Condensates in QCD from Dual Models

Andrey V. ZAYAKIN

LMU, München and ITEP, Moscow

"Aspects of Duality", Zakopane, June 21st, 2008

Outline of the talk

- From Chiral Perturbation Theory to AdS/CFT
  - Mass Spectra and Condensates in χPT
  - AdS/CFT with flavours
  - Condensates from Holography
  - Mass Spectra from Holography
- From AdS/CFT to Chiral Perturbation Theory
  - Pion-Photon Mixing

Condensate in Field-Theoretical Models

The Chiral Perturbation Theory(χPT) result of *Smilga and Shushpanov[1997]* is: for weak fields,

$$\langle\bar{q}q\rangle_H = \langle\bar{q}q\rangle_0\left(1+\frac{|eH|\ln 2}{16\pi^2 f_\pi^2}\right),$$

and for strong fields

$$\langle\bar{q}q\rangle_H \sim |eH|^{\frac{3}{2}} e^{-\frac{\pi}{2}\sqrt{\frac{\pi}{2\alpha_s|eH|}}}$$

Condensate in Nambu—Jona-Lasinio model was calculated by *Klevansky, Lemmer[1989]*:

$$\langle\bar{q}q\rangle_H = \left(1+c\frac{e^2H^2}{(\langle\bar{q}q\rangle_0)^{\frac{4}{3}}}\right).$$

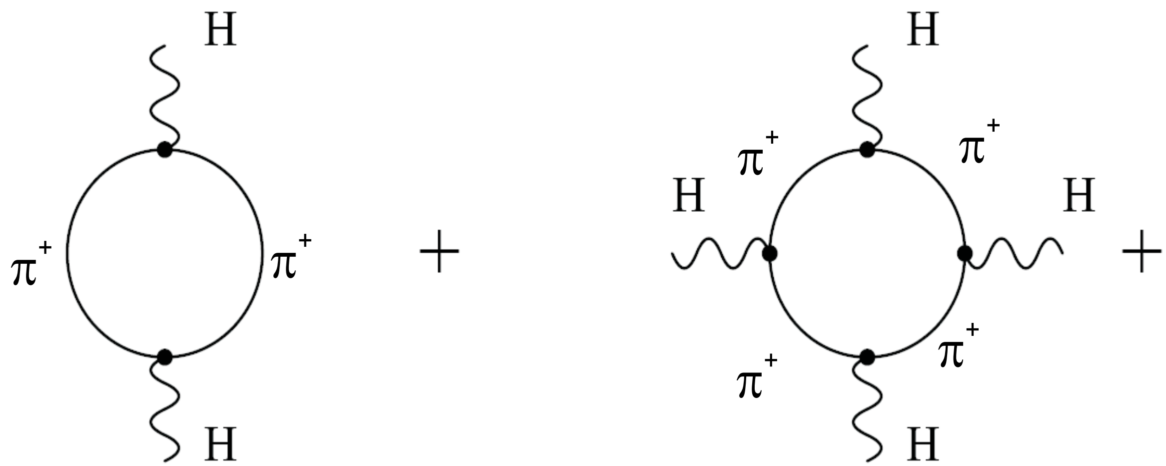
Important points to remember:

- χPT result is non-analytic like $\sqrt{F^2}$. This is the signature of the massless pions in loops.
- Suppression by $\frac{1}{N_c}$:

$$\frac{\delta\langle\bar{q}q\rangle_H}{\langle\bar{q}q\rangle_0} \sim \frac{1}{N_c}$$

Condensate via Euler—Heisenberg

What are the diagrams from which the linear result comes from? These are resummed one-loop diagrams with pions in the loops.



These diagrams contain IR singularities in the chiral limit. When resummed and differentiated over $m_q \sim m_\pi^2$, they yield a finite answer linear in H.

Beyond the Leading Order

Result by *Agasian, Shushpanov[1999]* Next-order corrections behave like

$$\langle\overline{\psi}\psi\rangle_{(H)}|_{NLO} = -\langle\overline{\psi}\psi\rangle_{(0)}\frac{(eH)^2}{(4\pi f_\pi)^4}\left[(\bar{l}_6 - \bar{l}_5)\left(\ln\frac{eH}{\mu^2} + C\right) - \frac{160(4\pi)^4}{3}d^r(\mu)\right]$$

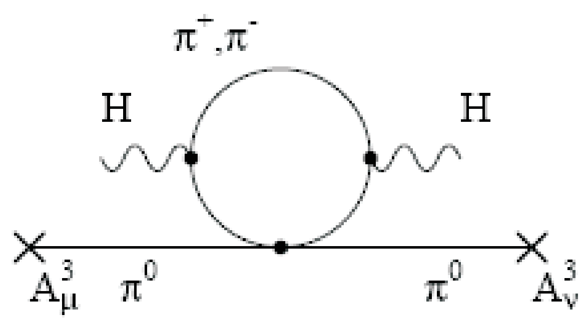
Where does the $\ln$ come from? Mass of pion in chiral limit is modified by terms like $\frac{F^2}{f_\pi^2}$. By analog with finite-temperature massless theories, a power expansion is substituted by a mixed power-logarithm expansion.

m_{π^0}

Important: we stress here that m_{π^0} and m_{π^+} behave themselves drastically differently. Here we speak about m_{π^0}.

$$m_\pi^2(H) = m_\pi^2(H)\left(1 - \frac{eH\ln 2}{16\pi^2 f_\pi^2}\right)$$

This result has the same Euler—Heisenberg nature as the result for condensates. It comes from resummed diagrams like this



where $\pi^\pm$ are running in the loops

AdS_5 Geometry

For review see Gubser et al. [1999].

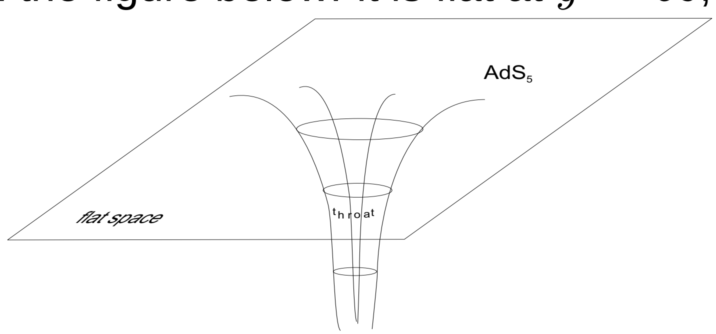
There is pack of N_c copies $D3$ branes, all placed into the same place in ten-dimensional spacetime. The branes being heavy act as a source term for (super)gravity equations of motion. When solved, they yield the $AdS_5 \times S^5$ metric with equal radii of the sphere and the AdS part

$$ds^2 = R^2 \left(\frac{dz^2 + dx^2}{z^2} + d\Omega_5^2 \right).$$

For $y = \frac{R^2}{z}$ the metric is re-written as

$$ds^2 = \left(1 + \frac{R^4}{y^4}\right)^{-\frac{1}{2}} dx^2 + \left(1 + \frac{R^4}{y^4}\right)^{\frac{1}{2}} \left(dy^2 + y^2 d\Omega_5^2\right)$$

which is illustrated in the figure below: it is flat at $y \to \infty$, and looks like a “throat” at $y \to 0$



Basics on AdS/CFT correspondence

For review see *D'Hoker, Freedman[2002]*, *Gubser et al. [1999]*. The AdS/CFT conjecture is:

$$Z_{SYM}[J] = Z_{string}[\Phi]$$

where partition function $Z_{SYM}[J]$ is calculated in presence of four-dimensional currents J, coupled to some operators O

$$Z_{SYM}[J] \equiv \langle e^{-(S+\sum JO_J)} \rangle,$$

and $Z_{string}[\Phi_{\partial AdS}]$ is defined as partition function of supergravity, where each field Φ^J in supergravity has its counterpart J — a current on the gauge theory side, which is related to the boundary value of the five-dimensional field as

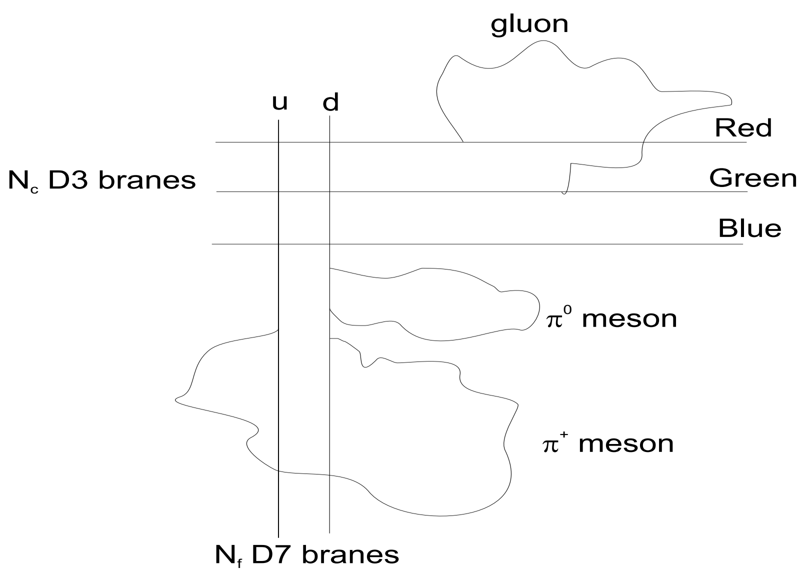
$$\Phi^J_{\partial AdS} \sim J.$$

Some examples of correspondence:

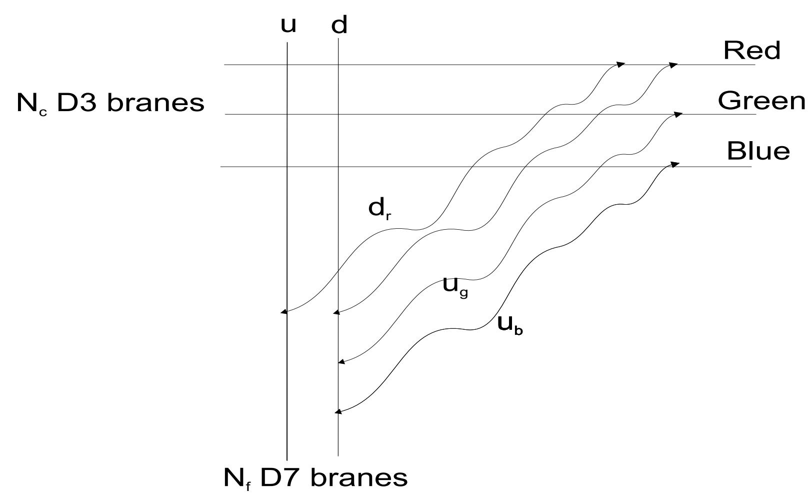
- Dilaton field ϕ in supergravity is dual to operator $\mathrm{tr}F^2$ in SYM.
- Graviton field $h_{\mu\nu}$ is dual to energy-momentum current $T_{\mu\nu}$ in SYM.

AdS/CFT with flavours

For review see: *Aharony, [2002]*; *Mateos [2007]*. General idea of introducing flavour into AdS/CFT is illustrated in the two pictures below:



(a)



(b)

We require $N_c \gg N_f$, for otherwise the stack of N_f $D7$ branes will deform the metric essentially. This is known as “quenched approximation”.

Holography with Flavours, External Fields

- *Karch, Katz [2002]*: Adding flavour to AdS/CFT.
- *Babington et al. [2004]*: Constable—Myers deformation.
- *Filev et al.[2007]*: Flavoured large N gauge theory in an external magnetic field.
- *Erdmenger, Meyer, Shock [2007]*: Pure AdS background in external fields.
- *Bergman et al. [2008]*: Phase transitions in Sakai/Sugimoto models due to electromagnetic fields:
- *Johnson, Kundu [2008]*: External Fields and Chiral Symmetry Breaking in the Sakai-Sugimoto Model.
- *Kim et al.[2008]*: Pair production in Sakai—Sugimoto model.

For a review see *Erdmenger et al. [2007]*.

Why Constable—Myers?

- There is no spontaneous chiral symmetry breaking in pure AdS, and the condensate is zero. However, there will be a condensate at non-zero Kalb—Ramond field B (see *Erdmenger, Meyer, Shock [2007]*).
- There is spontaneous symmetry breaking in AdS deformed a la Constable—Myers.

There are several reasons to study Constable—Myers background as a candidate for non-AdS/non-CFT correspondence, for it provides:

- Spontaneous CSB
- Conformal symmetry breaking
- Supersymmetry breaking

Constable—Myers Geometry

The Constable —Myers metric is organized as

$$ds^2 = H^{-\frac{1}{2}} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\frac{\delta}{4}} dx^2 + H^{\frac{1}{2}} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\frac{2-\delta}{4}} \frac{w^4 - b^4}{w^4} \sum_{i=1}^{6} dw_i^2,$$

where $w^2 = \rho^2 + L^2$, $\rho^2 = w_1^2 + w_2^2 + w_3^2 + w_4^2$, $L^2 = w_5^2 + w_6^2$,

$$H = \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta} - 1$$

and the dilaton is

$$e^{2\phi} = e^{2\phi_0} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\Delta}$$

and a C_4 form field

$$C_{(4)} = -\frac{1}{4} H^{-1} dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3,$$

with conditions imposed upon deformation parameters $\Delta^2 + \delta^2 = 10,\ \delta = \frac{1}{2b^4}$.

D3/D7 Model in Constable—Myers

$D7$ brane does not change the metric in the quenched approximation. The dynamics of the brane is described by a Dirac—Born—Infeld action

$$S_{D7} = \mu_7 \int d^8\xi \sqrt{\det_{\alpha,\beta}\left(2\pi\alpha' B_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta} + g_{\mu\nu}\frac{\partial X^\mu}{\partial\xi^\alpha}\frac{\partial X^\nu}{\partial\xi^\beta}\right)} + \int d^8\xi C_4 \wedge F \wedge B$$

D3 and D7 branes run as follows:

	0	1	2	3	4	5	6	7	8	9
D3	+	+	+	+	-	-	-	-	-	-
D8	+	+	+	+	+	+	+	+	-	-

$\xi_1 \dots \xi_8$ — internal coordinates of the world-volume. Let us search for an embedding like $w_5 = w(\rho), w_6 = 0$, where $\rho = \sqrt{w_1^2 + w_2^2 + w_3^2 + w_4^2}$. With such an Ansatz and in the metric given above, the DBI action is organized as

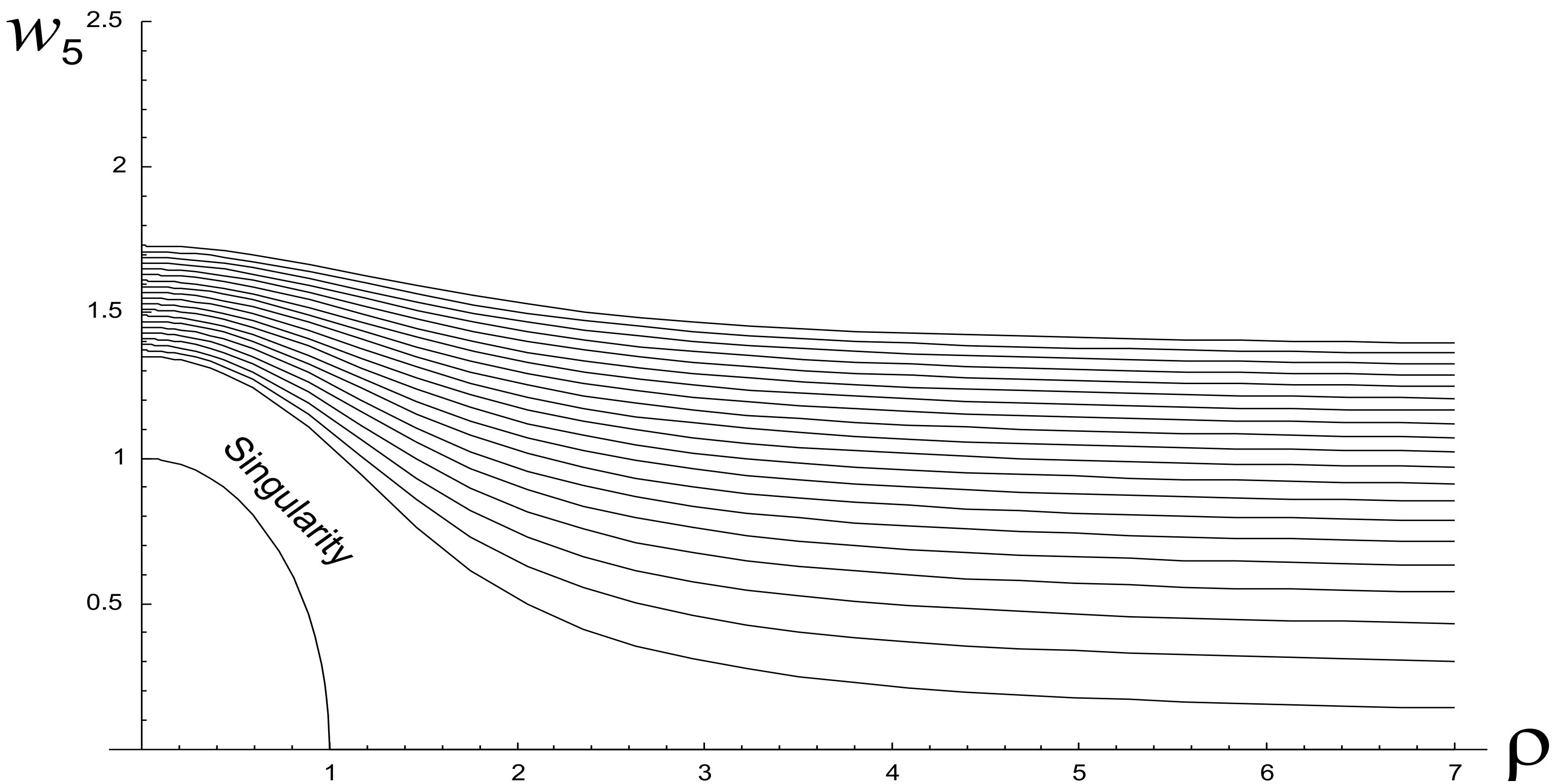
$$S = -\mu_7 \int d^8\xi G(\rho, w)\sqrt{1 + w'^2(\rho)}\sqrt{1 + B^2/g_{11}^2} + S_{\text{Chern—Simons}},$$

where $G(\rho, w) = \rho^3 \frac{((\rho^2+w^2)^2+b^4)((\rho^2+w^2)^2-b^4)}{(\rho^2+w^2)^4} e^{2\phi}$. The equations of motion will look like

$$\frac{d}{d\rho}\left(\frac{Gw'}{\sqrt{1+w'^2}}\sqrt{1 + B^2/g_{11}^2}\right) - \sqrt{1 + w'^2}\frac{d}{dw}\left(G\sqrt{1 + B^2/g_{11}^2}\right) = 0$$

Classical Solutions

Here we show how the classical solutions behave in our setting



A family of embeddings of the spectator D7 brane into Constable—Myers background.
The asymptotes of solutions will yield us vacuum parameters.

Condensate and Quark Masses

General AdS/CFT prescription *(D'Hoker, Freedman [2002])*: For a 10-dimensional bulk field ϕ the solution to field equations is a combination of two modes

$$\phi = \begin{cases} z_0^{\Delta}, \text{normalizable} \\ z_0^{4-\Delta}, \text{non-normalizable} \end{cases}$$

The boundary values of the non-normalizable mode in the bulk are related to the field values on the boundary as

$$J(\vec{x}) = \lim_{z_0 \to 0} z_0^{4-\Delta} \phi(z_0, \vec{x})$$

and thus a correlator of fields

$$< O(x)_J O_J(0) >= \frac{\delta^2 S_{bulk}}{\delta J(x) \delta J(0)}.$$

The normalizable mode of the field ϕ_J is related to the VEV of operator O_J.

Dependence of condensate on mass

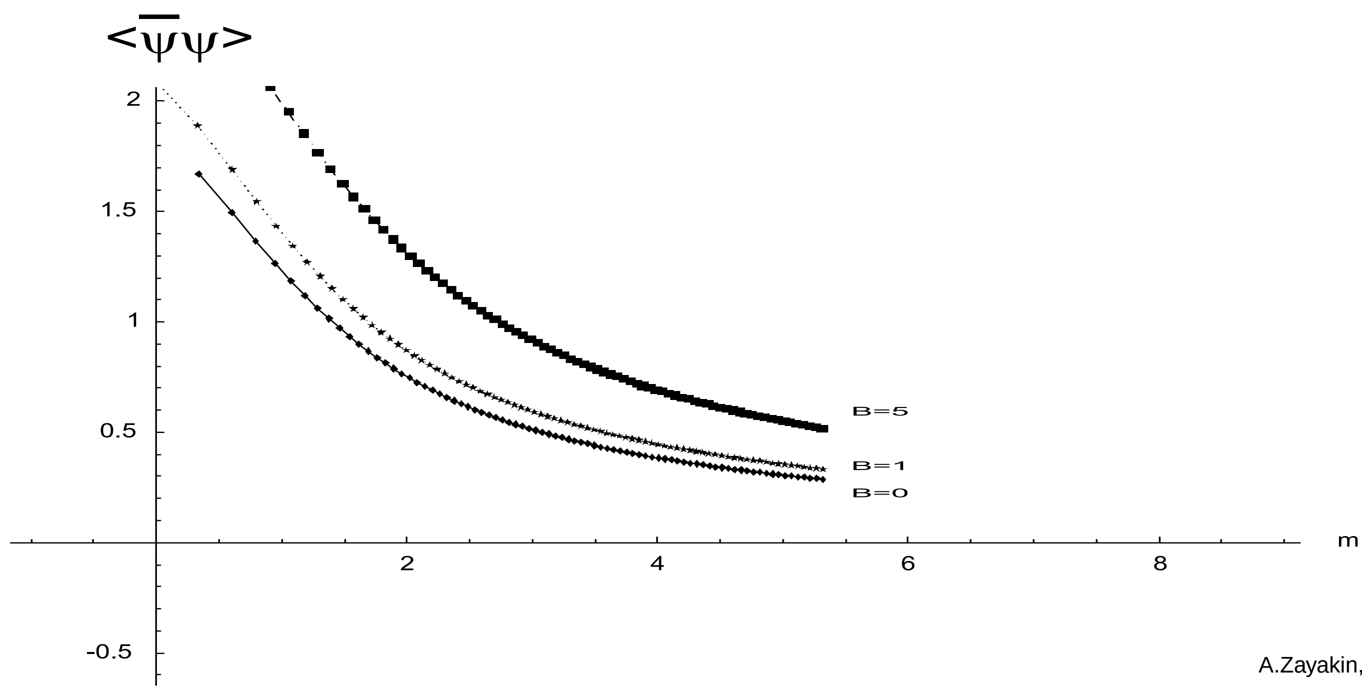
For D3/D7 the classical solution contains information on quark mass and vacuum condensate:

$$w(\rho) = m + \frac{c}{\rho^2}.$$

The parameters m and c correspond to quark mass and chiral condensate:

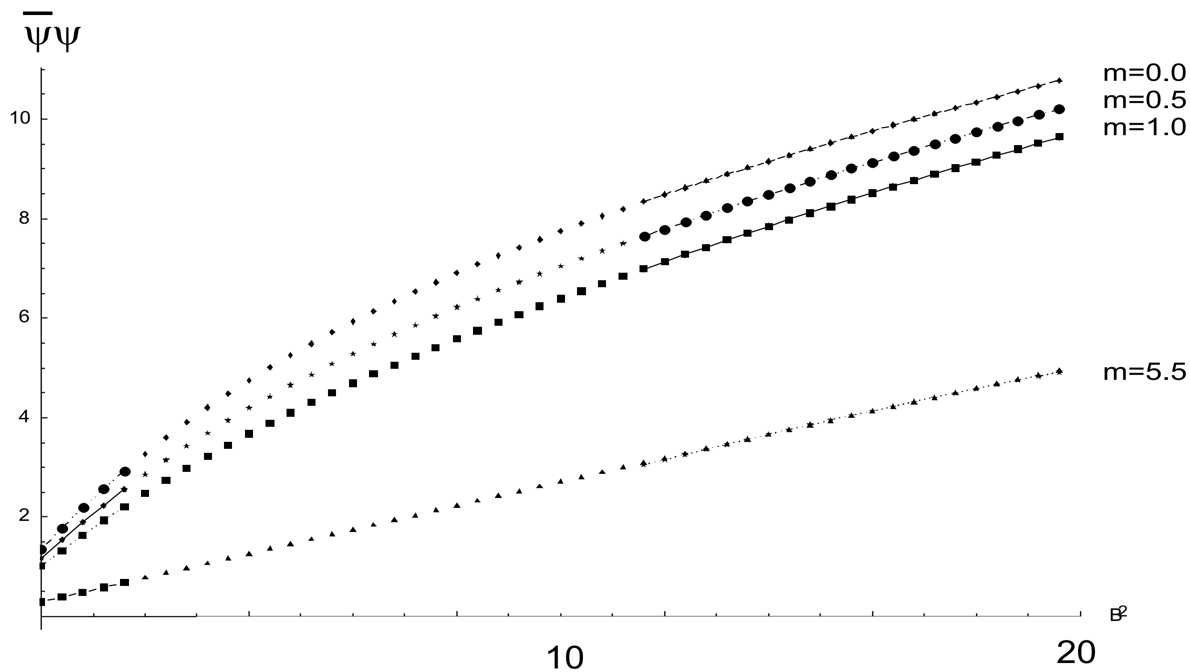
$$m_q = \frac{m}{2\pi\alpha'},$$
$$\langle \bar{q}q \rangle = \frac{c}{(2\pi\alpha')^3},$$

Physical solutions are found by imposing $w'(0) = 0$ and $w(0) = w_0 = const$. By studying a family of solutions one can obtain for any magnetic field a set of $(m, <\overline{\psi}\psi>)$ pairs.



Different regimes

Approximation by either linear or quadratic dependence is valid for large or small fields respec



Condensate dependence on the square of the magnetic field; continuous lines show approximation for small and large field limits:

$$\langle \overline{q}q \rangle_B \sim \begin{cases} \langle \overline{q}q \rangle_{B=0}(1 + c_1(m)B^2),\ B \ll 1 \\ \langle \overline{q}q \rangle_{B=0}(1 + c_2(m)B), B \gg 1. \end{cases}$$

Fluctuations

Meson fields are fluctuations around classical solutions. Their eigenvalues are meson masses. Following correspondence can be made:

Goldstone meson analogous to η' (massless at $N_c \to \infty$)	$\delta\phi$
Non-goldstone scalar meson	δL
Vector meson	δA_μ

The equations for e.g. $f(\rho) = \delta L(\rho)$ will look like:

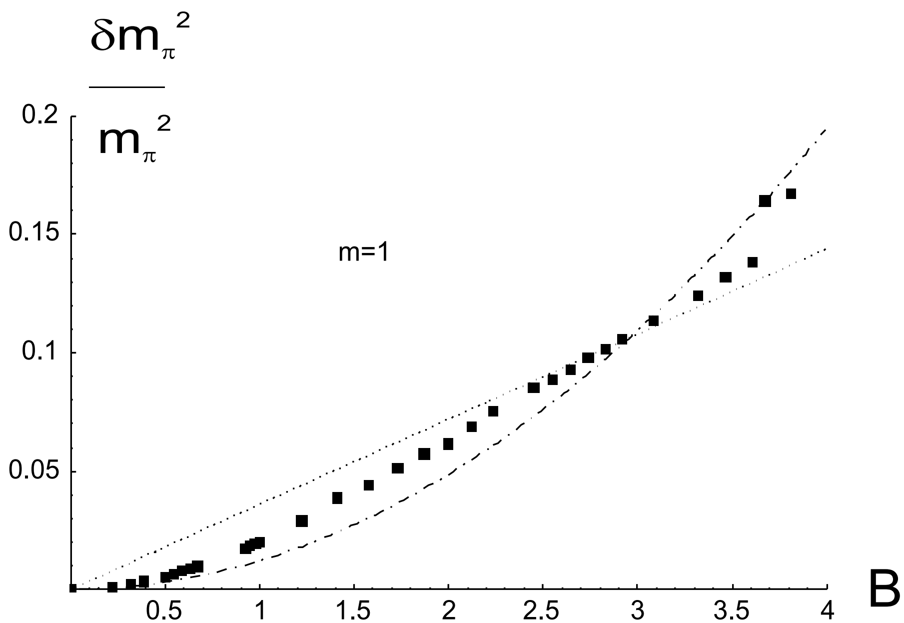
$$\frac{d}{d\rho}\left[\frac{G\sqrt{1+B^2/g_{11}^2}}{\sqrt{1+w'^2}}\partial_\rho f(\rho)\right] + M^2\frac{G\sqrt{1+B^2/g_{11}^2}}{\sqrt{1+w'^2}} H\left(\frac{(\rho^2+w^2)^2+b^4}{(\rho^2+w^2)^2-b^4}\right)^{\frac{1-\delta}{2}}\frac{(\rho^2+w^2)^2-b^4}{(\rho^2+w^2)^2}f(\rho)$$
$$-\sqrt{1+w'^2}\sqrt{1+B^2/g_{11}^2}\frac{4b^4\rho^3}{(\rho^2+w^2)^5}\left(\frac{(\rho^2+w^2)^2+b^4}{(\rho^2+w^2)^2-b^4}\right)^{\frac{\Delta}{2}}\left(2b^4-\Delta(\rho^2+w^2)^2\right)f(\rho)=0.$$

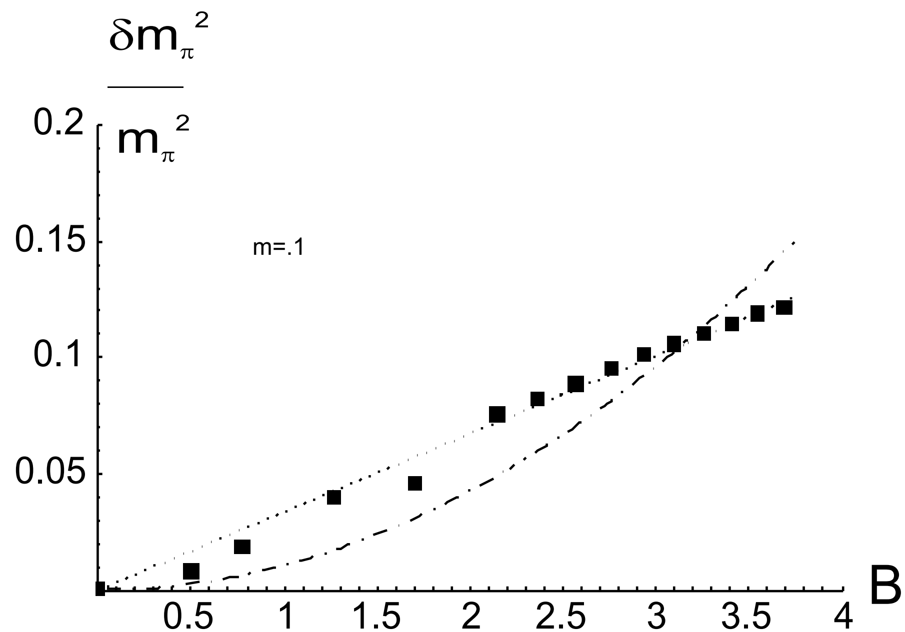
where boundary conditions are:

$$\begin{cases} f'(0) = 0 \\ f(\rho)|_{\rho\to\infty} \to \frac{1}{\rho^2}. \end{cases}$$

Mass Spectra

Below we show $\frac{m_\pi^2(B)-m_\pi^2(0)}{m_\pi^2(0)}$, obtained numerically. Note that approximation of it by expected linear or quadratic dependence is not satisfactory, unlike some other power laws

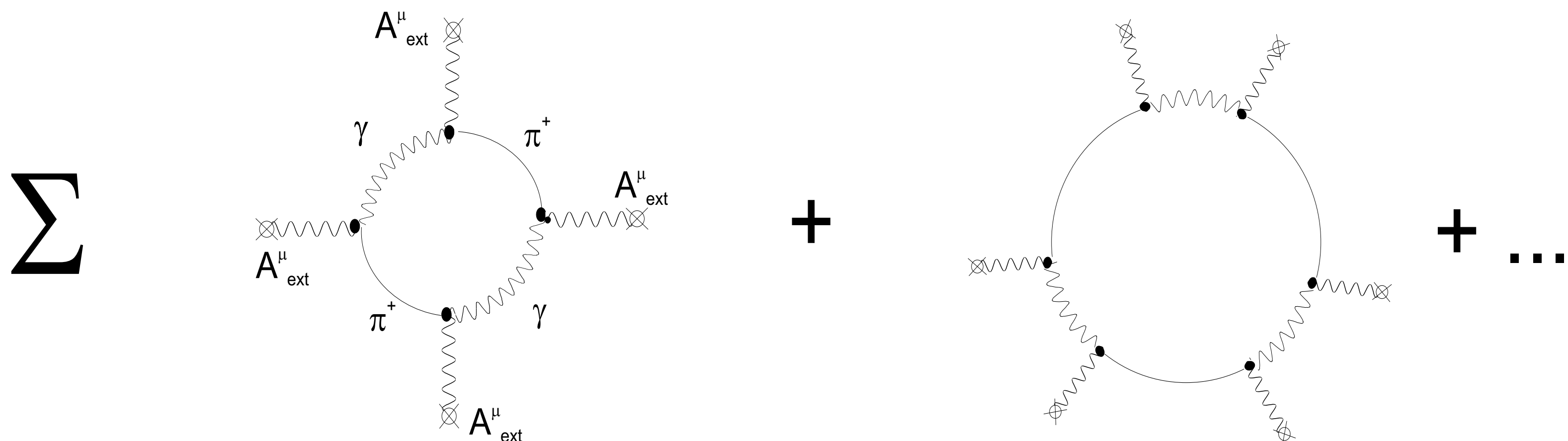




Spectra of m_π^2 as functions of B. Thin lines on the left ($m = 1$) and right ($m = 0.1$) plots show interpolation $\delta m_\pi^2 = \alpha B$ and $\delta m_\pi^2 = \alpha B^2$. One can see that neither of these interpolations is satisfactory.

Back to χPT

The previous sections presented us a puzzle: chiral perturbation theory predicts $\langle\overline{\psi}\psi\rangle \sim eH$, whereas $\langle\overline{\psi}\psi\rangle \sim (eH)^2$. One possible explanation would be to say that AdS/CFT has nothing to do with QCD an reproduces a kind of Nambu—Jona-Lasinio dynamics. However, χPT happened to have some unexplored objects even at one-loop level.



This diagram represents pion-photon mixing in an external field. The pion-photon vertex is roughly speaking the anomalous term

$$\mathcal{L}_{int} = -\frac{N_c e^2}{24\pi^2 f_\pi} F\tilde{F}\pi^0,$$

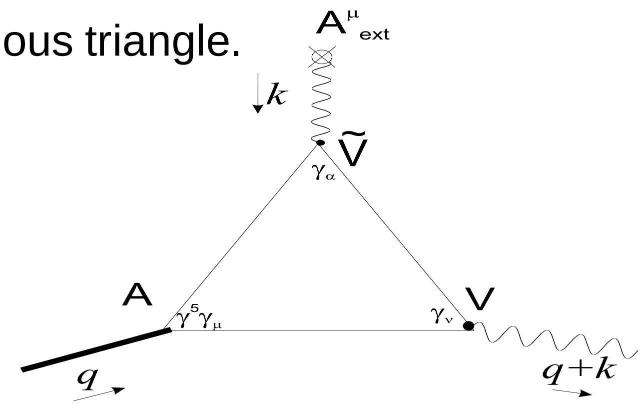
taken off-shell. Amazingly, the vacuum energy contribution of pion-photon mixing, shown in the diagram above, has never been studied within χPT.

Anomalous Triangle

Pion-photon mixing vertex comes from the following correlator *[Vainshtein 2002]*

$$T_{\mu\nu} = i \int d^4 x e^{iqx} \langle | T \left(j_\mu(x) j^5_\nu(0) \right) | \rangle,$$

associated with the anomalous triangle.



It has both a perturbative and a non-perturbative part coming from OPE. For vertex $\pi\gamma\gamma$:

$$c_1 \pi^0(q) q^\sigma \tilde{f}^{ext}_{\sigma\mu} A^\mu(q) + c_2 \pi^0(q) \frac{q^\sigma \tilde{f}^{ext}_{\sigma\mu}}{q^2} A^\mu(q)$$

π^0 is pion field, q its four-momentum, $\tilde{f}$ dual external field strength, and c_1, c_2 are given by OPE as:

$$c_1 = \frac{N_c e^2 \mathrm{tr}(V\tilde{V}A)}{2\pi^2 f_\pi}, \quad c_2 = \frac{4\mathrm{tr}(mAV\tilde{V})\chi < \bar{\psi}\psi >}{f_\pi}$$

where $V, \tilde{V}$ and A are flavour matrices, χ is vacuum magnetic susceptibility. Constant c_2 is responsible for essentially non-perturbative dynamics, as it contains vacuum parameters.

Pion-Photon Mixing

Here we show results for condensate contributions coming form pion-photon mixing:

$$\frac{\delta\langle\overline{\psi}\psi\rangle_H}{<\bar{\psi}\psi>_0} = \frac{1}{f_\pi^2}\frac{1}{16\pi^2}c_1^2H^2\ln\frac{c_1^2H^2}{\mu^2} + \frac{1}{8\pi^2}\frac{4A_uV_u\tilde{V}_u\chi}{f_\pi}c_1H^2\ln\frac{c_1^2H^2}{\mu^2}$$

The peculiar things about this result are:

- It depends as $\left(\frac{1}{N_c}\right)^0$, unlike the result of Smilga—Shishpanov, which is organized as $\left(\frac{1}{N_c}\right)^1$. Thus pion-photon mixing, unlike pure pion loop, is in the leading order in $\frac{1}{N_c}$.
- Roughly speaking, the field dependence is $\delta\langle\overline{\psi}\psi\rangle_H \sim H^2$

These two features surprisingly coincide with the AdS/CFT result. AdS/CFT by definition yields the leading-order in $\frac{1}{N_c}$, and we have already seen that the field dependence is organized as H^2 at least for small fields.

Conclusions

- AdS/CFT with flavour (D3/D7 model) provides a quadratically growing chiral condensate.
- This behaviour is not at all noticed in χPT literature.
- Motivated by this discrepancy, we have found and evaluated a one-loop diagram in χPT, which has been overlooked so far.
- Its behaviour reproduces qualitatively the AdS/CFT result:
  - Leading-order in $\frac{1}{N_c}$
  - $\delta < \overline{\psi}\psi > \sim H^2$

We do not claim that AdS/CFT result corresponds to the said diagram exactly. Rather, we want to illustrate the fruitfulness of bringing the ideas of χPT and of AdS/CFT together. The χPT question induces a search for an AdS/CFT, which motivates search for overlooked structures in χPT.

Perspectives

- Meson decay constants from holography
- Validity check of Gell-Mann—Oakes—Renner relation in holography.
- Dependence on temperature, chemical potential, external electric and magnetic field.
- Search of more realistic backgrounds to reproduce QCD

Work in progress in these directions.

Acknowledgements

I am grateful to the Organizers of the workshop “Aspects of duality” for the wonderful opportunity to learn a lot of new stuff and to present a part of my work.

Thanks to

- J. Rafelski
- A. Gorsky
- J. Erdmenger
- R. Meyer
- M. Haack
- I. Kirsch.