Four point functions in $N = 1$ SCFT

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N=1 SCFT

- $N = 1$ superconformal generators form algebra ($m \in \mathbb{Z}, k \in \mathbb{Z} + \frac{1}{2}$):

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12} m \left(m^2 - 1\right) \delta_{m+n}, \\ [L_m, S_k] &= \frac{m-2k}{2} S_{m+k}, \\ \{S_k, S_l\} &= 2L_{k+l} + \frac{c}{3}\left(k^2 - \frac{1}{4}\right)\delta_{k+l} \end{aligned}$$

- analogously for anti-holomorphic sector;

$$\left[L_m, \bar{L}_n\right] = 0, \quad \left\{S_k, \bar{S}_l\right\} = 0$$

- highest weight representation - supermodule $\mathcal{V}_{\Delta,c}$
- highest weight state ν_Δ:

$$\mathrm{L}_m \left|\nu_\Delta\right\rangle = \mathrm{S}_k \left|\nu_\Delta\right\rangle = 0, \quad m \in \mathbb{N}, k \in \mathbb{N} - \frac{1}{2}, \qquad \mathrm{L}_0 \left|\nu_\Delta\right\rangle = \Delta \left|\nu_\Delta\right\rangle$$

$$S_{-\frac{1}{2}} \left|\nu_\Delta\right\rangle = \left|*\nu_\Delta\right\rangle, \quad \mathrm{L}_0 \left|*\nu_\Delta\right\rangle = \left(\Delta + \frac{1}{2}\right) \left|*\nu_\Delta\right\rangle$$

States and operators

- descendant states on level f:

$$\nu_{\Delta,KM} \;\equiv\; S_{-k_i}\ldots S_{-k_1} L_{-m_j}\ldots L_{-m_1}\nu_\Delta\,,$$

  where $|K|+|M| = k_1+\cdots+k_i+m_1+\cdots+m_j = f$.
- supermodule

$$\mathcal{V}_\Delta \;=\; \mathcal{V}_\Delta^+ \oplus \mathcal{V}_\Delta^-\,,\quad \mathcal{V}_\Delta^+ \;=\; \bigoplus_{m\in\mathbb{N}\cup\{0\}} \mathcal{V}_\Delta^m\,,\quad \mathcal{V}_\Delta^- \;=\; \bigoplus_{k\in\mathbb{N}-\frac{1}{2}} \mathcal{V}_\Delta^k\,,$$

  where $\mathcal{V}_\Delta^\pm$ eigenspaces of parity operator $(-1)^{2(L_0-\bar{L}_0)}$
- scalar product: $\langle\nu\,|\nu\rangle = 1,\quad L_n^\dagger = L_{-n},\quad \langle\nu| = |\nu\rangle^\dagger$
- space of states

$$\mathcal{H} = \bigoplus_{\Delta,\bar{\Delta}} \mathcal{V}_\Delta \otimes \mathcal{V}_{\bar{\Delta}}$$

- states - operators correspondence:

$$\left|\xi_\Delta \otimes \bar{\xi}_{\bar{\Delta}}\right\rangle \qquad \leftrightarrow \qquad \lim_{z,\bar{z}\to 0} \phi_{\xi_\Delta,\bar{\xi}_{\bar{\Delta}}}(z,\bar{z})\,|0\rangle$$

Three point functions

Conformal Ward identities determine the form of 3-point functions:

$$\left\langle \xi_3 \otimes \bar{\xi}_3 \right| \phi_{\xi_2,\bar{\xi}_2}(z,\bar{z}) \left| \xi_1 \otimes \bar{\xi}_1 \right\rangle = z^{\Delta_3(\xi_3)-\Delta_2(\xi_2)-\Delta_1(\xi_1)} \bar{z}^{\bar{\Delta}_3(\bar{\xi}_3)-\bar{\Delta}_2(\bar{\xi}_2)-\bar{\Delta}_1(\bar{\xi}_1)}$$

$$\times \rho\left(\begin{smallmatrix}\Delta_3 & \Delta_2 & \Delta_1\\ \infty & 1 & 0\end{smallmatrix}\right)(\xi_3,\xi_2,\xi_1)\,\rho\left(\begin{smallmatrix}\bar{\Delta}_3 & \bar{\Delta}_2 & \bar{\Delta}_1\\ \infty & 1 & 0\end{smallmatrix}\right)(\bar{\xi}_3,\bar{\xi}_2,\bar{\xi}_1)\, C_{321}$$

- type of constant $C, \tilde{C}$ depends on total chiral parity of descendant states ξ_i
- structure constants:

$$C_{321} \equiv \langle \phi_{\nu_3,\bar{\nu}_3}(\infty)\phi_{\nu_2,\bar{\nu}_2}(1,1)\phi_{\nu_1,\bar{\nu}_1}(0,0)\rangle$$

$$\tilde{C}_{321} \equiv \langle \phi_{\nu_3,\bar{\nu}_3}(\infty)\phi_{*\nu_2,*\bar{\nu}_2}(1,1)\phi_{\nu_1,\bar{\nu}_1}(0,0)\rangle$$

- 3-point conformal blocks are polynomials in weights Δ_i, fully determined by symmetry

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$$\times \rho\left({}^{\Delta_3}_{\infty}\,{}^{\Delta_2}_{1}\,{}^{\Delta_1}_{0}\right)(\xi_3,\xi_2,\xi_1)\, \rho\left({}^{\bar{\Delta}_3}_{\infty}\,{}^{\bar{\Delta}_2}_{1}\,{}^{\bar{\Delta}_1}_{0}\right)(\bar{\xi}_3,\bar{\xi}_2,\bar{\xi}_1)\, \tilde{C}_{321}$$

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Four point functions

Any 4-point function

$$\langle\phi_{\xi_4,\bar{\xi}_4}(z_4,\bar{z}_4)\phi_{\xi_3,\bar{\xi}_3}(z_3,\bar{z}_3)\phi_{\xi_2,\bar{\xi}_2}(z_2,\bar{z}_2)\phi_{\xi_1,\bar{\xi}_1}(z_1,\bar{z}_1)\rangle$$

can be expressed in terms of 8 basic 4-point functions of primary fields:

$$\langle\phi_{\nu_4,\bar{\nu}_4}(\infty)\,\phi_{\nu_3,\bar{\nu}_3}(1,1)\phi_{\nu_2,\bar{\nu}_2}(z,\bar{z})\,\phi_{\nu_1,\bar{\nu}_1}(0,0)\rangle\,,$$

$$\langle\phi_{\nu_4,\bar{\nu}_4}(\infty)\,\phi_{*\nu_3,*\bar{\nu}_3}(1,1)\phi_{*\nu_2,*\bar{\nu}_2}(z,\bar{z})\,\phi_{\nu_1,\bar{\nu}_1}(0,0)\rangle\,,$$

$$\langle\phi_{\nu_4,\bar{\nu}_4}(\infty)\,\phi_{*\nu_3,\bar{\nu}_3}(1,1)\phi_{\nu_2,*\bar{\nu}_2}(z,\bar{z})\,\phi_{\nu_1,\bar{\nu}_1}(0,0)\rangle\,,$$

$$\ldots$$

where

$$z=\frac{(z_2-z_1)(z_4-z_3)}{(z_3-z_1)(z_4-z_2)}$$

Four-point functions and blocks

- Identity operator: $\mathbf{1} = \sum_p \sum_{f=|I|=|J|} |\nu_{\Delta_p,I}\rangle \left[G^f_{c,\Delta_p}\right]^{IJ} \langle \nu_{\Delta_p,J}|$,
- Gram matrix: $\left[G^f_{c,\Delta}\right]_{IJ} = \langle\, \nu^f_I \mid \nu^f_J \,\rangle_{c,\Delta}$
- Factorization of 4-point function:

$$\begin{aligned}
&\langle \nu_4 \otimes \bar{\nu}_4 |\, \phi_{\nu_3,\bar{\nu}_3}(1,1) \phi_{\nu_2,\bar{\nu}_2}(z,\bar{z})\, | \nu_1 \otimes \bar{\nu}_1 \rangle \\
&\quad = \sum_p \sum_{f=|I|=|J|} \langle \nu_4 \otimes \bar{\nu}_4 |\, \phi_{\nu_3,\bar{\nu}_3}(1,1) | \nu_{p,I} \otimes \bar{\nu}_{p,\bar{I}} \rangle \left[G^f_{c,\Delta_p}\right]^{IJ} \\
&\qquad \times \left[\bar{G}^f_{c,\bar{\Delta}_p}\right]^{\bar{I}\bar{J}} \langle \nu_{p,J} \otimes \bar{\nu}_{p,\bar{J}} | \phi_{\nu_2,\bar{\nu}_2}(z,\bar{z})\, | \nu_1 \otimes \bar{\nu}_1 \rangle = \,?
\end{aligned}$$

- 3-point function with ξ_p (ζ_p) - even (odd) descendant:

$$\left\langle \xi_p \otimes \bar{\xi}_p \right|\, \phi_{\nu_2,\bar{\nu}_2}(z,\bar{z})\, | \nu_1 \otimes \bar{\nu}_1 \rangle = \rho\!\left({}^{\Delta_p}_{\infty}\,{}^{\Delta_2}_{z}\,{}^{\Delta_1}_{0}\right)\!(\xi_p, \nu_2, \nu_1)\, \rho\!\left({}^{\bar{\Delta}_p}_{\infty}\,{}^{\bar{\Delta}_2}_{\bar{z}}\,{}^{\bar{\Delta}_1}_{0}\right)\!(\bar{\xi}_p, \bar{\nu}_2, \bar{\nu}_1)\, C_{p21}$$

$$\left\langle \zeta_p \otimes \bar{\zeta}_p \right|\, \phi_{\nu_2,\bar{\nu}_2}(z,\bar{z})\, | \nu_1 \otimes \bar{\nu}_1 \rangle = \rho\!\left({}^{\Delta_p}_{\infty}\,{}^{\Delta_2}_{z}\,{}^{\Delta_1}_{0}\right)\!(\zeta_p, \nu_2, \nu_1)\, \rho\!\left({}^{\bar{\Delta}_p}_{\infty}\,{}^{\bar{\Delta}_2}_{\bar{z}}\,{}^{\bar{\Delta}_1}_{0}\right)\!(\bar{\zeta}_p, \bar{\nu}_2, \bar{\nu}_1)\, \tilde{C}_{p21}$$

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&\quad= \sum_p \sum_{f=|I|=|J|} \langle\nu_4\otimes\bar\nu_4|\,\phi_{\nu_3,\bar\nu_3}(1,1)|\nu_{p,I}\otimes\bar\nu_{p,\bar I}\rangle \left[G^f_{c,\Delta_p}\right]^{IJ} \\
&\qquad\times \left[\bar G^f_{c,\bar\Delta_p}\right]^{\bar I\bar J} \langle\nu_{p,J}\otimes\bar\nu_{p,\bar J}|\phi_{\nu_2,\bar\nu_2}(z,\bar z)\,|\nu_1\otimes\bar\nu_1\rangle = ?
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$$\left\langle \zeta_p\otimes\bar\zeta_p\right|\phi_{\nu_2,\bar\nu_2}(z,\bar z)\,|\nu_1\otimes\bar\nu_1\rangle = \rho^{\Delta_p}_{\infty}\,{}^{\Delta_2}_{z}\,{}^{\Delta_1}_{0}(\zeta_p,\nu_2,\nu_1)\,\rho^{\bar\Delta_p}_{\infty}\,{}^{\bar\Delta_2}_{\bar z}\,{}^{\bar\Delta_1}_{0}(\bar\zeta_p,\bar\nu_2,\bar\nu_1)\,\tilde C_{p21}$$

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\end{aligned}
$$

$$
\begin{aligned}
&= \sum_p C_{43p}\; C_{p21} \left| \mathcal{F}^{1}_{\Delta_p} \begin{bmatrix} \Delta_3 & \Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix} (z) \right|^2 \\
&+ \sum_p \tilde C_{43p}\; \tilde C_{p21} \left| \mathcal{F}^{\frac{1}{2}}_{\Delta_p} \begin{bmatrix} \Delta_3 & \Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix} (z) \right|^2
\end{aligned}
$$

NS superconformal blocks

- The even blocks (4 types)

$$\mathcal{F}^{1}_{\Delta}\begin{bmatrix} {}_{-}\Delta_3 & {}_{-}\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix}(z) \;=\; z^{\Delta - {}_{-}\Delta_2 - \Delta_1}\left(1 + \sum_{m\in\mathbb{N}} z^m F^m_{c,\Delta}\begin{bmatrix} {}_{-}\Delta_3 & {}_{-}\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix}\right),$$

- The odd blocks (4 types)

$$\mathcal{F}^{\frac{1}{2}}_{\Delta}\begin{bmatrix} {}_{-}\Delta_3 & {}_{-}\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix}(z) \;=\; z^{\Delta - {}_{-}\Delta_2 - \Delta_1}\sum_{k\in\mathbb{N}-\frac{1}{2}} z^k F^k_{c,\Delta}\begin{bmatrix} {}_{-}\Delta_3 & {}_{-}\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix},$$

- where the coefficients of expansion:

$$F^f_{c,\Delta}\begin{bmatrix} {}_{-}\Delta_3 & {}_{-}\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix} = \sum_{|I|=|J|=f} \rho^{\Delta_4\ \ \Delta_3\ \ \Delta}_{\infty\ \ 1\ \ 0}(\nu_4, {}_{-}\nu_3, \nu^I_\Delta)\left[G^f_{c,\Delta}\right]^{IJ}\rho^{\Delta\ \ \Delta_2\ \ \Delta_1}_{\infty\ \ 1\ \ 0}(\nu^J_\Delta, {}_{-}\nu_2, \nu_1)$$

Blocks' properties

- completely determined by the conformal symmetry,
- the form of general superconformal block is unknown,
- polynomials in external weights Δ_i,
- as function of c blocks have poles in c_{rs},
- as function of Δ blocks have poles in Δ_{rs},

The recursive methods of approximate analytic determination of blocks are based on recurrence relations for blocks' coefficients:

$$F^{f}_{c,\Delta}\left[{}^{\Delta_3}_{\Delta_4}\,{}^{\Delta_2}_{\Delta_1}\right] = \mathrm{h}^{f}_{c,\Delta}\left[{}^{\Delta_3}_{\Delta_4}\,{}^{\Delta_2}_{\Delta_1}\right] + \sum_{\substack{1\leqslant rs\leqslant 2f\\ r+s\in 2\mathbb{N}}} \frac{\mathcal{R}^{f}_{c,\,rs}\left[{}^{\Delta_3}_{\Delta_4}\,{}^{\Delta_2}_{\Delta_1}\right]}{\Delta-\Delta_{rs}(c)}\,,$$

where the residue:

$$\mathcal{R}^{f}_{c,\,rs}\left[{}^{\Delta_3}_{\Delta_4}\,{}^{\Delta_2}_{\Delta_1}\right] \sim F^{f-\frac{rs}{2}}_{c,\Delta_{rs}+\frac{rs}{2}}\left[{}^{\Delta_3}_{\Delta_4}\,{}^{\Delta_2}_{\Delta_1}\right]$$

- *more effectively*: block as a power series of the elliptic coordinate *nome* $q = e^{i\pi\tau},\ \tau = \frac{iK(1-z)}{K(z)}$

Large Δ behavior

The asymptotic behavior of blocks for $\Delta \to \infty$ can be deduced from:

1. classical limit of superconformal blocks
   - determine how asymptotic depends of external weights, c and Δ,
   - can be calculate in supersymmetric Liouville theory

$$
\begin{aligned}
\mathcal{F}_{\Delta}^{1,\frac{1}{2}}\begin{bmatrix} \_\Delta_3 & \_\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix}(z) \quad &= \quad (16q)^{\Delta-\frac{c-3/2}{24}} \; z^{\frac{c-3/2}{24}-\Delta_1-\_\Delta_2} \; (1-z)^{\frac{c-3/2}{24}-\_\Delta_2-\_\Delta_3} \\
&\times \quad \theta_3^{\frac{c-3/2}{2}-4(\Delta_1+\_\Delta_2+\_\Delta_3+\Delta_4)} \; \mathcal{H}_{\Delta}^{1,\frac{1}{2}}\begin{bmatrix} \_\Delta_3 & \_\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix}(q),
\end{aligned}
$$

$$
\mathcal{H}_{\Delta}^{1,\frac{1}{2}}\begin{bmatrix} \_\Delta_3 & \_\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix}(q) \quad = \quad g_{--}^{1,\frac{1}{2}}(q) + \sum_{m,n} \frac{h_{mn}^{1,\frac{1}{2}}\begin{bmatrix} \_\Delta_3 & \_\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix}(q)}{\Delta - \Delta_{mn}},
$$

$$
h_{mn}^{1,\frac{1}{2}}\begin{bmatrix} \_\Delta_3 & \_\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix}(q) \sim H_{\Delta_{mn}+\frac{mn}{2}}^{1,\frac{1}{2}}\begin{bmatrix} \_\Delta_3 & \_\Delta_2 \\ \Delta_4 & \Delta_1 \end{bmatrix}(q).
$$

2. normalization: the functions $g_{--}^{1,\frac{1}{2}}(q)$ don't depend of external weights and c - can be set by superconformal blocks that have known form.

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2. normalization: the functions $g_{--}^{1,\frac{1}{2}}(q)$ don't depend of external weights and c - can be set by superconformal blocks that have known form.

Superconformal blocks for $c = \frac{3}{2}$ and $\Delta_0 = \frac{1}{8}$

$$\mathcal{F}^{1}_{\Delta_p}\left[\begin{smallmatrix}\Delta_0 & \Delta_0\\ \Delta_0 & \Delta_0\end{smallmatrix}\right](z) = [z(1-z)]^{-\frac{1}{4}}\ (16q)^{\Delta_p}\,\theta_3^{-2}(q)\,\theta_3(q^2),$$

$$\mathcal{F}^{\frac{1}{2}}_{\Delta_p}\left[\begin{smallmatrix}\Delta_0 & \Delta_0\\ \Delta_0 & \Delta_0\end{smallmatrix}\right](z) = [z(1-z)]^{-\frac{1}{4}}\ \frac{(16q)^{\Delta_p}}{\Delta_p}\,\theta_3^{-2}(q)\,\theta_2(q^2),$$

$$\mathcal{F}^{1}_{\Delta_p}\left[\begin{smallmatrix}\Delta_0 & *\Delta_0\\ \Delta_0 & \Delta_0\end{smallmatrix}\right](z) = [z(1-z)]^{-\frac{3}{4}}\ (16q)^{\Delta_p}\,\theta_3^{-4}(q)\,\theta_3(q^2),$$

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- Jacobi functions:

$$\theta_3(q) = \sum_{-\infty}^{\infty} q^{n^2}, \quad \theta_2(q) = \sum_{-\infty}^{\infty} q^{(n+\frac{1}{2})^2}$$

$$z = \frac{\theta_2^4(q)}{\theta_3^4(q)}, \quad q(z) = e^{i\pi\tau},\ \tau = i\frac{K(1-z)}{K(z)},$$

Superconformal blocks for $c = \frac{3}{2}$ and $\Delta_0 = \frac{1}{8}$

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$$\mathcal{F}^{1}_{\Delta_p}\begin{bmatrix}*\Delta_0 & *\Delta_0 \\ \Delta_0 & \Delta_0\end{bmatrix}(z) = z^{-\frac{3}{4}}(1-z)^{-\frac{5}{4}}\,(16q)^{\Delta_p}\,\frac{\theta_3(q^2)}{\theta_3^6(q)}\left(1 - \frac{q}{\Delta_p}\,\theta_3^{-1}(q)\,\frac{\partial\theta_3(q)}{\partial q} + \frac{\theta_2^4(q)}{4\Delta_p}\right),$$

$$\mathcal{F}^{\frac{1}{2}}_{\Delta_p}\begin{bmatrix}*\Delta_0 & *\Delta_0 \\ \Delta_0 & \Delta_0\end{bmatrix}(z) = -z^{-\frac{3}{4}}(1-z)^{-\frac{5}{4}}\,(16q)^{\Delta_p}\,\frac{\theta_2(q^2)}{\theta_3^6(q)}\,\Delta_p\left(1 - \frac{q}{\Delta_p}\,\theta_3^{-1}(q)\,\frac{\partial\theta_3(q)}{\partial q} + \frac{\theta_2^4(q)}{4\Delta_p}\right).$$

Conclusions

- We defined the superconformal blocks,
- derived recursive methods for determination of the blocks
  1. analyze classical limit of superconformal blocks (in SLFT),
  2. calculate the form of blocks in special case $c = \frac{3}{2}$ and $\Delta_1 = \frac{1}{8}$.

Now we are working on analytic properties and recurrence relations for superconformal blocks in Ramond sector of $N = 1$ SCFT.