Finite size giant magnons and interactions

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arXiv:0801.2064 [hep-th].

Giant magnons

- Consider a string moving in $\mathbb{R} \times S^2 \subset \mathrm{AdS}_5 \times S^5$.

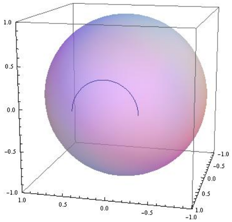
Hofman–Maldacena limit

$$E, J \to \infty \qquad p = \text{fixed}$$
$$E - J = \text{fixed} \qquad \lambda = \text{fixed}$$

- We can now relax the level matching condition $\sum p \in 2\pi\mathbb{Z}$ and consider single magnon excitations.

Giant magnons

- Use conformal gauge: $\frac{dJ}{dx} = \text{const} \rightarrow$ the worldsheet becomes infinitely long.
- Fundamental excitations are local on worldsheet, but macroscopic in space-time.
- Dispersion relation: $E - J = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}} \approx \frac{\sqrt{\lambda}}{\pi}\sin\frac{p}{2}$
- Compare with spin-chain magnon.



Finite size corrections

- The leading order corrections for finite J to the single giant magnon dispersion relation were calculated by Arutyunov, Frolov and Zamaklar, as a soliton solution in uniform gauge.

$$\Delta(E - J) = -\frac{4}{e^2}\frac{\sqrt{\lambda}}{\pi}\sin^3\frac{p}{2}e^{-\mathcal{R}},$$

$$\mathcal{R} = 2\frac{J}{E - J} + ap\cot\frac{p}{2}$$

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- The result is dependent on the gauge parameter a.
- We can get a gauge independent result (corresponding to $a=0$) by considering a giant magnon with momentum $p = 2\pi m/M$ on an orbifold $\mathbb{R}\times S^2/\mathbb{Z}_M$.

  [Astolfi, Forini, Grignani and Semenoff]

Multi-magnon states

For $J \to \infty$ we can construct multi-magnon states in two simple ways:

1. Put them on separate patches, separated by an infinite worldsheet.
2. Put several magnons on the same patch, in a scattering state.

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For finite J there is always a finite separation between the magnons, so any multi-magnon state is interacting.

Finite gap equations

- Encodes the spectrum of a classical string solution in terms of a differential on a Riemann surface.
- The differential is specified as a density along the square root cuts and log cuts (condensates).
- The density is constant on the condensates, but is given as solutions of an integral equation on the cuts:

$$\fint \frac{\rho(x)dx}{z-x} = f_i(E, J) \qquad z \in \mathcal{C}_i$$

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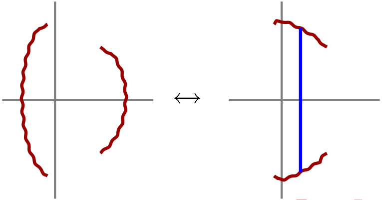
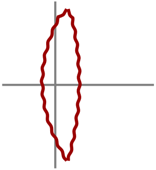
$$\fint \frac{\rho(x)\,dx}{z-x} = f_i(E, J) \qquad z \in \mathcal{C}_i$$

- We can re-cut the surface by reconnecting the branch points in another way, but we may need to add some extra condensates.



Finite gap equations

- A giant magnon is described by a two-cut solution in the singular limit where the end points of the cuts merge. [Vicedo]



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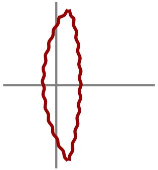
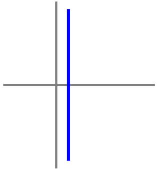


- It can also be described by a single condensate. [Minahan, Tirziu, Tseytlin]



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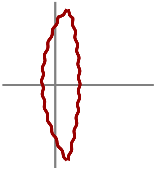

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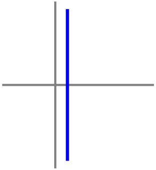
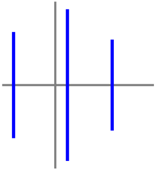
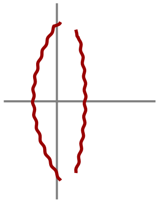


- A multi-magnon state is given by a set of condensates.



Finite size corrections from finite gap equations

- A finite size giant magnon is described by a two cut solution for which the endpoints *almost* coincide.



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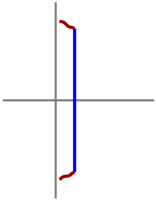
- A finite size giant magnon is described by a two cut solution for which the endpoints *almost* coincide.
- This is equivalent to a condensate with small cuts attached to the ends.



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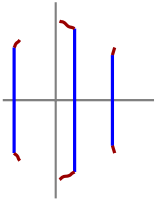
- A finite size giant magnon is described by a two cut solution for which the endpoints *almost* coincide.
- This is equivalent to a condensate with small cuts attached to the ends.
- A multi-magnon state is given by a number of such condensate – cut configurations.



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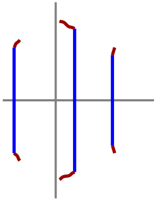
- A N magnon configuration is described by a genus $2N-1$ Riemann surface.



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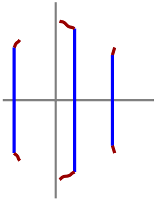
- A N magnon configuration is described by a genus $2N-1$ Riemann surface.
- In general we would expect hyper-elliptic solutions.



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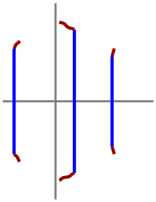

- The density for the cut $\mathcal{C}_i$ is governed by

$$-\!\!\!\!\!\!\int_{\mathcal{C}_i} \frac{\rho(x)}{z-x} = --\!\!\!\!\!\!\int_{\text{other}} \frac{\rho(x)}{z-x} + \cdots ,$$

where $z \in \mathcal{C}_i$ and the integral on the right is over all other cuts and condensates.

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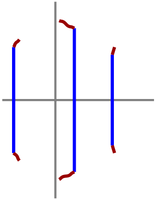

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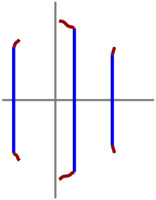

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where $z \in \mathcal{C}_i$ and the integral on the right is over all other cuts and condensates.

- Since the cuts are small, we can treat all integrands except for the closest condensate as constant in z.
- The problem is reduced to a one-cut problem which gives an algebraic solution.

Finite size corrections from finite gap equations

- For M magnons with momenta p_i, energy E_i

$$\Delta(E_i - J) = -\frac{4}{e^2}\frac{\sqrt{\lambda}}{\pi}\sin^3\frac{p_i}{2}\,e^{-2\frac{J}{E_i - J}} \times \prod_{k\neq i}^{m}\frac{\sin^2\frac{p_i+p_k}{4}}{\sin^2\frac{p_i-p_k}{4}}\,e^{-2\frac{E_k-J}{E_i-J}}\,.$$

This is singular as $p_i \to p_j$.

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  This is singular as $p_i \to p_j$.

- For M magnons with $p = 2\pi m/M$

$$\Delta(E - J) = -\frac{4}{e^2}\frac{\sqrt{\lambda}}{\pi}\sin^3\frac{p}{2}\,e^{-2\frac{J/M}{E-J}}\,.$$

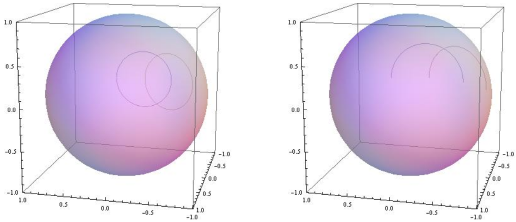
  This agrees exactly with the result for one magnon on a $\mathbb{Z}_M$ orbifold.

Pohlmeyer reduction

- The classical equations of motion for the $O(3)$ sigma model – i.e. for the gauge fixed string – together with the Virasoro constraints are equivalent to the sine-Gordon equation.
- The giant magnon solution corresponds to the fundamental solitonic solution – the kink.

Multi-magnon states from sine-Gorgon

Interacting two-magnon states can be constructed from kink–kink and kink–anti-kink scattering solutions to the sine-Gordon equation.



Finite size solutions can be constructed by considering periodic generalizations of these solutions. [Klose and McLoughlin]

Summary

- For infinite J, multi-magnon state can be either non-interacting or interacting.
- For finite J, multi-magnon states are always interacting.
- Explicit finite size interacting two-magnon states have been constructed from two-phase solutions to the sine-Gordon equation.
- The leading order finite size corrections for interacting states with *any* number of magnons have been computed using the finite gap equations.
- The finite gap computation can straightforwardly be extended to two-spin magnons on $\mathbb{R} \times S^3$.

Outlook

- Can the sine-Gordon calculations be generalized to describe magnons on $\mathbb{R} \times S^3$? This would give a nice check of the finite gap results, but would require finding periodic solutions to the complex sine-Gordon model.
- Can the algebraic curve formalism be used to explicitly reconstruct the string solutions from the finite gap results?

Thank you!