Hydrodynamic Flow of the Quark-Gluon Plasma and Gauge/Gravity Correspondence (lecture 3)

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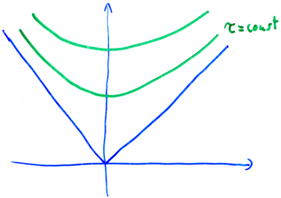
1. **Boost-invariant flow – reminder of lectures 1+2 by Robi**
2. **Going beyond perfect fluid**
   - Viscosity, relaxation time etc.
3. **Going beyond boost-invariance**
   - General hydrodynamic equations from AdS/CFT
4. **Going beyond hydrodynamics**
   - Modeling a heavy-ion collision
   - A 1+1 dimensional toy model
   - The 3+1 dimensional problem
5. **Physics in the expanding plasma**
   - Fundamental flavours and mesons in AdS/CFT
   - Fundamental flavours and mesons in an expanding plasma system
6. **Summary**



Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.

- Pass to proper-time/spacetime rapidity coordinates $(\tau, y, x_1 . x_2)$.
- In a conformal theory, $T^\mu_\mu = 0$ and $\partial_\mu T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$.
- $\varepsilon(\tau)$ is the energy density at mid-rapidity.

Previous lectures $\longrightarrow$ late-time asymptotics of $\varepsilon(\tau)$.
Here $\longrightarrow$ subasymptotic behaviour of $\varepsilon(\tau)$.

What is the physics of $\varepsilon(\tau)$?

- Weak coupling – free streaming

$$\varepsilon(\tau) = \frac{1}{\tau}$$

- Perfect fluid assumption

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}}$$

- Fluid with viscosity $\eta = \frac{\eta_0}{\tau}$

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} \left(1 - \frac{2\eta_0}{\tau^{\frac{2}{3}}} + \dots\right)$$

- Second order viscous hydrodynamics: η, τ_Π:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} \left(1 - \frac{2\eta_0}{\tau^{\frac{2}{3}}} + \frac{B(\eta, \tau_\Pi)}{\tau^{\frac{4}{3}}} + \dots\right)$$

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How to determine $\varepsilon(\tau)$?

- Consider $\varepsilon(\tau) = 1/\tau^s + \ldots$
- Construct the dual geometry

$$\boxed{\varepsilon(\tau)} \longrightarrow \boxed{ds^2 = \frac{g_{\mu\nu}(z,\tau)dx^\mu dx^\nu + dz^2}{z^2}}$$

- Take the scaling limit $\tau \to \infty$, $z \to \infty$ keeping $v = \frac{z}{\tau^{\frac{s}{4}}}$ fixed
- The curvature invariant $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ is nonsingular in the scaling limit only for $s = \frac{4}{3}$
- This corresponds to perfect fluid behaviour
- The resulting geometry is the evolving black hole described in previous lectures

Is this an exact perfect fluid?

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- At subleading order we find 4^{th} order pole singularities in the curvature

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- Set $\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}}\left(1 - \frac{2A}{\tau^r}\right)$
- Solve for geometry and compute the curvature [Nakamura,Sin;RJ]

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- The singular terms may cancel with each other only when $r = \frac{2}{3}$ and $A = 2^{-\frac{1}{2}}3^{-\frac{3}{4}}$
- This correspond exactly to corrections coming from viscosity with the numerical coefficient exactly corresponding to $\eta/s = \frac{1}{4\pi}$ [See Son's lecture].
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- Go to higher order [Heller,RJ]

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- $R_3(v)$ has 4^{th} order poles which can be cancelled by a definite choice of B which fixes the relaxation time τ_Π.
- B is uniquely fixed. The value of τ_Π *depends* on the type of 2^{nd} order hydrodynamic theory used to describe $\varepsilon(\tau)$. Subsequent work fixed uniquely this theory → Son's lectures
- After fixing $R_3(v)$ there remains a *logarithmic* singularity. This is probably due to a pathology of Fefferman-Graham coordinate expansion → see talk by M. Heller

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Assumptions

- We picked boost-invariant setup with full transverse symmetry
- Energy-momentum tensor completely expressed in terms of $\varepsilon(\tau)$

AdS/CFT computation

- Construct dual geometry – solve Einstein's equations
- Fix $\varepsilon(\tau)$ from nonsingularity

Link with hydrodynamics

- Take $\varepsilon(\tau)$ from AdS/CFT
- Plug it into phenomenological hydrodynamic equations
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Question

- Can one lift the symmetry assumptions?
- Is it possible to see hydrodynamic equations more directly?

The approach of [Bhattacharyya,Hubeny,Minwalla,Rangamani]

- Start from a static black hole with fixed temperature T which describes a fluid at rest, $u^\mu = (1,0,0,0)$ with constant energy density
- Perform a boost to obtain a uniform fluid moving with constant velocity u^μ
- The resulting metric (in Eddington-Finkelstein coordinates) is

$$ds^2 = -2u_\mu dx^\mu dr - r^2 \left(1 - \frac{T^4}{\pi^4 r^4}\right) u_\mu u_\nu dx^\mu dx^\nu + r^2(\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

where $r = \infty$ corresponds to the boundary, $r = T/\pi$ is the horizon while $r = 0$ is the position of the singularity.

Promote T and u^μ to (slowly-varying) functions of x^μ

Caveat: The metric is no longer an exact solution of Einstein's equations

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$$ds^2 = -2u_\mu dx^\mu dr - r^2 \left(1 - \frac{T^4}{\pi^4 r^4}\right) u_\mu u_\nu dx^\mu dx^\nu + r^2(\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

where $r = \infty$ corresponds to the boundary, $r = T/\pi$ is the horizon while $r = 0$ is the position of the singularity.

Promote T and u^μ to (slowly-varying) functions of x^μ

Caveat: The metric is no longer an exact solution of Einstein's equations

- Perform an expansion of the Einstein equations in gradients of spacetime fields.
- Find corrections to the metric at first and second order
- Require nonsingularity to fix integration constants
- Read off the resulting energy-momentum tensor $T_{\mu\nu}$
- $T_{\mu\nu}$ is expressed in terms u^μ and T and their derivatives

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4 (\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3 \sigma^{\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2\, T^{\mu\nu}_{2b} + (2 - \log 2)\left(\frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

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The most interesting (and most difficult) open problems are beyond the reach of hydrodynamics.

Key questions:

- why is thermalization/isotropisation so fast?
- can we understand the thermalization time?
- how does thermalization occur?

Modeling a heavy-ion collision

Find a model for the projectile

- Assume no dependence on transverse coordinates
- The configuration should depend only on *one* light cone coordinate
- Tracelessness and conservation of energy-momentum tensor leads to

$$T_{--} = f(x^-) \qquad \text{all other components vanish}$$

- The dual geometry can be found *exactly* [RJ,Peschanski]

$$ds^2 = \frac{-2dx^+dx^- + z^4 f(x^-) dx^{-2} + dx_\perp^2 + dz^2}{z^2}$$

- When $f(x^-) = \mu\delta(x^-)$ we are dealing with a shock-wave

Consider the collision of two such shockwaves

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A $1+1$ dimensional toy model

- [Kajantie,Louko,Tahkokallio] considered a collision in a 1+1 dimensional CFT dual to 2+1 dimensional gravity
- Advantage (but also a disadvantage...) is that 2+1 gravity is often easy to solve exactly
- Consider two shockwaves (or more general projectiles) coming on two light-cone directions:

$$(*) \qquad T_{--} = f(x^-) \qquad T_{++} = g(x^+)$$

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Problem: The projectiles pass through each other and after the collision are unaffected by the presence of each other

Reason: The distribution $(*)$ is the most general one possible in a 1+1D CFT. No place for nontrivial dynamics of thermalization etc.

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The 3+1 dimensional problem

- In order to study more realistic physics we have to tackle the full 3+1 dimensional setup
- No longer solvable
- The existence of exact plane-wave collision metrics in 4D general relativity does not help – one more coordinate!
- Pioneering work by Grumiller, Romatschke: small time expansion – but problems with energy positivity
- Different expansion: Albacete, Kovchegov, Taliotis
- Entropy production through a trapped surface formation [Gubser,Pufu,Yarom] (somewhat different setup)

Lots of open questions remain!

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Physics in the expanding plasma

It is interesting to ask questions about gauge theory physics influenced by the expanding and cooling plasma system.

- Use the 'moving black-hole' geometry instead of the usual AdS_5 or *AdS black hole*.
- New feature: explicit time-dependence
- Study the physics of flavours and mesons

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Fundamental flavours and mesons in AdS/CFT

- Consider $\mathcal{N} = 4$ SYM + one additional flavour ($\mathcal{N} = 2$)
- No chiral symmetry breaking!
- AdS/CFT description:
  Embed a $D7$ brane in the geometry
- Lightest mesons $\equiv$ fluctuations of the $D7$ brane embedding (or $D7$ gauge fields)

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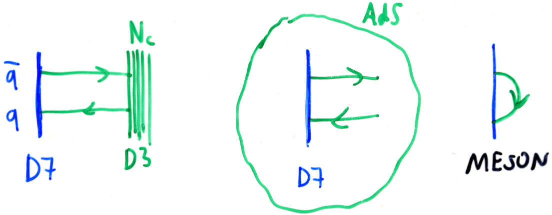

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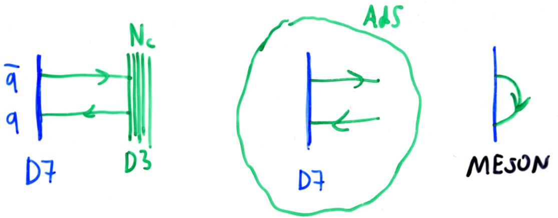

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Finite temperature case (static plasma)

- Embed a $D7$ brane in the black hole geometry
- Two types of embeddings:
  1. 'Minkowski embeddings' : the $D7$ brane does not reach the horizon – heavy quarks – stable mesons
  2. 'Black hole embeddings' : the $D7$ brane touches the horizon – light quarks – mesons dissociate
- Procedure:
  1. Fix the current quark mass m by boundary conditions for the embedding at the boundary
  2. Solve for the embedding from DBI EOM
  3. Read off $\langle\bar{\psi}\psi\rangle$ from subasymptotics of the embedding
  4. Study fluctuations of the embedding

$$\delta\phi(x^\mu,\rho) = e^{ik_\mu x^\mu} f_{k_\mu}(\rho)$$

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Finite temperature case (static plasma)

- Embed a $D7$ brane in the black hole geometry
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Fundamental flavours and mesons in an expanding plasma system

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- Use late time expansion ⟶ 'Minkowski embedding'
- Embed $D7$ brane into plasma geometry (including viscosity)

$$y_6(\rho, \tau) = m - \frac{f_1(\rho)}{\tau^{\frac{8}{3}}} + \frac{f_2(\rho)}{\tau^{\frac{10}{3}}} + \dots$$

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$$\langle \bar{\psi}\psi \rangle \propto \frac{\varepsilon^2(\tau)}{m^5}$$

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