# Bulk viscosity in the gauge-string duality

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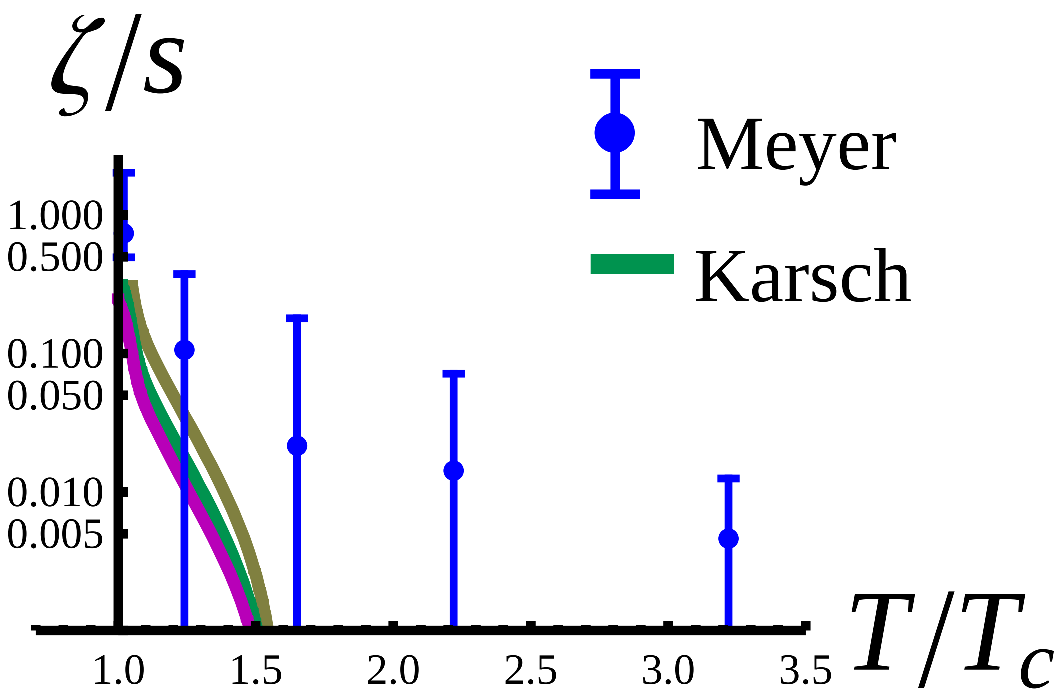
Based on work with S. S. Gubser, A. Nellore and S. S. Pufu

[Gubser et al. 2008ab]

Zakopane, June 2008

# Bulk viscosity spike

Lattice studies indicate that the bulk viscosity to entropy density ration $\zeta/s$ of the quark-gluon plasma increases sharply in the vicinity of the critical temperature [Kharzeev and Tuchin 2007; Karsch et al. 2007; Meyer 2008].



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We must consider a deformation of $\mathcal{N} = 4$ SYM.

# Finite temperature duality

Replace $AdS$ with $AdS$-Schwarzschild, an asymptotically $AdS$ black hole solution of Einstein's equations.

- This geometry has metric

$$ds^2 = \frac{L^2}{z^2}\left(-h(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{h(z)}\right)$$

$$h(z) = 1 - \left(\frac{z}{z_H}\right)^4.$$

  With event horizon at $z = z_H$.

- The black hole has Hawking temperature $T = 1/(\pi z_H)$

# Scalar sourced backgrounds

We consider the backgrounds with non-zero scalar fields in the bulk. The relevant action is

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\left(\partial\phi\right)^2 - V\left(\phi\right)\right) .$$

We will look at geometries of the form

$$ds^2 = e^{2A(r)}\left(-h(r)dt^2 + d\vec{x}^2\right) + \frac{e^{2B(r)}}{h(r)}dr^2 \qquad \phi = \phi(r) .$$

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The dual field theories are not conformal $\Rightarrow$ they can have non-zero $\zeta$!

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For small $\phi$

$$V(\phi) \approx -\frac{12}{L^2} + \frac{1}{2} m^2 \phi^2\,.$$

And near the boundary

$$\phi(r) \approx C_1 e^{(\Delta-4)A(r)} + C_2 e^{-\Delta A(r)}\,,$$

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where $\Delta(\Delta - 4) = m^2 L^2$.
$C_1$ controls deformation of $\mathcal{N} = 4$ given by

$$\mathcal{L} = \mathcal{L}_{\rm SYM} + \Lambda^{4-\Delta} \mathcal{O}_\phi$$

# Kubo formulas for viscosities

Bulk and shear viscosities can be computed from stress-energy tensor correlators through the Kubo formulas

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,G^R_{12,12}(\omega)\,,$$

$$\zeta = -\lim_{\omega\to 0}\frac{1}{9\omega}\mathrm{Im}\,G^R_{ii,jj}(\omega)\,.$$

Where

$$G^R_{ij,kl}(\omega) \equiv -i\int dt\, d^3x\, e^{i\omega t}\theta(t)\langle[T_{ij}(t,\vec{x}),T_{kl}(0,0)]\rangle\,.$$

# Computing correlators I

$T_{ij}$ couples to metric perturbations $h_{ij}$ - to compute its correlators we use the fact that in the strong coupling limit

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We are interested in $\vec{k} = 0$ correlators, so it is enough to consider

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}(r,t) \qquad h_{\mu\nu}(r,t) = h_{\mu\nu}(r)e^{i\omega t} \,.$$

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To compute retarded correlator we keep only boundary term, schematically

$$G^R = J\left[h_{ij}\right](r = r_{\rm bdy}) .$$

This is the prescription of [Son and Starinets 2002].

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Outgoing solutions ($+$) would give advanced correlator.

Action has $U(1)$ symmetry with conserved current $\mathcal{F} = \mathrm{Im}\, \mathcal{J}$. Imaginary part of correlator is given by

$$\mathrm{Im}\, G_R = \mathcal{F}$$

# Shear viscosity

- Relevant metric perturbations are $h_{12} = e^{2A} H_{12}$.

$$H_{12}'' + \left(4A' - B' + \frac{h'}{h}\right) H_{12}' + \frac{e^{-2A+2B}}{h^2} \omega^2 H_{12} = 0$$

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- We obtain expected result $\eta/s = 1/4\pi$.

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- They couple to $h_{00}$, $h_{55}$ and $\delta\phi$.
- Can decouple them in gauge $r = \phi$.

$$
\begin{aligned}
H_{11}'' &= \left(-\frac{1}{3A'} - 4A' + 3B' - \frac{h'}{h}\right) H_{11}' + \\
&\quad + \left(-\frac{e^{2A-2B}\omega^2}{h^2} + \frac{h'}{6hA'} - \frac{h'B'}{h}\right) H_{11}
\end{aligned}
$$

# Choosing the scalar potential

As shown in [Gubser and Nellore 2008], if we take

$$V(\phi) = -\frac{12}{L^2}\cosh(\gamma\phi) + \frac{b}{L^2}\phi^2 \qquad \gamma \approx 0.606 \qquad b \approx 2.057\,,$$

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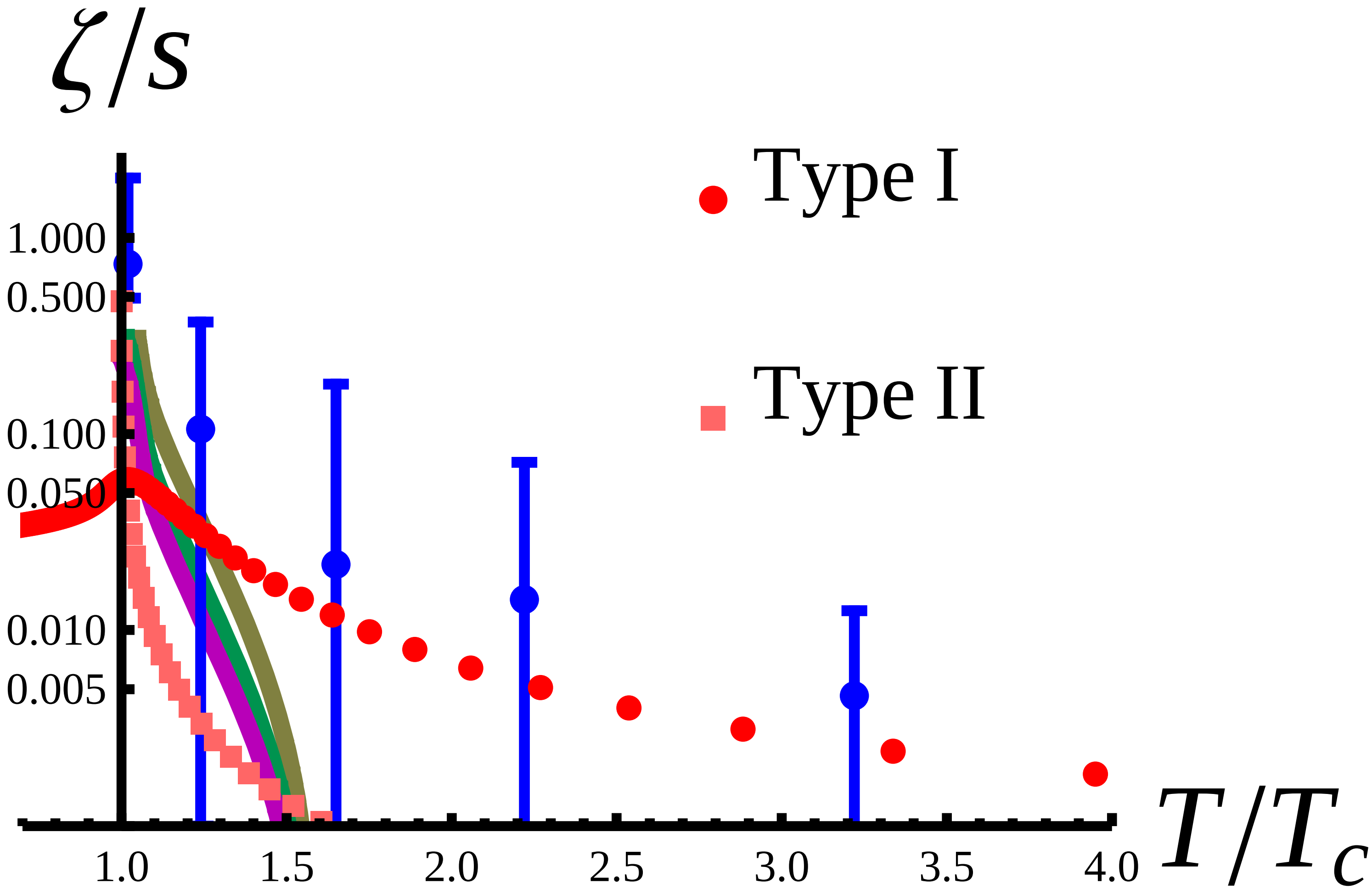
We also consider the type II potential

$$V(\phi) = -\frac{12}{L^2}\left(1+\phi^2\right)^{\frac{1}{4}}\cosh\left(\sqrt{\frac{2}{3}}\phi\right) + \frac{b}{L^2}\phi^2 \qquad b \approx 6.86\,.$$

# Bulk viscosity results



# Viscosity bound

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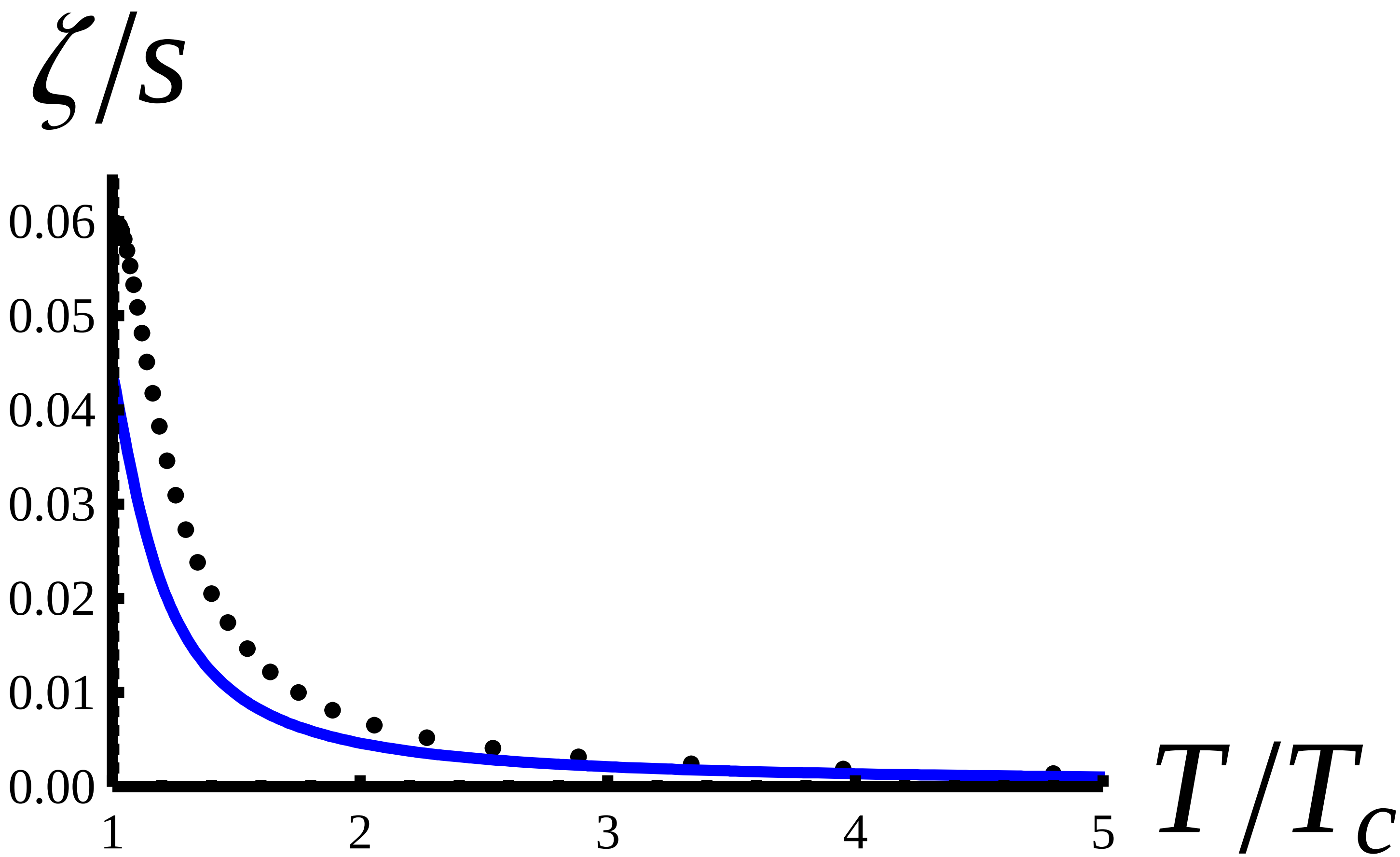

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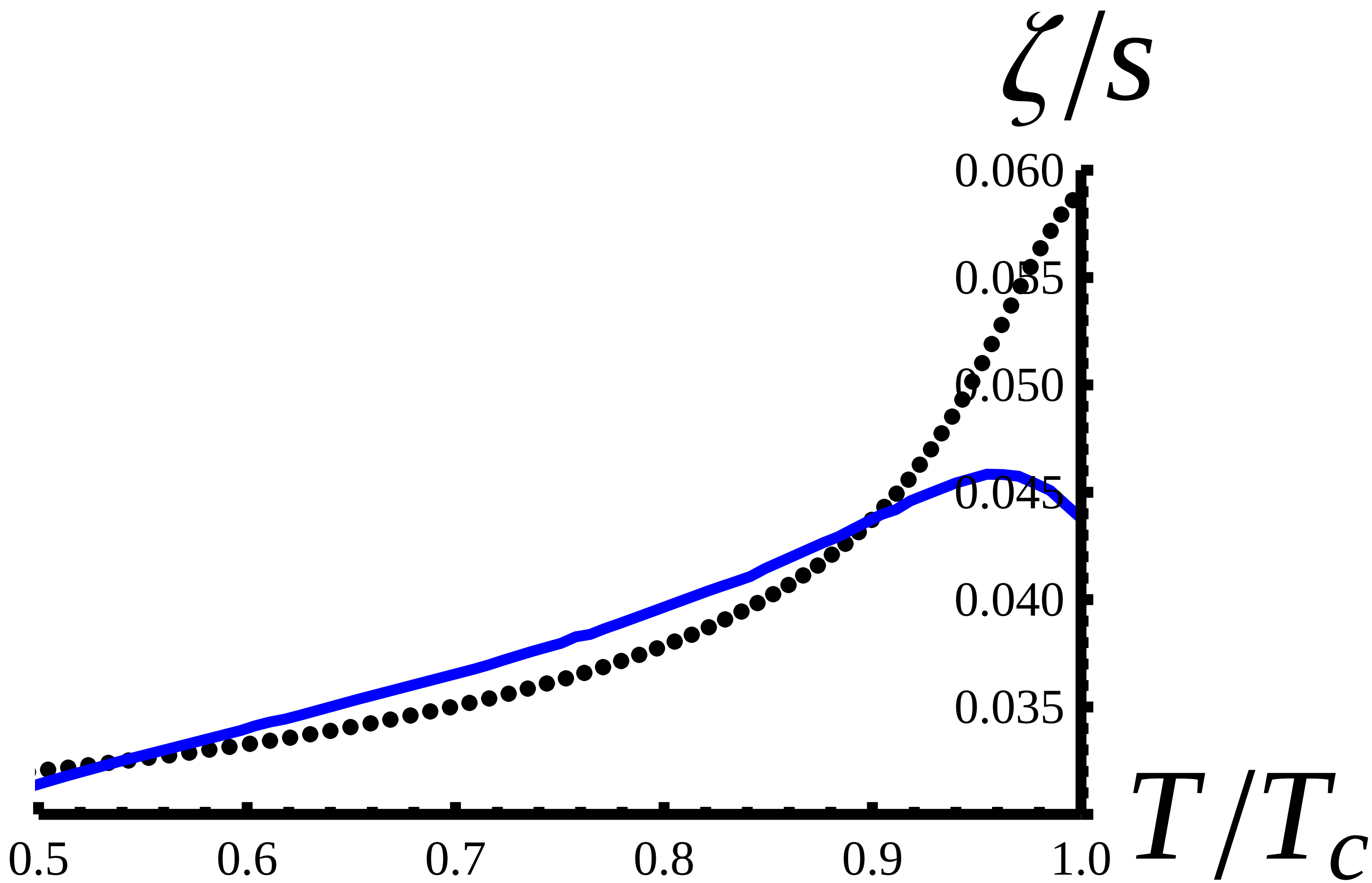

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