On two-point correlation functions in AdS/QCD

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References

This talk is mainly based on the work by A.K., [hep-th] 0801.4215, Phys.Rev.D77 No.12

Model under consideration was developed in J. Erlich, E. Katz, D. T. Son, M. Stephanov, Phys.Rev.Lett. 95 (2005) 261602. [arXiv:hep-ph/0501128] and L. Da Rold, A. Pomarol, Nucl.Phys. B721 (2005) 79-97. [arXiv:hep-ph/0501218]

Table of contents

Introduction

The AdS/QCD models are the way to apply methods of the AdS/CFT correspondence to describe QCD in strong coupling regime.
The feature of AdS/CFT is the relation between AdS curvature radius and t'Hooft constant in the gauge theory.

$$\frac{R^4}{4\pi\alpha'^2} = \lambda' = N_c g_{ym}^2$$

Our aim is to compute some two-point correlation functions in QCD in the simplest model with hard wall cut of the AdS and reveal their dependence of parameters of the QCD, such as number of colors N_c and t'Hooft constant λ'

The model

- We consider AdS_5 space with the hard wall, placed at some radial coordinate z_m

$$ds^2 = \frac{R^2}{z^2}(-dz^2 + dx^\mu dx_\mu); \qquad 0 < z \leq z_m$$

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- The fields in AdS are in correspondence with QCD currents:

$$\begin{aligned} L^a_\mu &\leftrightarrow \bar{q}_L \gamma^\mu t^a q_L \\ R^a_\mu &\leftrightarrow \bar{q}_R \gamma^\mu t^a q_R \\ \left(\frac{2}{z}\right) X^{\alpha\beta} &\leftrightarrow \bar{q}^\alpha_R q^\beta_L \end{aligned}$$

here t^a are generators of $SU(N_f)$ in the adjoint representation. $X^{\alpha\beta}$ is bifundamental in $SU_L(N_f)xSU_R(N_f)$ and we take $N_f = 2$ in the case of two light quarks.

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- Boundary conditions at z_m are: $\partial_z L(z_m) = 0 \;\; ; \;\; \partial_z R(z_m) = 0.$

The model

The action is

$$S = \int d^5x \sqrt{g}\, Tr \left\{ \Lambda^2 (|DX|^2 + \frac{3}{R^2}|X|^2) - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

where

$$D_B X = \partial_B X - \imath L_B X + \imath X R_B$$

$$L(R) = L(R)^a t^a$$

$$F_{(L)BD} = \partial_B L_D - \partial_D L_B - \imath [L_B, L_D],$$

and we introduce the normalization constant Λ of field X

The model

The classical solution for $X^{\alpha\beta}$ has the form

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3.$$

By the AdS/CFT conjecture, one relates M to the quark mass matrix, and Σ to the VEV of operator $\langle \bar{q}q \rangle$, i.e. quark condensates. We choose normalization such as $M = m\mathbf{1}; \Sigma = \sigma\mathbf{1}$, assuming the equality of quark masses.

It is convenient to decompose:

$$X = X_0 e^{\imath 2\pi^a (t^a)} = \mathbf{1}\frac{v(z)}{2} e^{\imath 2\pi^a t^a} \qquad v(z) = mz + \sigma z^3$$

One can see, that in the quadratic order X interacts only with axial field $2A = L - R$ and not with vector one $(2V = R + L)$. That means X breaks chiral symmetry.

The model

We take the transverse gauge for V_μ and decompose.A_μ on longitudinal and transverse parts.:

$$\partial_\mu V_\mu = 0 \qquad A_\mu = A_{\perp\mu} + \partial_\mu \phi$$

One can relate the pseudoscalar current $\bar{q}\gamma_5 q$ with axial vector current $\bar{q}\gamma_5\gamma_\mu q$ via:

$$\partial_\mu \left(\bar{q}\gamma_5\gamma_\mu q\right) = 2m\left(\bar{q}\gamma_5 q\right)$$

The model

This allows us to write down convenient table of correspondence:

$$V_\mu \leftrightarrow \bar{q}\gamma^\mu q = J_V$$
$$A_\mu \leftrightarrow \bar{q}\gamma_5\gamma^\mu q = J_A$$
$$\frac{Q^2}{2m}\phi \leftrightarrow \bar{q}\gamma_5 q = J_\pi$$

Using this correspondance we can compute some current correlators in QCD via AdS/CFT recipe, for example:

$$\langle J_V(q_1) J_V(q_2)\rangle = \frac{\delta}{\delta V_0(q_1)} \frac{\delta}{\delta V_0(q_2)} S(V_{classic})|_{V_0=0}.$$

$$V_0(q) = V_{classic}(q, z)|_{z=0}$$

Parameter fixing

Equations of motion:

$$\left[\partial_z\left(\frac{1}{z}\partial_z V_\mu^a\right)+\frac{q^2}{z}V_\mu^a\right]_\perp=0$$

$$\left[\partial_z\left(\frac{1}{z}\partial_z A_\mu^a\right)+\frac{q^2}{z}A_\mu^a-\frac{R^2g_5^2\Lambda^2v^2}{z^3}A_\mu^a\right]_\perp=0$$

$$\partial_z\left(\frac{1}{z}\partial_z\phi^a\right)+\frac{R^2g_5^2\Lambda^2v^2}{z^3}(\pi^a-\phi^a)=0$$

$$-q^2\partial_z\phi^a+\frac{R^2g_5^2\Lambda^2v^2}{z^2}\partial_z\pi^a=0$$

Parameter fixing

Equation for V is exactly solvable and gives

$$V(Q,z) = -V_0(Q)\frac{1}{I_0(Qz_m)}Qz[K_0(Qz_m)I_1(Qz) - I_0(Qz_m)K_1(Qz)]$$

The variation of metric with respect to boundary value V_0 is

$$\delta S_V = -\int d^4x \frac{R}{g_5^2}\left[\delta V_\mu \frac{\partial_z V_\mu}{z}\right]_{z=\epsilon}$$

And the result for current correlator is:

$$\langle J^a_{V\mu}(q) J^b_{V\nu}(q)\rangle = \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V(q^2)$$

Parameter fixing – z_m

Here

$$\Pi_V(Q^2) = -\frac{R}{g_5^2}\frac{\Big(K_0(Qz_m) - I_0(Qz_m)[ln(Q\epsilon/2) + \gamma]\Big)}{I_0(Qz_m)}$$

The poles of euclidean correlator correspond to masses of bound states, so we can fix z_m by the ρ-meson mass:

$$I_0(\imath M_\rho z_m) = 0 \qquad \Longrightarrow \qquad z_m = \frac{2.4}{M_\rho}$$

Parameter fixing – g_5

In the large Q^2 limit we have

$$\Pi_V(Q^2) = -\frac{R}{2g_5^2} lnQ^2\epsilon^2$$

This result can be compared with the QCD sum rules leading term:

$$\Pi_V(Q^2) = -\frac{N_c}{24\pi^2} lnQ^2\epsilon^2$$

And this fixes g_5

$$\boxed{\frac{g_5^2}{R} = \frac{12\pi^2}{N_c}}$$

Parameter fixing – Λ

To compute correlator of J_π we find solutions for coupled ϕ and π near the boundary

$$\phi(z) = \phi_0(q) Qz K_1(Qz).$$

$$\pi(z) = -\phi_0(q) \frac{Q^2}{g_5^2 R^2 \Lambda^2 m^2} Qz K_1(Qz).$$

The variation of action with respect to $\phi_0(q)$ gives

$$\delta S_\pi = \int d^4x \ \ \frac{R}{g_5^2} \left[\delta\partial_\mu\phi \frac{\partial_z \partial_\mu \phi}{z} \right]_{z=\epsilon} - \Lambda^2 R^3 \left[\delta\pi \frac{v^2}{z^3} \partial_z \pi \right]_{z=\epsilon}$$

Parameter fixing – Λ

And we get for correlator:

$$\langle J_\pi(q), J_\pi(q)\rangle = 2\frac{R}{g_5^2}\frac{1}{g_5^2 R^2 \Lambda^2} Q^2 ln(Q^2\epsilon^2)$$

Comparison with the sum rules leading term

$$\langle J_\pi(q), J_\pi(q)\rangle_{QCD} = \frac{N_c}{16\pi^2} Q^2 ln(Q^2\epsilon^2)$$

gives the value of Λ

$$\boxed{\Lambda^2 = \frac{8}{3}\frac{1}{g_5^2 R^2} = \frac{2N_c}{9\pi^2}\frac{1}{R^3}}$$

Parameter fixing – σ

It remains us to fix relation between σ and condensate.
In QCD

$$\langle \bar{q}q \rangle = \frac{\delta \varepsilon_{QCD}}{\delta m_q}|_{m_q=0}$$

In AdS it corresponds to:

$$\langle \bar{q}q \rangle = \frac{\delta \ S(X_0)}{\delta m}\bigg|_{m=0} = 3R^3\Lambda^2\sigma = \frac{2N_c}{3\pi^2}\sigma$$

Results – Π_A

Now all parameters are fixed. And the action looks:

$$S = \frac{N_c}{12\pi^2}\int d^5x \left\{ -\frac{1}{4z}(F_A^2 + F_V^2) + \frac{4}{3z^3}v(z)^2(\partial\pi - A)^2 + \frac{4}{z^5}v(z)^2 \right\}$$

Solving EOM for A_μ in large Q^2 limit we can obtain the axial current correlator, where OPE terms emerge from $1/Q^2$ corrections of solution:

$$\Pi_A(Q^2) = -\frac{N_c}{24\pi^2}\left[lnQ^2 + \frac{128}{15}\frac{\sigma^2}{Q^6} - \frac{64}{9}\frac{\sigma m}{Q^4} \right]$$

Results – Π_{LR}

The interesting object is "left-right" correlator $\Pi_{LR} = \Pi_A - \Pi_V$

$$\Pi_{LR} = -\frac{N_c}{9\pi^2}\left[\frac{16}{5}\frac{\sigma^2}{Q^6} - \frac{8}{3}\frac{\sigma m_q}{Q^4}\right]$$

Note here, that it has not powers of R, namely it has the order λ'^0. If we denote coefficients in this formla as f and ρ, we find that at $\lambda' \to \infty$ our calculation predicts:

$$f(\lambda') \sim \rho(\lambda') \sim \lambda'^0$$

while at weak coupling regime(sum rules):

$$\rho(\lambda') \sim \lambda'^0 \qquad f(\lambda') = -4\pi\alpha_s \sim \lambda'.$$

Results – f_π

We can also obtain the value of f_π, using the relation:

$$\Pi_A(Q)|_{Q\to 0} = \frac{f_\pi^2}{Q^2}$$

For this purpose we compute solution for A_μ at $Q = 0$ and get:

$$f_\pi^2 = -\frac{R}{g_5}\frac{\partial_z a(z)}{z}|_{z=0,Q=0} =$$
$$\approx \frac{R}{g_5} 2.16 \sigma^{2/3} = \frac{N_c}{12\pi^2} 2.16 \left(\frac{3\pi^2}{2}\frac{\langle \bar{q} q\rangle}{N_c}\right)^{2/3} \sim 40 Mev$$

Do not coincide with expected 140 Mev

Problems and solutions

- Meson masses do not demonstrate Regge behavior
- Coefficients in OPE of correlators differ from sum rules
- Incorrect value of f_π

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- – Modification of scalar potential

Conclusion

The model under consideration has several free parameters, but still has some predictive power. It gives qualitatively satisfactory results, but numbers differ. Study of such simple model gives an insight on common features of AdS/QCD and proposes modifications, needed to obtain more realistic results.

Thank you for your attention!