$$

M_P will play a role very similar to f_π

$\mathcal{R}$ contains two derivatives of the dynamical variable $g_{\mu\nu}$

$$\mathcal{R}_{\mu\nu} = \partial_\nu \Gamma^\alpha_{\mu\alpha} - \partial_\alpha \Gamma^\alpha_{\mu\nu} + \Gamma^\alpha_{\beta\nu}\Gamma^\beta_{\mu\alpha} - \Gamma^\alpha_{\beta\alpha}\Gamma^\beta_{\mu\nu}$$

$$\Gamma^\gamma_{\alpha\beta} = \frac{1}{2} g^{\gamma\rho}\left(\partial_\beta g_{\rho\alpha} + \partial_\alpha g_{\rho\beta} - \partial_\rho g_{\alpha\beta}\right)$$

$$\mathcal{R} \sim \partial\partial g$$

In the chiral language, the Einstein-Hilbert action is $\mathcal{O}(p^2)$ (i.e. most relevant).

Symmetry

Based on promoting a global symmetry (Lorentz)

$$
\begin{aligned}
x'^a &= \Lambda^a{}_b x^b \\
\eta_{ab} &= \Lambda^c{}_a \Lambda^d{}_b \eta_{cd}
\end{aligned}
$$

to a local one

$$
\begin{aligned}
x'^\mu = x'^\mu(x) \quad &\longrightarrow \quad dx'^\mu = \Lambda^\mu{}_\nu(x) dx^\nu \\
\bar{\Lambda}_\mu{}^\nu(x) \quad &\equiv \quad [\Lambda^\mu{}_\nu(x)]^{-1} \\
\Lambda^\mu{}_\nu \bar{\Lambda}_\rho{}^\nu \quad &= \quad \delta^\mu_\rho
\end{aligned}
$$

This can be done if the metric is allowed to be a coordinate dependent field transforming as

$$
\begin{aligned}
g'_{\mu\nu}(x') &= \bar{\Lambda}_\mu{}^\alpha \bar{\Lambda}_\nu{}^\beta g_{\alpha\beta}(x) \\
d\tau^2 &= g'_{\mu\nu}(x') dx'^\mu dx'^\nu = g_{\alpha\beta}(x) dx^\alpha dx^\beta
\end{aligned}
$$

Fields transform as scalars, vectors, etc., under this change

$$
\begin{aligned}
\phi'(x') &= \phi(x) \\
A'^\mu(x') &= \Lambda^\mu{}_\nu(x) A^\nu(x)
\end{aligned}
$$

Why Einstein-Hilbert

Arguably, these considerations alone, in particular relevance in the RG sense (and not renormalizability) are the ones that single out Einstein-Hilbert action (in front e.g. of $\mathcal{R}^2$).

Einstein-Hilbert action has all the ingredients for being an effective theory describing the long distance properties of some unknown dyamics

Are gravitons just Goldstone bosons of some (Lorentz) unbroken symmetry?

Quantum corrections in gravity

Analogous to the weak field expansion in pion physics

$$U = I + i\frac{\pi(x)}{f_\pi} + ...$$

one writes

$$\begin{aligned} g_{\mu\nu} &\equiv \eta_{\mu\nu} + \kappa h_{\mu\nu} \\ g^{\mu\nu} &= \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda} h_\lambda^{\ \nu} + \ldots \end{aligned}$$

so $\kappa \leftrightarrow \frac{1}{f_\pi}$

Curvatures:

$$\begin{aligned} R_{\mu\nu} &= \frac{\kappa}{2}\left[\partial_\mu\partial_\nu h^\lambda_{\ \lambda} + \partial_\lambda\partial^\lambda h_{\mu\nu} - \partial_\mu\partial_\lambda h^\lambda_{\ \nu} - \partial_\lambda\partial_\nu h^\lambda_{\ \mu}\right] + \mathcal{O}(h^2) \\ R &= \kappa\left[\Box h^\lambda_{\ \lambda} - \partial_\mu\partial_\nu h^{\mu\nu}\right] + \mathcal{O}(h^2) \end{aligned}$$

indices are raised and lowered with $\eta_{\mu\nu}$. This can be done around any fixed background space time metric.

Gauge fixing and field equations

Green function do not exist without a gauge choice and it is most convenient to use harmonic gauge

$$\partial^\lambda h_{\mu\lambda} = \frac{1}{2}\partial_\mu h^\lambda_\lambda$$

The field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu}, \qquad \sqrt{g}T^{\mu\nu} \equiv -2\frac{\delta}{\delta g_{\mu\nu}}\left(\sqrt{g}\mathcal{L}_m\right)$$

reduce in this gauge to

$$\Box h_{\mu\nu} = -16\pi G\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^\lambda_\lambda\right)$$

The momentum space propagator is relatively simple in this gauge. Around Minkowski:

$$iD_{\mu\nu\alpha\beta} = \frac{i}{q^2 + i\epsilon}P_{\mu\nu,\alpha\beta} \qquad P_{\mu\nu,\alpha\beta} \equiv \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right]$$

In addition one needs to include the gauge-fixing and ghost part

$$\mathcal{L}_{gf} = \sqrt{\bar{g}}\left\{\left(D^\nu h_{\mu\nu} - \frac{1}{2}D_\mu h^\lambda_\lambda\right)\left(D_\sigma h^{\mu\sigma} - \frac{1}{2}D^\mu h^\sigma_\sigma\right)\right\} \; \mathcal{L}_{gh} = \sqrt{\bar{g}}\eta^{*\mu}\left[D_\lambda D^\lambda \bar{g}_{\mu\nu} - R_{\mu\nu}\right]\eta^\nu$$

It is plain that perturbative calculations in quantum gravity are manifestly difficult.

Divergences

The following two results are well known

$$\mathcal{L}^{(div)}_{1loop} = -\frac{1}{16\pi^2\epsilon}\left\{\frac{1}{120}\bar{R}^2 + \frac{7}{20}\bar{R}_{\mu\nu}\bar{R}^{\mu\nu}\right\}$$

(t Hooft and Veltman)

$$\mathcal{L}^{(div)}_{2loop} = -\frac{209\kappa^2}{5760(16\pi^2)}\frac{1}{\epsilon}\bar{R}^{\alpha\beta}{}_{\gamma\delta}\bar{R}^{\gamma\delta}{}_{\eta\sigma}\bar{R}^{\eta\sigma}{}_{\alpha\beta}$$

(Goroff and Sagnotti)

It is less well appreciated that the two results are on a different footing. The result of ’t Hooft and Veltman

– is gauge dependent

– vanishes when the field equation in empty space are used

– gives a net divergence when $T_{\mu\nu} \neq 0$, but the result is, in principle, incomplete.

The one-loop counterterms computed by ’t Hooft and Veltman are largely irrelevant from the point of view of effective lagrangians (they vanish on shell).

de Sitter space-time

In de Sitter space

$$S = \frac{1}{16\pi G}\int dx\sqrt{-g}(\mathcal{R} - 2\Lambda)$$

$$\Gamma^{div}_{eff} = -\frac{1}{16\pi^2\epsilon}\int dx\sqrt{-g}[c_1\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + c_2\Lambda^2 + c_3\mathcal{R}\Lambda + c_4\mathcal{R}^2].$$

The constants c_i are actually gauge dependent and only a combination of them is gauge invariant.

Using the equations of motion (in absence of matter) $\mathcal{R}_{\mu\nu} = g_{\mu\nu}\Lambda$, the previous equation reduces to the (gauge-invariant) on-shell expression

$$\Gamma^{div}_{eff} = \frac{1}{16\pi^2\epsilon}\int dx\sqrt{-g}\frac{29}{5}\Lambda^2.$$

If we set $\Lambda = 0$ above, we get the well-known 't Hooft and Veltman divergence

$$\Gamma^{div}_{eff} = -\frac{1}{16\pi^2\epsilon}\int dx\sqrt{-g}[\frac{7}{20}\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \frac{1}{120}\mathcal{R}^2].$$

Counterterms and power counting

Exactly as the chiral lagrangian Einstein-Hilbert requires an infinite number of counterterms

$$\mathcal{L} = M_P^2\sqrt{-g}\mathcal{R} + \alpha_1\sqrt{-g}\mathcal{R}^2 + \alpha_2\sqrt{-g}(\mathcal{R}_{\mu\nu})^2 + \alpha_3\sqrt{-g}(\mathcal{R}_{\mu\nu\alpha\beta})^2 + \ldots$$

The divergences can be absorbed by redefining the coefficients just as before

$$\alpha_i \to \alpha_i + \frac{c_i}{\epsilon}$$

The expansion parameter is a tiny number in normal circumstances

$$p^2/16\pi M_P^2$$

or

$$\nabla^2/16\pi^2 M_P^2, \qquad \mathcal{R}/16\pi^2 M_P^2$$

The most effective of all effective actions!!

Why we need genuine loop effects

Consider

$$\mathcal{L} = \frac{2}{\kappa^2} R + cR^2 + (matter)$$

The equation of motion is

$$\Box h + \kappa^2 c^2 \Box\Box h = (8\pi G T)$$

The Green function for this equation has the form

$$\begin{aligned} G(x) &= \int \frac{d^4 q}{(2\pi)^4} \frac{e^{iq\cdot x}}{q^2 + \kappa^2 c q^4} \\ &= \int \frac{d^4 q}{(2\pi)^4} \left[\frac{1}{q^2} - \frac{1}{q^2 + 1/\kappa^2 c} \right] e^{-iq\cdot x} \end{aligned}$$

Leading to a correction to Newton's law

$$V(r) = -G m_1 m_2 \left[\frac{1}{r} - \frac{e^{-r/\sqrt{\kappa^2 c}}}{r} \right]$$

Experimental bounds indicate $c < 10^{74}$. If c was a reasonable number there would be no effect on any observable physics at terrestrial scales.

Note that if $c \sim 1, \sqrt{\kappa^2 c} \sim 10^{-35} m$. The curvature is so small that R^2 terms are irrelevant at ordinary scales

Why we need genuine loop effects II

However using the full solution of the wave equation is *not* compatible with the effective lagrangian philosophy (higher orders in κ are sensitive to higher curvatures we have not considered). The leading behaviour of the correction is

$$\frac{e^{-r/\sqrt{\kappa^2 c}}}{r} \rightarrow 4\pi\kappa^2 c\delta^3(\vec{r})$$

$$\frac{1}{q^2+\kappa^2 c q^4} = \frac{1}{q^2} - \kappa^2 c + \cdots$$

Thus

$$V(r) = -Gm_1 M_2 \left[\frac{1}{r} + 128\pi^2 G c\delta^3(\vec{x})\right]$$

Totally unobservable, even as a matter of principle.

Of course, apart from the divergences there are finite pieces and *non-local* pieces since in DR we get at the one-loop level

$$\frac{1}{\epsilon} + \log\frac{-p^2}{\mu^2}$$

Or, in position space $\frac{1}{\epsilon} + \log\frac{\nabla^2}{\mu^2}, \qquad \nabla = \text{ covariant derivative.}$

Non-localities are due to the propagation of massless non-conformal modes, such as the graviton itself.

Quantum corrections to Newton law

Let us use 'chiral counting' arguments to derive the relevant quantum corrections to Newton law (up to a constant)

Propagator at tree level: $\frac{1}{p^2}$

One-loop corrections: $\frac{1}{p^2}(1 + A\frac{p^2}{M_P^2} + B\frac{p^2}{M_P^2}\log p^2)$

Consider the interaction with an static source ($p^0 = 0$) and let us Fourier transform

$$\int d^3x \exp(i\vec{p}\vec{x})\frac{1}{p^2} \sim \frac{1}{r} \qquad \int d^3x \exp(i\vec{p}\vec{x})1 \sim \delta(\vec{x})$$

$$\int d^3x \exp(i\vec{p}\vec{x}) \log p^2 \sim \frac{1}{r^3}$$

Thus the corrections are of the form

$$\frac{GMm}{r}(1 + C\frac{G\hbar}{r^2} + \ldots)$$

We note that

$$\left[\frac{Gm}{c^2}\right] = L, \qquad \left[\frac{G\hbar}{c^3}\right] = L^2$$

so C is a pure number.

The inclusion of matter

A long controversy regarding the value of C exist in the literature (Donoghue, Muzinich, Vokos, Hamber, Liu, Bellucci, Khriplovich, Kirilin, Holstein, Bjerrum-Bohr,...)

The commonly accepted result is obtained by considering the inclusion of *quantum* matter fields (a scalar field actually) and considering all type of loops

Feynman rules

$$
\begin{aligned}
\tau_{\mu\nu} &= -\frac{i\kappa}{2}\left(p_\mu p'_\nu + p'_\mu p_\nu - g_{\mu\nu}[p\cdot p' - m^2]\right) \\
\tau_{\eta\lambda,\rho\sigma} &= \frac{i\kappa^2}{2}\Big\{ I_{\eta\lambda,\alpha\delta} I^{\delta}{}_{\beta,\rho\sigma}\left(p^\alpha p'^\beta + p'^\alpha p^\beta\right) \\
&\quad -\frac{1}{2}\left(\eta_{\eta\lambda} I_{\rho\sigma,\alpha\beta} + \eta_{\rho\sigma} I_{\eta\lambda,\alpha\beta}\right) p'^\alpha p^\beta \\
&\quad -\frac{1}{2}\left(I_{\eta\lambda,\rho\sigma} - \frac{1}{2}\eta_{\eta\lambda}\eta_{\rho\sigma}\right)[p\cdot p' - m^2]\Big\}
\end{aligned}
$$

with

$$
I_{\mu\nu,\alpha\beta} \equiv \frac{1}{2}[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}]
$$

The inclusion of matter II

$$\mathcal{L}_{RR} = \frac{1}{3849\pi^3 r^3}(42R_{\mu\nu}R^{\mu\nu} + R^2)$$

$$\mathcal{L}_{RT} = -\frac{\kappa}{8\pi^2 r^3}(3R_{\mu\nu}T^{\mu\nu} - 2RT)$$

$$\mathcal{L}_{TT} = \frac{\kappa^2}{60\pi r^3}T^2$$

Using the equation of motion

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu}$$

$$\Rightarrow \quad \mathcal{L}_{total} = -\frac{\kappa^2}{60\pi r^3}(138T_{\mu\nu}T^{\mu\nu} - 31T^2)$$

The final result is positive: gravity is more atractive at long distances

$$C = \frac{41}{10\pi}$$

What happens for *classical* matter, e.g. a cloud of dust, is in my view still an open problem.

Power counting in effective gravity

- 3-graviton coupling: $\sim \kappa q^2$
- 4-graviton coupling: $\sim \kappa^2 q^2$
- (On-shell) matter– 1-graviton coupling: $\sim \kappa m^2$
- (On-shell) matter– 2-graviton coupling: $\sim \kappa^2 m^2$
- Graviton propagator: $\sim \frac{1}{q^2}$
- Matter propagator $\sim \frac{1}{mq}$

If we iterate the 4-graviton vertex to produce a one loop diagram we obtain schematically

$$\mathcal{M}_{loop} \sim \kappa^4 \int \frac{d^4 l}{(2\pi)^4} \frac{(l-p_1)^2 (l-p_2^2)^2}{l^2 (l-q)^2}$$

If this loop integral is regularized dimensionally, which does not introduce powers of any new scale, the integral will be represented in terms of the exchanged momentum to the appropriate power. Thus we have

$$\mathcal{M}_{loop} \sim \kappa^4 q^4$$

Power counting in effective gravity II

When matter fields are included in loops the situation is more subtle The tree level result is

$$\mathcal{M}_{tree} = \kappa^2 \cdot \frac{m_1^2 m_2^2}{q^2}$$

Iterating this to form a loop

$$\mathcal{M}_{loop} \sim \kappa^4 m_1^4 m_2^4 \cdot \int d^4 l \cdot \frac{1}{m_1(l+p)} \cdot \frac{1}{m_2(l+p')} \cdot \frac{1}{(l+q')^2} \cdot \frac{1}{(l+q)^2}$$

which by the same reasoning is

$$\mathcal{M}_{loop} \sim \kappa^4 \cdot \frac{m_1^3 m_2^3}{q^2} \sim \kappa^2 \cdot \frac{m_1^2 m_2^2}{q^2} \cdot \kappa^2 m_1 m_2$$

Here the expansion parameter appears as $\kappa^2 m^2$ This issue has been studied by Donoghue

$$A_{(N_m,N_g)} \sim q^D$$

$$D = 2 - \frac{N_E^m}{2} + 2N_L - N_V^m + \sum_n (n-2) N_V^g[n] + \sum_l l \cdot N_V^m[l]$$

If we disregard matter vertices this is *identical* to Weinberg's result for chiral theories However it is dangerous the negative N_V^m term appearing in D. Although no general proof exists yet, Donoghue has been able to prove cancellation of the dangerous terms at the one-loop level except for the terms leading to $1/r$ corrections (classical, non-linear)

The use of equations of motion

In chiral lagrangians they allow to get rid of redundant operators

$$U\Box U^\dagger - (\Box U)U^\dagger = 0$$

$$\mathrm{Tr}\ U\Box U^\dagger \to 0$$

Notice that in gravity, the equation of motion mixes terms of different ‘chiral’ order

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu} - g_{\mu\nu}\Lambda$$

For instance, it is incorrect to use

$$R_{\mu\nu} = g_{\mu\nu}\Lambda$$

in ’t Hooft and Veltman calculation. It just does not reproduce the de Sitter result.

Cosmological implications

We are concerned about (universal) quantum corrections to the Einstein-Hilbert lagrangian

$$\frac{1}{16\pi^2 M_{pl}^2}\mathcal{R}[\log\nabla^2]\mathcal{R}$$

There are two reasons why such apparently hopelessly small corrections might be relevant in a cosmological setting

– Curvature was much larger at early stages of the universe: in a de Sitter universe $\mathcal{R}\sim H^2$, $H^2 = 8\pi G V_0/3$, $H \leq 10^{13}$ GeV (present value is 10^{-42} GeV).

– Logarithmic non local term corresponds to an interaction between geometries that is long-range in time, an effect that does not have an easy classical interpretation.

– These non-localities are unrelated to $f(\mathcal{R})$ models. They are real and unambigous.

Somewhat related (?) non-localities (but at the two loop level) were studied by Tsamis and Woodard long ago. They slow down the rate of inflation.

Our conventions:

$$S = \frac{1}{16\pi G}\int dx\sqrt{-g}(\mathcal{R}-2\Lambda) + S_{matter}, \qquad R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Quantum corrections

Quantum corrections to the Einstein-Hilbert action were originally computed by 't Hooft and Veltman in the case of vanishing cosmological constant, and by Chistensen and Duff for a de Sitter background. The key ingredient we shall need is the divergent part of the one-loop effective action. Setting $d = 4 + 2\epsilon$

$$\Gamma^{div}_{eff} = -\frac{1}{16\pi^2\epsilon}\int dx\sqrt{-g}[c_1\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + c_2\Lambda^2 + c_3\mathcal{R}\Lambda + c_4\mathcal{R}^2].$$

The constants c_i are actually gauge dependent and only a combination of them is gauge invariant. Using the equations of motion (in absence of matter) $\mathcal{R}_{\mu\nu} = g_{\mu\nu}\Lambda$, the previous equation reduces to the (gauge-invariant) on-shell expression

$$\Gamma^{div}_{eff} = \frac{1}{16\pi^2\epsilon}\int dx\sqrt{-g}\frac{29}{5}\Lambda^2.$$

If we set $\Lambda = 0$ above, we get the well-known 't Hooft and Veltman divergence, that in the so-called minimal gauge is

$$\Gamma^{div}_{eff} = -\frac{1}{16\pi^2\epsilon}\int dx\sqrt{-g}[\frac{7}{20}\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \frac{1}{120}\mathcal{R}^2].$$

If the equations of motion are used in the absence of matter this divergence is absent.

E.o.M. in the presence of non-local terms

For the sake of discussion, we shall consider here a simplified effective action that includes only terms containing the scalar curvature

$$S = \kappa^2 \left(\int dx \sqrt{-g} \mathcal{R} + \tilde{\alpha} \int dx \sqrt{-g} \mathcal{R} \ln(\nabla^2/\mu^2) \mathcal{R} + \tilde{\beta} \int dx \sqrt{-g} \mathcal{R}^2 \right)$$
$$\equiv \kappa^2 \left(S_1 + \tilde{\alpha} S_2 + \tilde{\beta} S_3 \right),$$

where $\kappa^2 = M_P^2/16\pi = 1/16\pi G$. μ is the subtraction scale. The coupling $\tilde{\beta}$ is μ dependent in such a way that the total action S is μ-independent.

– The value of $\tilde{\beta}$ is actually dependent on the UV structure of the theory (it contains information on all the modes -massive or not- that have been integrated out)

– The value of $\tilde{\alpha}$ is unambiguous: it depends only on the IR structure of gravity (described by the Einstein-Hilbert Lagrangian) and the massless (nonconformal) modes. In conformal time

$$g_{\mu\nu} = a^2(\tau) \eta_{\mu\nu}, \ \mathcal{R} = 6 \frac{a''(\tau)}{a^3(\tau)}, \ \sqrt{-g} = a^4(\tau).$$

We first obtain the variation of the local part

$$\frac{\delta S_1}{\delta a(\tau)} = 12 a'' \qquad \frac{\delta S_3}{\delta a(\tau)} = 72 \left(-3 \frac{(a'')^2}{a^3} - 4 \frac{a' a'''}{a^3} + 6 \frac{(a')^2 a''}{a^4} + \frac{a^{(4)}}{a^2} \right)$$

Computing the non-local part

To obtain the variation of the non-local (logarithmic piece) we need to compute

$$\langle x|\log\nabla^2|y\rangle$$

where in conformal coordinates

$$\nabla^2 = a^{-3}\Box a + \frac{1}{6}\mathcal{R}$$

To the order we are computing we can neglect the $\mathcal{R}$ term in the previous equation and commute the scale factor a with the flat d'Alembertian

$$\nabla^2 = \left(\frac{a}{a_0}\right)^{-2}\Box$$

Where $a_0 = a(0)$. With this rescaling (absorbable in $\tilde{\beta}$), at $\tau = 0$ the d'Alembertian in conformal space matches with the Minkowskian one.

We can now separate S_2 in turn into a local and a genuinely non-local piece

$$S_2 = \int dx\sqrt{-g}\,\left(-2\mathcal{R}\ln(a)\mathcal{R} + \mathcal{R}\ln(\Box/\mu^2)\mathcal{R}\right) \equiv S_2^I + S_2^{II}.$$

$$\frac{\delta S_2^I}{\delta a(\tau)} = -72\left\{\frac{(a')^2a''}{a^4}[12\ln a - 10] + \frac{a'a'''}{a^3}[-8\ln a + 4] + \frac{(a'')^2}{a^3}[-6\ln a + 2] + \frac{a^{(4)}}{a^2}2\ln a\right\}$$

Computing the non-local part II

Finally we have to compute

$$\langle x|\ln\Box|y\rangle = \lim_{\epsilon\longrightarrow 0}\frac{1}{\epsilon}\langle x|\Box^{\epsilon}|y\rangle - \frac{1}{\epsilon}\langle x|y\rangle$$

The delta function is in one-to-one correspondence with the counterterm. The Green function we are interested will be

$$\sim \frac{1}{|x-y|^{4+2\epsilon}}$$

After integration of $\vec{x}-\vec{y}$ we get

$$\sim \frac{1}{|t-t'|^{1+2\epsilon}}$$

So

$$S_2^{II} = 36\int d\tau\,\frac{a''(\tau)}{a(\tau)}\int_0^{\tau} d\tau'\,\frac{1}{\tau-\tau'}\,\frac{a''(\tau')}{a(\tau')}$$

The variation of S_2^{II} is

$$\frac{\delta S_2^{II}}{\delta a(\tau)} = 36\left\{\left[2a^{-3}(\tau)\left(a'(\tau)\right)^2 - 2a^{-2}(\tau)a''(\tau)\right]\int_0^{\tau} d\tau'\,\frac{1}{\tau-\tau'}\,\frac{a''(\tau')}{a(\tau')}\right.$$

$$\left.-2a^{-2}(\tau)a'(\tau)\frac{\partial}{\partial\tau}\left(\int_0^{\tau} d\tau'\,\frac{1}{\tau-\tau'}\,\frac{a''(\tau')}{a(\tau')}\right) + a^{-1}(\tau)\frac{\partial^2}{\partial\tau^2}\left(\int_0^{\tau} d\tau'\,\frac{1}{\tau-\tau'}\,\frac{a''(\tau')}{a(\tau')}\right)\right\}$$

Next ...

In the spirit of effective Lagrangians we would obtain first the lowest order equation of motion from S_1 and plug it in $\tilde{\alpha}(S_2^I + S_2^{II}) + \tilde{\beta} S_3$

Quantum corrections act as an external driving force superimposed to Einstein equations. This procedure of course gives trivially a zero additional contribution here as neither matter nor a cosmological constant have been considered.

QG effects in de Sitter universe

The relevant one-loop corrected effective action is

$$S = \frac{1}{16\pi G}\int dx\sqrt{-g}(\mathcal{R}-2\Lambda) + \frac{1}{16\pi^2}\int dx\sqrt{-g}\frac{29}{5}\Lambda\ln\frac{\nabla^2}{\mu^2}\Lambda + \text{local terms of } \mathcal{O}(p^4)$$

$$\equiv \kappa^2\left(\int dx\sqrt{-g}(\mathcal{R}-2\Lambda) + \tilde{\alpha}S_2\right).$$

$$\tilde{\alpha} = \frac{G}{\pi}\times\frac{29}{5}$$

We split S_2 in two parts

$$S_2^I = -2\int dx\sqrt{-g}\Lambda^2\ln(a), \qquad S_2^{II} = \int dx\sqrt{-g}\Lambda\ln(\Box/\mu^2)\Lambda,$$

and obtain the corresponding variations following the method outlined

$$\frac{\delta S_2^I}{\delta a(\tau)} = -2\Lambda^2 a^3(\tau)\left[4\ln(a(\tau))+1\right] \qquad \frac{\delta S_2^{II}}{\delta a(\tau)} = 2\Lambda^2 a(\tau)\int_0^\tau d\tau' a^2(\tau')\frac{\mu^{-2\epsilon}}{|\tau-\tau'|^{1+2\epsilon}}.$$

Next ...

The equation of motion will be

$$12a''(\tau) - 8\Lambda a^3(\tau) + \tilde{\alpha}\frac{\delta S_2}{\delta a(\tau)} = 0$$

which at lowest order is just

$$12a''(\tau) - 24H^2 a^3(\tau) = 0, \qquad H^2 = \Lambda/3$$

The lowest order solution (with $a(0) = 1$) is

$$a_I(\tau) = \frac{1}{1 - H\tau}$$

The final step is to plug the 0-th order solution $a_I(\tau)$ into the variation of S_2 and recalculate the solution for $a(\tau)$.

Solving the evolution equation

Note that we use a perturbative procedure is of course only valid as long as the correction is small compared to the unperturbed solutions.

We introduce a variable s defined $a_I(\tau) = e^s$. Then s counts the number of e-folds

$$\frac{\delta S_2^I}{\delta a(\tau)} = -2\Lambda^2 e^{3s}\left[4s+1\right] \qquad \frac{\delta S_2^{II}}{\delta a(\tau)} = 2\Lambda^2 e^s I(s)$$

and the equation of motion reads

$$e^{2s}a''(s) + e^{2s}a'(s) - 2a^3(s) = \frac{3}{2}\tilde{\alpha}H^2\left(-e^{3s}(1+4s) + e^s I(s)\right),$$

where I is

$$I(s) = \ln\left(\frac{\mu}{H}(1-e^{-s})\right)e^{2s} + e^s(1 - e^s - se^s),$$

and the equation to solve is

$$e^{2s}a''(s) + e^{2s}a'(s) - 2a^3(s) = \frac{3}{2}\tilde{\alpha}H^2\left[-(5s+2)e^{3s} + e^{2s} + e^{3s}\ln\left(\frac{\mu}{H}(1-e^{-s})\right)\right]$$

Note that $\tilde{\alpha}$ appears only in the combination $\tilde{\alpha}H^2$. Since there are H large uncertainties in H in practice only the sign of $\tilde{\alpha}$ is relevant. In addition, there is some ambiguity associated to the choice of the renormalization scale that appears in the combination $\ln(\mu/H)$

Numerical results

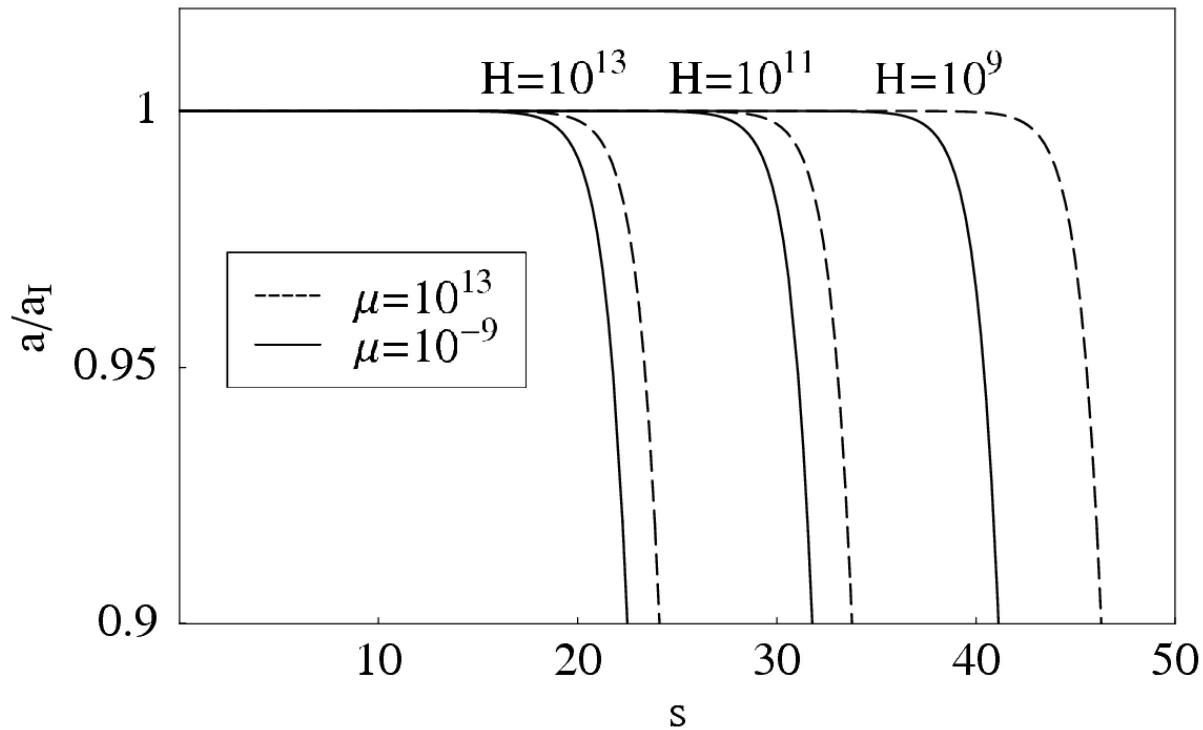

Figure 2: The scale factor relative to the inflationary expansion for different values of μ and H (all units are GeV). We can see that the curves present a very similar behaviour for the different values shown, though a higher value of H leads earlier to deviations from the usual inflationary expansion. Higher values of μ also have this effect, which is larger as H increases. In fact, if we considered values of μ/H large enough (but not relevant physically), the logarithm term would become dominant and the deviation would be positive.

Space quantum correlations

Let us now assume that $a = a(\tau, \vec{x})$. Then

$$\frac{\delta S_2^{II}}{\delta a(\tau, \vec{x})} \sim \Lambda^2 a(\tau, \vec{x}) \int_0^\tau d\tau' d^3\vec{y} a^2(\tau', \vec{y}) \frac{\mu^{-2\epsilon}}{|x - y|^{4+2\epsilon}}.$$

This corresponds to new correlations of a quantum nature between different points.

Gravity as a Goldstone phenomenon

We have given arguments why the Einstein-Hilbert action could be viewed as an effective one

– Dimensionful coupling constant ($M_{pl} \sim f_\pi$)

– Derivative couplings ($\sqrt{-g}\mathcal{R} \sim g\partial\partial g$)

– Action based on RG criteria of relevance, not on renormalizability (unlike Yang-Mills)

– Power counting anologous to ChPT

– Massless quanta ($\pi \leftrightarrow g_{\mu\nu}$)

– Obvious global symmetry to be broken (Lorentz)

As an entertainment we shall investigate a formulation inspired as much as possible in the chiral symmetry breaking of QCD

– No *a priori* metric, only affine connection is needed (parallelism)

– Lagrangian is manifestly independent of the metric

– Breaking is triggered by fermion condensate

Chiral Symmetry Breaking

A successful model for QCD is the so-called chiral quark model. Consider the matter part lagrangian of QCD with massless quarks (2 flavours)

$$\mathcal{L} = i\bar{\psi}\ \partial\!\!\!/ \psi = i\bar{\psi}_L\ \partial\!\!\!/ \psi_L + i\bar{\psi}_R\ \partial\!\!\!/ \psi_R$$

This theory has a global $SU(2) \times SU(2)$ symmetry that forbids a mass term M

However after chiral symmetry breaking pions appear and they must be included in the effective theory. Then it is possible to add the following term

$$-M\bar{\psi}_L U \psi_R - M\bar{\psi}_R U^\dagger \psi_L$$

invariant under the full global symmetry

$$\psi_L \to L\psi_L, \qquad \psi_R \to R\psi_R, \qquad U \to LUR^\dagger$$

Chiral symmetry breaking is characterized by the presence of a fermion condensate

$$< \bar{\psi}\psi > \neq 0$$

To determine whether the condensate is zero or not one is to solve a 'gap'-like equation in some modelization of QCD, or on the lattice.

Integrating out the fermions reproduces the chiral effective lagrangian

Spontaneous Lorentz breaking

There is only one possible term bilinear in fermions that is invariant under Lorentz $\times$ *Diff*

$$\bar{\psi}_a \gamma^a \nabla_\mu \psi^\mu$$

To define ∇ we only need an affine connection

$$\nabla_\mu \psi^\mu = \partial_\mu \psi^\mu + i\omega_\mu^{ab} \sigma_{ab} \psi^\mu + \Gamma^\nu_{\mu\nu} \psi^\mu$$

Note that no metric is needed at all to define the action if we assume that ψ^μ behaves as a contravariant spinorial vector density under *Diff*

We would like to find a non trivial condensate

$$< \bar{\psi}_a \psi^\mu > \sim e_a^\mu$$

We have to include some dynamics to trigger symmetry breaking and make sure the lagrangian is hermitian

$$S_I = \int d^4x ((\bar{\psi}_a \psi^\mu + \bar{\psi}^\mu \psi_a) B_\mu^a + c \det(B_\mu^a))$$

Note that the interaction one also behaves as a density thanks to one of the Levi-Civita symbol hidden in the determinant of B.

The effective action

We shall consider the above model for $D = 2$ for simplicity.

Note the peculiar 'free' kinetic term $\gamma^a \otimes k_\mu$

$$M = \begin{pmatrix} B_{11} & k_1 & B_{12} & k_2 \\ k_1 & B_{11} & k_2 & B_{12} \\ B_{21} & -ik_1 & B_{22} & -ik_2 \\ ik_1 & B_{21} & ik_2 & B_{22} \end{pmatrix}$$

We define

$$M = i\gamma^a \otimes \nabla_\mu \qquad \Delta = MM^\dagger$$

We want to compute

$$W = -\frac{1}{2}\int_0^\infty \frac{dt}{t} \text{tr} \left\langle x | e^{-t\Delta} | x \right\rangle$$

$$\left\langle x | e^{-t\Delta} | x \right\rangle = \frac{1}{t^{D/2}} \int \frac{d^D k}{(2\pi)^D} \text{tr} \left[e^{-k^2\gamma^a\gamma^b + i\sqrt{t}(\gamma^a \mathcal{D}^b_\mu k_\mu + \mathcal{D}^a_\mu k_\mu \gamma^b) + t\mathcal{D}^a_\mu \mathcal{D}^b_\mu} \right]$$

$$= \frac{1}{t^{D/2}} \int \frac{d^D k}{(2\pi)^D} e^{-tB_c^2} \text{tr} \left[e^{-k^2\gamma^a\gamma^b + i\sqrt{t}(\gamma^a \mathcal{D}^b_\mu k_\mu + \mathcal{D}^a_\mu k_\mu \gamma^b) + t(\mathcal{D}^a_\mu \mathcal{D}^b_\mu + B_c^2 \delta^a_\mu \delta^b_\mu)} \right]$$

Note that

$$e^{-k^2\gamma^a\gamma^b} = \delta^{ab} - \frac{1}{D}\gamma^a\gamma^b + \frac{1}{D}\gamma^a\gamma^b e^{-Dk^2} \equiv P^{ab} + \frac{1}{D}\gamma^a\gamma^b e^{-Dk^2}$$

The gap equation

Let us first consider the case where there is no connection at all ($w_\mu(x) = 0$). We can then use homogeneity and isotropy arguments to look for constant solutions of the gap equation associated to

$$V_{eff} = c\det(B^a_\mu) + 2\int \frac{d^n k}{(2\pi)^n}\mathrm{Tr}\,(\log(-\gamma^a k_\mu + B^a_\mu))$$

The extremum of V_{eff} are found from

$$cn\epsilon_{aa_2\ldots.a_n}\epsilon^{\mu\mu_2\ldots.\mu_n}B^{a_2}_{\mu_2}\ldots.B^{a_n}_{\mu_n} + 2\mathrm{tr}\int \frac{d^n k}{(2\pi)^n}(-\gamma\otimes k + B)^{-1}|^\mu_a = 0$$

Notice that the equations are invariant under the permutation

$$B_{ij} \to B_{\sigma(i)\sigma(j)}, k_i \to k_{\sigma(i)}, \sigma\epsilon S_2$$

Notice also that the equations of motion show that

$$< \bar\psi_a\psi^\mu >\sim \epsilon_{ab}\epsilon^{\mu\nu}B^b_\nu$$

The ‘gap equation’ to solve for constant values of B_{ij} is

$$cB_{ij} - \frac{1}{16\pi}B_{ij}\log\frac{\det B}{\mu} = 0$$

A logarithmic divergence has been absorbed in c.

The effective action II

The next step is to consider $w_\mu(x) \neq 0$ and $B^a_\mu = \delta^a_\mu B_c + b^a_\mu$. This requires the evaluation of the effective action.

The heat-kernel expansion at order t^0 and t^1 gives

$$W = -\frac{1}{2}\int_0^\infty \left[\frac{e^{-tB_c^2}}{t^{D/2}} \frac{2}{(D4\pi)^{D/2}} (1 + t \left(-\frac{(B^a_\mu)^2}{D} + B_c^2 + \frac{(B^a_\mu)^2}{D^2} \right) \right]$$

$$= \frac{(B^a_\mu)^2}{32\pi} \left(\frac{2}{\epsilon} - \gamma - \log B_c^2 + \log 4\pi + 1 \right) + \frac{2B_c^2}{16\pi})$$

$w_\mu(x)$ drops from the effective action at this order in t and hence the solution of the gap equation is exact (at this order) even for a non-trivial connection

At the next order (t^4) one gets terms with four fields/derivatives. We expect as a result of the e.o.m. things like

$$w^{ab}_\mu \sim B^a_\nu \partial B^b_\nu + \ldots$$

What next

The previous example is all too trivial but it shows perfectly the general ideas.

Once a “n-bein” has been dynamically generated, one is allowed to write a terms that (in 4d) will look like

$$-M\bar{\psi}_a e^a_\mu \psi^\mu + h.c.$$

M is a parameter with dimensions of mass that on dimensional grounds must appear in four domensions.

In 4 dimensions the affine connection and the “n-bein” have both to be determined dynamically too. We expect that they will result in giving the connection that is compatible with the metric.

We expect

$$M^2_{pl} \sim \frac{M^2}{16\pi^2} \log \frac{\mu}{M}$$

Manifest breaking of Lorentz symmetry

Let us consider (for the moment just as a theoretical possibility) the possibility of explicit breaking of Lorentz breaking by means of a time-like constant axial vector. Consider electromagnetism in such a background

$$\mathcal{L} = \mathcal{L}_{\rm INV} + \mathcal{L}_{\rm LIV}$$

$$\mathcal{L}_{\rm INV} = -\,\frac{1}{4}\,F^{\alpha\beta}F_{\alpha\beta} + \bar{\psi}[\not\partial - e\,\not{A} - m_e]\psi \qquad \mathcal{L}_{\rm LIV} = \frac{1}{2}\,m_\gamma^2\,A_\mu\,A^\mu + \,\frac{1}{2}\,\eta_\alpha A_\beta \widetilde{F}^{\,\alpha\beta}$$

It will be useful for us to keep $m_\gamma > 0$ for the time being. Otherwise gauge invariance is manifest.

E.o.M.:

$$\left\{g^{\,\lambda\nu}\left(k^2 - m_\gamma^2\right) + i\,\varepsilon^{\,\lambda\nu\alpha\beta}\,\eta_\alpha\,k_\beta\right\}\tilde{A}_\lambda(k) = 0$$

We can build two complex and space-like chiral polarization vectors $\varepsilon_\pm^\mu(k)$which satisfy the orthonormality relations

$$-\,g_{\,\mu\nu}\,\varepsilon_\pm^{\mu\,*}(k)\,\varepsilon_\pm^\nu(k) = 1 \qquad g_{\,\mu\nu}\,\varepsilon_\pm^{\mu\,*}(k)\,\varepsilon_\mp^\nu(k) = 0$$

In addition we have

$$\varepsilon_T^\mu(k) \sim k^\mu \qquad \varepsilon_L^\mu(k) \sim k^2\eta^\mu - k^\mu\eta\cdot k$$

They fulfill

$$g_{\,\mu\nu}\,\varepsilon^{\mu\,*}_{\;A}(k)\,\varepsilon^{\nu}_{\;B}(k) \;=\; g_{\,AB} \qquad g^{\,AB}\,\varepsilon^{\mu\,*}_{\;A}(k)\,\varepsilon^{\nu}_{\;B}(k) \;=\; g^{\,\mu\nu}$$

Different physics in different frames

The polarization vectors of positive and negative chirality are solutions of the vector field equations if and only if

$$k_{\pm}^{\mu} = (\omega_{\mathbf{k}\,\pm}, \mathbf{k}) \qquad \omega_{\mathbf{k}\,\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta|\mathbf{k}|} \qquad \varepsilon_{\pm}^{\mu}(\mathbf{k},\eta) = \varepsilon_{\pm}^{\mu}(k_\pm) \quad \left(\, k_{\pm}^{0} = \omega_{\mathbf{k}\pm}\,\right)$$

In order to avoid problems with causality we want $k_\pm^2 > 0$. For photons of negative chirality this can happen iff

$$|\mathbf{k}| < \frac{m_\gamma^2}{\eta} \equiv \Lambda_{\,\gamma}$$

for $m_\gamma = 0$ they cannot exist. As is known to everyone the processes $e^- \to e^-\gamma$ or $\gamma \to e^+e^-$ cannot occur. However here physics is different in different frames and for the latter process

$$\omega_{\,\mathbf{k}\,\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta\,|\mathbf{k}|} = \sqrt{\mathbf{p}^2 + m_e^2} + \sqrt{(\mathbf{p}-\mathbf{k})^2 + m_e^2}$$

Possible iff

$$|\mathbf{k}| \geq \frac{4m_e^2}{\eta} \equiv k_{\,\mathrm{th}} \qquad (m_\gamma = 0)$$

The electron-positron pairs will be created with a large momentum.

$$\sum |\mathcal{M}_{\,+}(k,p,q)|^2 = \alpha\,\theta\,(|\mathbf{k}| - k_{\mathrm{th}}) \times \;\frac{16\pi}{\mathbf{k}^2}\,\left\{(p\cdot k_+)\left(p\cdot k_+ + \mathbf{k}^2\right) + \eta\,\mathbf{p}^2\,|\mathbf{k}|\right\}$$

Astroparticle consequences

What if η^α represents a oscillating *axion* background?

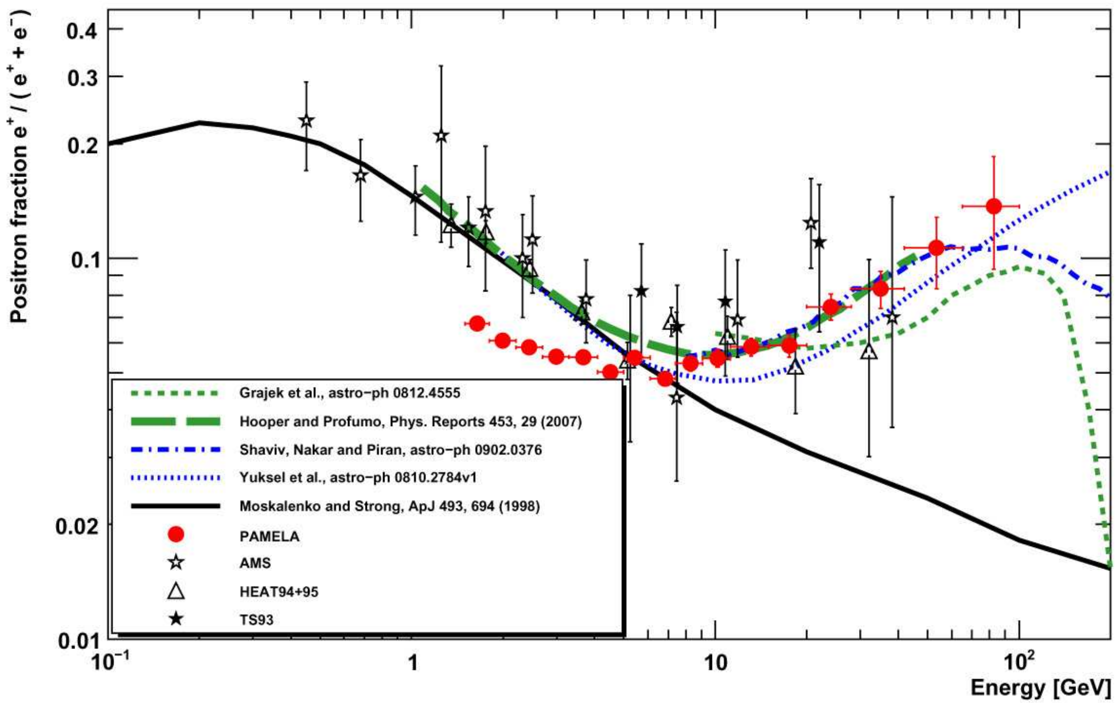

Figure 3: Positrons detected by the PAMELA mission

Summary

– We have analyzed the relevance of the non-local quantum corrections due to the virtual exchange of gravitons and other massless modes to the evolution of the cosmological scale factor in FRW universes.

– Effect is largest in a de Sitter universe with a large cosmological constant.

– The effects are locally absolutely tiny, but they lead to a noticeable secular effect that slows down the inflationay expansion.

– In a matter dominated universe the effect is a lot smaller, and it appears to be of the opposite sign. Quantum effects seem to enhance the expansion rate in this case.

– These effects have no classical analogy.

– The results presented here are not 'just another model'. Quantum gravity non-local loop corrections exist.

– It would be very interesting to compute the space correlations that these logarithmic terms introduce.

– A toy model where gravitons appear as a Goldstone phenomenos has been constructed. The model has no metric whatsoever originally.

– A model with explicit Lorentz breaking (due to the æther-like nature of an axion background has rather exotic effects