Partons and jets at strong coupling (II)

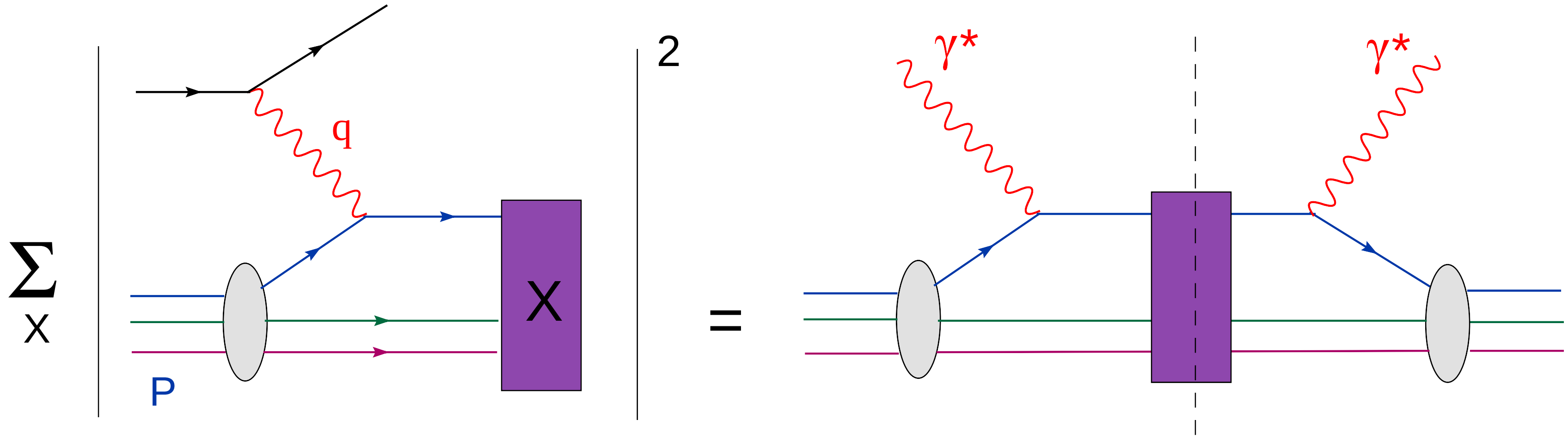
Edmond Iancu
IPhT Saclay & CNRS

Outline

- Lecture I : Partons and jets in QCD at weak coupling
  - Introduction & Motivations
  - The situation at weak coupling (pQCD, phenomenology)
- Lecture II : A high–energy current in AdS/CFT
  - Invitation towards strong coupling
    - ▷ A lesson from OPE
    - ▷ DIS off a plasma
  - Methodology (black hole, wave equations)
  - The vacuum problem as a warm up
    - ▷ The UV/IR correspondence
- Lecture III : $\mathcal{R}$–current in a strongly–coupled plasma
  - Results & Physical discussion
  - General consequences for high–energy scattering

DIS: Current–current correlator

$$\sigma_{\gamma^* p}(x, Q^2) = \frac{4\pi^2\alpha_{\mathrm{em}}}{Q^2} F_2(x, Q^2) \propto \mathrm{Im}\,\Pi_2(q)$$



$$\Pi_{\mu\nu}(q) = \int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\,\langle P\,|\mathrm{T}\,\{J_\mu(x)J_\nu(0)\}\,|\,P\rangle$$

- Hadronic polarization tensor for space–like momenta:
  2 scalar functions $\Pi_{1,2}(x, Q^2)$
- OPE : Parton picture is recovered in the large–Q^2 limit

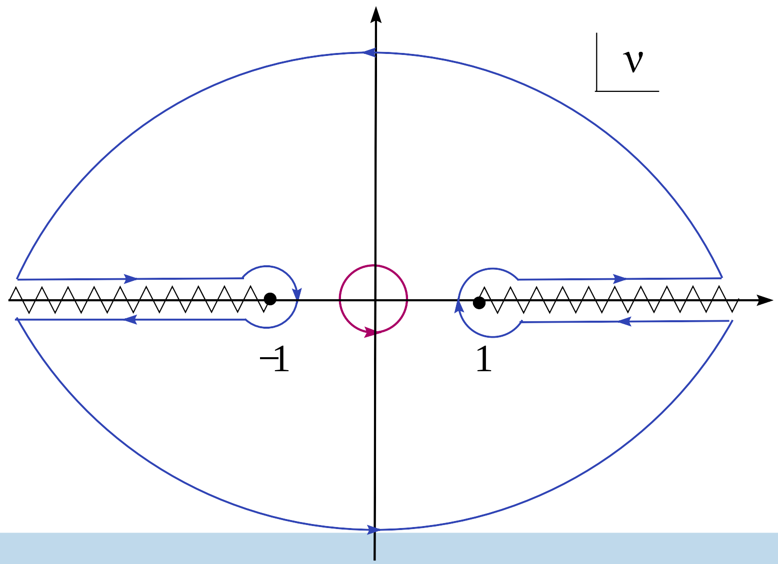
OPE for DIS ... in a nutshell

$$\int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, \mathrm{T}\left\{J_\mu(x)J_\nu(0)\right\} = \sum_{j,n} \tilde{C}_{j,n}(Q^2)\left[\mathcal{O}_{j,n}(0)\right]_{Q^2}$$

$j = 2, 4, 6, \ldots$: spin; n : operator index for a given spin

$$\Pi_2(x, Q^2) = i\sum_{j,n} A_{j,n}\, C_{j,n}(Q^2)\, \frac{1}{x^{j-1}}$$: valid for $x \equiv \dfrac{Q^2}{2P\cdot q} \gg 1$

- Contour integration in the complex $\nu \equiv 1/x$ plane



$$\int \frac{\mathrm{d}\nu}{2\pi i}\, \frac{1}{\nu^{j-2}}\, \Pi_2(\nu, Q^2)$$

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- Moments of the structure function F_2 :

$$M_j(Q^2) \equiv \int_0^1 \mathrm{d}x\, x^{j-2}\, F_2(x, Q^2) = \sum_{j,n} A_{j,n}\, C_{j,n}(\Lambda^2)\left(\frac{\Lambda^2}{Q^2}\right)^{(\tau_{j,n}-2)/2}$$

$\tau_{j,n} = \Delta_{j,n} - j$ = 'twist' ; $\Delta_{j,n} = d_{j,n} + \gamma_{j,n}$

$\gamma_{j,n}$ = anomalous dimension of the operator $\mathcal{O}_{j,n}$

Leading–twist approximation

$$M_j(Q^2) \equiv \int_0^1 \mathrm{d}x\, x^{j-2}\, F_2(x,Q^2) = \sum_{j,n} A_{j,n}\, C_{j,n}(\Lambda^2) \left(\frac{\Lambda^2}{Q^2}\right)^{(\tau_{j,n}-2)/2}$$

- Large $Q^2 \Longrightarrow$ leading–twist (or 'twist–two') operators

  classical twist $\equiv d_{j,n} - j = 2 \Longrightarrow \tau_{j,n} - 2 = \gamma_{j,n} \sim \mathcal{O}(\alpha_s N_c)$

$$\mathcal{O}_{j,f}^{\mu_1\mu_2\cdots\mu_j} = \bar{q}_f\, \gamma^{\mu_1}\,(iD^{\mu_2})\ldots(iD^{\mu_j})\, q_f \qquad \text{(symmetrized \& traceless)}$$

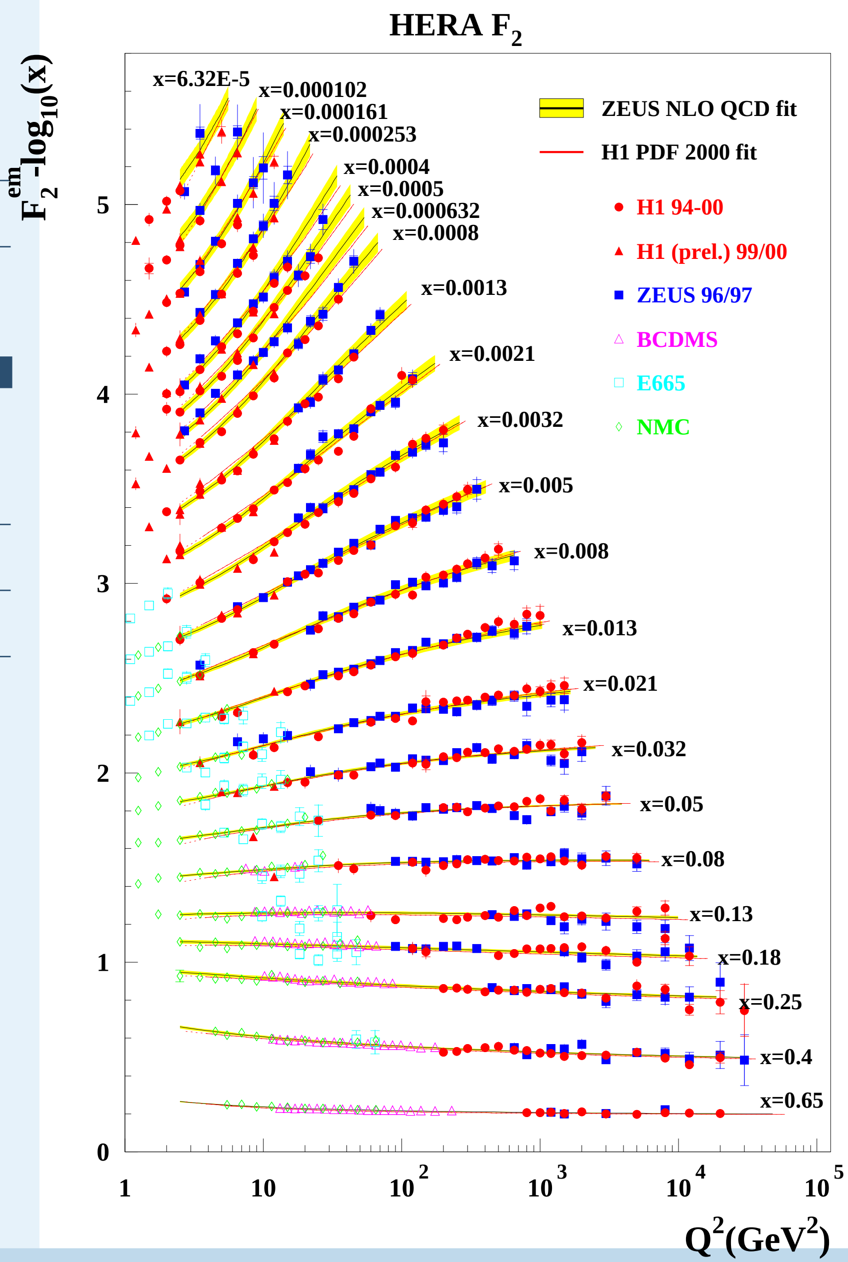
$$\mathcal{O}_{j,g}^{\mu_1\mu_2\cdots\mu_j} = F^{\mu_1\nu}\,(iD^{\mu_2})\ldots(iD^{\mu_{j-1}})\, F_\nu^{\mu_j}$$

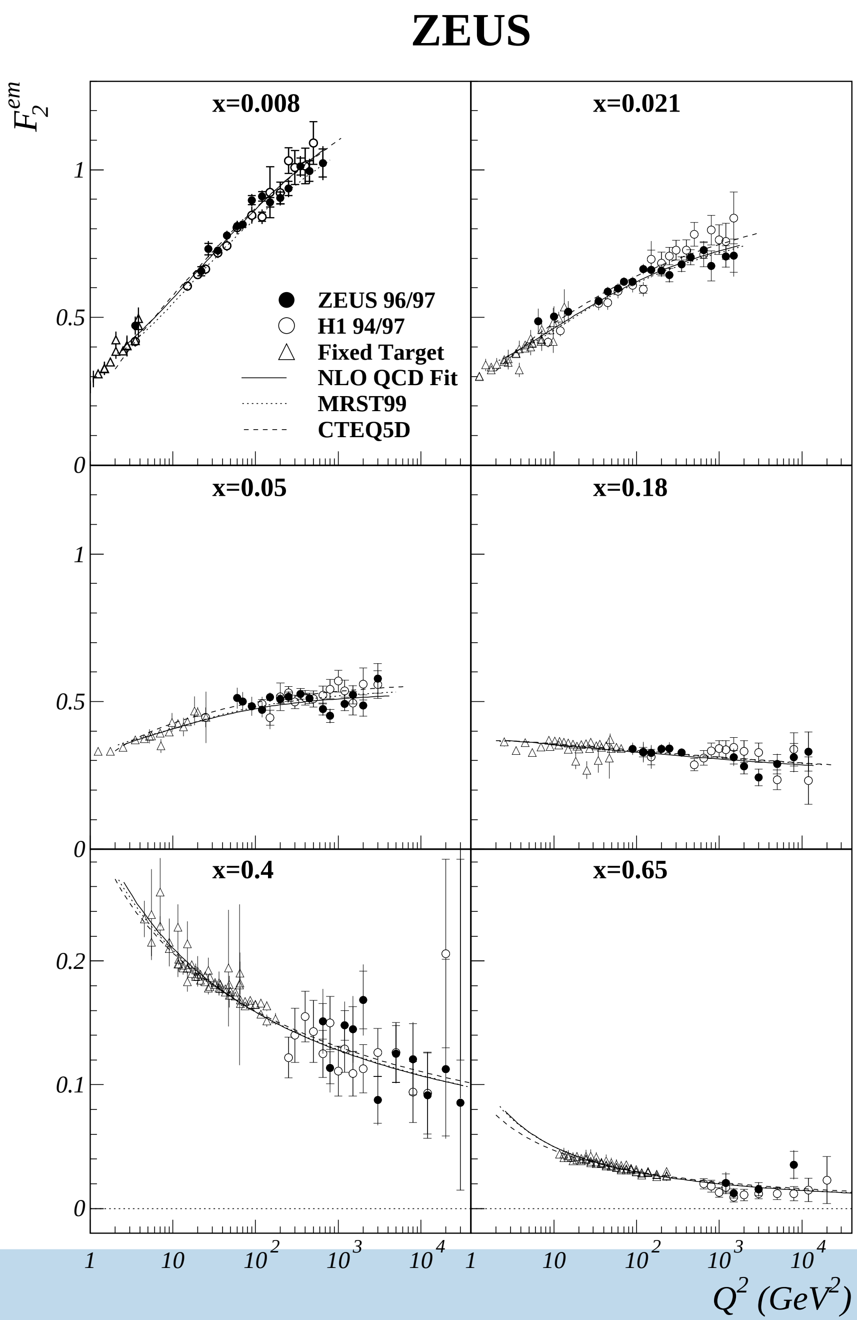
- All the $\gamma_{j,n}$ are positive $\Longrightarrow$ $M_j(Q^2) \to 0$ as $Q^2 \to \infty$
- ... except for 2 which vanish (conservation laws) :

$$J_f^\mu = \bar{q}_f\, \gamma^\mu\, q_f \Longrightarrow \int \mathrm{d}x \left[q_f(x,Q^2) - \bar{q}_f(x,Q^2)\right] = N_f$$

$$T^{\mu\nu} = \sum_f \mathcal{O}_{2,f}^{\mu\nu} + \mathcal{O}_{2,g}^{\mu\nu} \Longrightarrow \int \mathrm{d}x\, x\left[g(x,Q^2) + q(x,Q^2) + \bar{q}(x,Q^2)\right] = 1$$

F_2 increases at small x, but decreases at large x





Partons in perturbative QCD: A summary

- For a given, 'hard', resolution scale $Q^2 \gg \Lambda^2_{\rm QCD}$
  - parton picture makes sense (twist–2 operators)
  - most partons live at small $x \ll 1$
  - these small–x partons are predominantly gluons
- ... yet, the total energy (or longitudinal momentum) is carried by the few partons surviving at large $x \sim \mathcal{O}(1)$

$$\int_0^1 \mathrm{d}x\, F_2(x, Q^2) \,=\, const. \qquad \text{as} \quad Q^2 \to \infty$$

As $x \to 0$, F_2 rises 'only' like $F_2(x, Q^2) \sim x^{-\lambda}$ with $\lambda \sim 0.3$

- $Q^2 \lesssim Q_s^2(x) \sim 1/x^\lambda$: gluon saturation

$$n(x, Q^2) \sim 1/(g^2 N_c) = const. \Longrightarrow F_2(x, Q^2) \approx const.$$

Towards strong coupling

- OPE holds for generic values of $\lambda \equiv g^2 N_c$, and implies

$$M_j(Q^2) \equiv \int_0^1 \mathrm{d}x\, x^{j-2}\, F_2(x,Q^2) = \sum_{j,n} A_{j,n}\, C_{j,n}(\Lambda^2) \left(\frac{\Lambda^2}{Q^2}\right)^{(\tau_{j,n}-2)/2}$$

- Weak coupling : $\tau_{j,n} - 2 = \gamma_{j,n} \sim \lambda \ll 1$ for $\tau = 2$

  $\Longrightarrow$ twist $\tau = 2$ operators dominate at large Q^2

- Strong coupling : $\Delta \sim \tau \sim \gamma \sim \lambda^{1/4} \gg 1$ for $\tau = 2$

  (Gubser, Klebanov, Polyakov, 2002)

- Protected operators: τ_p is finite and $\sim \mathcal{O}(1)$ as $\lambda \to \infty$

- The protected operators dominate high–Q^2 DIS when $\lambda \gg 1$

  (Polchinski, Strassler, 2002)

- Generally: non–partonic operators (twist $\tau_p > 2$)

  $\Longrightarrow$ scattering off a 'hadron' as a whole

Towards strong coupling

- The structure functions are strongly suppressed at large Q^2:

$$M_j \sim (\Lambda^2/Q^2)^{\Delta-1} \quad \text{for} \quad j \geq 3$$

$\Delta \geq 4$: lowest–dimension operator which can create the hadron

- Probability for a hadronic fluctuation with size $1/Q \ll 1/\Lambda$
- Where have all the partons gone ?!
- Recall: $M_2 = \int \mathrm{d}x\, F_2(x, Q^2) = const.$ as $Q^2 \to \infty$
  - required by energy–momentum conservation
  - possible since $T^{\mu\nu}$ is a protected operator with twist $\tau_p = 2$

  $\Longrightarrow F_2(x, Q^2)$ has support only at very small x

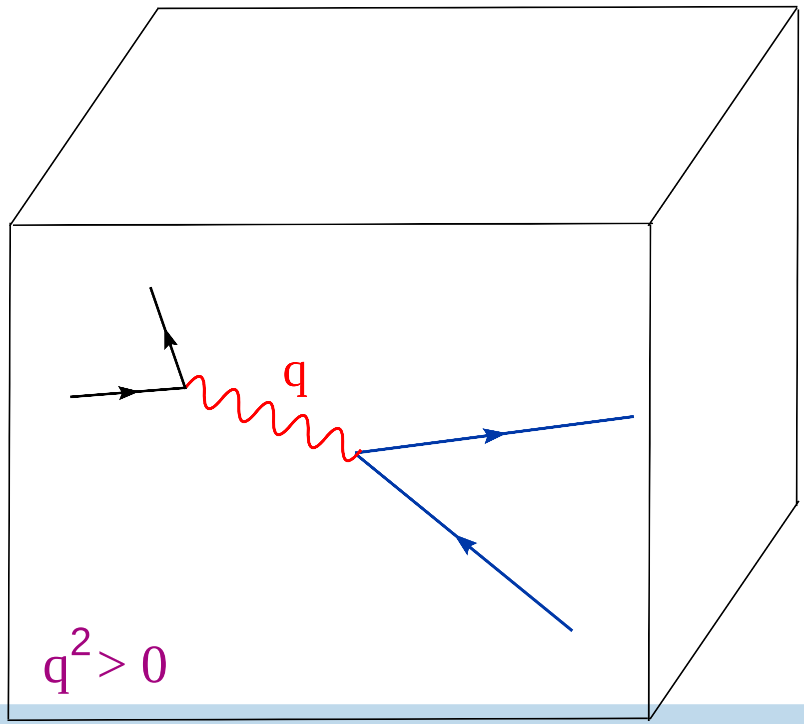
- ‘All partons have been branching down to very small x’
- $\gamma_{j,n} \sim \lambda^{1/4} \gg 1$: the moments of the splitting functions

Electromagnetic current in a plasma

- Retarded polarization tensor: thermal expectation value

$$\Pi_{\mu\nu}(q) \equiv \int \mathrm{d}^4x \, \mathrm{e}^{-iq\cdot x} \, i\theta(x_0) \, \langle \, [J_\mu(x), J_\nu(0)] \, \rangle_T$$

- 'Hard probe' : large virtuality $|q^2| \gg T^2$
- Time–like current ($q^2 > 0$) : jet production & their subsequent interactions in the plasma



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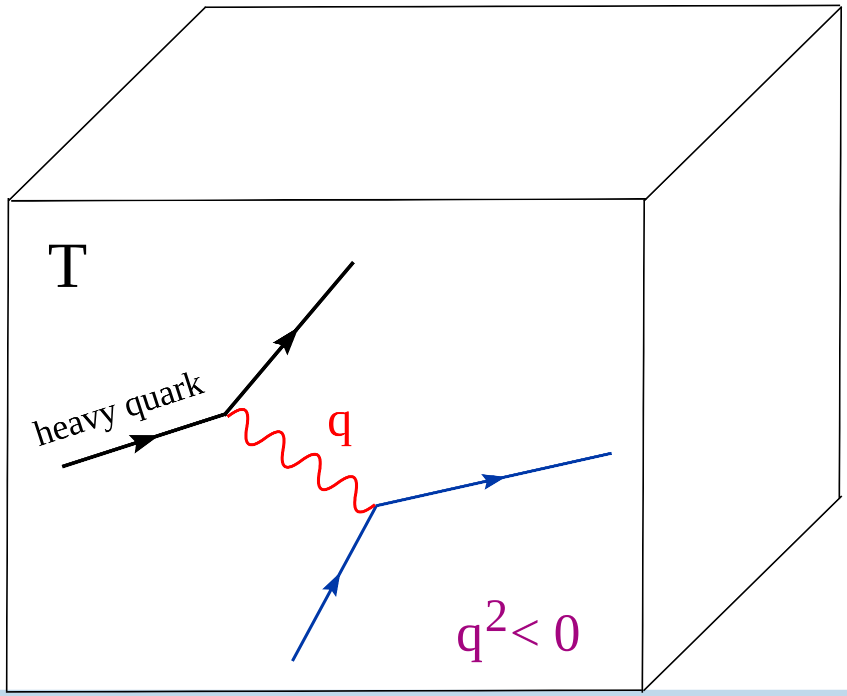
- 'Hard probe' : large virtuality $|q^2| \gg T^2$
- Space–like current ($q^2 < 0$) : DIS & parton structure



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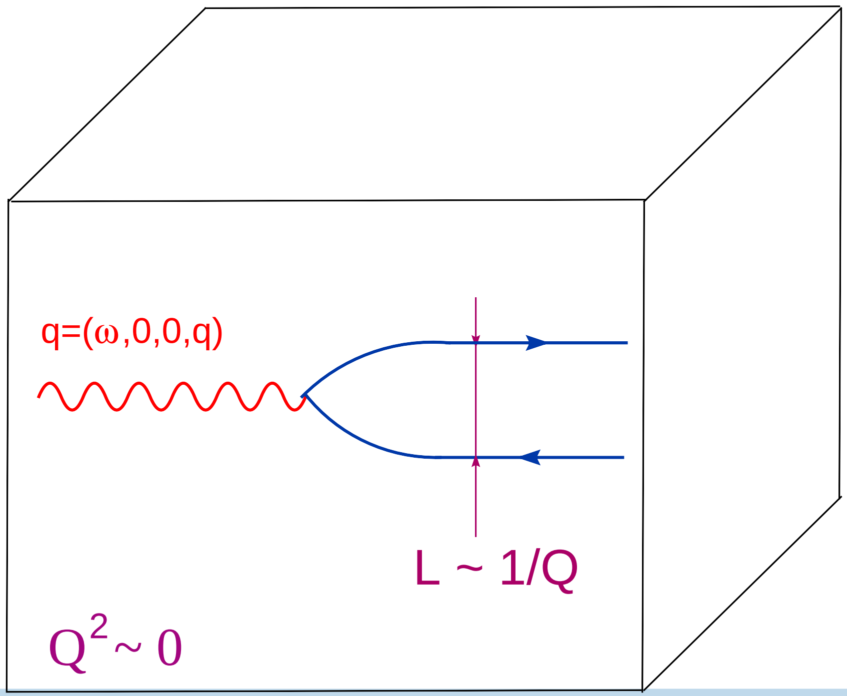
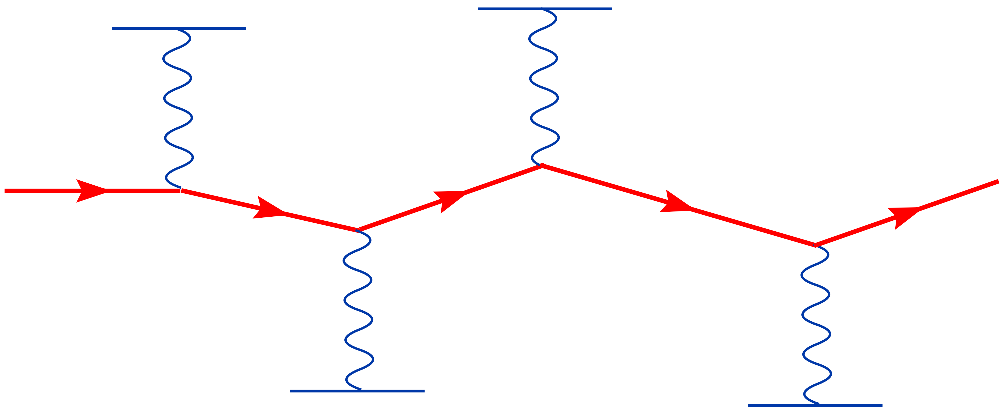
- 'Hard probe' : large virtuality $|q^2| \gg T^2$
- High–energy current ($Q^2 \approx 0$) : 'meson' in the plasma



Transverse momentum broadening

- A parton (say, heavy quark) undergoes multiple scattering (random kicks) off the plasma constituents



$$\frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}t} \equiv \hat{q} \simeq \alpha_s N_c \frac{xg(x,Q^2)}{N_c^2-1}$$

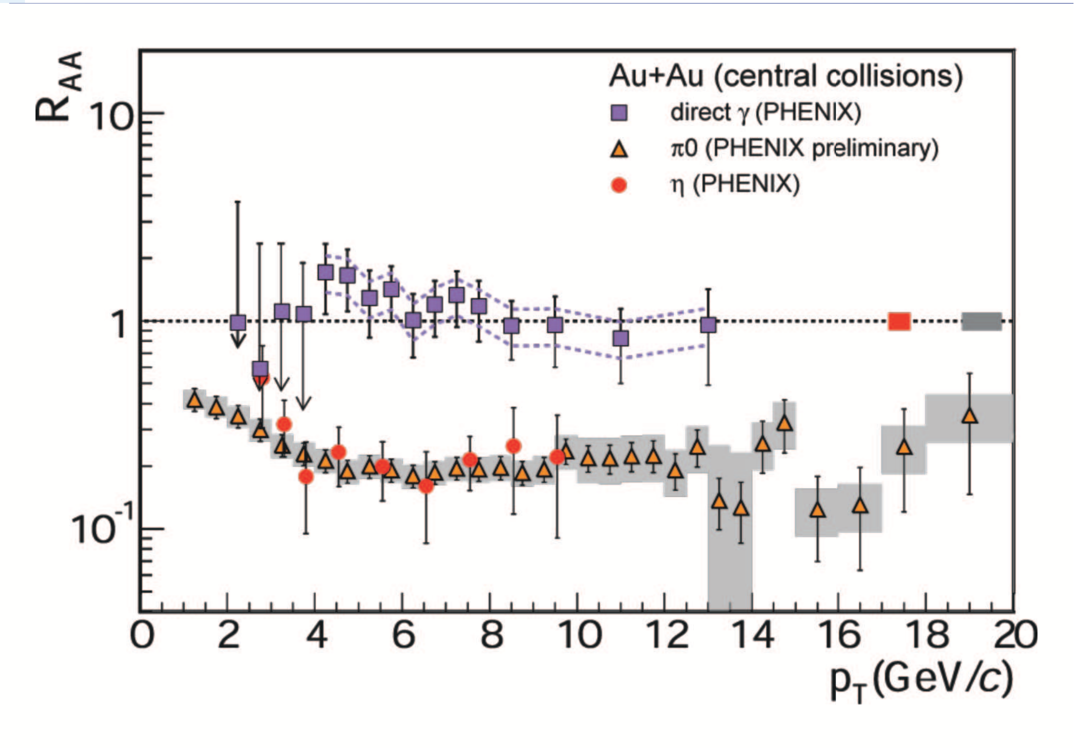
- $xg(x,Q^2)$: gluon distribution per unit volume in the medium
- Weakly–coupled QGP : incoherent sum of the gluon distributions produced by thermal quarks and gluons

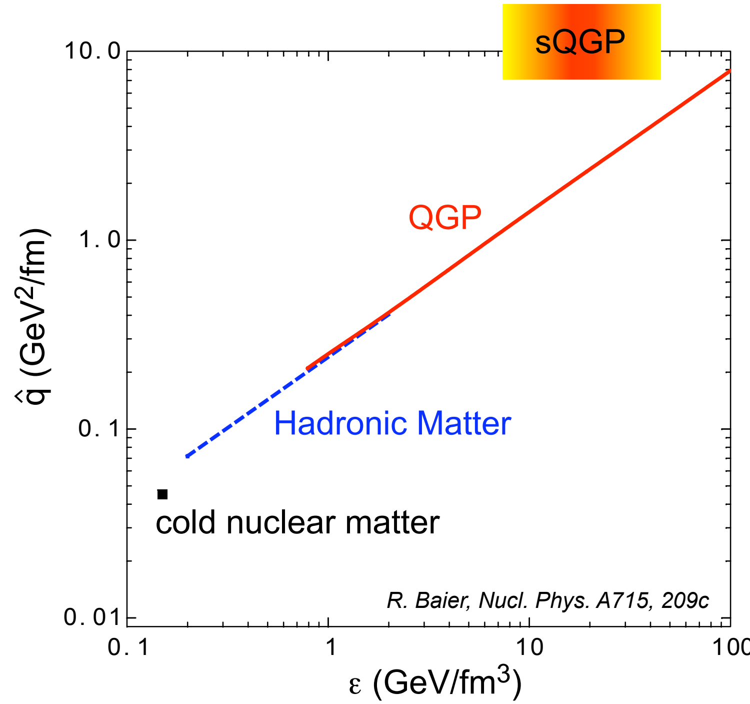
$$xg(x,Q^2) \simeq n_q(T)\,xG_q + n_g(T)\,xG_g\,, \text{ with } n_{q,g}(T) \propto T^3$$

Nuclear modification factor

- How to measure $\hat{q}$? Compare AA collisions at RHIC to pp

$$R_{AA}(p_\perp) \equiv \frac{Yield(A+A)}{Yield(p+p)\ \times\ A^2}$$





- RHIC data seem to prefer $\hat{q} \simeq 10$ GeV^2/fm, which is too large to be accounted for by weakly–coupled QGP (??)

DIS off a plasma: weak vs. strong coupling

- Moral: At weak coupling, there is no need to separately study DIS off a plasma

  Plasma structure functions = thermal superpositions of the structure functions separately produced by the 'quasiparticles' (weakly interacting quarks and gluons)

- At strong coupling, we do not know the structure of the plasma in terms of 'quasiparticles' (if any)

  ... but this is not crucial !

- AdS/CFT gives us a prescription for directly computing DIS off the strongly coupled $\mathcal{N} = 4$ SYM plasma

  A classical wave problem in the AdS_5 Black Hole geometry

AdS/CFT at finite temperature

- $\mathcal{N}=4$ SYM at finite temperature $\Longleftrightarrow$ type IIB string theory in $AdS_5 \times S^5$–Schwarzschild geometry

$$\mathrm{d}s^2 = \frac{r^2}{R^2}(-f(r)\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2) + \frac{R^2}{r^2 f(r)}\mathrm{d}r^2 + R^2\mathrm{d}\Omega_5^2$$

where $f(r) = 1 - (r_0^4/r^4)$ and r_0 = the BH horizon

- 'A Black Hole in the radial dimension of AdS_5'

(a Black D–Brane: homogeneous in the physical 4D)

- The correspondence (for parameters) :

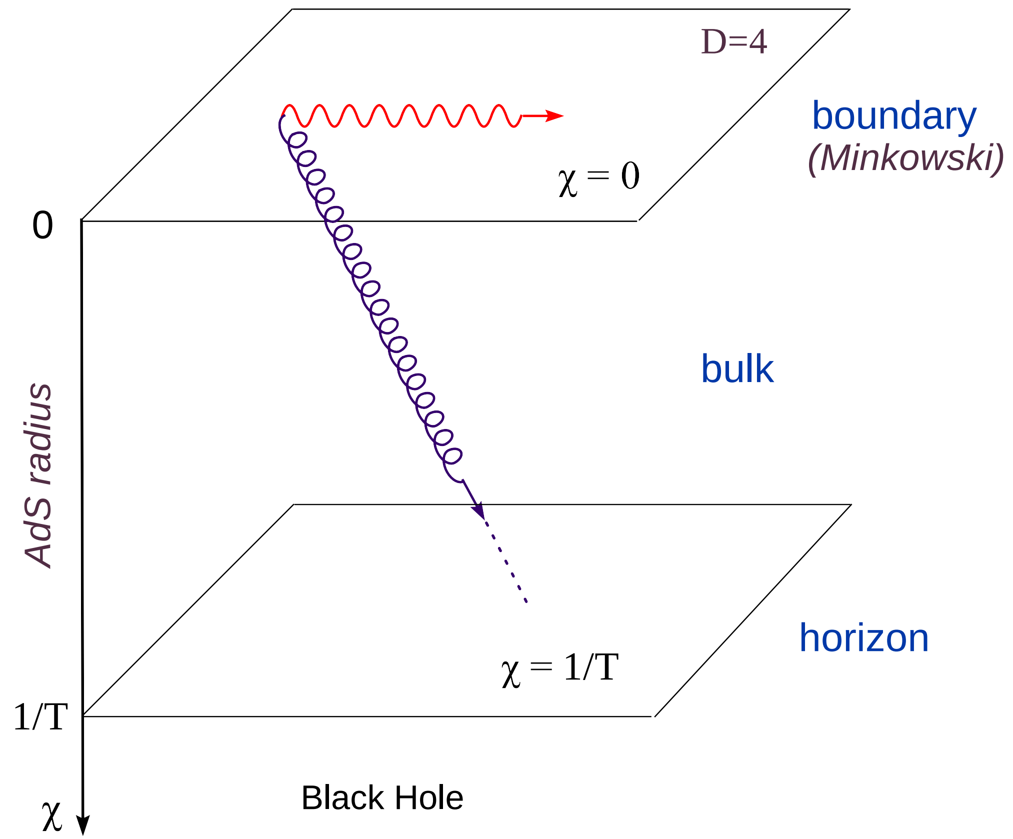
$$4\pi g_s = g^2\,, \qquad (R/l_s)^4 = g^2 N_c \equiv \lambda\,, \qquad r_0/R^2 = \pi T$$

- 'Strong coupling limit': $\lambda \to \infty$ for fixed $g^2 \ll 1$ ($N_c \to \infty$)

$\longrightarrow$ 'supergravity approximation' (classical gravity)

The AdS_5 Black Hole

- Other useful coordinates: $u \equiv (r_0/r)^2$ and $\chi \equiv R^2/r$
- The BH horizon : $r = r_0$, or $u = 1$, or $\chi = 1/(\pi T)$
- The Minkowski boundary : $r \to \infty$, or $u = 0$, or $\chi = 0$



$\mathcal{R}$–current in AdS/CFT

- ‘Electromagnetic’ current in $\mathcal{N}=4$ SYM : the $\mathcal{R}$–current
  - pick one of the $U(1)$ subgroups of the global SO(4) $\mathcal{R}$–symmetry and gauge it
  - some of the (adjoint) fermion and scalar fields of $\mathcal{N}=4$ SYM will carry this $\mathcal{R}$–charge
- Conserved $\mathcal{R}$–current J_μ which couples to the $\mathcal{R}$–photon A_μ
- $A_\mu(t,\boldsymbol{x})$: a perturbation on the Minkowski boundary of AdS_5
- $A_m(t,\boldsymbol{x},\chi)$, with $m=(t,x,y,z,\chi)$: the Maxwell field induced by this perturbation in the bulk of AdS_5
- $\mathcal{R}$–current J_μ in $\mathcal{N}=4$ SYM $\Longleftrightarrow$ wave $A_m(t,\boldsymbol{x},\chi)$ in AdS_5
- In practice: Solve Maxwell equations in AdS_5 (BH) geometry subjected to appropriate boundary conditions

Retarded polarization tensor

- Maxwell equation in curved space (AdS_5 BH)

$$\partial_m\left(\sqrt{-g}g^{mn}g^{pq}F_{nq}\right) = 0 \qquad \text{where} \qquad F_{mn} = \partial_m A_n - \partial_n A_m$$

- Plane–waves current and fields :

$$J_\mu(x) = n_\mu \mathrm{e}^{-i\omega t+ikz} \Longrightarrow A_\mu(t,\boldsymbol{x},\chi) = \mathrm{e}^{-i\omega t+ikz} A_\mu(\chi)$$

- Boundary conditions
  - $A_\mu(\chi=0) = A_\mu^{(0)}, \qquad A_\chi(\chi=0) = 0$
  - regularity at $\chi \to \infty$ (vacuum, i.e., $T=0$), or
  - no reflected wave returning to the boundary ('outgoing')

    $A_m \propto \exp\{-i\omega(t - v_\chi \chi)\}$ with positive radial velocity v_χ

- The retarded polarization tensor :

$$\Pi_{\mu\nu}(\omega,k) = \frac{\partial^2 \mathcal{S}_{\rm cl}}{\partial A_\mu^{(0)} \partial A_\nu^{(0)}}$$

Classical action

$$S = -\frac{N^2}{64\pi^2 R} \int d^4x \, d\chi \sqrt{-g} \, g^{mp} g^{nq} F_{mn} F_{pq}$$

- Gauge choice: $A_\chi = 0$
- After using EOM : $S = \int d^4x \, \mathcal{S} = \Delta V \, \Delta t \, \mathcal{S}$

$$\mathcal{S} = \frac{N^2}{32\pi^2} \left[\frac{1}{\chi} \Big(- A_0 \partial_\chi A_0^* + f A_3 \partial_\chi A_3^* + f A_i \partial_\chi A_i^* \Big) \right]_{\chi=0} \qquad (i = 1, 2)$$

  - Sensitive to the fields near the boundary alone ($\chi \to 0$)
  - Finite temperature: the contribution of the horizon ($\chi = \chi_0 \equiv 1/\pi T$) has been discarded (cf. lectures by Son)
  - Imaginary part due to the outgoing–wave b.c. at large χ
  - The imaginary part is independent of χ (check !)
- Recall: Im $\Pi_{\mu\nu}$ yields the cross–section

Equations of motion: $D^m F_{mn} = 0$

- Second–order differential eqs. in χ with parameters ω, q, r_0
- 2 types of modes: longitudinal ($A_{0,3}$) and transverse ($A_{1,2}$)
- Dimensionless variables: $\tilde{\chi} = (2\pi T)\chi$, $\tilde{\omega} = \omega/(2\pi T)$, etc.
- Focus on the longitudinal sector, omit tilde's
- $D^\nu F_{\nu\chi} = 0 \implies A_3' \; = \; -(\omega/fk)A_0'$ (with $A_\mu' \equiv \partial A_\mu/\partial\chi$)

$\implies$ only one independent eq. for A_0' or, preferably,

$$\psi(\chi) \; \equiv \; \frac{2}{\sqrt{\chi}} \, \frac{\partial A_0}{\partial \chi}$$

- Boundary condition:

$$\chi \, \frac{\partial}{\partial \chi} \, \frac{\psi}{\sqrt{\chi}} \bigg|_{\chi \to 0} \; = \; 2k\Big(kA_0^{(0)} + \omega A_3^{(0)}\Big)$$

Longitudinal wave equation

$$\psi'' + \frac{1}{4\chi^2}\psi + \frac{\omega^2 - k^2 f}{f^2}\psi + \frac{f'}{f}\left(\psi' - \frac{1}{\chi}\psi\right) = 0$$

- **Zero–temperature limit (vacuum):** $f = 1 \Longrightarrow$

$$\psi'' + \frac{1}{4\chi^2}\psi + q^2\psi = 0 \qquad \text{with} \qquad q^2 \equiv \omega^2 - k^2$$

▷ Lorentz invariance is manifest !

- **Finite temperature:** Long–range gravitational interactions

$$f = 1 - \left(\frac{\chi}{\chi_0}\right)^4 = 1 - \left(\frac{r_0}{r}\right)^4 \quad \text{where} \quad \chi_0 = 2 \quad \& \quad r_0 = \pi R^2 T$$

hence:

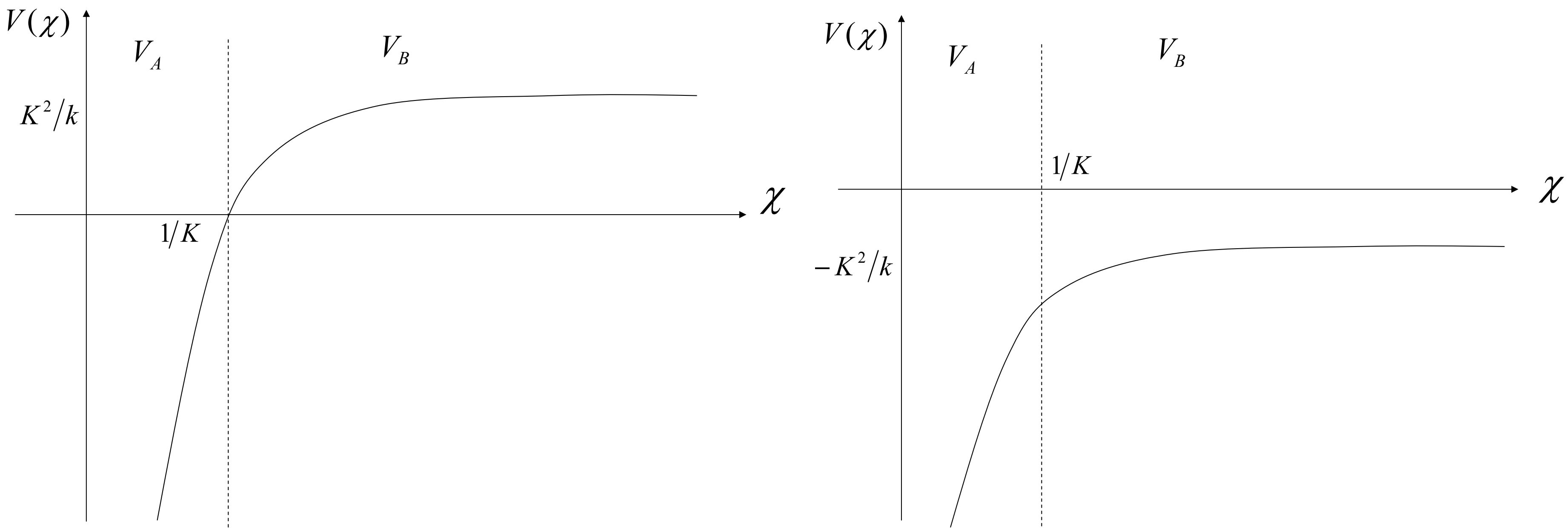
$$(\omega^2 - k^2 f) - q^2 = \frac{k^2\chi^4}{16} \propto \frac{k^2 R^8 T^4}{r^4} \propto G\,\frac{k^2(N_c^2 T^4)}{r^4}$$

The vacuum case as a warm up

$$-\psi'' + \left(-\frac{1}{4\chi^2} \pm Q^2\right)\psi = 0 \quad \text{with} \quad Q^2 \equiv |q^2| = |\omega^2 - q^2|$$

upper sign: space–like ($q^2 < 0$); lower sign: time–like ($q^2 > 0$)

- "Time–independent Schrödinger eq." : $-\psi'' + V_\pm(\chi)\psi = 0$



space–like   time–like

Space–like current in the vacuum

- Potential barrier

  $\Longrightarrow$ the wave is trapped near the boundary, at $\chi \lesssim 1/Q$

- General solution: $\psi(\chi) = \sqrt{\chi}\,\big(c_1 \mathrm{K}_0(Q\chi) + c_2 \mathrm{I}_0(Q\chi)\big)$
  - regularity at infinity $\Longrightarrow c_2 = 0$
  - BC at $\chi = 0 \Longrightarrow c_1 = -2k\big(kA_0^{(0)} + \omega A_3^{(0)}\big)$

    ▷ Recall: $\mathrm{K}_0 \approx -\ln(z/2) - \gamma$ when $z \ll 1$

- The longitudinal piece of the classical action:

$$\mathcal{S} = -\frac{N^2}{64\pi^2}\big(kA_0^{(0)} + \omega A_3^{(0)}\big)^2 \left[\ln Q^2 + 2\ln\chi + const.\right]_{\chi=0}$$

- ‘Holographic renormalization’ : subtract the divergence (plus the constant term, for convenience)

- Vacuum polarization tensor: fixed by current conservation

Time–like current in the vacuum

- No potential barrier

  $\Longrightarrow$ the wave can move out to arbitrarily large χ

- General solution: $\psi(\chi) = \sqrt{\chi}\left(c_1 \mathrm{J}_0(Q\chi) + c_2 \mathrm{N}_0(Q\chi)\right)$
  - outgoing–wave BC at large $\chi \Longrightarrow c_2 = ic_1$
  - BC at $\chi = 0 \Longrightarrow c_1 = -i\pi k\left(kA_0^{(0)} + \omega A_3^{(0)}\right)$
  - $\mathrm{H}_0^{(1)} = \mathrm{J}_0 + i\mathrm{N}_0$: Hankel function

$$\psi(t,\chi) \propto \mathrm{e}^{-i(\omega t - Q\chi)} \quad \text{when} \quad \chi \gg 1/Q$$

- 'Holographic renormalization' (as before)

- Vacuum polarization tensor: $q^\mu \Pi_{\mu\nu}(q) = 0$

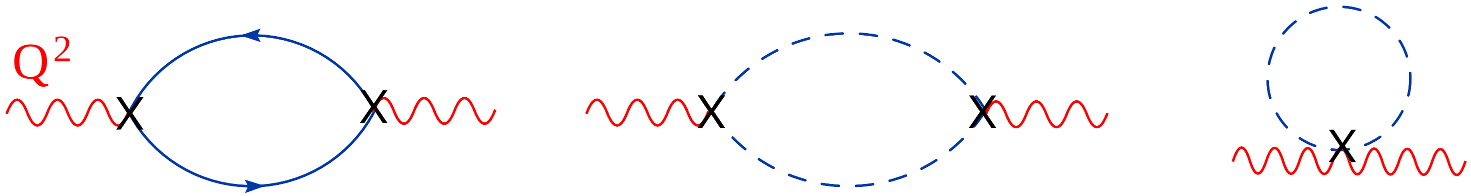
$$\Pi_{\mu\nu}(q) = \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{N_c^2 Q^2}{32\pi^2}\left(\ln\frac{Q^2}{\mu^2} - i\pi\Theta(q^2)\mathrm{sgn}(\omega)\right)$$

The vacuum polarization tensor

- The complete AdS/CFT result in the strong coupling limit:

$$\Pi_{\mu\nu}(q) = \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \frac{N_c^2 Q^2}{32\pi^2} \left(\ln \frac{Q^2}{\mu^2} - i\pi\Theta(q^2)\mathrm{sgn}(\omega)\right)$$

- The same as the respective result at one–loop level !



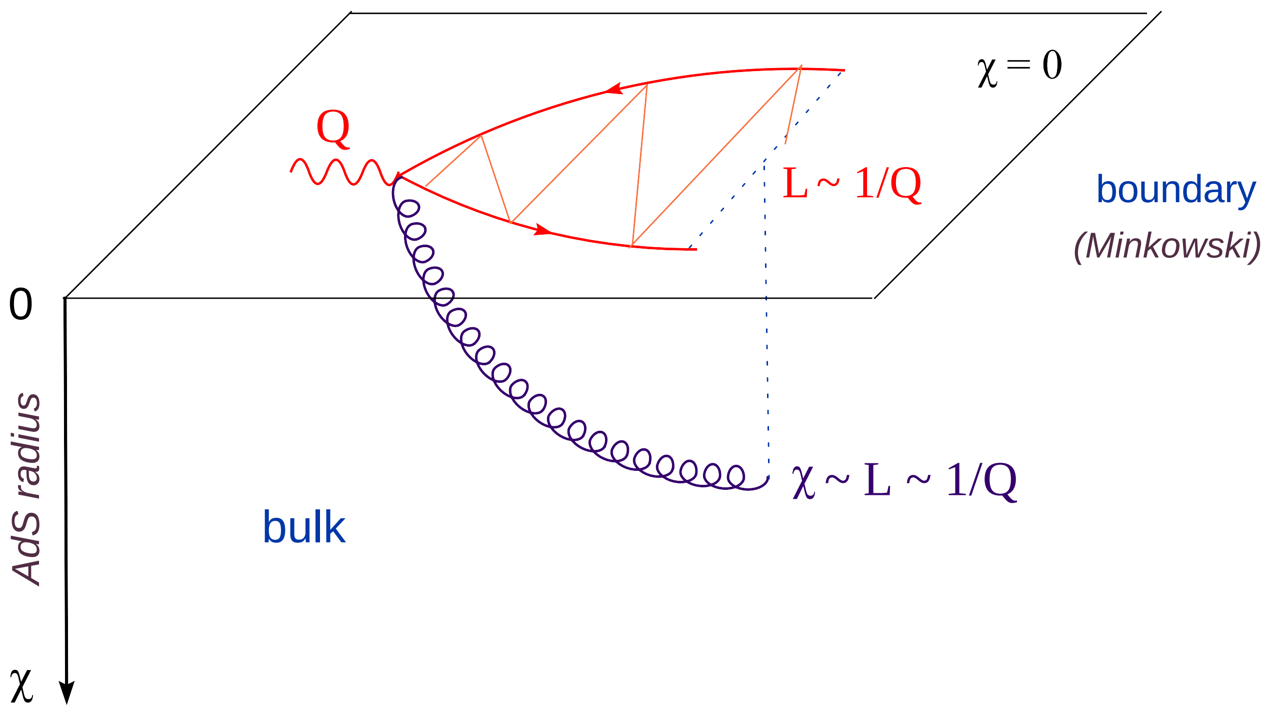
▷ Non–renormalization property due to SUSY

- Time–like: imaginary part due to decay into massless fields
- The space–time picture at 1–loop level is well understood

... and can be used to interpret the AdS/CFT calculation !

The UV/IR correspondence (1)

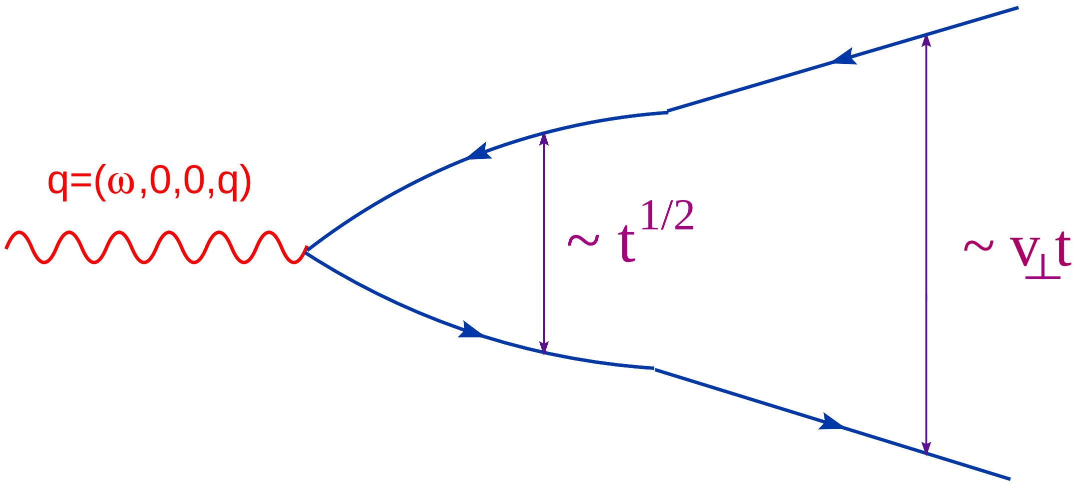
- Radial dimension $\chi \longleftrightarrow$ transverse size L of the partonic fluctuation in the gauge theory



- Early times $t \lesssim \omega/Q^2$: the transverse size of the fluctuation grows via diffusion ($L \sim \sqrt{t}$), up to a size $L \sim 1/Q$
- So does the position χ of the wave packet in AdS_5 !

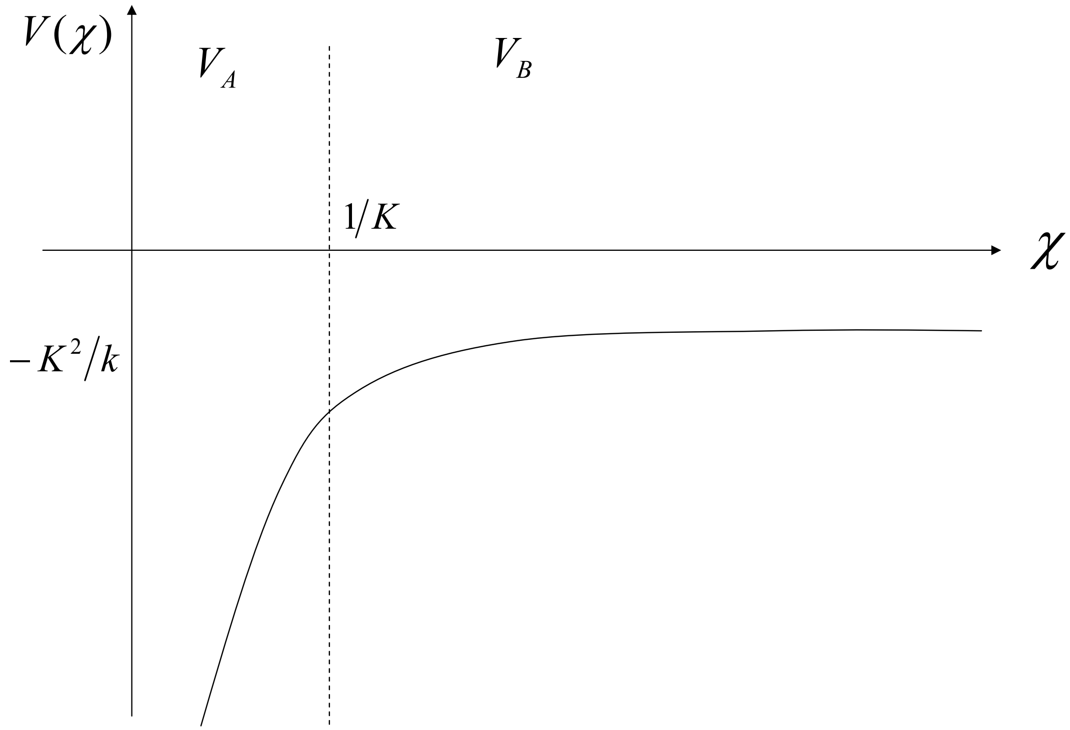
The UV/IR correspondence (2)

- Time–like current at late times $t \gtrsim \omega/Q^2$:
  - the fluctuation decays into a pair of massless partons, which undergo free streaming



  - common longitudinal velocity $v_z = k/\omega$
  - transverse velocity $v_\perp = \sqrt{1 - v_z^2}$
- The transverse size of the pair grows like $L(t) \simeq 2v_\perp t$

The UV/IR correspondence (3)



- The Maxwell wave in AdS_5 at late times $t \gtrsim \omega/Q^2$:

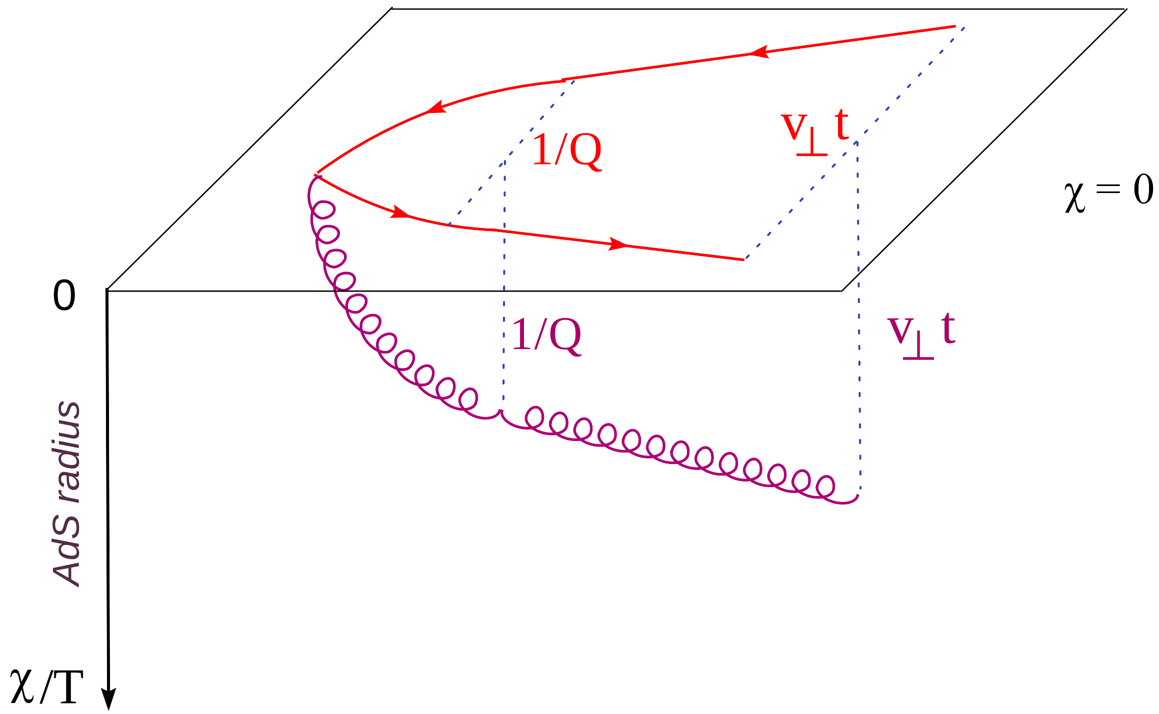
free streaming with radial velocity $v_\chi = \sqrt{1 - v_z^2}$

$$\psi(t, z, \chi) \propto \mathrm{e}^{-i(\omega t - kz - Q\chi)} \implies \chi(t) \sim \frac{Q}{\omega} t = \sqrt{1 - v_z^2}\, t$$

The UV/IR correspondence (4)

- All such results are consistent with the UV/IR duality :

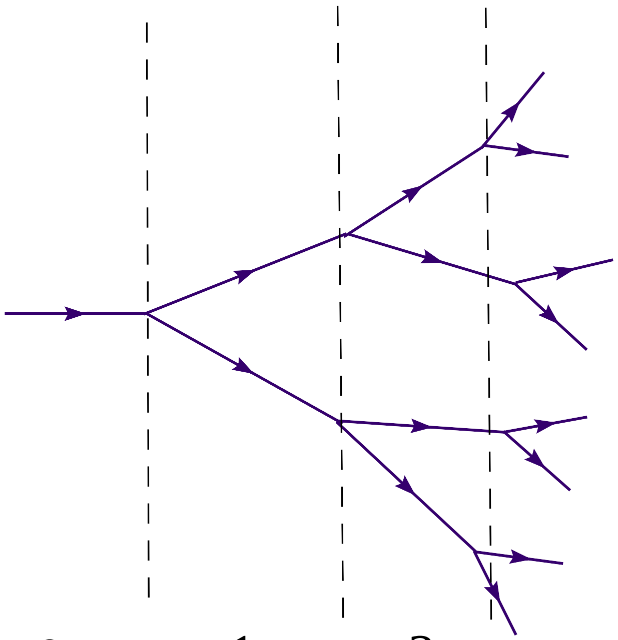
$$\chi \equiv \frac{R^2}{r} \longleftrightarrow L \sim \frac{1}{Q}$$



- Important for the physical interpretation of AdS/CFT results

Parton branching at strong coupling

- No reason why branching should stop at 2 parton level !
- No reason to favour special corners of phase–space !



$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

$$Q_n \sim \frac{Q_{n-1}}{2} \sim \frac{Q}{2^n}$$

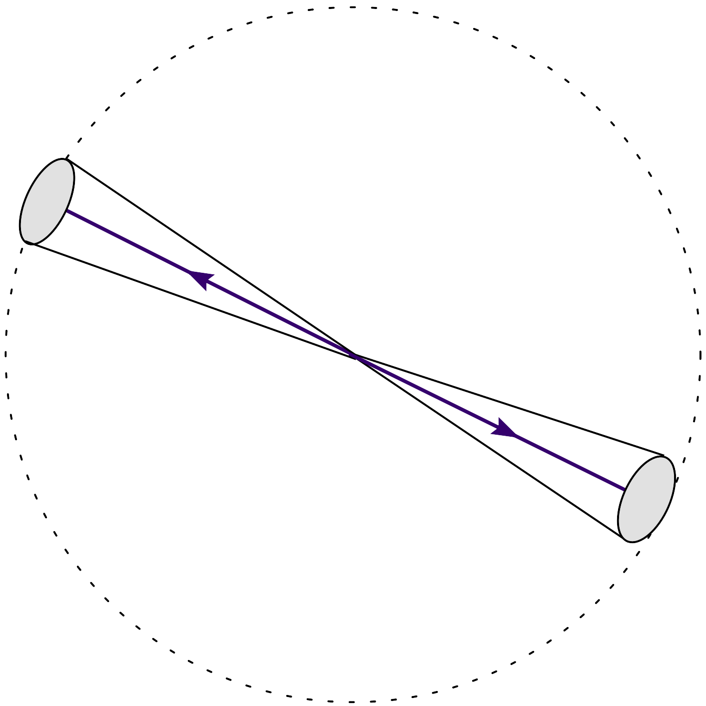
$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

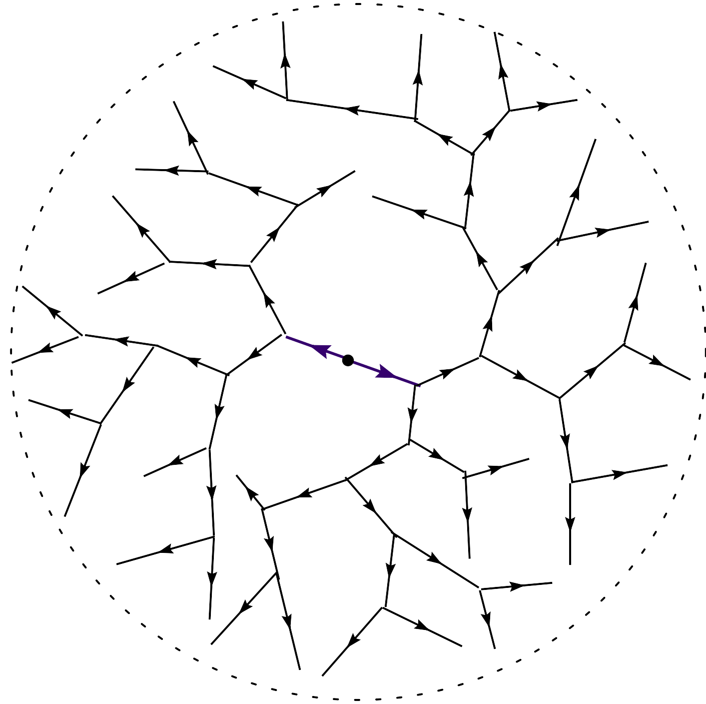
$$\frac{Q_n}{\omega_n} \sim \frac{Q}{\omega} = \frac{1}{\gamma}$$

$$\frac{Q_n - Q_{n-1}}{\Delta t_n} \sim -\frac{Q}{\omega} Q_n^2 \qquad \Longrightarrow \qquad \frac{\mathrm{d}Q(t)}{\mathrm{d}t} \simeq -\frac{Q^2(t)}{\gamma}$$

- $L(t) \sim 1/Q(t) \implies L(t) \sim t/\gamma = \sqrt{1-v_z^2}\, t$ ✓

e^+e^- at strong coupling





- Infrared cutoff $\Lambda \longrightarrow$ splitting continues down to $Q \sim \Lambda$
- In the COM frame $\longrightarrow$ spherical distribution
  (similar conclusion by Hofman and Maldacena, 2008)
- No jets in e^+e^- annihilation at strong coupling !