Negative radiation pressure <u>two-component field case</u>

Plan

T. Romańczukiewicz (P. Forgács, J. Karkowski, A. Lukács)

June 15th, 2008

T. Romańczukiewicz NRP – vortices









T. Romańczukiewicz NRP - vortices

(日)

What is the negative radiation pressure Example in ϕ^4

(日)

E.

Outline



Introduction

- What is the negative radiation pressure
- Example in ϕ^4

2 Goldstone's model

3 Conclusions

T. Romańczukiewicz NRP – vortices

What is the negative radiation pressure Example in ϕ^4

Negative radiation pressure

Negative radiation pressure in *classical* field theories is a phenomenon when an object (i.e. topological defect) instead of being pushed away by radiation is being pulled toward the source of radiation.

In all cases there is some kind of mechnism which transforms part of the energy of incoming wave into another wave which carries more momentum.

Examples

- kinks in 1+1 D in ϕ^4 theory, and small modifications of ϕ^4
- oscillons (pseudobreathers) in 1+1 D, probably also in higher dimensions
- vortices in abelian Higgs model
- vortices in Goldstone's model (?)

What is the negative radiation pressure Example in ϕ^4

(日)

Negative radiation pressure

Negative radiation pressure in *classical* field theories is a phenomenon when an object (i.e. topological defect) instead of being pushed away by radiation is being pulled toward the source of radiation.

In all cases there is some kind of mechnism which transforms part of the energy of incoming wave into another wave which carries more momentum.

Examples

- kinks in 1+1 D in ϕ^4 theory, and small modifications of ϕ^4
- oscillons (pseudobreathers) in 1+1 D, probably also in higher dimensions
- vortices in abelian Higgs model
- vortices in Goldstone's model (?)

What is the negative radiation pressure Example in ϕ^4

・ロ・ ・ 四・ ・ 回・ ・ 回・ ・

Negative radiation pressure

Negative radiation pressure in *classical* field theories is a phenomenon when an object (i.e. topological defect) instead of being pushed away by radiation is being pulled toward the source of radiation.

In all cases there is some kind of mechnism which transforms part of the energy of incoming wave into another wave which carries more momentum.

Examples

- kinks in 1+1 D in ϕ^4 theory, and small modifications of ϕ^4
- oscillons (pseudobreathers) in 1+1 D, probably also in higher dimensions
- vortices in abelian Higgs model
- vortices in Goldstone's model (?)

What is the negative radiation pressure Example in ϕ^4

Scattering in 1+1 d:

- general scattering: force **pushing** the object $F = |R^2|A^2q^2$
- kink in ϕ^4 is transparent
- linear approximation: F = 0
- nonlinearity introduces waves with 2ω. They have larger momentum/energy ratio:

$$r_{\omega} = \sqrt{\omega^2 - 4}/\omega < r_{2\omega} = \sqrt{\omega^2 - 1}/\omega.$$

• A surplus of momentum pushes the kink **towards** the source of radiation $F \sim A^4$



(日)

What is the negative radiation pressure Example in $\phi^{\rm 4}$

Scattering in 1+1 d:

- general scattering: force **pushing** the object $F = |R^2|A^2q^2$
- kink in ϕ^4 is transparent
- linear approximation: F = 0
- nonlinearity introduces waves with 2ω. They have larger momentum/energy ratio:

$$r_{\omega} = \sqrt{\omega^2 - 4}/\omega < r_{2\omega} = \sqrt{\omega^2 - 1}/\omega.$$

• A surplus of momentum pushes the kink **towards** the source of radiation $F \sim A^4$



< □ > < □ > < □ > < □ > < □ > <

э

What is the negative radiation pressure Example in ϕ^4

Scattering in 1+1 d:

- general scattering: force **pushing** the object $F = |R^2|A^2q^2$
- kink in ϕ^4 is transparent
- linear approximation: F = 0
- nonlinearity introduces waves with 2ω. They have larger momentum/energy ratio:

$$r_{\omega} = \sqrt{\omega^2 - 4}/\omega < r_{2\omega} = \sqrt{\omega^2 - 1}/\omega.$$

• A surplus of momentum pushes the kink **towards** the source of radiation $F \sim A^4$



< □ > < □ > < □ > < □ > < □ > <

What is the negative radiation pressure Example in ϕ^4

Scattering in 1+1 d:

- general scattering: force **pushing** the object $F = |R^2|A^2q^2$
- kink in ϕ^4 is transparent
- linear approximation: F = 0
- nonlinearity introduces waves with 2ω. They have larger momentum/energy ratio:

$$r_{\omega} = \sqrt{\omega^2 - 4}/\omega < r_{2\omega} = \sqrt{\omega^2 - 1}/\omega.$$

• A surplus of momentum pushes the kink **towards** the source of radiation $F \sim A^4$



・ロット 小型マネ ロマ

What is the negative radiation pressure Example in ϕ^4

Scattering in 1+1 d:

- general scattering: force **pushing** the object $F = |R^2|A^2q^2$
- kink in ϕ^4 is transparent
- Iinear approximation: F = 0
- nonlinearity introduces waves with 2ω. They have larger momentum/energy ratio:

$$r_{\omega} = \sqrt{\omega^2 - 4}/\omega < r_{2\omega} = \sqrt{\omega^2 - 1}/\omega.$$

 A surplus of momentum pushes the kink towards the source of radiation *F* ~ *A*⁴



・ロ・ ・ 四・ ・ 回・ ・ 回・

Vortex A toy model Linearization

Outline



Conclusions

T. Romańczukiewicz NRP – vortices

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Vortex A toy model Linearization

Goldstone's model is described by following Lagrangian

$$\mathcal{L}=rac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi^{*}-rac{1}{2}\left(\phi\phi^{*}-1
ight)^{2}$$

(日)

Vortex A toy model Linearization

Goldstone's model is described by following Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi^* - \frac{1}{2} \left(\phi \phi^* - 1 \right)^2$$

hackican hat



Vortex A toy model Linearization

Goldstone's model is described by following Lagrangian

$$\mathcal{L}=rac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi^{*}-rac{1}{2}\left(\phi\phi^{*}-1
ight)^{2}$$

Small perturbations around the vacuum manifold can be divided into two sectors

- massless Goldstone's mode excitation along the valley costs no energy
- massive amplitude mode excitation perpendicular to the valley

Vortex A toy model Linearization

Goldstone's model is described by following Lagrangian

$$\mathcal{L}=rac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi^{*}-rac{1}{2}\left(\phi\phi^{*}-1
ight)^{2}$$

Small perturbations around the vacuum manifold can be divided into two sectors

- massless Goldstone's mode excitation along the valley costs no energy
- massive amplitude mode excitation perpendicular to the valley



Vortex A toy model Linearization

Goldstone's model is described by following Lagrangian

$$\mathcal{L}=rac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi^{*}-rac{1}{2}\left(\phi\phi^{*}-1
ight)^{2}$$

Small perturbations around the vacuum manifold can be divided into two sectors

- massless Goldstone's mode excitation along the valley costs no energy
- massive amplitude mode excitation perpendicular to the valley



・ロ・ ・ 四・ ・ ヨ・ ・ ヨ・ ・

The Lagrangian leads to equation of motion

$$\ddot{\phi} - \Delta \phi + 2\phi(\phi \phi^* - 1) = 0. \tag{1}$$

As usual the static, linear (along z-axis) vortex solution has the form

$$\phi_s(r,\theta) = f(r)e^{iN\theta}.$$
 (2)

э

This leads to an equation for profile

$$f'' + \frac{1}{r}f' - \frac{N}{r^2}f - 2f\left(f^2 - 1\right) = 0,$$
(3)

where $f(\infty) = 1$ (minimum of energy) and f(0) = 0 (smoothness of field – topological zero).

Vortices with winding number N larger than 1 are unstable so we assume N = 1. It is quite easy to obtain an asymptotic form of f for large values of r:

$$f(r \to \infty) = 1 - \frac{1}{4}r^{-2} - \frac{9}{32}r^{-4} - \frac{161}{128}r^{-6} - \frac{24661}{2048}r^{-8} + \mathcal{O}(r^{-10}).$$
(4)

The Lagrangian leads to equation of motion

$$\ddot{\phi} - \Delta \phi + 2\phi(\phi \phi^* - 1) = 0. \tag{1}$$

As usual the static, linear (along z-axis) vortex solution has the form

$$\phi_s(r,\theta) = f(r)e^{iN\theta}.$$
 (2)

3

This leads to an equation for profile

$$f'' + \frac{1}{r}f' - \frac{N}{r^2}f - 2f\left(f^2 - 1\right) = 0,$$
(3)

where $f(\infty) = 1$ (minimum of energy) and f(0) = 0 (smoothness of field – topological zero).

Vortices with winding number N larger than 1 are unstable so we assume N = 1. It is quite easy to obtain an asymptotic form of f for large values of r:

$$f(r \to \infty) = 1 - \frac{1}{4}r^{-2} - \frac{9}{32}r^{-4} - \frac{161}{128}r^{-6} - \frac{24661}{2048}r^{-8} + \mathcal{O}(r^{-10}).$$
(4)

Vortex A toy model Linearization

How do vortices interact with radiation?

Goldstone's mode pushes the vortex

$$\phi(\mathbf{x}, \mathbf{y} = \mathbf{L}, t) = (f(\mathbf{r}) + i\mathbf{A}\cos(\omega t))e^{i\theta}$$

amplitude mode pulls the vortex (NRP)

$$\phi(x, y = L, t) = (f(r) + A\cos(\omega t))e^{i\theta}$$

• combination of the above two pushes the vortex also a little sideways (*Magnus force*)

$$\phi(x, y = L, t) = (f(r) + A \exp(i\omega t))e^{i\theta}$$

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Vortex A toy model Linearization

How do vortices interact with radiation?

Goldstone's mode pushes the vortex

$$\phi(\mathbf{x}, \mathbf{y} = \mathbf{L}, t) = (f(\mathbf{r}) + i\mathbf{A}\cos(\omega t))e^{i\theta}$$

amplitude mode pulls the vortex (NRP)

$$\phi(\mathbf{x},\mathbf{y}=L,t)=(f(r)+A\cos(\omega t))\mathrm{e}^{\mathrm{i}\theta}$$

• combination of the above two pushes the vortex also a little sideways (*Magnus force*)

$$\phi(x, y = L, t) = (f(r) + A \exp(i\omega t))e^{i\theta}$$

Vortex A toy model Linearization

How do vortices interact with radiation?

Goldstone's mode pushes the vortex

$$\phi(\mathbf{x}, \mathbf{y} = \mathbf{L}, t) = (f(\mathbf{r}) + i\mathbf{A}\cos(\omega t))e^{i\theta}$$

amplitude mode pulls the vortex (NRP)

$$\phi(\mathbf{x}, \mathbf{y} = \mathbf{L}, t) = (f(\mathbf{r}) + \mathbf{A}\cos(\omega t))e^{i\theta}$$

• combination of the above two pushes the vortex also a little sideways (*Magnus force*)

$$\phi(\mathbf{x}, \mathbf{y} = \mathbf{L}, t) = (f(\mathbf{r}) + \mathbf{A}\exp(\mathrm{i}\omega t))\mathrm{e}^{\mathrm{i}\theta}$$

Vortex A toy model Linearization

Are vortices reflectionless?

They most likely are not! There must be some other mechanism standing behind the NRP.

Excitations around vacuum have two components: massless Goldstone's mode and massive amplitude field.

Vortex A toy model Linearization

Are vortices reflectionless?

They most likely are not! There must be some other mechanism standing behind the NRP.

Excitations around vacuum have two components: massless Goldstone's mode and massive amplitude field.



・ロ・ ・ 四・ ・ ヨ・ ・ ヨ・ ・

Vortex A toy model Linearization

Let us consider 1+1 d model. A wave with mass m_1 and amplitude *A* hitting the kink after transition is transformed into another wave with different mass m_2 and amplitude *B*. For simplicity let us assume R = 0.



$$\frac{1}{2}A^2\omega k_1 = \frac{1}{2}B^2\omega k_2 \Rightarrow B^2 = \frac{k_1}{k_2}A^2$$

where $k_i^2 = \omega^2 - m_i^2$. The force acting on the kink is equal to:

$$F = -rac{1}{2}A^2k_1^2 + rac{1}{2}B^2k_2^2 = rac{1}{2}A^2k_1(k_2 - k_1).$$

If $k_2 > k_1$ (or other words: $m_2 < m_1$) the force pushes the kink against the wave and we have another example of the NRP.





・ロン ・雪 ・ ・ ヨ ・

э

Vortex A toy model Linearization

Let us consider 1+1 d model. A wave with mass m_1 and amplitude *A* hitting the kink after transition is transformed into another wave with different mass m_2 and amplitude *B*. For simplicity let us assume R = 0.

From energy conservation law we get:

$$\frac{1}{2}A^2\omega k_1 = \frac{1}{2}B^2\omega k_2 \Rightarrow B^2 = \frac{k_1}{k_2}A^2$$

where $k_i^2 = \omega^2 - m_i^2$. The force acting on the kink is equal to:

$$F = -\frac{1}{2}A^2k_1^2 + \frac{1}{2}B^2k_2^2 = \frac{1}{2}A^2k_1(k_2 - k_1).$$

If $k_2 > k_1$ (or other words: $m_2 < m_1$) the force pushes the kink against the wave and we have another example of the NRP.



Vortex A toy model Linearization

Example 1

A toy model of two interacting fields:

$$\mathcal{L} = \mathcal{L}_{\phi^4} + rac{1}{2} \left(\psi_t^2 - \psi_x^2 - m^2 \psi^2
ight) + \mathcal{L}_{init}$$

where

$$\mathcal{L}_{int} = rac{1}{2}\kappa(\phi^2-1)\psi.$$

 $m_{\phi} = 2.$



(日)

Vortex A toy model Linearization

Example 1 cont.

We consider scattering of the ψ over the kink. In limit for small coupling constant κ and small amplitudes *A* we obtain that

where $k = \sqrt{\omega^2 - m^2}$ and $q = \sqrt{\omega^2 - 4}$ are wave numbers of ψ and ϕ fields respectivily.

Note that our approximation fails when $k \approx q$ that is when $m \approx 2$. Using Noether theorem we can calculate energy and momentum balance.

From that we can obtain the force acting on the kink.

Vortex A toy model Linearization

Exaple 1 cont.

$$F = \frac{1}{2}A^2q\left(T^2(q-k) - R^2(q+k)\right)$$

Note that when q < k *i.e.* m < 2 the force is always negative (or at most zero) whatever the coefficients R and T are.

When m > 2 the direction in which the kink accelerate can be determined only after substitution the values of R and T. The negative radiation pressure appears when

$$T^2 > rac{q+k}{q-k}R^2$$

This inequality can be rewritten (when $R \neq 0$) as

$$\frac{\sinh \frac{q-k}{2}\pi}{\sinh \frac{q+k}{2}\pi} > \frac{q+k}{q-k}$$

which is true for all q > k > 0. The force vanishes only when R = 0 that is when $\omega^2 = \frac{3m^2}{2}$.

Vortex A toy model Linearization

Exaple 1 cont.

$$F = \frac{1}{2}A^2q\left(T^2(q-k) - R^2(q+k)\right)$$

Note that when q < k *i.e.* m < 2 the force is always negative (or at most zero) whatever the coefficients R and T are.

When m > 2 the direction in which the kink accelerate can be determined only after substitution the values of R and T.

The negative radiation pressure appears when

$$T^2 > rac{q+k}{q-k}R^2.$$

This inequality can be rewritten (when $R \neq 0$) as

$$\frac{\sinh \frac{q-k}{2}\pi}{\sinh \frac{q+k}{2}\pi} > \frac{q+k}{q-k}$$

which is true for all q > k > 0. The force vanishes only when R = 0 that is when $\omega^2 = \frac{3m^2}{2}$.

Vortex A toy model Linearization

Exaple 1 cont.

$$F=\frac{1}{2}A^2q\left(T^2(q-k)-R^2(q+k)\right)$$

Note that when q < k *i.e.* m < 2 the force is always negative (or at most zero) whatever the coefficients R and T are.

When m > 2 the direction in which the kink accelerate can be determined only after substitution the values of R and T.

The negative radiation pressure appears when

$$T^2 > rac{q+k}{q-k}R^2$$

This inequality can be rewritten (when $R \neq 0$) as

$$rac{\sinhrac{q-k}{2}\pi}{\sinhrac{q+k}{2}\pi}>rac{q+k}{q-k}$$

which is true for all q > k > 0. The force vanishes only when R = 0 that is when $\omega^2 = \frac{3m^2}{2}$.

Vortex A toy model Linearization



Vortex A toy model Linearization

Further examples

- Vortices in Goldstone's mode: NRP for scattering amplitude wave
- Abelian Higgs model vortices:
 - NRP for scattering amplitude wave
 - NRP for scattering vector field (for $\beta = m_s/m_v < 0.3$)

Vortex A toy model Linearization

Further examples

- Vortices in Goldstone's mode: NRP for scattering amplitude wave
- Abelian Higgs model vortices:

NRP for scattering amplitude wave
NRP for scattering vector field (for β = m_s/m_v < 0.3)

Vortex A toy model Linearization

Further examples

- Vortices in Goldstone's mode: NRP for scattering amplitude wave
- Abelian Higgs model vortices:
 - NRP for scattering amplitude wave
 - NRP for scattering vector field (for $\beta = m_s/m_v < 0.3$)



To the static vortex we add a small, cylindrically-symmetric perturbation $\phi = \phi_s + \delta \phi$. The appropriate equation has the form:

$$\ddot{\delta\phi} - \Delta\delta\phi + 2(2f^2 - 1)\delta\phi + 2f^2 e^{2iN\theta}\delta\phi^* = 0.$$
(5)

One can seek the solution of the form:

$$\delta\phi = \sum_{m=-\infty}^{\infty} e^{i(N+m)\theta} \left(e^{i\omega t} \boldsymbol{s}_m^+ + e^{-i\omega t} \boldsymbol{s}_m^- \right).$$
(6)

The equations for s_m^{\pm} can be obtained by plugging (6) into (5) and changing the summing variables from *m* to -m in some terms:

$$\begin{bmatrix} \mathbf{D}_m & 2f^2 \\ 2f^2 & \mathbf{D}_{-m} \end{bmatrix} \begin{bmatrix} \mathbf{s}_m^+ \\ \mathbf{s}_{-m}^{-*} \end{bmatrix} = \omega^2 \begin{bmatrix} \mathbf{s}_m^+ \\ \mathbf{s}_{-m}^{-*} \end{bmatrix} =: \mathbf{L} \begin{bmatrix} \mathbf{s}_m^+ \\ \mathbf{s}_{-m}^{-*} \end{bmatrix},$$
(7)

where

$$\mathbf{D}_m := -\partial_{rr} - \frac{1}{r}\partial_r + \frac{(N+m)^2}{r^2} + 2(2f^2 - 1).$$
(8)

It is more convenient to introduce the following variables:

$$\begin{bmatrix} a_m \\ g_m \end{bmatrix} := \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} s_m^+ \\ s_{-m}^{-*} \end{bmatrix}.$$
(9)

The equation for new functions is simply

$$\begin{bmatrix} \mathbf{D}_{a} & \frac{2Nm}{r^{2}} \\ \frac{2Nm}{r^{2}} & \mathbf{D}_{g} \end{bmatrix} \begin{bmatrix} \mathbf{a}_{m} \\ \mathbf{g}_{m} \end{bmatrix} = \omega^{2} \begin{bmatrix} \mathbf{a}_{m} \\ \mathbf{g}_{m} \end{bmatrix},$$
(10)

where

$$\mathbf{D}_a = -\partial_{rr} - \frac{1}{r}\partial_r + \frac{N^2 + m^2}{r^2} + 2(3f^2 - 1),$$
 (11a)

$$\mathbf{D}_g = -\partial_{rr} - \frac{1}{r}\partial_r + \frac{N^2 + m^2}{r^2} + 2(f^2 - 1).$$
 (11b)

The physical interpretation is now more clear. a_m describes a field which far away from the vortex core looks like a field with mass $m_a^2 = 4$ and g_m describes a massless field (the Goldstone's mode).

It is more convenient to introduce the following variables:

$$\begin{bmatrix} a_m \\ g_m \end{bmatrix} := \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} s_m^+ \\ s_{-m}^{-*} \end{bmatrix}.$$
(9)

The equation for new functions is simply

$$\begin{bmatrix} \mathbf{D}_{a} & \frac{2Nm}{r^{2}} \\ \frac{2Nm}{r^{2}} & \mathbf{D}_{g} \end{bmatrix} \begin{bmatrix} \mathbf{a}_{m} \\ \mathbf{g}_{m} \end{bmatrix} = \omega^{2} \begin{bmatrix} \mathbf{a}_{m} \\ \mathbf{g}_{m} \end{bmatrix},$$
(10)

(日) (圖) (E) (E) (E)

where

$$\mathbf{D}_{a} = -\partial_{rr} - \frac{1}{r}\partial_{r} + \frac{N^{2} + m^{2}}{r^{2}} + 2(3f^{2} - 1),$$
 (11a)

$$\mathbf{D}_g = -\partial_{rr} - \frac{1}{r}\partial_r + \frac{N^2 + m^2}{r^2} + 2(f^2 - 1).$$
 (11b)

The physical interpretation is now more clear. a_m describes a field which far away from the vortex core looks like a field with mass $m_a^2 = 4$ and g_m describes a massless field (the Goldstone's mode).

Crossections for $\omega = 3.0$ for different positive values of *m* together with fitted functions (for large *r*) of type $f(x) = A + Be^{-Cm}$.



T. Romańczukiewicz NRP -

NRP - vortices

E.



Vortex A toy model Linearization

• For Goldstone's mode the values are:

 $egin{array}{rcl} A_g = & -0.00028 & \pm & 0.00016 \ B_g = & 0.7982 & \pm & 0.0059 \ C_g = & 0.2856 & \pm & 0.0012 \end{array}$

 A_g should be positive (all $\sigma > 0$) so $A_g = 0$ with accuracy better than 3 three digits. Exponentially decaying crossections are very common.

• For amplitude mode the values are: $A_g = 0.04187 \pm 0.00077$ $B_g = 0.226 \pm 0.022$ $C_g = 0.265 \pm 0.016$ Note that A_g is significantly larger than $0 \Rightarrow$ infinite crossection?

Our assumption according the function could be wrong. Infinite crossection is nothing unusual (compare with Coulomb scattering).



Vortex A toy model Linearization

• For Goldstone's mode the values are:

$$egin{array}{rcl} A_g = & -0.00028 & \pm & 0.00016 \ B_g = & 0.7982 & \pm & 0.0059 \ C_g = & 0.2856 & \pm & 0.0012 \end{array}$$

 A_g should be positive (all $\sigma > 0$) so $A_g = 0$ with accuracy better than 3 three digits. Exponentially decaying crossections are very common.

• For amplitude mode the values are:

 $egin{array}{rcl} A_g = & 0.04187 & \pm & 0.00077 \ B_g = & 0.226 & \pm & 0.022 \ C_g = & 0.265 & \pm & 0.016 \end{array}$

Note that A_g is significantly larger than $0 \Rightarrow$ infinite crossection?

Our assumption according the function could be wrong. Infinite crossection is nothing unusual (compare with Coulomb scattering).

Vortex A toy model Linearization

Mass of the vortex is infinite and diverges as

$$M_R = M_0 + \pi \log R,$$

where M_0 is finite and $R \gg 1$ is a radius within which we integrate energy density.

If the vortex would accelerate with finite acceleration the total crossection should also diverge as

 $\sigma_{aa}^2 \sim \log R \sim \log m$

which implies

$$\sigma_{aa}(m) = \frac{A_3}{\sqrt{m}}$$

which also gives a nice fit ($A_3 = 0.1991 \pm 0.0034$).

・ロ・ ・ 四・ ・ ヨ・ ・ ヨ・ ・

э

Vortex A toy model Linearization

Mass of the vortex is infinite and diverges as

$$M_R = M_0 + \pi \log R,$$

where M_0 is finite and $R \gg 1$ is a radius within which we integrate energy density.

If the vortex would accelerate with finite acceleration the total crossection should also diverge as

$$\sigma_{aa}^2 \sim \log R \sim \log m$$

which implies

$$\sigma_{aa}(m) = \frac{A_3}{\sqrt{m}}$$

which also gives a nice fit ($A_3 = 0.1991 \pm 0.0034$).

Vortex A toy model Linearization

Mass of the vortex is infinite and diverges as

$$M_R = M_0 + \pi \log R,$$

where M_0 is finite and $R \gg 1$ is a radius within which we integrate energy density.

If the vortex would accelerate with finite acceleration the total crossection should also diverge as

$$\sigma_{aa}^2 \sim \log R \sim \log m$$

which implies

$$\sigma_{aa}(m) = \frac{A_3}{\sqrt{m}}$$

which also gives a nice fit ($A_3 = 0.1991 \pm 0.0034$).

・ロ ・ ・ 日 ・ ・ 日 ・ ・ 日 ・

Vortex A toy model Linearization

There is something wrong in our approach.

- We expected $\sigma_{ag} > \sigma_{aa}$ in order to explain NRP and we obtained finite σ_{ag} and infinite σ_{aa} .
- Even if the topological zero was accelerating towards the source of radiation the scattering for the whole field implies the radiation pressure is **positive**.
- Conclusion: vortex does not interact with radiation as a rigid body!

Vortex A toy model Linearization

There is something wrong in our approach.

- We expected $\sigma_{ag} > \sigma_{aa}$ in order to explain NRP and we obtained finite σ_{ag} and infinite σ_{aa} .
- Even if the topological zero was accelerating towards the source of radiation the scattering for the whole field implies the radiation pressure is **positive**.
- Conclusion: vortex does not interact with radiation as a rigid body!

Vortex A toy model Linearization

There is something wrong in our approach.

- We expected $\sigma_{ag} > \sigma_{aa}$ in order to explain NRP and we obtained finite σ_{ag} and infinite σ_{aa} .
- Even if the topological zero was accelerating towards the source of radiation the scattering for the whole field implies the radiation pressure is **positive**.
- Conclusion: vortex does not interact with radiation as a rigid body!

Vortex A toy model Linearization

Justification (not a proof!):

- Eigenfunctions for large values of *m* are flat until they reach their first maximum and a zero shortly after.
- The first zero of is always larger than $r_0 > m/k$.
- For small distances *R* from the vortex core only small values of $m \leq 2kR$ needs to be consider.



<ロ> < 回 > < 回 > < 回 > < 回 >

Conclusion

Field inside a small tube can undergo a NRP, however further away from the core the positive RP emerge. This causes a **stress** between the vortex core and its asymptotic cloud.

Vortex A toy model Linearization

Justification (not a proof!):

- Eigenfunctions for large values of *m* are flat until they reach their first maximum and a zero shortly after.
- The first zero of is always larger than $r_0 > m/k$.
- For small distances *R* from the vortex core only small values of $m \lesssim 2kR$ needs to be consider.



<ロ> < 回 > < 回 > < 回 > < 回 >

Conclusion

Field inside a small tube can undergo a NRP, however further away from the core the positive RP emerge. This causes a **stress** between the vortex core and its asymptotic cloud.

Vortex A toy model Linearization

Justification (not a proof!):

- Eigenfunctions for large values of *m* are flat until they reach their first maximum and a zero shortly after.
- The first zero of is always larger than $r_0 > m/k$.
- For small distances *R* from the vortex core only small values of *m* ≤ 2*kR* needs to be consider.



< □ > < □ > < □ > < □ > < □ >

Conclusion

Field inside a small tube can undergo a NRP, however further away from the core the positive RP emerge. This causes a **stress** between the vortex core and its asymptotic cloud.

Vortex A toy model Linearization

Path of the topological zero of the vortex. First the vortex is accelerating towards the radiation. Then the rest of the field drags the vortex core.



T. Romańczukiewicz NI

Outline



Goldstone's model



T. Romańczukiewicz NRP – vortices

イロト イロト イヨト イヨト



- Due to the difference in masses between massless Goldstone's mode and massive amplitude mode **the core** of the vortex undergoes NRP when is hit with amplitude wave. $F \sim A^2$ unlike in ϕ^4 model where $F \sim A^2$.
- For larger distances the asymptotic field with infinite mass is pushed by the radiation
- a stress is created which finally drags the vortex
- Abelian Higgs model vortices are much more compact objects (often dubbed the *local* vortices). There is a true NRP (needs more calculations).

・ロン ・雪 と ・ ヨ と



- Due to the difference in masses between massless Goldstone's mode and massive amplitude mode **the core** of the vortex undergoes NRP when is hit with amplitude wave. $F \sim A^2$ unlike in ϕ^4 model where $F \sim A^2$.
- For larger distances the asymptotic field with infinite mass is pushed by the radiation
- a stress is created which finally drags the vortex
- Abelian Higgs model vortices are much more compact objects (often dubbed the *local* vortices). There is a true NRP (needs more calculations).

・ロン ・雪 と ・ ヨ と



- Due to the difference in masses between massless Goldstone's mode and massive amplitude mode **the core** of the vortex undergoes NRP when is hit with amplitude wave. $F \sim A^2$ unlike in ϕ^4 model where $F \sim A^2$.
- For larger distances the asymptotic field with infinite mass is pushed by the radiation
- a stress is created which finally drags the vortex
- Abelian Higgs model vortices are much more compact objects (often dubbed the *local* vortices). There is a true NRP (needs more calculations).

・ロ・ ・ 四・ ・ 回・ ・ 回・ …

э.

Significance of the NRP

- Acts as effective attractive interaction between a defect and perturbation or excited defects. Instead of F ~ e^{-aR} we have F ~ R^{-D}.
- After collision defect can anihilate into radiation.
- Can speed up a process of colapse of system of defects.

Significance of the NRP

- Acts as effective attractive interaction between a defect and perturbation or excited defects. Instead of F ~ e^{-aR} we have F ~ R^{-D}.
- After collision defect can anihilate into radiation.
- Can speed up a process of colapse of system of defects.

Significance of the NRP

- Acts as effective attractive interaction between a defect and perturbation or excited defects. Instead of F ~ e^{-aR} we have F ~ R^{-D}.
- After collision defect can anihilate into radiation.
- Can speed up a process of colapse of system of defects.

Τ.R. (φ⁴)

Acta Phys. Pol., B 35, 523 (2004)

P. Forgacs, A. Lukacs, T.R. (ϕ^4)

Phys. Rev., D77, 125012 (2008)

T.R. (Toy model)

soon in arXives (2008)



P. Forgacs, A. Lukacs, J. Karkowski, T.R. (ϕ^4) in preparation

T.F

http://prunus.if.uj.edu.pl/UCP/

< □ > < □ > < □ > < □ > < □ > <

Τ.R. (φ⁴)

Acta Phys. Pol., B 35, 523 (2004)

P. Forgacs, A. Lukacs, T.R. (ϕ^4)

Phys. Rev., D77, 125012 (2008)

T.R. (Toy model)

soon in arXives (2008)



P. Forgacs, A. Lukacs, J. Karkowski, T.R. (ϕ^4) in preparation

T.R.

http://prunus.if.uj.edu.pl/UCP/

・ロン ・雪 ・ ・ ヨ ・

크

Τ.R. (φ⁴)

Acta Phys. Pol., B 35, 523 (2004)

P. Forgacs, A. Lukacs, T.R. (ϕ^4)

Phys. Rev., D77, 125012 (2008)

T.R. (Toy model)

soon in arXives (2008)



P. Forgacs, A. Lukacs, J. Karkowski, T.R. (ϕ^4) in preparation

T.R.

http://prunus.if.uj.edu.pl/UCP/

・ロン ・雪 ・ ・ ヨ ・

크