# On a consistent AdS/CFT description of boost-invariant plasma

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Zakopane, 20.06.08

Based on `arXiv: 0805.3774 [hep-th]`

**Relativistic Heavy Ion Collider**

- creation of Quark Gluon Plasma
- strongly coupled dynamical medium
- ab initio transport coefficients (hydrodynamics)?

**AdS/CFT**

- non-perturbative gauge theory dynamics?
- gravity dual of time-dependent phenomena?
- $\mathcal{N} = 4$ SYM transport properties at $\lambda \to \infty$

**Basics**

- long-wavelength effective theory
- vast reduction of # degrees of freedom
  - velocity $\mathrm{u}^\mu(x)$ constrained by $\mathrm{u}^\mu \mathrm{u}_\mu = -1$
  - temperature $\mathrm{T}(x)$
- slow changes $\rightarrow$ gradient expansion
- expansion parameter $\frac{1}{\mathrm{L}\cdot\mathrm{T}}$
  ($\mathrm{T}$ is temperature, $\mathrm{L}$ is characteristic length-scale)

**Gradient expansion**

- definition of the energy-momentum tensor

$$T^{\mu\nu} = \epsilon\cdot\mathrm{u}^\mu\mathrm{u}^\nu + \mathrm{p}\cdot\Delta^{\mu\nu} - \eta\cdot\left(\Delta^{\mu\lambda}\nabla_\lambda\mathrm{u}^\nu + \Delta^{\nu\lambda}\nabla_\lambda\mathrm{u}^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla^\lambda\mathrm{u}_\lambda\right) + \ldots$$

- EOMs $\nabla_\mu T^{\mu\nu} = 0$ + equation of state (e.g. $\epsilon = 3p$)

**Symmetries**

- $1+1$ D dynamics ($x^0$ and $x^1$)
- translational and rotational symmetries in $\perp$ plane
- boost-invariance = no $y$-dependence
  ($x^0 = \tau \cdot \cosh y$ and $x^1 = \tau \cdot \sinh y$)

**The energy-momentum tensor**

- due to symmetries and conservation depends only on $\epsilon(\tau)$
- gradient expansion to the second order

$$\epsilon(\tau) = \left(\frac{N_c^2}{2\pi^2}\right) \frac{1}{\tau^{4/3}} \left\{ 1 - 2\eta_0 \frac{1}{\tau^{2/3}} + \left[\frac{3}{2}\eta_0^2 - \frac{2}{3}\left(\eta_0 \tau_\Pi^0 - \lambda_1^0\right)\right] \frac{1}{\tau^{4/3}} + \cdots \right\}$$

where

- $\eta_0$ - shear viscosity coefficient
- $\tau_\Pi^0$ - relaxation time coefficient
- $\lambda_1^0$ - new constant introduced in `0712.2451 [hep-th]`

**Gauge theory side**

$$T_{\mu\nu} = \begin{pmatrix} \epsilon(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3\epsilon'(\tau) - \tau^2\epsilon(\tau) & 0 & 0 \\ 0 & 0 & \epsilon(\tau) + \frac{1}{2}\tau\epsilon'(\tau) & 0 \\ 0 & 0 & 0 & \epsilon(\tau) + \frac{1}{2}\tau\epsilon'(\tau) \end{pmatrix}$$

AdS/CFT dictionary relates $T_{\mu\nu}$ to the bulk metric

**Gravity dual**

- the most general metric ansatz takes the form

$$\mathrm{d}s^2 = \frac{-e^{a(\tau,z)}\mathrm{d}\tau^2 + \tau^2 e^{b(\tau,z)}\mathrm{d}y^2 + e^{c(\tau,z)}\mathrm{d}x_\perp^2 + \mathrm{d}z^2}{z^2}$$

- it reflects all the symmetries of $T_{\mu\nu}$ and solves

$$R_{MN} - \frac{1}{2}RG_{MN} - 6G_{MN} = 0$$

**Reminder:**

$$\mathrm{d}s^2 = \frac{-e^{a(\tau,z)}\mathrm{d}\tau^2 + \tau^2 e^{b(\tau,z)}\mathrm{d}y^2 + e^{c(\tau,z)}\mathrm{d}x_\perp^2 + \mathrm{d}z^2}{z^2}$$

**Gravitational gradient expansion:**

$$a(\tau,z) = a_0\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{2/3}}a_1\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{4/3}}a_2\left(\frac{z}{\tau^{1/3}}\right) + \ldots$$

$$b(\tau,z) = b_0\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{2/3}}b_1\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{4/3}}b_2\left(\frac{z}{\tau^{1/3}}\right) + \ldots$$

$$c(\tau,z) = c_0\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{2/3}}c_1\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{4/3}}c_2\left(\frac{z}{\tau^{1/3}}\right) + \ldots$$

$$\mathcal{R}^2(\tau,z) = \mathcal{R}_0^2(\frac{z}{\tau^{1/3}}) + \frac{1}{\tau^{2/3}}\mathcal{R}_1^2(\frac{z}{\tau^{1/3}}) + \frac{1}{\tau^{4/3}}\mathcal{R}_2^2(\frac{z}{\tau^{1/3}}) + \ldots$$

**This is AdS counterpart of hydrodynamics**

$$\epsilon(\tau) = \left(\frac{N_c^2}{2\pi^2}\right)\frac{1}{\tau^{4/3}}\left\{1-2\eta_0\frac{1}{\tau^{2/3}}+\left[\frac{3}{2}\eta_0^2-\frac{2}{3}(\eta_0\tau_\Pi^0-\lambda_1^0)\right]\frac{1}{\tau^{4/3}}+\cdots\right\}$$

# Something's wrong with the world today

- integration constant responsible for $\tau_{\Pi}^{0}$ and $\lambda_{1}^{0}$ is fixed by requiring cancelation of all poles at $v = 3^{1/4}$
- there is always a logarithmic contribution

$$\mathcal{R}_{MNOP}\mathcal{R}^{MNOP} = \text{finite} + \left(\frac{1}{\tau^{2/3}}\right)^{3} 8 \cdot 2^{1/2} 3^{3/4} \log\left(3^{1/4} - v\right) + \dots$$

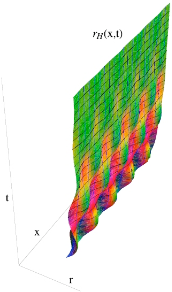
- $\log(3^{1/4} - v)$ shows in higher order curvature invariants (e.g. $\mathcal{R}_{MN}\mathcal{R}^{MN}$, $\mathcal{R}_{MNOP}\mathcal{R}^{OPQS}\mathcal{R}_{QS}{}^{MN}$)
- cannot be cancelled by a finite number of massive fields
- present in the case of
  - KK of type IIB to $AdS_5 x S^5$ (1 massive field - $S^5$ radius)
  - Klebanov-Witten model (2 massive fields)

[hep-th/0703143], 0712.2025 [hep-th]

- boosted black brane (BB) with slowly-varying $\mathrm{b}$ and $\mathrm{u}^\mu$

$$\mathrm{ds}^2_{BB} = -2\mathrm{u}_\mu \mathrm{dx}^\mu \mathrm{d}r - r^2 \left(1 - \frac{1}{\mathrm{b}^4 r^4}\right) \mathrm{u}_\mu \mathrm{u}_\nu \mathrm{dx}^\mu \mathrm{dx}^\nu + P_{\mu\nu} + O\left(\frac{1}{L \cdot T}\right)$$

- completely regular construction
- hydrodynamics = long-wavelength distortions of BB horizon




0712.2456 [hep-th], 0803.2526 [hep-th]

## Boosting black brane

- $u^\mu = (\cosh y, \sinh y, 0, 0)$, $b \sim \tilde{\tau}^{1/3}$ leads to

$$ds^2_{BB} = 2\mathrm{d}\tilde{\tau}\mathrm{d}r + r^2\left\{-\left(1 - \frac{3}{4\tilde{\tau}^{4/3}r^4}\right)\mathrm{d}\tilde{\tau}^2 + \tilde{\tau}^2\mathrm{d}y^2 + \mathrm{d}x_\perp^2\right\}$$

## New boost-invariant ansatz

- this suggests the Ansatz

$$ds^2 = 2\mathrm{d}\tilde{\tau}\mathrm{d}r + r^2\left(-e^{\tilde{a}(\tilde{\tau},r)}\mathrm{d}\tilde{\tau}^2 + \tilde{\tau}^2 e^{\tilde{b}(\tilde{\tau},r)}\mathrm{d}y^2 + e^{\tilde{c}(\tilde{\tau},r)}\mathrm{d}x_\perp^2\right)$$

where

$$\begin{aligned}
\tilde{a}(\tilde{\tau}, r) &= \tilde{a}_0\left(r\cdot\tilde{\tau}^{1/3}\right) + \frac{1}{\tilde{\tau}^{2/3}}\tilde{a}_1\left(r\cdot\tilde{\tau}^{1/3}\right) + \frac{1}{\tilde{\tau}^{4/3}}\tilde{a}_2\left(r\cdot\tilde{\tau}^{1/3}\right) + \ldots \\
\tilde{b}(\tilde{\tau}, r) &= \tilde{b}_0\left(r\cdot\tilde{\tau}^{1/3}\right) + \frac{1}{\tilde{\tau}^{2/3}}\tilde{b}_1\left(r\cdot\tilde{\tau}^{1/3}\right) + \frac{1}{\tilde{\tau}^{4/3}}\tilde{b}_2\left(r\cdot\tilde{\tau}^{1/3}\right) + \ldots \\
\tilde{c}(\tilde{\tau}, r) &= \tilde{c}_0\left(r\cdot\tilde{\tau}^{1/3}\right) + \frac{1}{\tilde{\tau}^{2/3}}\tilde{c}_1\left(r\cdot\tilde{\tau}^{1/3}\right) + \frac{1}{\tilde{\tau}^{4/3}}\tilde{c}_2\left(r\cdot\tilde{\tau}^{1/3}\right) + \ldots
\end{aligned}$$

**Fefferman-Graham** $(\tau, z)$

- leading order solution

$$\mathrm{d}s^2_{JP} = \frac{1}{z^2}\left\{ -\frac{\left(1-\frac{z^4}{3\tau^{4/3}}\right)^2}{1+\frac{z^4}{3\tau^{4/3}}}\mathrm{d}\tau^2 + \left(1+\frac{z^4}{3\tau^{4/3}}\right)\left(\tau^2\mathrm{d}y^2 + \mathrm{d}x_\perp^2\right) + \mathrm{d}z^2 \right\}$$

- singular as $z \to 3^{1/4}\tau^{1/3}$

[hep-th/0512162]

**Eddington-Finkelstein** $(\tilde{\tau}, r)$

- leading order solution

$$\mathrm{d}s^2_{BB} = 2\mathrm{d}\tilde{\tau}\mathrm{d}r + r^2\left\{ -\left(1-\frac{3}{4\tilde{\tau}^{4/3}r^4}\right)\mathrm{d}\tilde{\tau}^2 + \tilde{\tau}^2\mathrm{d}y^2 + \mathrm{d}x_\perp^2 \right\}$$

- regular as $\frac{1}{r} \to \frac{\sqrt{2}}{3^{1/4}} \cdot \tilde{\tau}^{1/3}$

0805.3774 [hep-th]

$$\tilde{a}_1(\tilde{v}) = -\frac{2}{3}\cdot\frac{2\cdot 3^{-1/2}+2^{1/2}3^{-1/4}\tilde{v}+\tilde{v}^2}{\left(2^{1/2}3^{-1/4}+\tilde{v}\right)\cdot\left(2\cdot 3^{-1/2}+\tilde{v}^2\right)}$$

$$\tilde{b}_1(\tilde{v}) = \frac{\pi}{\sqrt{2}3^{3/4}} - \frac{\sqrt{2}}{3^{3/4}}\arctan\left(\frac{3^{1/4}}{\sqrt{2}}\tilde{v}\right) - \frac{2\sqrt{2}}{3^{3/4}}\log\tilde{v} + \\ + \frac{\sqrt{2}}{3^{3/4}}\log\left(\frac{\sqrt{2}}{3^{1/4}}+\tilde{v}\right) + \frac{1}{\sqrt{2}3^{3/4}}\log\left(\frac{2}{3^{1/2}}+\tilde{v}^2\right)$$

$$\tilde{c}_1(\tilde{v}) = -\frac{\pi}{2\sqrt{2}3^{3/4}} + \frac{1}{\sqrt{2}3^{3/4}}\arctan\left(\frac{3^{1/4}}{\sqrt{2}}\tilde{v}\right) + \\ + \frac{\sqrt{2}}{3^{3/4}}\log\tilde{v} - \frac{1}{\sqrt{2}3^{3/4}}\log\left(\frac{\sqrt{2}}{3^{1/4}}+\tilde{v}\right) + \\ - \frac{1}{2\sqrt{2}3^{3/4}}\log\left(\frac{2}{3^{1/2}}+\tilde{v}^2\right)$$

0805.3774 [hep-th]

**Main result:**

- gauge theory dynamics from AdS/CFT

$$\epsilon(\tau) = \left(\frac{N_c^2}{2\pi^2}\right) \frac{1}{\tau^{4/3}} \left\{1 - 2\eta_0 \frac{1}{\tau^{2/3}} + \left[\frac{3}{2}\eta_0^2 - \frac{2}{3}(\eta_0 \tau_\Pi^0 - \lambda_1^0)\right] \frac{1}{\tau^{4/3}} + \cdots \right\}$$

**Asymptotic behavior:**

- perfect fluid behavior from "non-singularity"
- temperature cools down as $\frac{1}{\tau^{1/3}}$

**Viscosity:**

- $\eta_0 = \frac{1}{2^{1/2} 3^{3/4}}$
- it saturates the bound $\left(\frac{\eta}{s} = \frac{1}{4\pi}\right)$

**Relaxation time and $\lambda_1^0$:**

- $\tau_\Pi = (2 - \log 2) / (2\pi T)$ and $\lambda_1 = \frac{\eta}{2\pi T}$

0712.2451 [hep-th]

**Good things:**

- dynamics @ strong coupling
- agrees with near-equilibrium methods
- regular metric (no blow-up)

**Open problems and future applications:**

- dynamical horizons
  (I.Booth, MPH, M.Spaliński - work in progress)
- early time dynamics
  (readdressing 0705.1234 [hep-th])
- dynamical D7 branes embeddings
  (extending 0709.3910 [hep-th] to black hole embeddings)
- other symmetric flows
  (cosmology?)