# Renomalization of twist-4 three quark operators

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  see e.g. works of *A.V. Belitsky, V.M.Braun, S.E.Derkachov, G.P.Korchemsky and A.N. Manashov*
- already for the twist-4 case much less is known

# Why, what and how

- however, recently there has been great activity in $\mathcal{N} = 4$ SYM theories and integrable spin chains
  see e.g. *N. Beisert, C. Kristjansen and M. Staudacher, Nucl.Phys.B664:131-184,2003.*
- in *N.Beisert (2004)* and *N.Beisert, G.Ferretti, R.Heise and K.Zarembo (2005)* it was shown (for all twists) that the 1-loop RGEs can be written in Hamiltonian form using the quadratic Casimir operator of the full conformal group $SO(4,2)$
- this suggest that there should be an general strategy/technique how to deal with higher-twist operators that are not quasi-partonic (minimal twist) in a consistent way for QCD applications

# Why, what and how

What this talk will try to cover:

- construction of an operator basis for 3-quark operator of twist four which is favourable in QCD applications
- the general form of the evolution kernels
- spectra of the twist-4 operators
- integral of motion in the case of chiral operators

# Why, what and how

What this talk will not cover:

- gluon operators and their mixing with quark operators
- spectra for gluon operators
- multiplicatively renormalizible operators and their relation with the usual baryon distribution amplitudes

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What this talk will not cover:

- gluon operators and their mixing with quark operators
- spectra for gluon operators
- multiplicatively renormalizible operators and their relation with the usual baryon distribution amplitudes
- How the calculations actually look like

# Notations

First some remarks on notation: it is very convenient to use the Spinor formalism, but here we only need:

$$q = \begin{pmatrix} \psi_\alpha \\ \bar\chi^{\dot\beta} \end{pmatrix}, \qquad \bar q = (\chi^\beta, \bar\psi_{\dot\alpha}) \qquad x^\mu \to x_\mu \sigma^\mu = x_{\alpha\dot\alpha}$$

where $\psi$ is the chiral quark field and $\bar\chi$ is the antichiral one

| | $\psi_+$ | $\psi_-$ | $\bar\chi_+$ | $\bar\chi_-$ |
|---|---|---|---|---|
| $j$ | 1 | 1/2 | 1 | 1/2 |
| $t$ | 1 | 2 | 1 | 2 |

$q_+ = \lambda^\alpha q_\alpha$ and $\bar\chi_- = \bar\chi_{\dot\alpha}\bar\mu^{\dot\alpha}$ $\qquad F^{\mu\nu} = \frac{i}{2}\left(\sigma^{\mu\nu}_{\alpha\beta} f^{\alpha\beta} - \bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}\bar f^{\dot\alpha\dot\beta}\right)$

# Twist-3 case

Have a look at twist-3 case
(*V.M.Braun, S.E.Derkachov, G.P.Korchemsky and A.N. Manashov, 1999*):
two independent non-local operators (twist=# fields ):

$$
\begin{aligned}
\mathcal{O}_1 &= \varepsilon^{ijk}\psi_+^{i,a}(a_1 z)\psi_+^{j,b}(a_2 z)\psi_+^{k,c}(a_3 z)\\
\mathcal{O}_2 &= \varepsilon^{ijk}\psi_+^{i,a}(a_1 z)\psi_+^{j,b}(a_2 z)\bar{\chi}_+^{k,c}(a_3 z)
\end{aligned}
$$

each kann be written as an expansion over local operators:

$$
\mathcal{O}_i = \sum_{N,q} \mathcal{P}(a_1, a_2, a_3)\mathbb{O}_{N,q}
$$

Operators $\mathbb{O}_{N,q}$ of different dimension will not mix under renormalization

# TWIST-3 CASE

the problem of finding the anomalous dimensions reduces to the eigenvalue problem

$$\mathcal{H}\begin{pmatrix}\mathbb{O}_{N,0}\\ \mathbb{O}_{N,1}\\ \vdots\end{pmatrix} = \gamma_{N,q}\begin{pmatrix}\mathbb{O}_{N,0}\\ \mathbb{O}_{N,1}\\ \vdots\end{pmatrix}$$

where $\mathcal{H}$ is the 1-loop integral kernel. The form of $\mathcal{H}$ is almost uniquely determined by conformal symmetry

# ONE-PARTICLE LIGHT-RAY OPERATORS

For higher-twist operator things are more difficult:

- more independent operators as we can now have not only "minus-quarks" but also derivatives i.e. in our notation $D_{+-}$, $D_{--}, \ldots$
- several non-trivial relations obscure the independent operators
- it is very advantageous to use only operators for the basis that have "nice" conformal properties
- at the end of the day: one-particle light-ray operator basis

# ONE-PARTICLE LIGHT-RAY OPERATORS

In the chiral case the operator basis with $T = 4, H = 1/2$ is simply:

$$
\begin{aligned}
Q_1(z_1, z_2, z_3) =& \epsilon^{ijk} \psi_-^{a,i}(a_1 z)\, \psi_+^{b,j}(a_2 z)\, \psi_+^{c,k}(a_3 z)\,, \\
Q_2(z_1, z_2, z_3) =& \epsilon^{ijk} \psi_+^{a,i}(a_1 z)\, \psi_-^{b,j}(a_2 z)\, \psi_+^{c,k}(a_3 z)\,, \\
Q_3(z_1, z_2, z_3) =& \epsilon^{ijk} \psi_+^{a,i}(a_1 z)\, \psi_+^{b,j}(a_2 z)\, \psi_-^{c,k}(a_3 z)\,.
\end{aligned}
$$

each minus field adds one additional unit of twist.

$i, j, k$ are color, $a, b, c$ flavour indices.

# ONE-PARTICLE LIGHT-RAY OPERATORS

For the mixed-chirality case with $T = 4, H = -1/2$ things look a bit different:

$$\mathcal{Q}_1(z_1, z_2, z_3) = \epsilon^{ijk}\, \psi_-^{a,i}(a_1 z)\, \psi_+^{b,j}(a_2 z)\, \bar{\chi}_+^{c,k}(a_3 z)\,,$$
$$\mathcal{Q}_2(z_1, z_2, z_3) = \epsilon^{ijk}\, \psi_+^{a,i}(a_1 z)\, \psi_-^{b,j}(a_2 z)\, \bar{\chi}_+^{c,k}(a_3 z)\,,$$

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\mathcal{Q}_3(z_1, z_2, z_3) &= -\frac{1}{2}\epsilon^{ijk}\, \psi_+^{i,a}(a_1 z)\, \psi_+^{j,b}(a_2 z)\, [D_{-+}\bar{\chi}_+^{k,c}](a_3 z)
\end{aligned}$$

simply putting $\bar{\chi}_-$ would give wrong quantum numbers

# Kernels

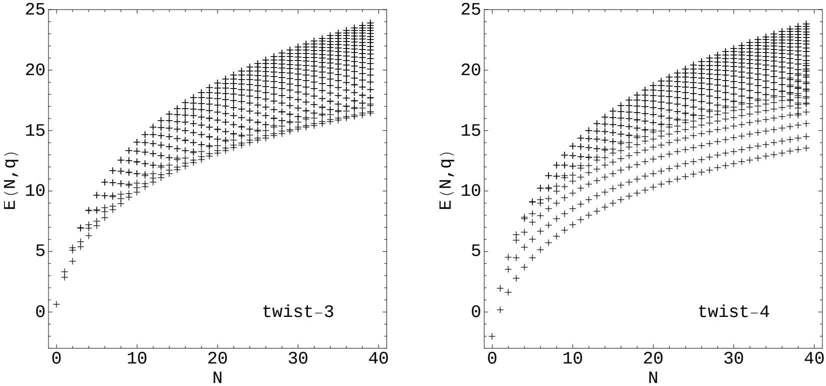
to 1-loop accuracy the general structure of the 2-to-2 kernels is prescribed by conformal symmetry:

$$[K^{i_1 i_2}_{j_1 j_2}\varphi](z_1, z_2) =$$
$$= \int_0^1 d\alpha \int_0^1 d\beta\, \bar{\alpha}^{i_1+j_1-2}\, \alpha^{i_2-j_2}\, \bar{\beta}^{i_2+j_2-2}\, \beta^{i_1-j_1}\, \kappa\left(\frac{\alpha\beta}{\bar{\alpha}\bar{\beta}}\right)\, \varphi(z_{12}^{\alpha}, z_{21}^{\beta})\,,$$

where $\kappa$ is some function.
Note that all kernels are given in coordinate space, following *I.I. Balitsky and V.M.Braun, QCD String Operators*.
This seems to be superior compared to a representation in momentum space.

## MIXED-CHIRALITY CASE

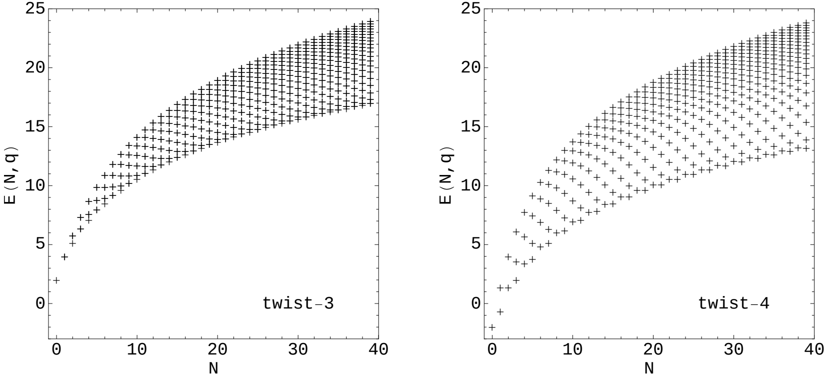
Spectrum of anomalous dimensions




The spectrum of twist-4 operators automatically contains the spectrum corresponding to twist three

# CHIRAL CASE

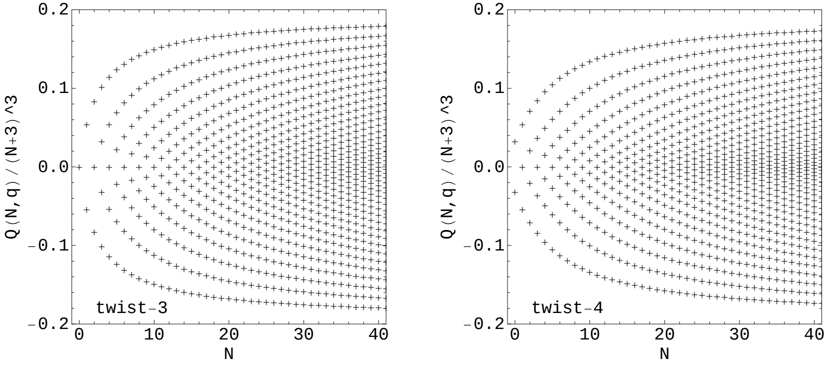
Spectrum of anomalous dimensions




Note that the spectrum (once twist-3 remnants are removed) is very regular, and indeed one can find an operator $\hat{Q}$ which commutes with the chiral Hamiltonian $\mathbb{H}^{chiral}$

# Integrability

The conserved charge $\hat{Q}$ can again be decomposed into the conserved charge $Q_3$ of the twist-3 case and a new part.



# INTEGRABILITY

The line of lowest eigenvalues is given by:

$$E^{tw-3}_{Q_3=0}(N) = \left(1 + \frac{1}{N_c}\right)\left\{4\left[\psi(N+3) - \psi(2)\right] - \frac{1}{2}\right\}, \qquad N - \text{even},$$

where $\psi(x)$ is the Euler $\psi$–function, and

$$E^{tw-4}_{Q_3=0}(N) = \left(1 + \frac{1}{N_c}\right)\left\{4\left[\psi\left(\frac{N+3}{2}\right) - \psi(2)\right] - \frac{1}{2}\right\}, \qquad N - \text{odd}.$$

The appearance of $(N+3)/2$ as argument of the Euler $\psi$–function is characteristic for systems involving half-integer conformal spins.
see *A.V.Belitsky, 1999* and *M.Beccaria and F.Catino, 2008*

# Conclusions

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- allows for a consistent calculation framework for non quasi-partonic operators
- spectrum of anomalous dimensions of twist-4 three quark operators and multiplicatively renormalizible operators (which are relevant in hard exclusive reactions involving a helicity flip)

# Conclusions

Thank you for your attention.

# BASIS

The complete basis for quarks and self-dual gluon field contains seven fields:

$$
\begin{aligned}
\psi_+^{(j,m)}(z) &= (D^{2\dot{1}})^{2j-2}(D^{1\dot{1}})^{2m}\psi_1(z)\,,\\
\psi_-^{(j,m)}(z) &= (D^{1\dot{2}})^{2j-1}(D^{1\dot{1}})^{2m}\psi_2(z)\,,\\
\bar{\chi}_+^{(j,m)}(z) &= (D^{1\dot{2}})^{2j-2}(D^{1\dot{1}})^{2m}\bar{\chi}_{\dot{1}}(z)\,,\\
\bar{\chi}_-^{(j,m)}(z) &= (D^{2\dot{1}})^{2j-1}(D^{1\dot{1}})^{2m}\bar{\chi}_{\dot{2}}(z)\,,\\
f_{++}^{(j,m)}(z) &= (D^{2\dot{1}})^{2j-3}(D^{1\dot{1}})^{2m}f_{11}(z)\,,\\
f_{--}^{(j,m)}(z) &= (D^{1\dot{2}})^{2j-1}(D^{1\dot{1}})^{2m}f_{22}(z)\,,\\
f_{+-}^{(1,m)}(z) &= (D^{1\dot{1}})^{2m}f_{12}(z)\,.
\end{aligned}
$$

With this construction kit one can build operators of arbitrary twist with good transformation properties under conformal transformations

# CONSERVED CHARGE – CONSTRUCTION

First observe that

$$S_{ik} = \partial_k (z_k - z_i) \equiv (\partial/\partial z_k)(z_k - z_i)\,.$$

acts as the intertwining operator between the representations $T^{j_k=1/2} \otimes T^{j_i=1}$ and $T^{j_k=1} \otimes T^{j_i=1/2}$
Define $3 \times 3$ matrix

$$[Q^{\pm}_{ik}]^{ik} = S_{ik}\,, \qquad [Q^{\pm}_{ik}]^{ki} = S_{ki}$$

and all other off-diagonal matrix elements being zero. For the diagonal matrix elements we put $[Q^{\pm}_{ik}]^{ii} = [Q^{\pm}_{ik}]^{kk} = \frac{1}{2}$ and

$$[Q^{+}_{ik}]^{jj} = \frac{1}{2} + S_{ik} \qquad [Q^{-}_{ik}]^{jj} = \frac{1}{2} + S_{ki}\,,$$

## CONSERVED CHARGE – CONSTRUCTION

The two-particle Casimir operators $\widehat{J}^2_{ik}$ can be written in terms of $Q^{\pm}_{ik}$ as

$$\widehat{J}^2_{ik} = \frac{1}{2}\{Q^{+}_{ik}, Q^{-}_{ik}\}$$

And finally:

$$\widehat{Q}_3 = \frac{i}{2}[\widehat{J}^2_{12}, \widehat{J}^2_{23}] \, .$$

The operator $\widehat{Q}_3$ commutes with the Hamiltonian $\mathcal{H}_q^{\psi\psi\psi}$:

$$[\widehat{Q}_3, \mathcal{H}_q^{\psi\psi\psi}] = 0$$

and defines, therefore, a nontrivial integral of motion.