EXPLORING AN S-MATRIX FOR GRAVITATIONAL COLLAPSE

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- **Classical collapse and Choptuik scaling**
- **Linearized gravity, scattering and effective action**
- **Solving effective equations of motion**

1. x-space soln :	**critical lines**
	closed trapped surfaces
	on shell action (scaling)
	multiplicity of gravitons
2. p-space soln :	**spectrum of gravitons**
	• complex trajectories

I. Classical gravitational collapse

Solve Einstein + field equations with a matter e.g.

$$R^{\mu\nu} + \frac{1}{2} R g^{\mu\nu} = 8\pi G T^{\mu\nu}, \quad \nabla_\mu \nabla^\mu \phi = 0 \tag{1}$$

Initial conditions

$$\phi(r, 0; p) \tag{2}$$

Generically: if $p < p^*$ no collapse – a dispersive phase
if $p > p^*$ collapse – a black hole phase

$\Rightarrow$ numerical solutions: close to the criticality solutions are self-similar (locally)

$$\phi_*(sr, st) = \phi_*(r, t) \tag{3}$$

and

$$M(p) = c(p - p^*)^\gamma \tag{4}$$

Choptuik, 1990

II. From Einstein-Hilbert action to an effective action

- **Linearize $\sqrt{-g}R$ to $O(h^4) \longrightarrow$ a mess**

  Sherk and Schwartz, Kaku, Aragone and Chela Flores '75

  Bengtsson, Cederval and Lindgren '83

- **Reorganize according to the kinematics of the high energy collisions**

  - exchanged gravitons (ones in the t - channel)

  - scattered gravitons (ones in the s - channel)

  - fix the gauge, keep only the modes (of $h_{\mu\nu}(x)$) relevant for the multi-regge kinematics

  - integrate over "no-man's" (heavy) modes

  - Lipatov's action (QCD and/or gravity) – still complicated

  Lipatov '91, Kirschner and Szymanowski '95

- **- Further simplifications: only eikonal diagrams with one emission**

  Amati,Ciafaloni, Veneziano '81,'88,...,'07

  - ACV action - manageable

II. Solving effective equations of motion in x-space

$$\begin{aligned}
\mathcal{A} &= 2\pi G s \int d^2\mathbf{x} \left[a(\mathbf{x})\bar{s}(\mathbf{x}) + \bar{a}(\mathbf{x})s(\mathbf{x}) - \frac{1}{2}\partial_i\bar{a}\partial_i a + \frac{(\pi R)^2}{2}(-(\partial_i\partial_i\phi)^2 + 2\phi\partial_i\partial_i\mathcal{H}) \right] \\
\partial_i\partial_i\mathcal{H} &= -\partial_i\partial_i a\partial_i\partial_i\bar{a} + \partial_i\partial_j a\partial_i\partial_j\bar{a}, \qquad i = 1, 2
\end{aligned}$$

$$R = 2G\sqrt{s}, \quad R_i(r) = 4GE_i(r), \quad r = |\mathbf{x}|, \quad R_i(\infty) = R \tag{5}$$

$$\int d^2\mathbf{x}s(\mathbf{x}) = 1, \quad R_i(r) = R\int^r d^2\mathbf{x}s(\mathbf{x}). \tag{6}$$

Equations of motion in the axially symmetric case

$$\ddot{\rho}(r) \;=\; \frac{1}{2}\frac{R_1(r)R_2(r)}{\rho^2(r)} \tag{7}$$

$$\rho(0) = 0, \quad \rho(r) \sim r^2, r \longrightarrow \infty \tag{8}$$

$$\rho(r) = r^2(1 - 2\pi\dot{\phi}(r)), \quad \dot{} = d/dr^2 \tag{9}$$

Parameters

Energy: $\sqrt{s} \leftrightarrow R$, **impact parameter: b**
Sources: sizes L_i,

Kinematics

- $b > R$ – **"normal" scattering and production of gravitons**
- $b < R$ **energy** $\sqrt{s}$ **within the radius** $2G\sqrt{s}$ – **??**
- **there may exist a critical value of** R/b, $(R/b)_c$
- **head-on scattering of two extended sources, vary** R/L
- **head-on scattering of a particle and a ring with radius** L

  - an axially symmetric version of particle-particle scattering at the impact parameter b=L

III. Momentum space

$$\mathcal{A} = \frac{Gs}{2}\int \frac{d^2\mathbf{k}}{\mathbf{k}^2}\Big[\beta_1(\mathbf{k})s_2(-\mathbf{k}) + \beta_2(\mathbf{k})s_1(-\mathbf{k}) \\ -\beta_1(\mathbf{k})\beta_2(-\mathbf{k}) - \frac{(\pi R)^2}{2}[-h(\mathbf{k})h(-\mathbf{k}) - h(-\mathbf{k})\mathcal{H}(\mathbf{k})]\Big] \tag{10}$$

Equations of motion

$$\begin{aligned} \beta_i(k^2) &= s_i(k^2) + \frac{R^2}{8}\int \frac{dk_1^2 dk_2^2}{k_1^2 k_2^2}\sqrt{\lambda(k_1^2, k_2^2, k^2)}h(k_1^2)\beta_i(k_2^2) \\ h(k^2) &= \frac{1}{4\pi^2}\int \frac{dk_1^2 dk_2^2}{k^2 k_1^2 k_2^2}\sqrt{\lambda(k_1^2, k_2^2, k^2)}\beta_1(k_1^2)\beta_2(k_2^2) \end{aligned} \tag{11}$$

$$\lambda(k_1^2, k_2^2, k^2) = 2k_1^2k_2^2 + 2k^2k_2^2 + 2k_1^2k^2 - k^4 - k_1^4 - k_2^4 \tag{12}$$

Solution in p-space

- **a) iteration**

- **b) as algebraic equations**

⇒ In both cases there is a critical value of R/L !

Algebraic solution works also in the "BH" phase and produces complex solutions, i.e. action.

Solution in x-space

solve equivalent 1-st order system

$$\dot{\rho}(r) = \sqrt{\sigma(r) - \frac{R_1(r)R_2(r)}{\rho}} \quad i.e. \quad \sigma \equiv \dot{\rho}^2 + \frac{R_1(r)R_2(r)}{\rho},$$
$$\dot{\sigma}(r) = \frac{1}{\rho(r)}\frac{d(R_1R_2)}{dr^2}, \tag{13}$$

with initial conditions

$$\rho(0) = 0 \quad , \quad \sigma(0) = \sigma_0, \tag{14}$$

and find a σ_0 such that $\sigma(Max(L_1, L_2)) = 1$.

(finite sources $\Rightarrow \sigma \overset{(2)}{=} const. \overset{(1)}{=} 1 \;\; r > Max(L_1, L_2)$)

$\Rightarrow$ **Such (real) solutions exist only for $R/L < (R/L)_c$!**

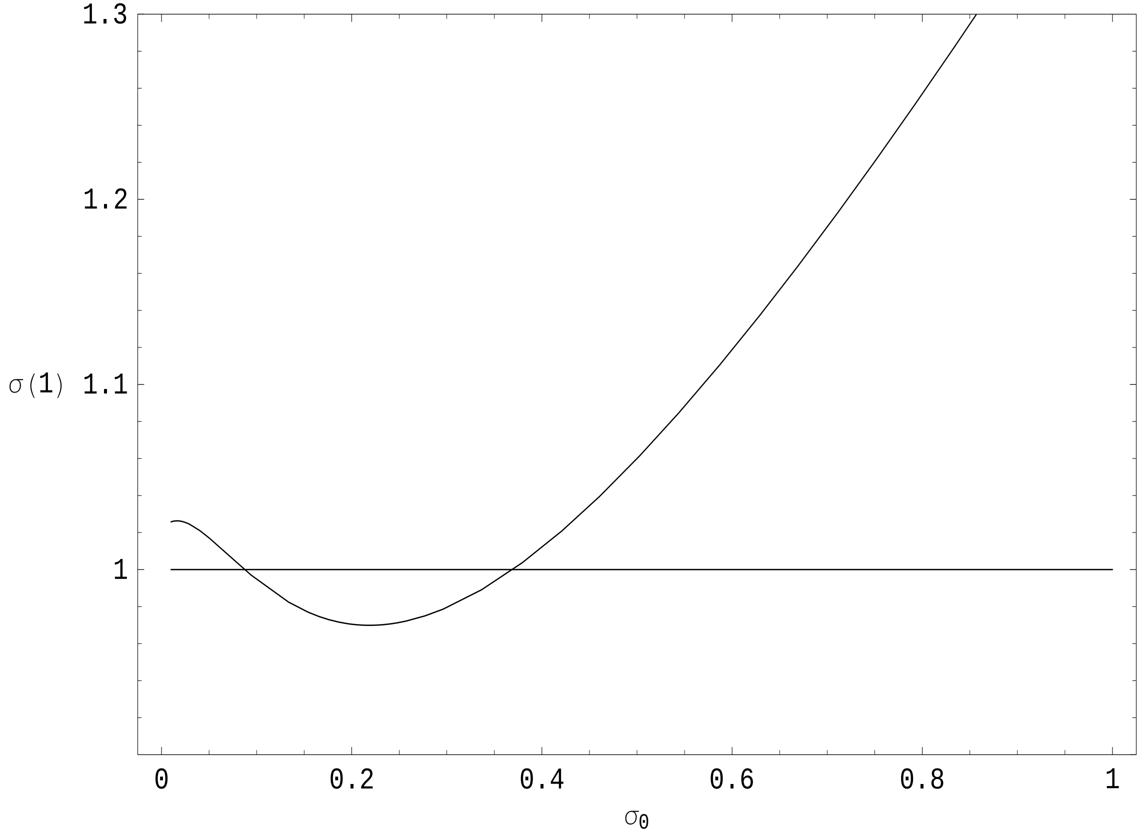


Figure 1:

d		0.5	1.0	1.6	2.5	4.0	
A-x		0.419	0.471	0.502	0.528	0.550	
A-p		0.429	0.476	0.499	0.501	0.477	
σ		0.01	0.1	0.2	0.3	3.0	
B-x		0.615	0.572	0.525	0.486	0.470	
B-p		0.058	0.436	0.501	0.489	0.476	
ρ	0.25	0.333	0.5	1.0	2.0	3.0	4.0
C-x	.810	.816	.821	.823	.821	.816	.810
C-p	.823	.833	.850	.841	.838	.840	.832

Table 1: $(R/L)_c$ for a range of sizes of the power-like and Gaussian sources: a comparison between configuration and momentum-space results. A, B and C label sources as discussed below. In the case C: $\rho = L_2/L_1$ and the critical value of the ratio $2R/(L_1+L_2)$ is shown.

A: "Lorentz" profile with width d $\qquad R_c|^B_{\sigma=0} = R_c|^A_{d=\infty} = 2^{1/2}3^{3/4} \cong .62$

$$s(r) = \frac{dL^4}{\pi(dL^4 + (1-d)r^4)^{3/2}}, \quad d < L$$

B: Central gaussian on a gaussian ring
C: Two gaussians with different sizes

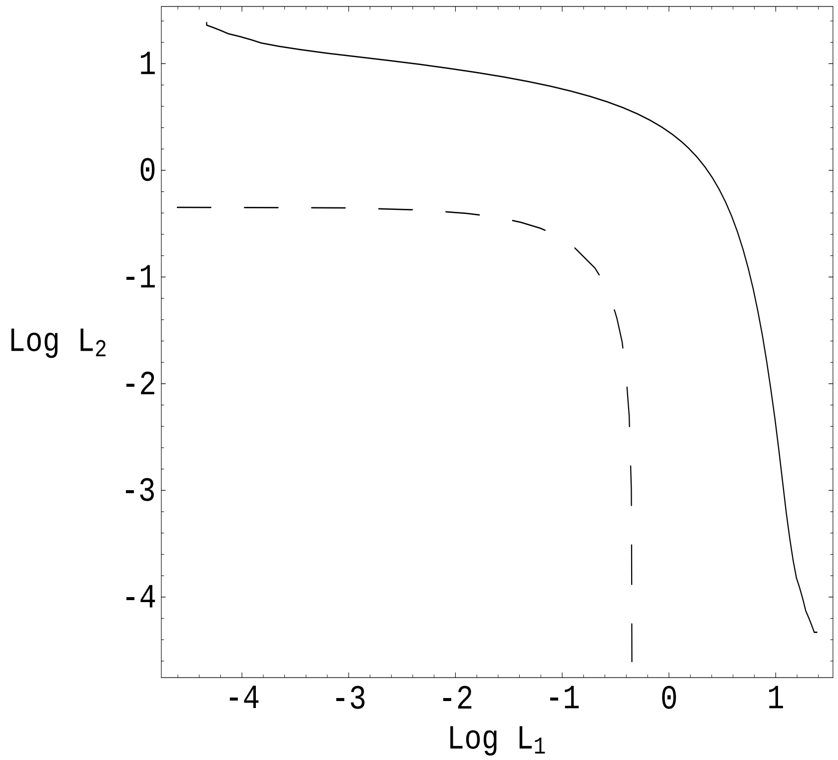


Figure 2: A critical line (solid) in the (L_1, L_2) plane. The lower bound (dashed) comes from the CTS criterion

Closed trapped surface

Both light rays, perpendicular to a two dimensional surface, move inside (ending on the singularity).

$$R_1(R_c)R_2(R_c) = R_c^2 \tag{15}$$

CTS provide a sufficient condition for existence of a BH

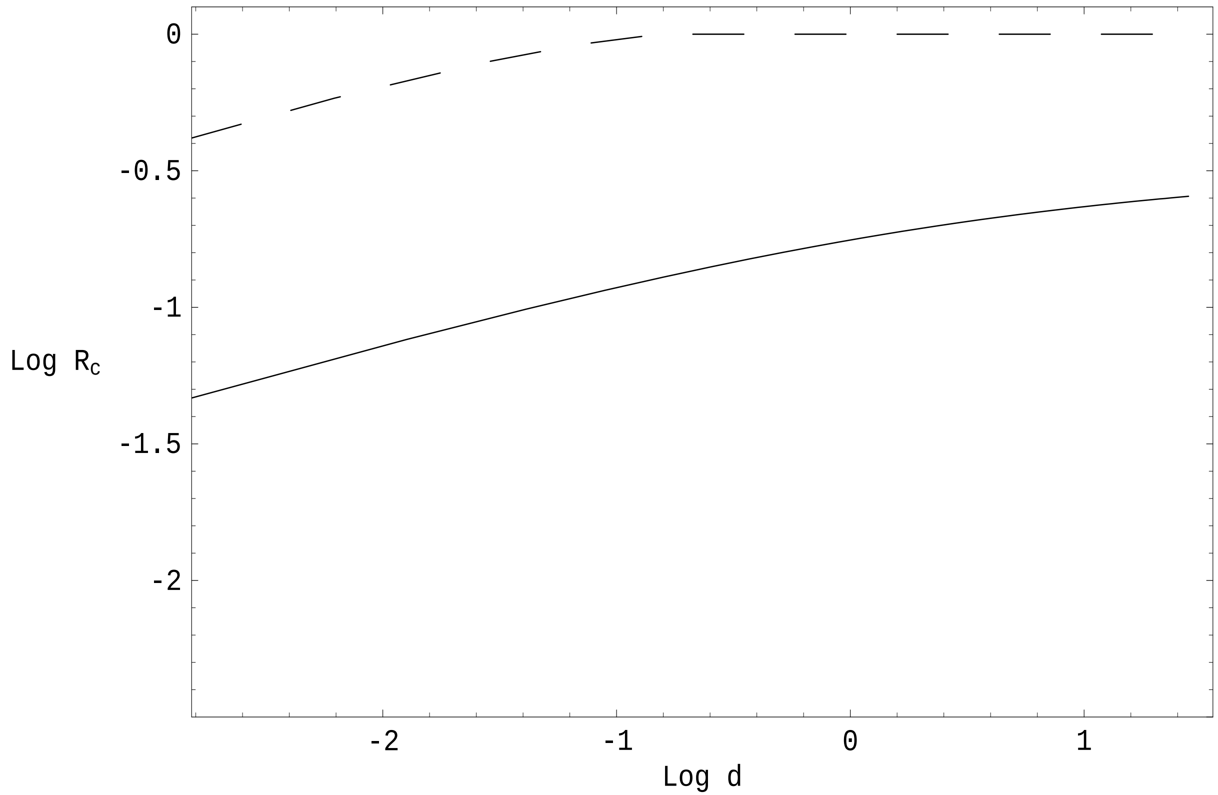


Figure 3: **A critical line (solid) in the $(R/L, d)$ plane for a $b = 0$ scattering of two lorentzian sources.**

III. The action around the critical point

The OEM action is IR divergent, however the derivative

$$\frac{\partial(\mathcal{A}/Gs)}{\partial R^2} = \frac{1}{R^4}\int dr^2(1-\dot{\rho})^2 = \frac{\pi^2}{R^3\sqrt{s}}\langle N\rangle\,, \tag{16}$$

is IR-finie.
The fit is consistent with

$$\langle N\rangle = c_0 + c_1(R_c - R)^{\frac{1}{2}}\,, \tag{17}$$

and confirms the behaviour

$$A(R) = A_0 + A_1(R_c - R) + A_2(R_c - R)^{3/2}\,, \tag{18}$$

consistent with the local nature of Choptuik scaling

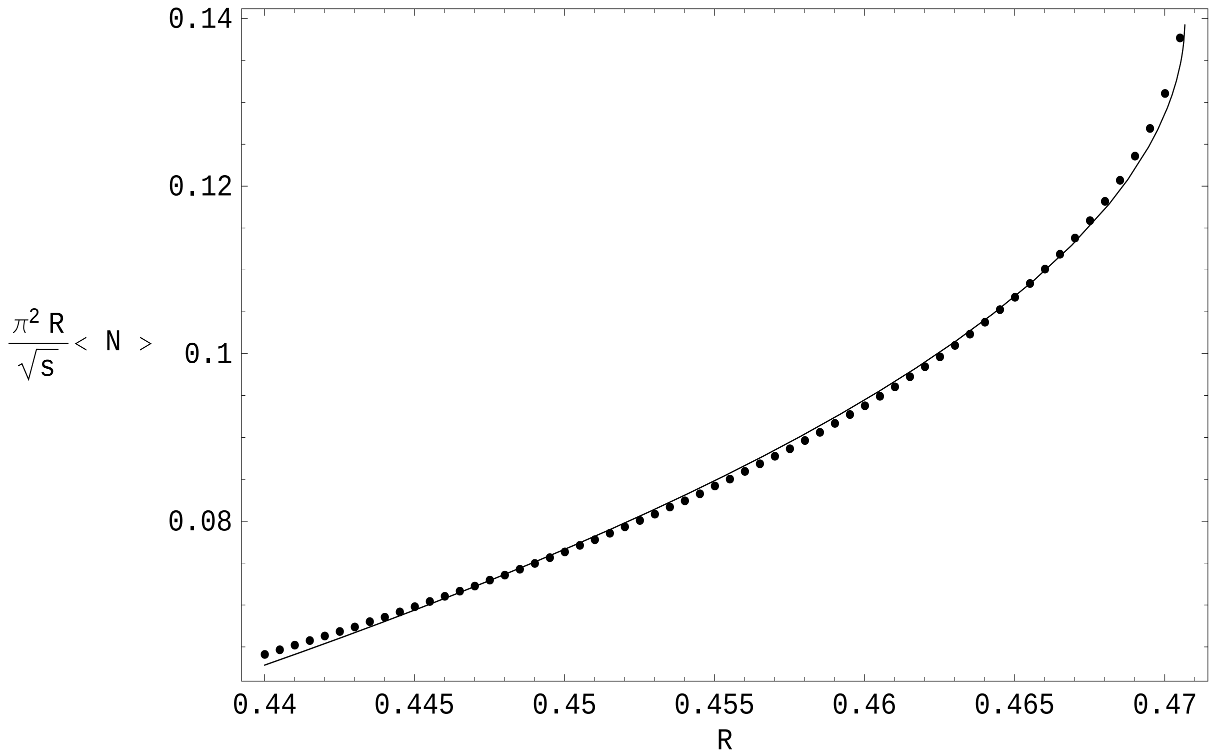


Figure 4: Total multiplicity of emitted gravitons and the fit $0.138 - .46(R_c - R)^{0.52}$, $R_c = .47067$.

IV. Spectrum of emitted gravitons

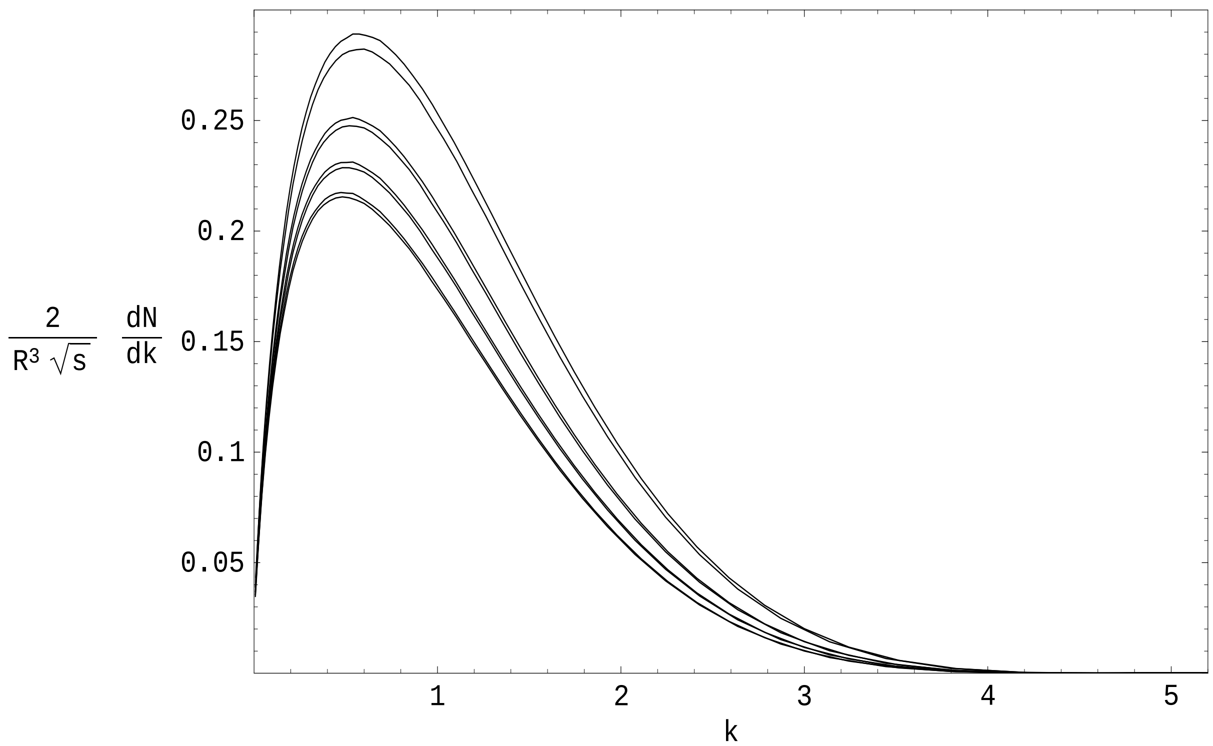


Figure 5: Spectrum of gravitons (A). R=.44,.45,.46,.47; $R_c = .47067, n = 60 - 70$.

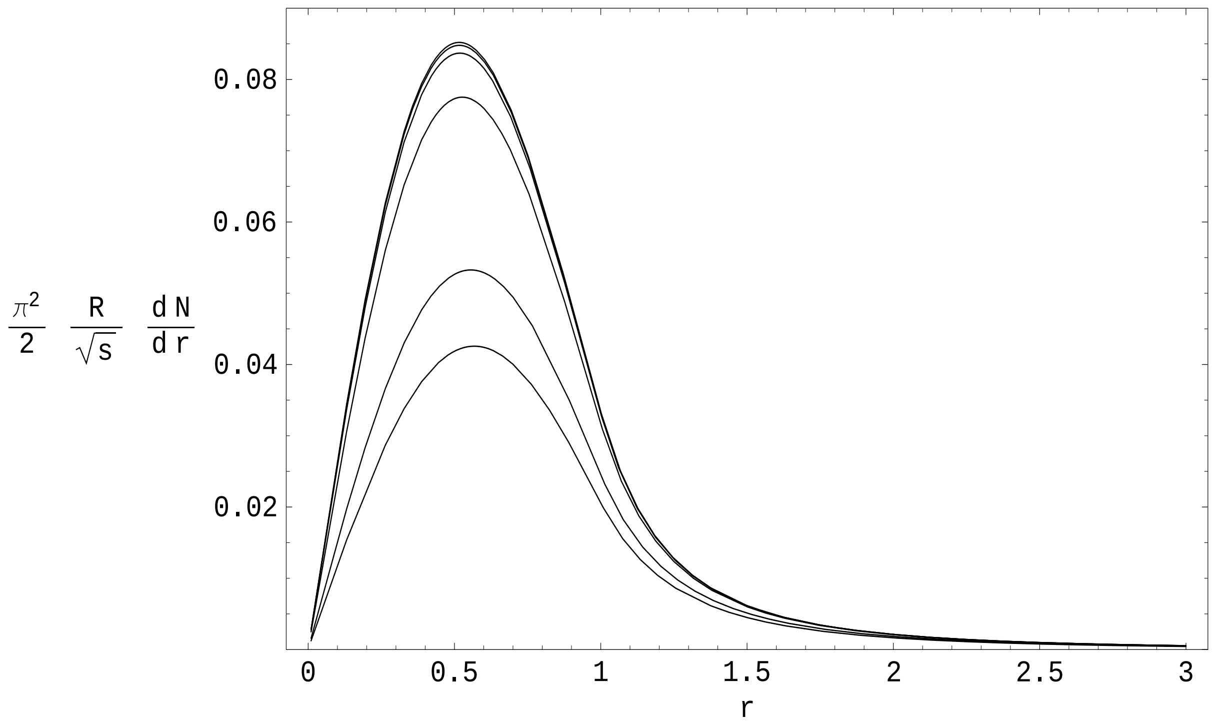


Figure 6: Profile in the x-space, $R = .45, .46, .47, .4706, .47064, .47065; R_c = .470673$

$\Rightarrow$ **The limiting spectrum is regular.**

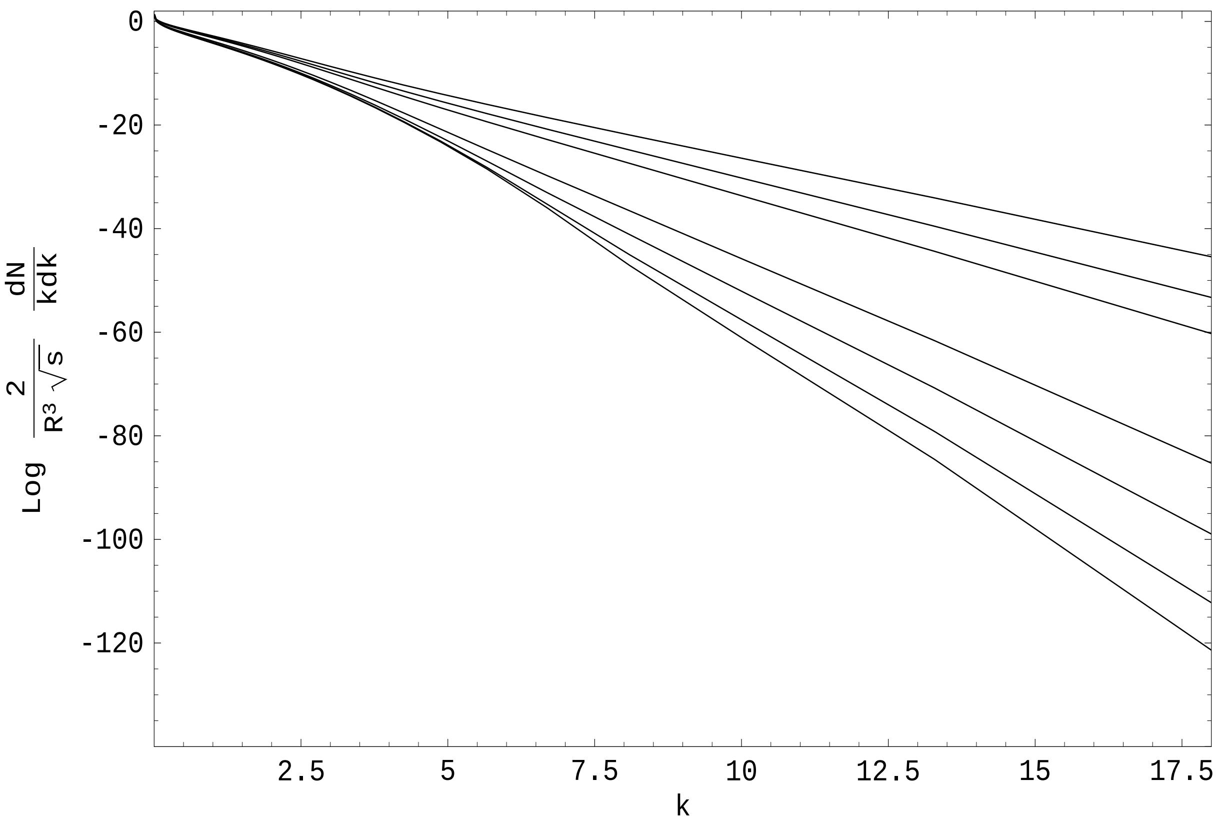
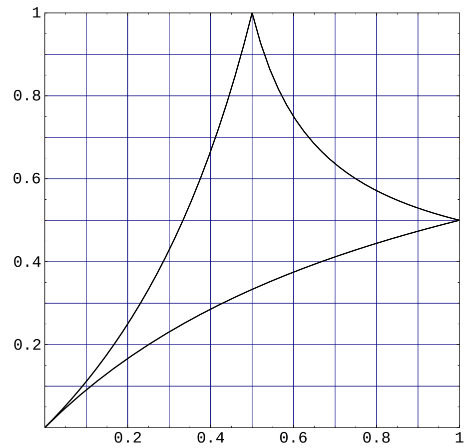


Figure 7: Looking for the "Hawking temperature" of emitted gravitons. R=.1,.2,.3,.4,.6,.8,.82,.83; $R_c = .841$

Beyond the critical point

Discretization

$$k \to v = \frac{1}{1+kL}, \qquad \Delta(k_1, k_2, k) \to T(v_1, v_2, v) \tag{19}$$



$$n = 6 \Rightarrow 3^6 = 729 \Rightarrow \text{solutions} \quad - \quad 2.5\ hrs \tag{20}$$

$R < R_c$ **only one stable solution – identical to the one from the recursion**
$R > R_c$ **also the second solution "conjugate" to the recursive one**

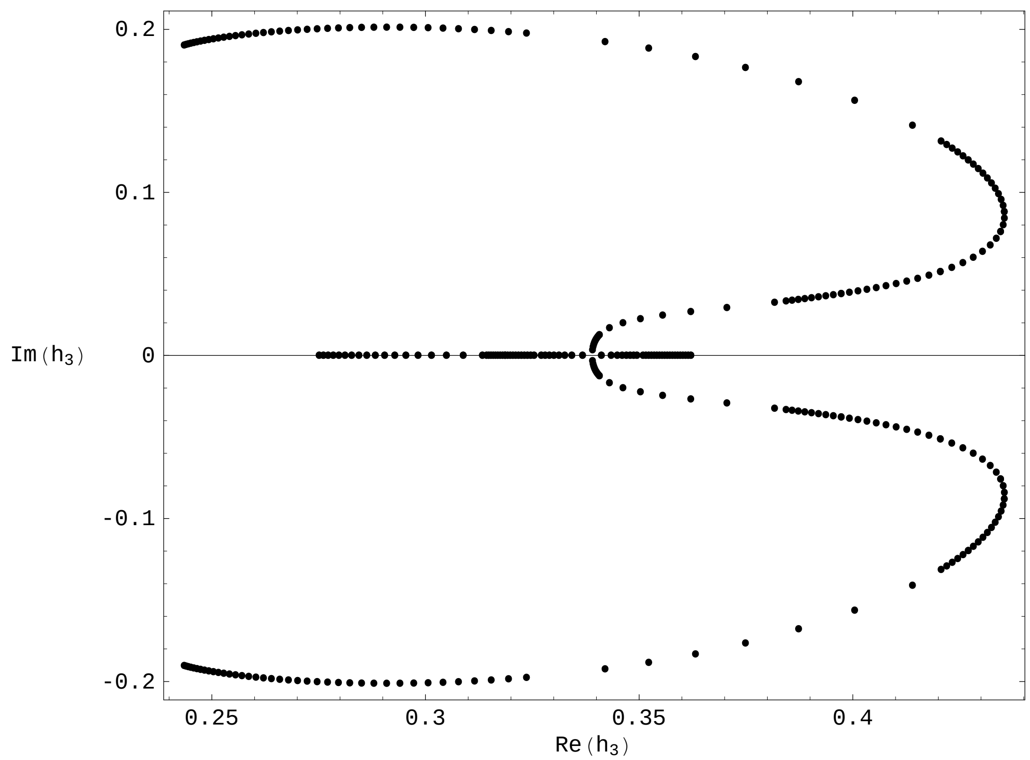


Figure 8: Two complex trajectories of $h(k_3)$ parametrized by R moving across R_c.

Conclusions

- **Eikonal-like summation of single emission diagrams is sufficient to capture nonlinearities needed for a collapse.**
- **The on mass shell action is singular $(R_c - R)^\gamma$.**

  The critical exponent $\gamma \cong 2\gamma_{Choptuik}$.
- **There exists a smooth limiting distribution of gravitons at R_{c-}.**
- **Imaginary action $\Rightarrow$ instabilities above R_c.**
- **Future: $\Rightarrow$ construct a stable state (****),**

  $\Rightarrow$ lift some of the approximations.