Partons and jets at strong coupling (III)

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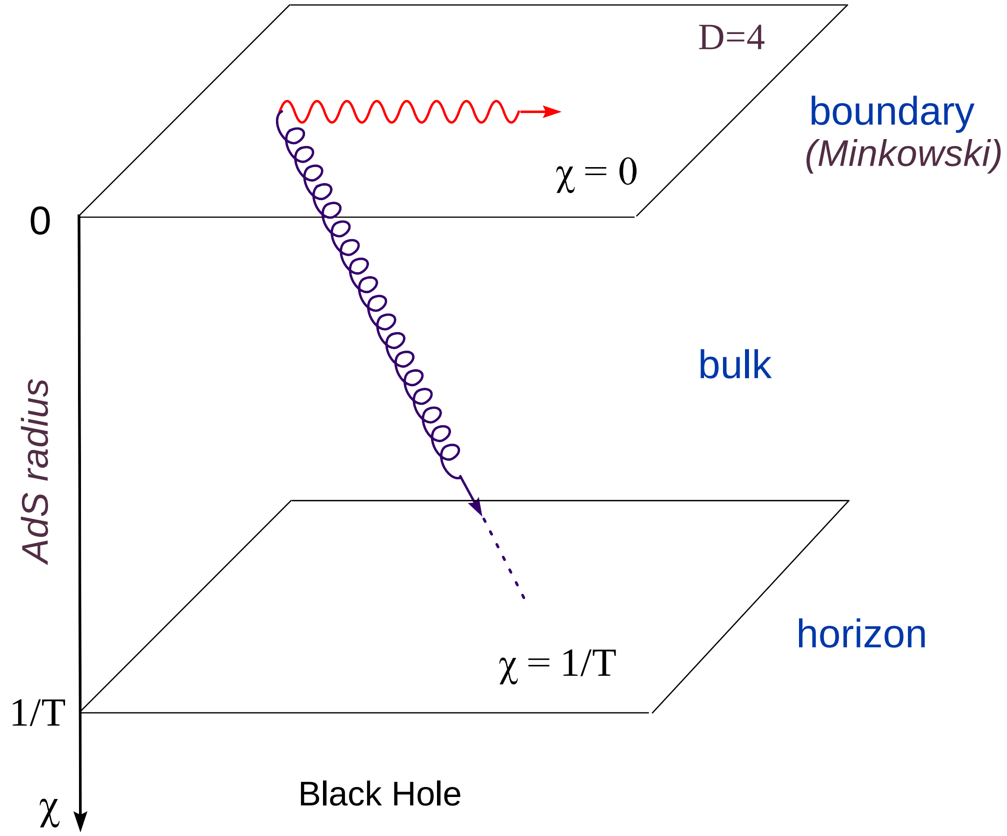
Outline

- Lecture I : Partons and jets in QCD at weak coupling
  - Introduction & Motivations
  - The situation at weak coupling (pQCD, phenomenology)
- Lecture II : A high–energy current in AdS/CFT
  - Invitation towards strong coupling
  - Methodology (black hole, wave equations)
  - The vacuum problem as a warm up
- Lecture III : $\mathcal{R}$–current in a strongly–coupled plasma
  - Results & Physical discussion
    - ▷ Medium induced branching
    - ▷ Parton saturation at strong coupling
    - ▷ Jet energy loss & momentum broadening
  - General consequences for high–energy scattering

Maxwell wave in a Black Hole

- $\mathcal{N}=4$ SYM at temperature T $\longleftrightarrow$ a Black Hole in the 'radial' dimension of AdS_5 : horizon at $\chi_0 = 1/\pi T$ or $\tilde{\chi}_0 = 2$
- Abelian current J_μ in 4D $\longleftrightarrow$ Maxwell wave falling into AdS_5



Electromagnetic current in a plasma

- Abelian $\mathcal{R}$–current : $J_\mu(x) \propto \mathrm{e}^{-i\omega t+ikz}$ with $q^2 = \omega^2 - k^2$
- Retarded polarization tensor: thermal expectation value

$$\Pi_{\mu\nu}(q) \equiv \int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\theta(x_0)\, \langle\, [J_\mu(x), J_\nu(0)]\, \rangle_T$$

- ‘Hard probe’ : $Q^2 \equiv |q^2| \gg T^2 \Longrightarrow$ short distances
- High energy : $\omega \simeq k \gg Q$ (most interesting)
- Im $\Pi_{\mu\nu}$: absorption of the current by the plasma
  - time–like current ($q^2 > 0$) : jets, meson
  - space–like current ($q^2 < 0$) : DIS & parton structure
- ‘The fall of the Maxwell wave in the Black Hole’
- The trajectory of this fall (+ the UV/IR correspondence)

$\Longrightarrow$ the physical mechanism responsible for absorption

Wave equation in the AdS BH

$$\psi'' + \frac{1}{4\chi^2}\psi + \frac{\omega^2 - k^2 f}{f^2}\psi + \frac{f'}{f}\left(\psi' - \frac{1}{\chi}\psi\right) = 0$$

$$f = 1 - \left(\frac{\tilde{\chi}}{\tilde{\chi}_0}\right)^4 \quad \text{where} \quad \tilde{\chi}_0 = 2 \quad \text{or} \quad \chi_0 = 1/\pi T$$

- The most interesting dynamics at high Q^2 takes place near the Minkowski boundary : $\chi \lesssim 1/Q \ll \chi_0$

  $\Longrightarrow$ one can replace $f \to 1$ everywhere except in the piece amplified by the energy :

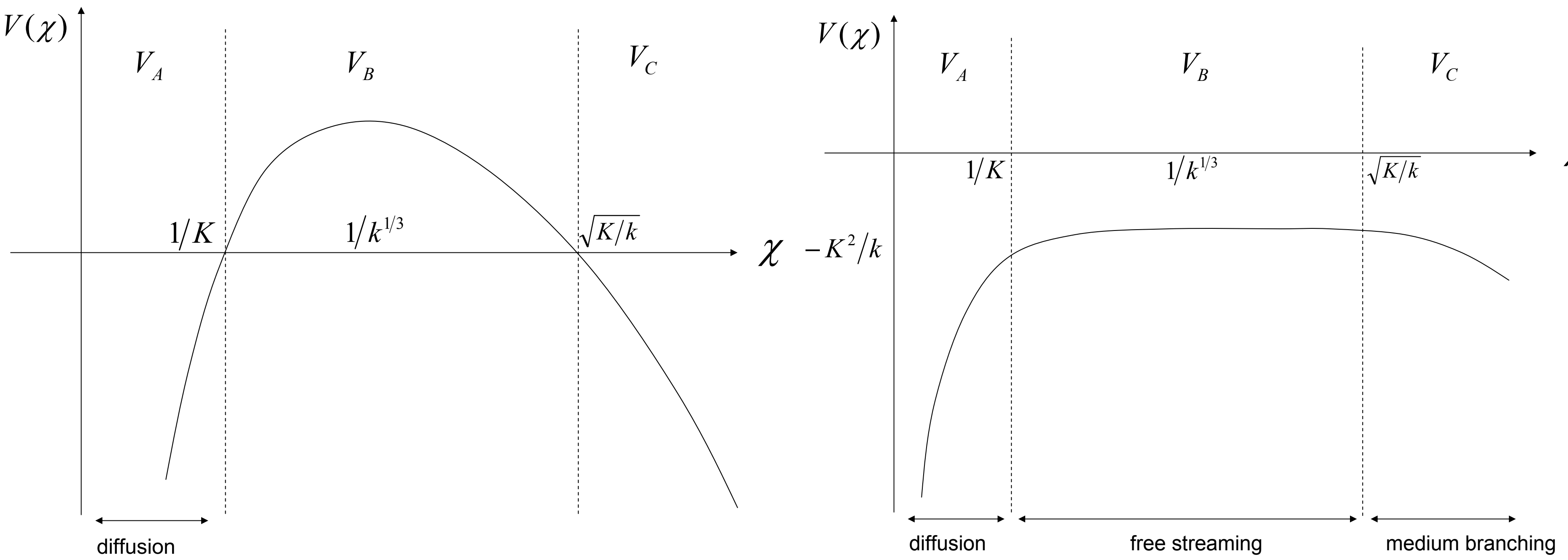
  $$\omega^2 - k^2 f = q^2 + k^2\left(\frac{\chi}{\chi_0}\right)^4 = \mp Q^2 + \frac{k^2\chi^4}{16}$$

  upper sign: space–like ($q^2 < 0$); lower sign: time–like ($q^2 > 0$)

- Competition between Q^2 (virtuality) and $k^2\chi^4$ (interactions in the plasma) $\Longrightarrow$ critical point $\chi_{\rm cr} \sim \sqrt{Q/k} \ll \chi_0$

Effective potential

- Effective “Schrödinger equation” : $-\psi'' + V_{\pm}(\chi)\psi \; = \; 0$



space–like

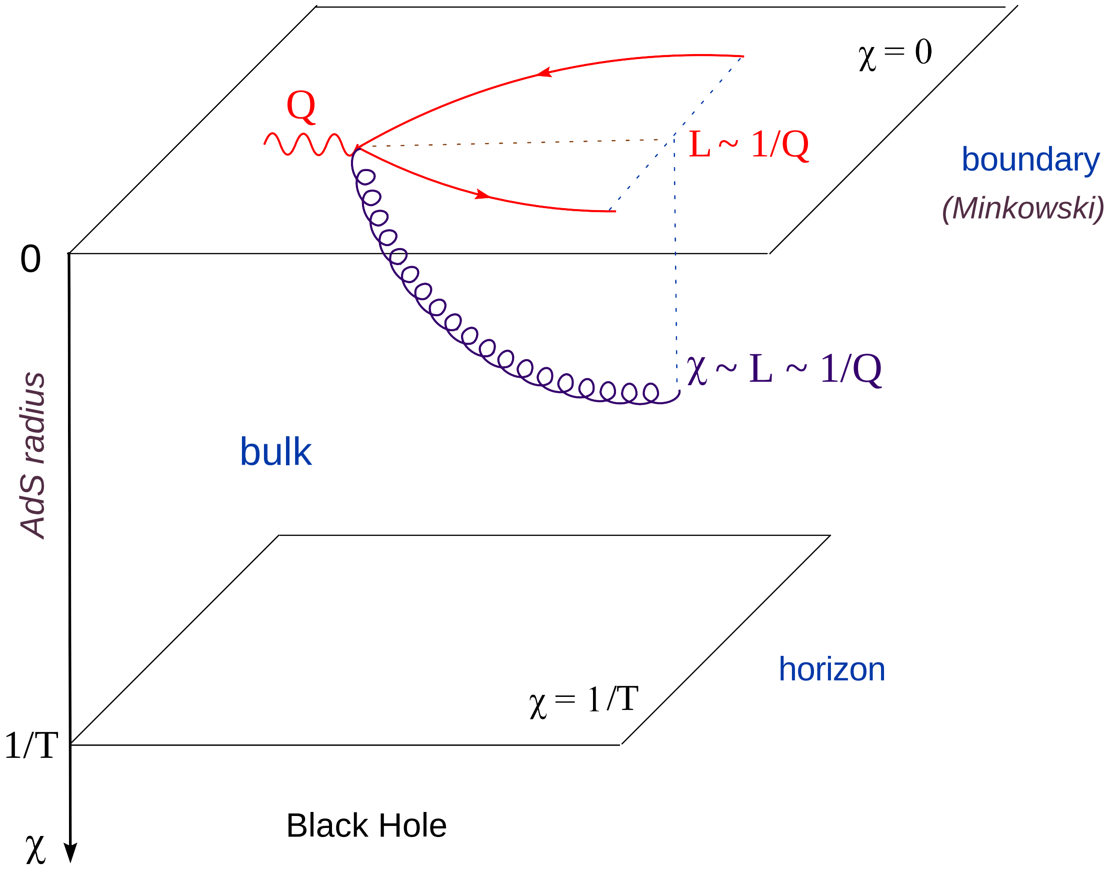
time–like

($K \;\equiv\; Q$ in all the figures)

- The potential becomes attractive at sufficiently large χ
  - small $\chi \ll \chi_{\rm cr}$: same dynamics as in the vacuum
  - larger χ : accelerated fall into the black hole

Space–like current

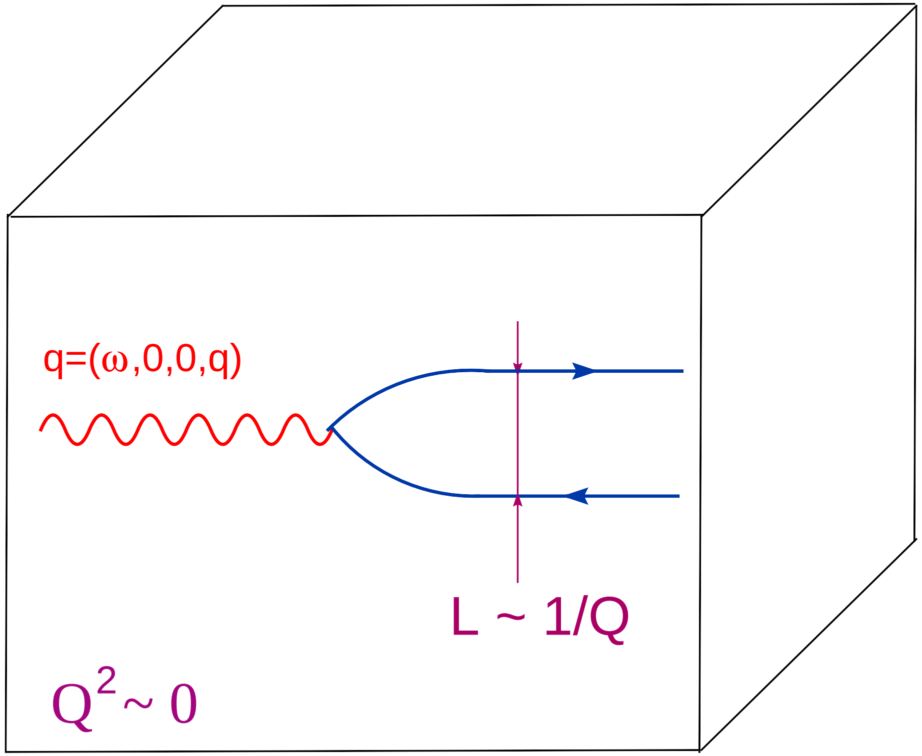
- After an early diffusion ($L \sim \sqrt{t}$) at times $t \lesssim \omega/Q^2$, the wave gets stuck near the boundary: $\chi \lesssim 1/Q \ll 1/T$



- No interaction with the BH ... except through tunneling

‘Perfect color transparency’

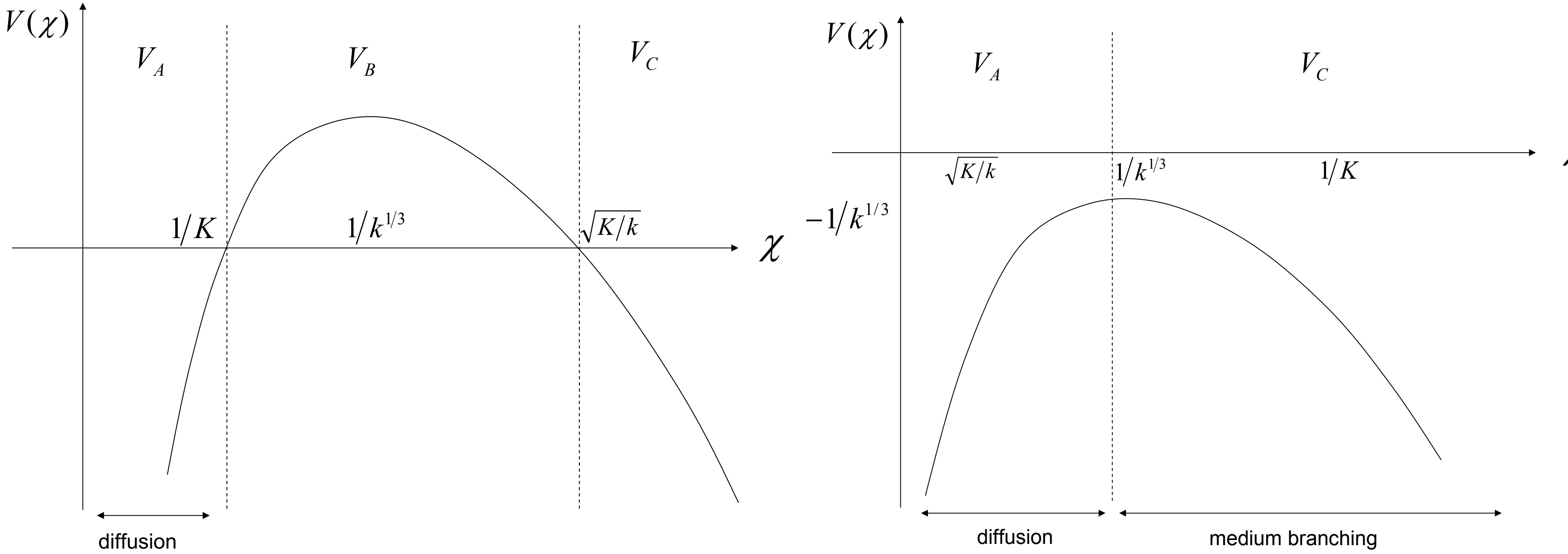
- A small color dipole (‘meson’) with size $L \sim 1/Q$ propagate through the strongly–coupled plasma without interactions !



- pQCD: the dipole cross–section vanishes too, but much more slowly : $\sigma_{\rm dipole} \sim 1/Q^2$ as $Q^2 \to \infty$ (twist–2 operator)

Increasing the energy

- By increasing the energy ω, the interactions in the plasma become stronger and stronger



moderate energy

high energy

($K \equiv Q$ in all the figures)

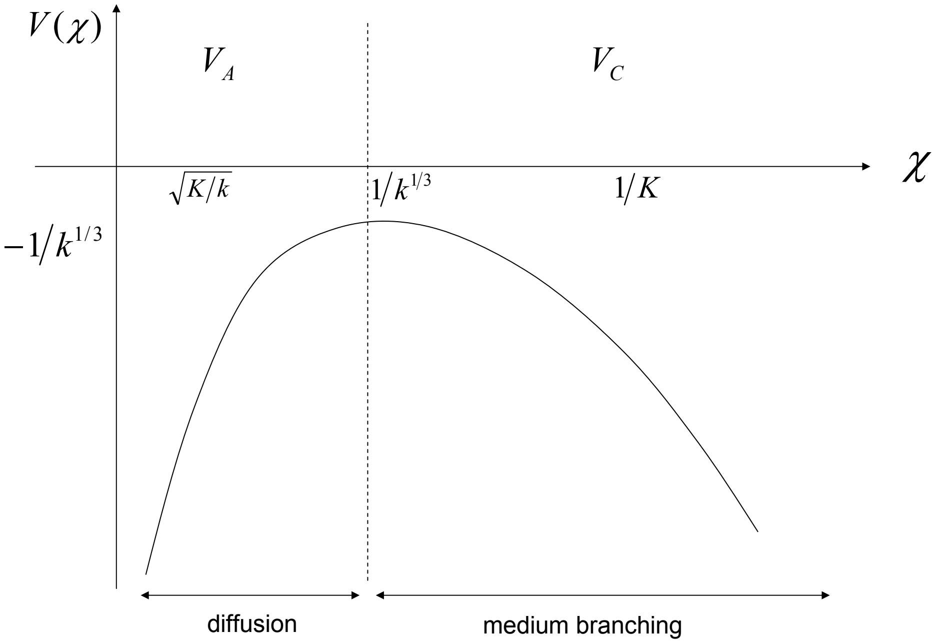
- The barrier disappears when the penetration length $\chi \sim 1/Q$ gets close to the 'point of no return' $\chi_{\rm cr} \sim \sqrt{Q/k}$

High energy

- This requires a minimal energy ω_s (for given Q^2 and T)

$$\frac{1}{Q} \sim \frac{1}{T}\sqrt{\frac{Q}{\omega}} \quad\Longrightarrow\quad \omega_s \sim \frac{Q^3}{T^2}$$

- For $\omega > \omega_s$, the wave is falling into the BH $\Longrightarrow$ energy loss



- The virtuality plays no role $\Longrightarrow$ the high–energy dynamics is the same for both space–like and time–like currents

Saturation momentum

$$\omega_s \sim \frac{Q^3}{T^2} \iff Q_s \simeq (\omega T^2)^{1/3} \simeq \frac{T}{x}$$

- Bjorken's x for a plasma (plasma rest frame) : $x = \frac{Q^2}{2\omega T}$
- More on the physical meaning of this condition
- On the supergravity side:

$$\underbrace{Q}_{\text{potential barrier}} \lesssim \underbrace{\frac{\omega T^2}{Q^2}}_{\text{gravitational potential}}$$

Saturation momentum

Outline

Current in the plasma
- Black Hole
- Current in a plasma
- EOM
- Potential
- Space–like
- Small meson
- High energy
- Saturation momentum
- Large x
- Trailing string
- Partons
- Saturation
- Time–like
- Early times
- Jets
- The fall
- Branching
- Energy loss
- Heavy Quark
- Momentum broadening

Conclusions

$$\omega_s \sim \frac{Q^3}{T^2} \iff Q_s \simeq (\omega T^2)^{1/3} \simeq \frac{T}{x}$$

- On the gauge theory side

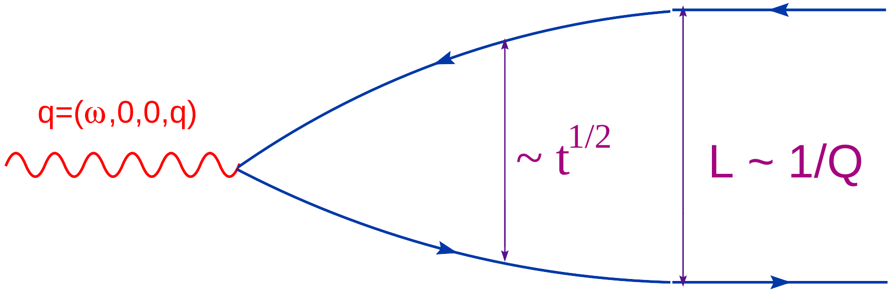
$$\underbrace{Q}_{\text{virtuality barrier}} \lesssim \underbrace{\frac{\omega}{Q^2}}_{\text{lifetime}} \times \underbrace{T^2}_{\text{plasma force}}$$

The partonic fluctuation must live long enough to feel the effects of the plasma

Saturation momentum

$$\omega_s \sim \frac{Q^3}{T^2} \iff Q_s \simeq (\omega T^2)^{1/3} \simeq \frac{T}{x}$$

- $L_s \sim 1/Q_s$: the minimal size for the dipole to feel the plasma



- Dipoles with size $L \gtrsim L_s$ cannot survive in the plasma.

$$L_s \sim \frac{1}{Q_s} \quad \& \quad \gamma \sim \frac{\omega}{Q} \implies L_s \sim \frac{1}{\sqrt{\gamma}T} = \frac{(1-v_z^2)^{1/4}}{T}$$

“meson screening length”, “limiting velocity”

[cf. Liu, Rajagopal, Wiedemann; Chernicoff et al; Caceres et al (2006)]

Large–x: No partons, no jets !

$$Q_s(x) \simeq \frac{T}{x} \quad \Longleftrightarrow \quad x_s(Q) \simeq \frac{T}{Q}$$

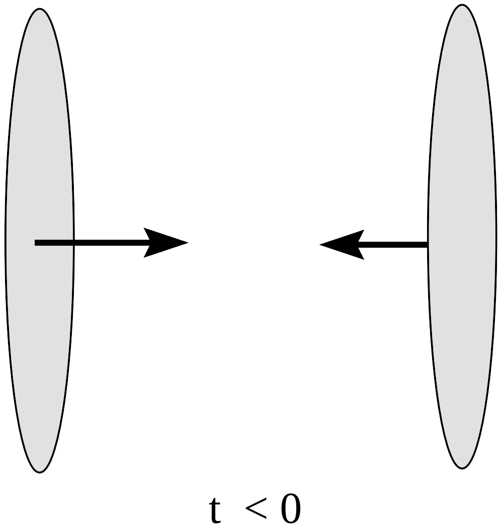
- For $Q > Q_s(x)$ or $x > x_s(Q)$: $F_2(x, Q^2) \approx 0$

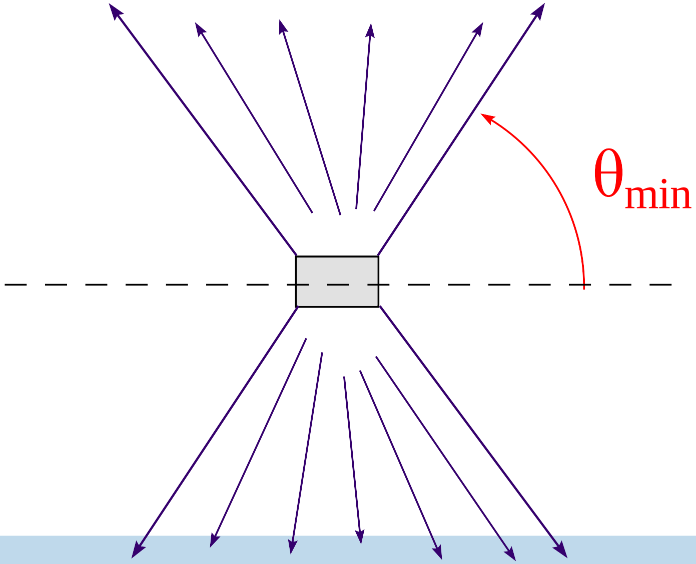
  $\Longrightarrow$ no partons with large momentum fractions $x > x_s$

- ‘All partons have branched down to small values of x’

  (Polchinki and Strassler, 02; Hatta, E.I., Mueller, 07; see below !)

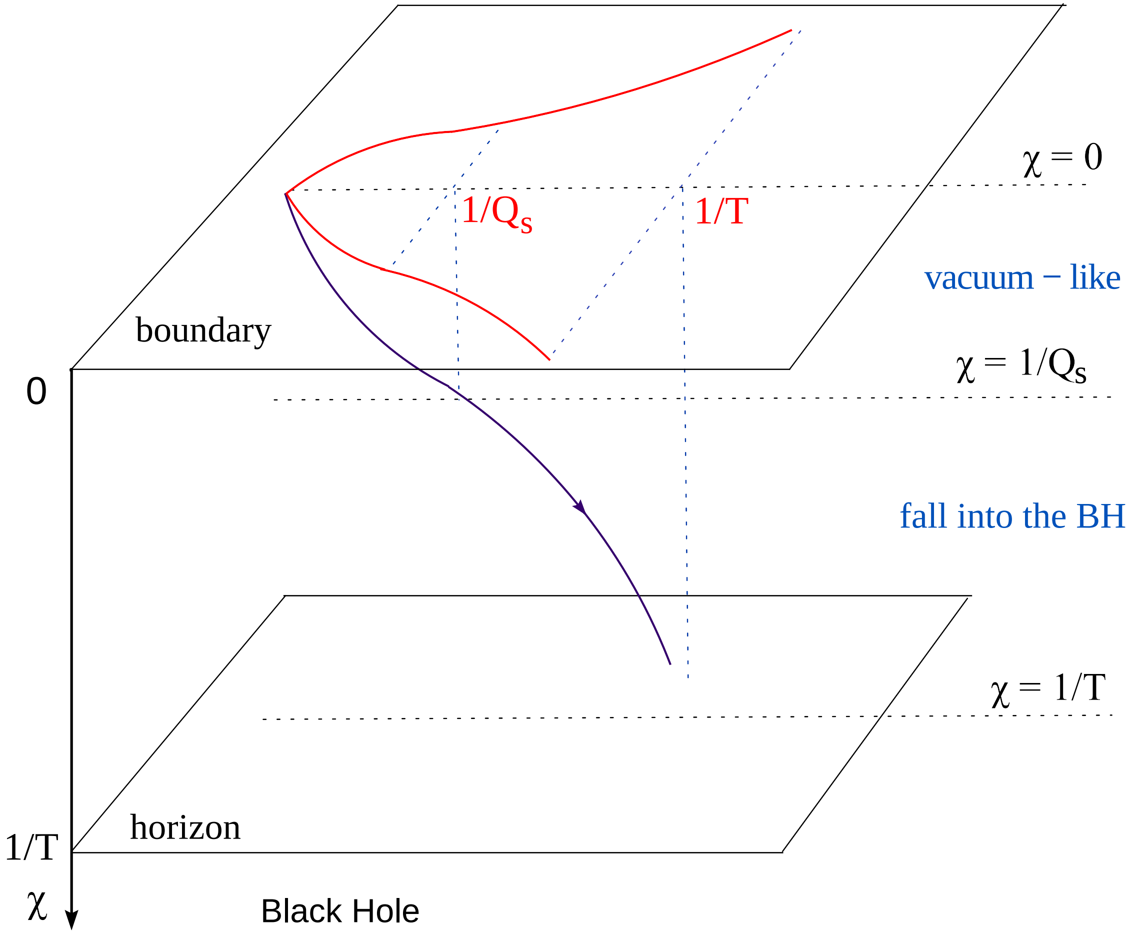
- No forward/backward jets in hadron–hadron collisions !





High energy: The fall

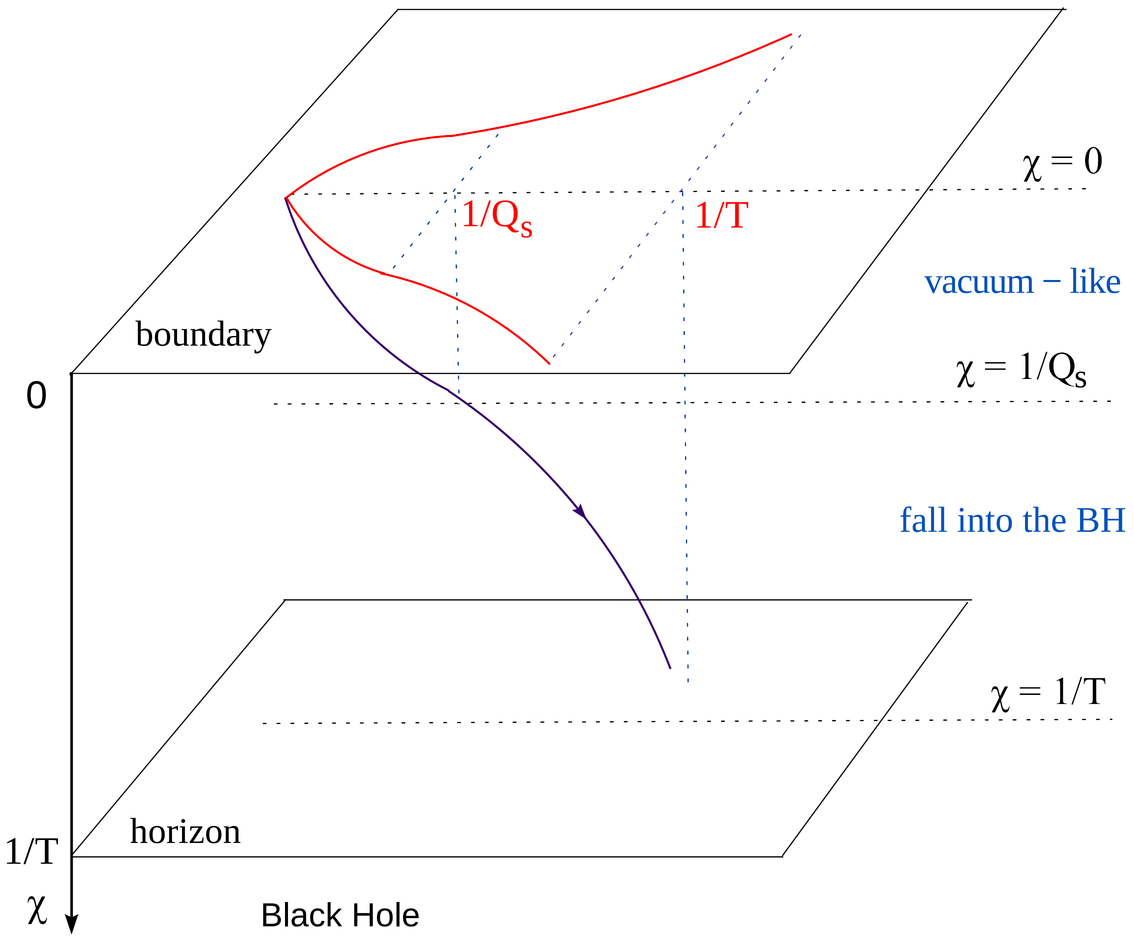
- For $Q > Q_s(x)$ or $x > x_s(Q)$: the wave diffuses up to $\chi \sim 1/Q_s$, then it falls along a massless geodesics



- The same as the 'trailing string' *(Herzog et al; Gubser, 06)*

The lifetime of a high–energy current

Lifetime of the current: $\Delta t \sim \dfrac{\omega}{Q_s^2} \sim \dfrac{1}{T}\left(\dfrac{\omega}{T}\right)^{1/3}$



- Same as the ‘gluon lifetime’ *(Gubser, Gulotta, Pufu, and Rocha,08)*

Finding the partons (plasma IMF)

- Total absorbtion of the current in the plasma:

  Unitarity limit (maximal possible scattering) for DIS

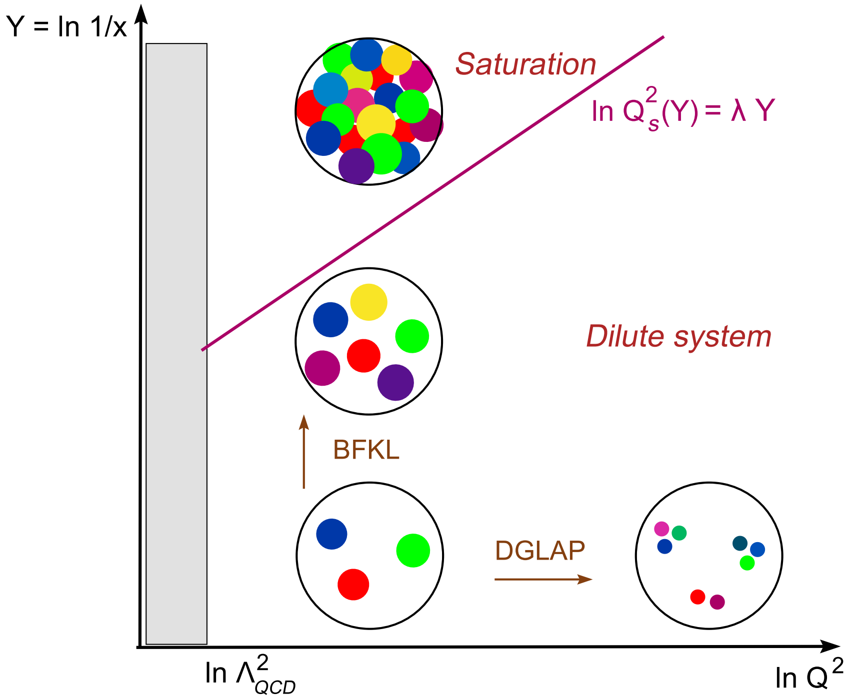
- Saturation line: $Q_s(x) \simeq T/x$ or $x_s(Q) \simeq TQ$

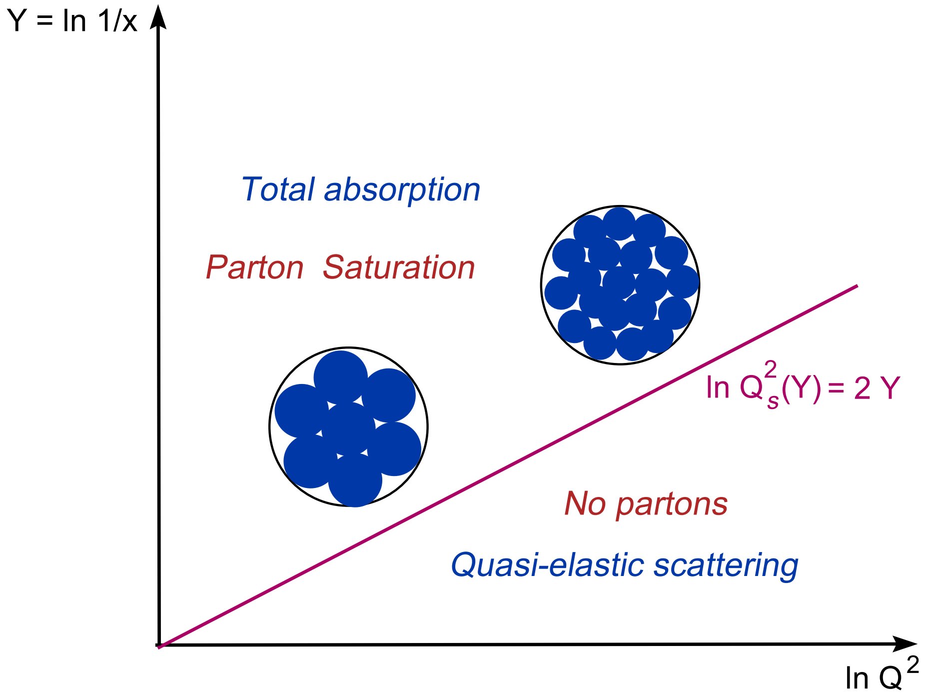
$$F_2(x, Q^2) \simeq x N_c^2 Q^2 \begin{cases} \exp\left\{-(x/x_s)^{1/2}\right\} & \text{for} \quad x \gg x_s \equiv T/Q \\ 1 & \text{for} \quad x \lesssim x_s \equiv T/Q, \end{cases}$$

- For given $Q \gg T$, all the partons have fallen at $x \lesssim x_s \ll 1$
- For $x \lesssim x_s$: occupation numbers of $\mathcal{O}(1) \Longrightarrow$ saturation
- Energy sum rule:

$$\mathcal{E} = T^2 \int_0^1 \mathrm{d}x\, F_2(x, Q^2) \sim T^2 \left[x F_2(x, Q^2) \right]_{x = x_s} \sim N_c^2 T^4 \ \checkmark$$

Saturation line: weak vs. strong coupling

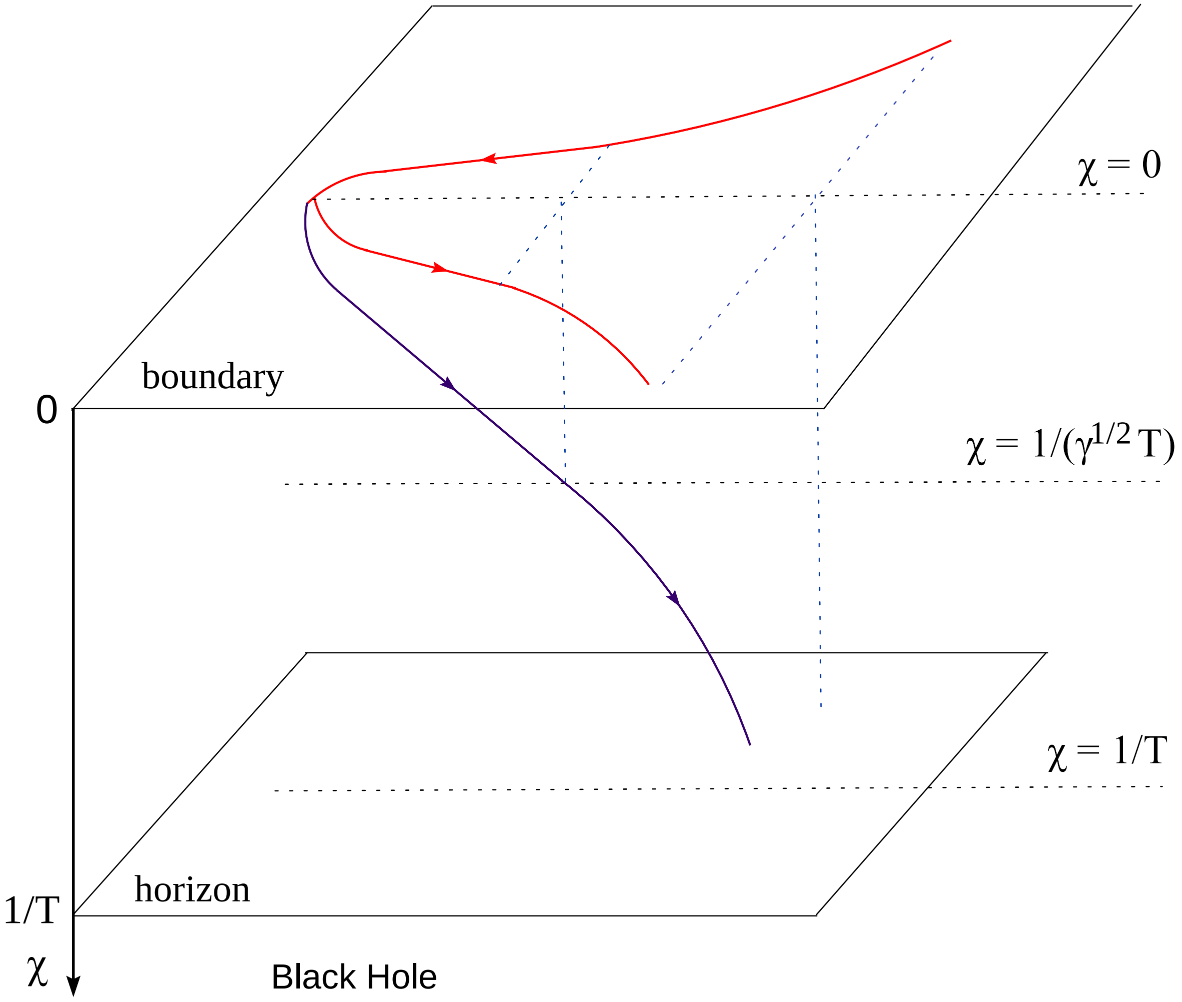




- Saturation exponent : $Q_s^2(x) \propto 1/x^{\lambda_s} \equiv \mathrm{e}^{\lambda_s Y}$
  - weak coupling (lowest order): $\lambda_s \approx 1.23\, g^2 N_c$
  - phenomenology & pQCD NLO: $\lambda_s \approx 0.3$
  - strong coupling (plasma): $\lambda_s = 2$ (graviton)

Time–like current at strong coupling

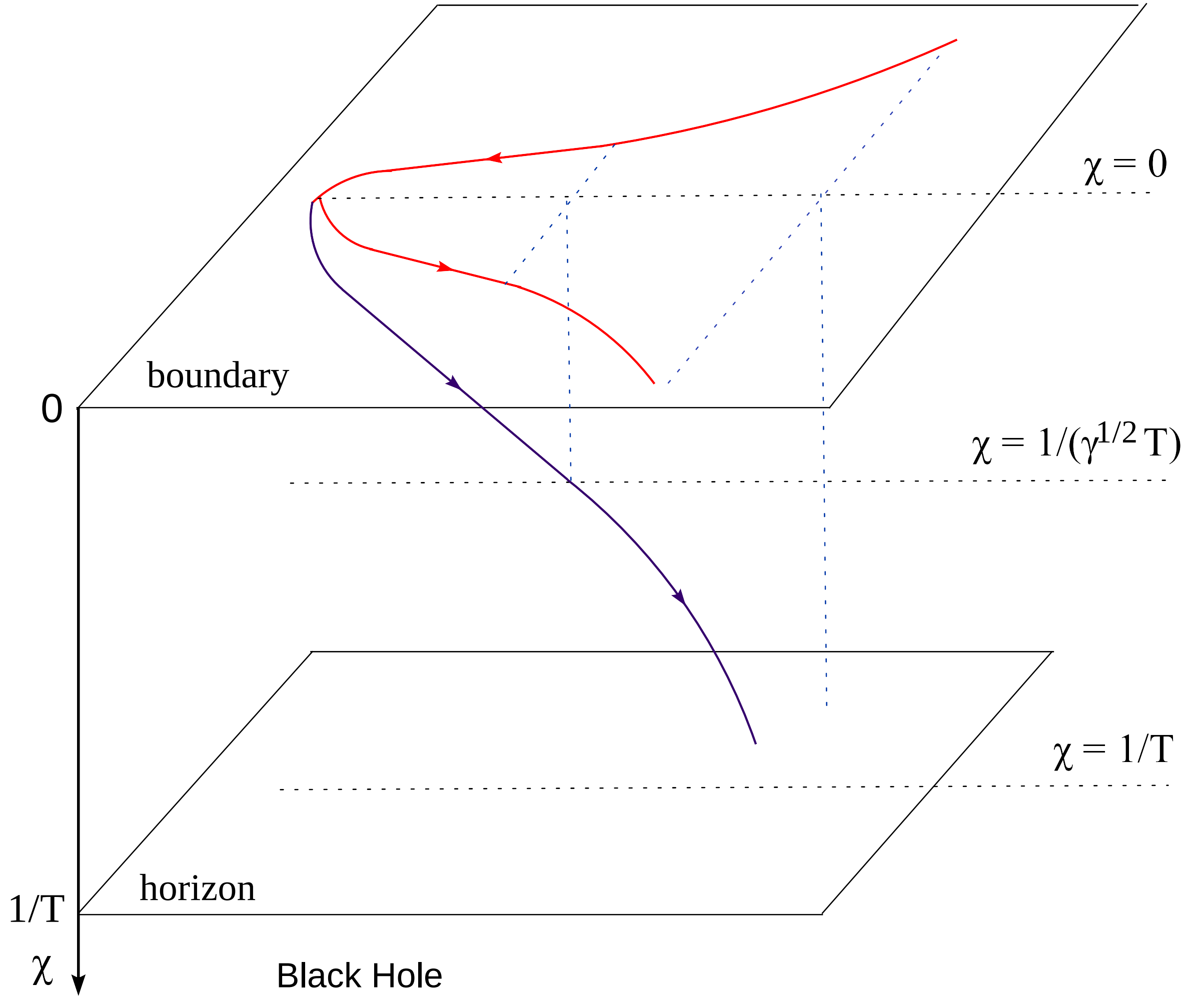
- A time–like current can decay (into partons of $\mathcal{N} = 4$ SYM) already in the vacuum



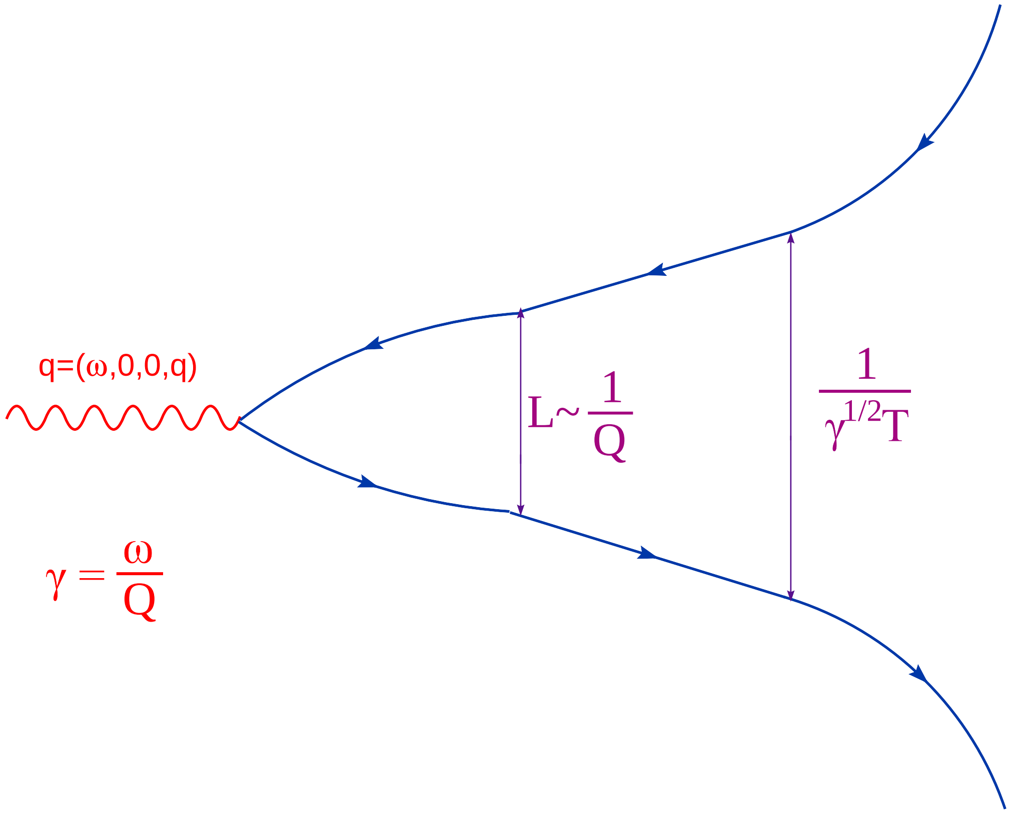
Early times: free streaming

- Early times/small size L :

  free expansion up to the critical size $L_{\rm cr} \sim 1/\sqrt{\gamma}\,T$



Jets in the plasma: **early times**

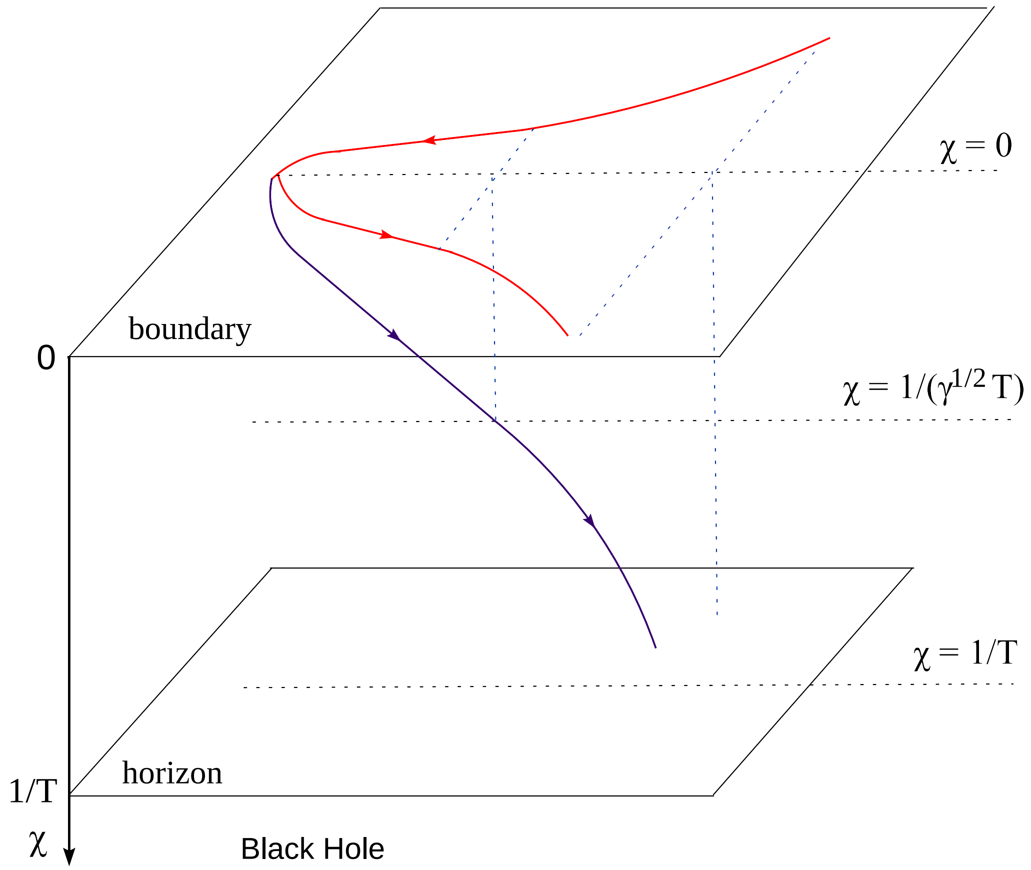


- Free streaming up to the point of no return:

$$\chi_{\rm cr} \sim \frac{1}{\sqrt{\gamma}} \quad \Longrightarrow \quad L_{\rm cr} \sim \frac{1}{\sqrt{\gamma}\,T} = \frac{(1-v_z^2)^{1/4}}{T}$$

Later times : falling into the black hole

- ‘trailing string’ *[Herzog, Karch, Kovtun, Kozcaz, Yaffe; Gubser, 2006]*

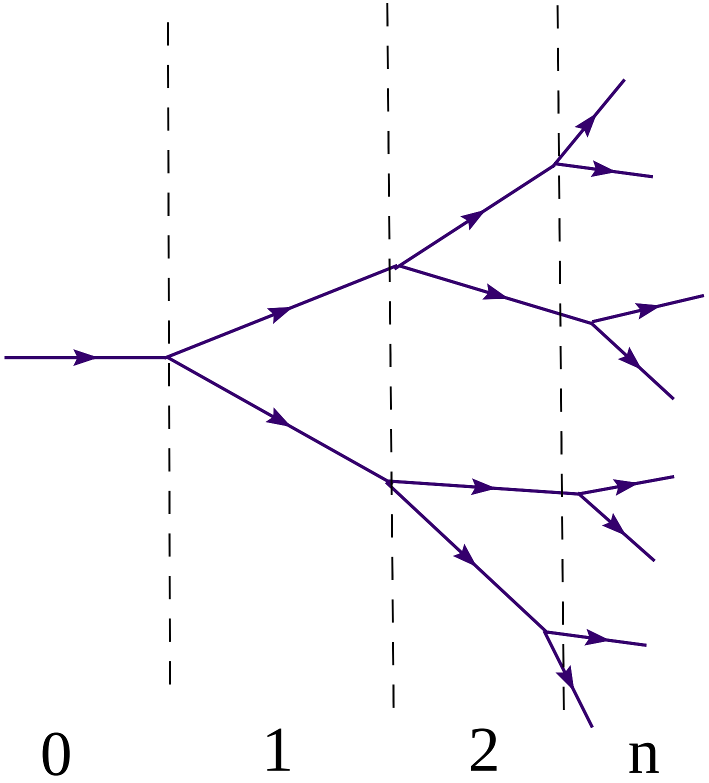


- Energy transfer from the partons to the plasma

Lifetime of the current/penetration length: $\Delta t \sim \Delta z \sim \dfrac{\sqrt{\gamma}}{T}$

Medium induced branching

- Universal energy loss mechanism, active at partonic level
  - no reason why branching should stop at 2 parton level
  - no reason to favour special corners of phase–space



$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

$$Q_n \sim \frac{Q_{n-1}}{2} \quad \text{(vacuum)}$$

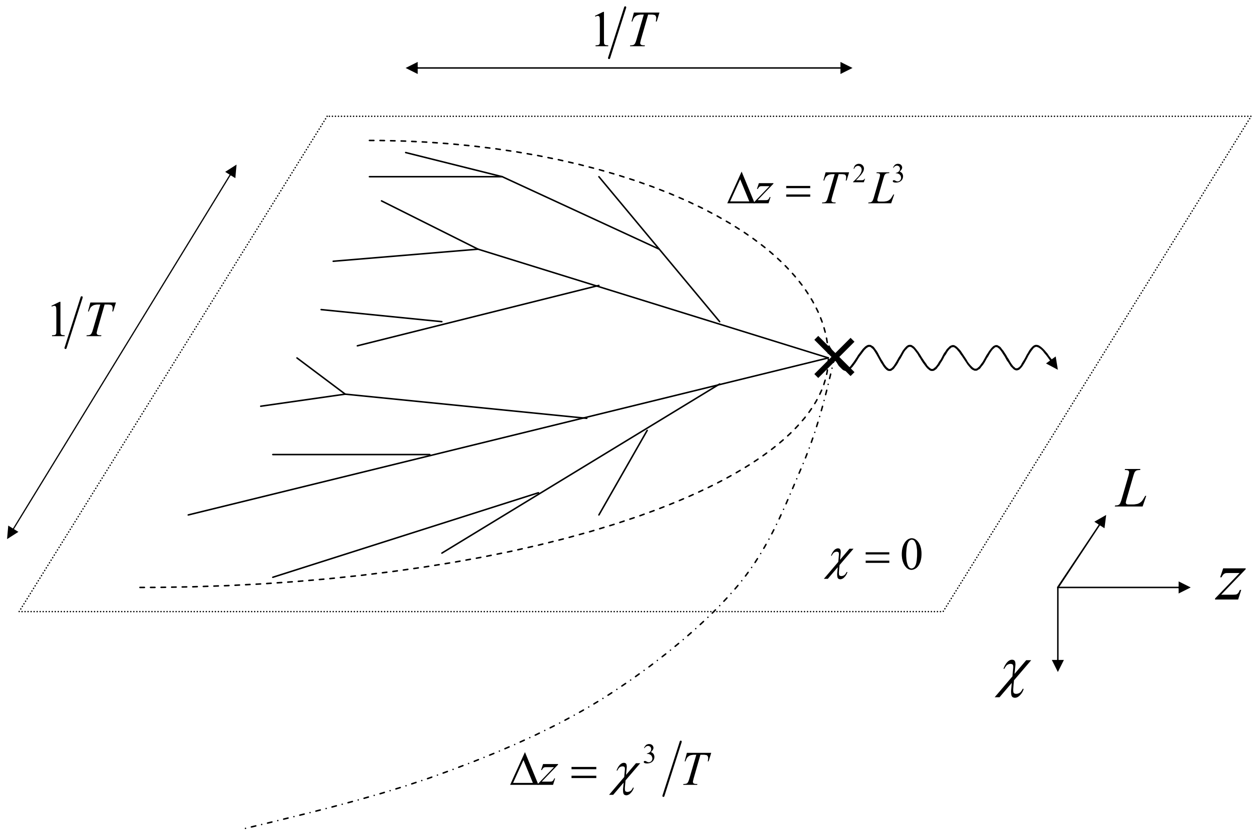
$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

$$\frac{\Delta Q_n}{\Delta t_n} \sim -T^2 \quad \text{(plasma)}$$

$\Longrightarrow \quad Q_n \sim (\omega_n T^2)^{1/3}$: saturation momentum at step n

Trailing string revisited

- The enveloping curve of the resulting partonic system coincides with the ‘trailing string’ *(Herzog et al; Gubser, 2006)*



- ... as it should by virtue of the UV/IR correspondence

Energy loss: massless parton

- Energy loss per branching:

$$\frac{\omega_n - \omega_{n-1}}{t_n - t_{n-1}} \sim -\frac{\omega_n}{\omega_n/Q_n^2} \sim -Q_n^2$$

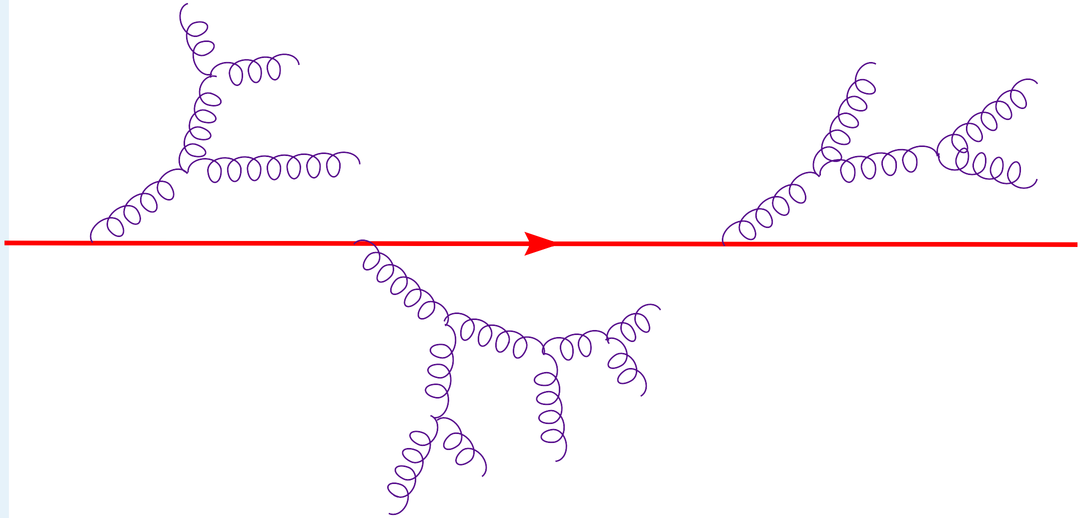
$$\Longrightarrow \quad -\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} \simeq Q_s^2(t) = (\omega T^2)^{2/3}$$

- Parton lifetime (the time to loose all its energy) :

$$\Delta t \sim \frac{1}{T}\left(\frac{\omega_0}{T}\right)^{1/3}$$

- Same as the 'gluon lifetime' *(Gubser, Gulotta, Pufu, and Rocha,08)*

Heavy Quark: Energy loss

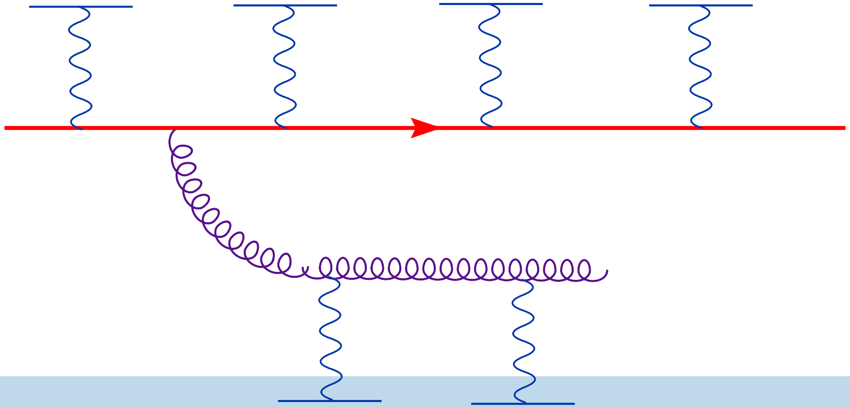


$$Q_s^2 \sim \left(t_{\rm br} T^2\right)^2 \sim \gamma\, T^2$$

$$-\frac{\mathrm{d}E}{\mathrm{d}t} \simeq \sqrt{\lambda}\,\frac{\omega}{(\omega/Q_s^2)} \simeq \sqrt{\lambda}\, Q_s^2$$

(Herzog et al; Gubser, 2006)

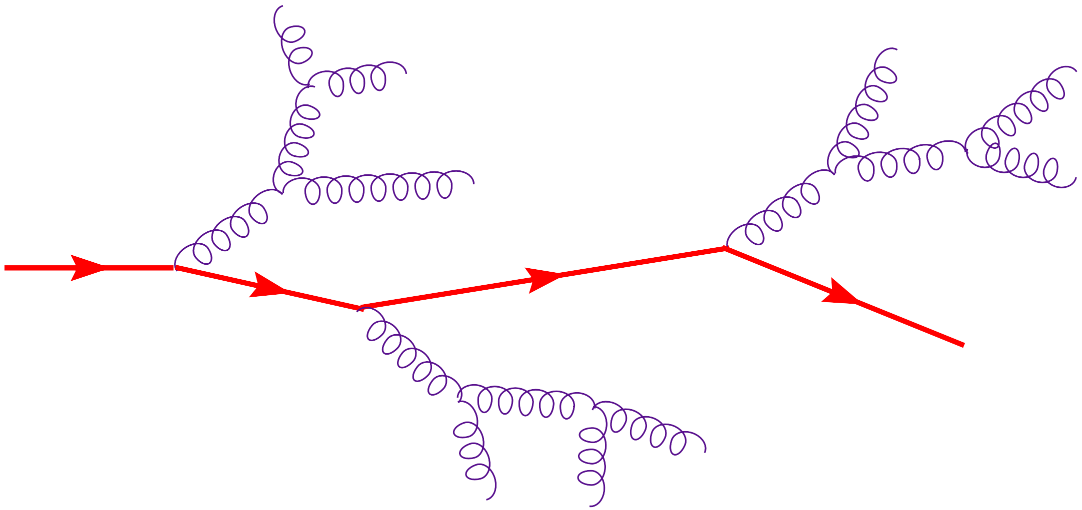
- pQCD : same formula, but with $\sqrt{\lambda} \to g^2 N_c$ and different Q_s



$$Q_s^2 \sim t_{\rm br}\, \hat{q}$$

(quenching param.)

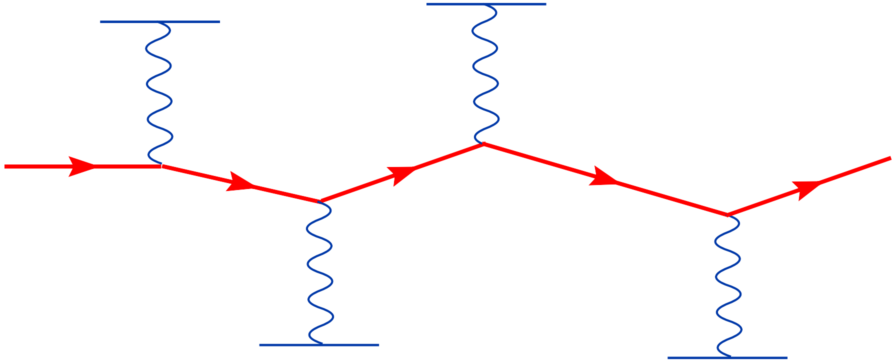
Transverse momentum broadening



$$\frac{\mathrm{d}p_\perp^2}{\mathrm{d}t} \sim \sqrt{\lambda}\, T^2 Q_s$$

Casalderrey-Solana, Teaney, 2006; Gubser, 2006; Dominguez et al, 2008

- pQCD : different physics ! thermal rescattering



$$\frac{\mathrm{d}p_\perp^2}{\mathrm{d}t} \sim \hat{q}$$

Conclusions

- Hard probes & high-energy physics appears to be quite different at strong coupling as compared to QCD
  - no forward/backward particle production in HIC
  - no jets in e^+e^- annihilation
  - jet momentum broadening dominated by in–medium parton branching, as opposed to thermal rescattering
- Not necessarily surprising: by asymptotic freedom, hard & high–energy physics in QCD is weakly coupled
- Are AdS/CFT methods useless for HIC ? Not necessarily so !
  - Transition from partons to fluid *(Gubser et al, Chesler and Yaffe)*
  - Some observables receive contributions from several scales, from soft to hard: use AdS/CFT in the soft sector

    Jet quenching: Liu, Rajagopal, Wiedemann, 06; Mueller, 08
  - Separation scale $Q_0 = ??$ Perhaps $2\pi T_c \sim 1$ GeV
  - ... and long–range properties: hydro, thermalization etc