Background Geometry in 4D Causal Dynamical Triangulations

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Outline

1. Path integral formulation of Quantum Gravity
2. Causal Dynamical Triangulations
3. Monte Carlo Simulations
4. Phase diagram
5. Background geometry

- The partition function of quantum gravity is defined as a formal integral over all geometries weighted by the Einstein-Hilbert action.
- To make sense of the gravitational path integral one uses the standard method of regularization - discretization.
- The path integral is written as a nonperturbative sum over all causal triangulations $\mathcal{T}$.
- Wick rotation is well defined due to global proper-time foliation.
- Using Monte Carlo techniques we can calculate expectation values of observables.

$$Z = \int \frac{\mathcal{D}_{\mathcal{M}}[g]}{Diff_{\mathcal{M}}} e^{iS^{EH}[g]} \quad \rightarrow \quad Z = \sum_{\mathcal{T}} \frac{1}{s(\mathcal{T})} e^{iS^{Regge}[g]}$$

Causal Dynamical Triangulations (CDT) is a background independent approach to quantum gravity.

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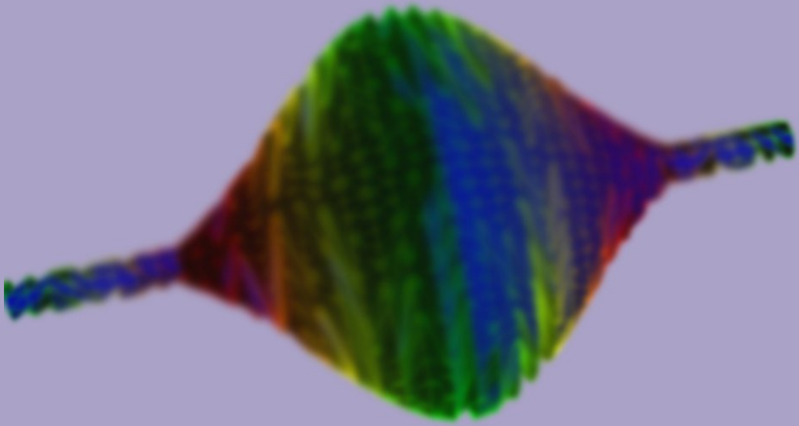
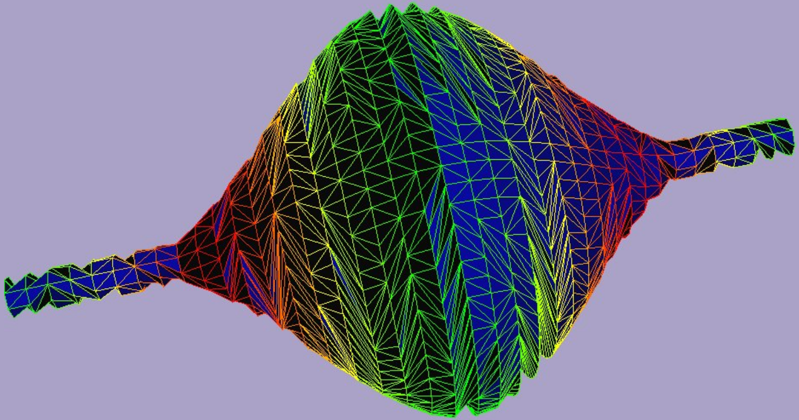
Dynamical Triangulations

A manifold with topology $S^3 \times S^1$ ...

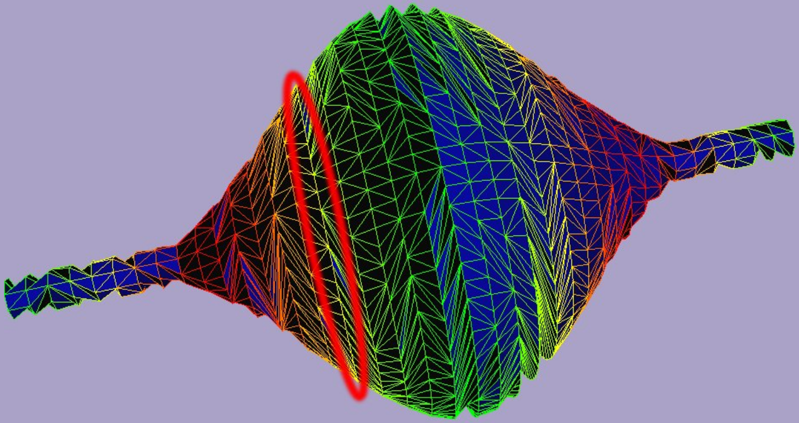


Dynamical Triangulations

. . . is discretized by gluing 4-simplices - triangulation

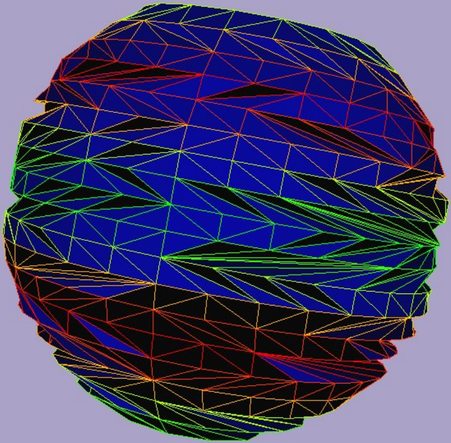


The spatial slices (of constant time) ...



Causal Dynamical Triangulations

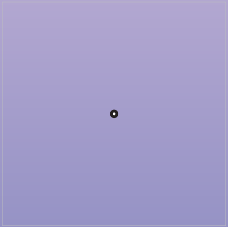
. . . have S^3 topology



Fundamental building blocks of CDT

- d-dimensional simplicial manifold can be obtained by gluing pairs of d-simplices along their $(d-1)$-faces.
- The metric is flat inside each d-simplex.
- Length of time links a_t and space links a_s is constant.
- The angle deficit (curvature) is localised at $(d-2)$-dimensional sub-simplices.

0D simplex - point



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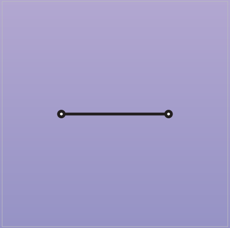
1D simplex - link



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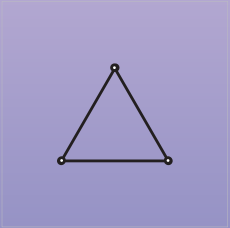
2D simplex - triangle



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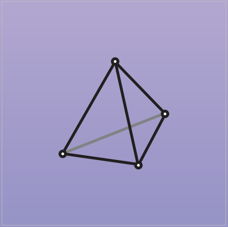
3D simplex - tetrahedron



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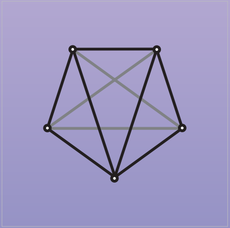
4D simplex - 4-simplex



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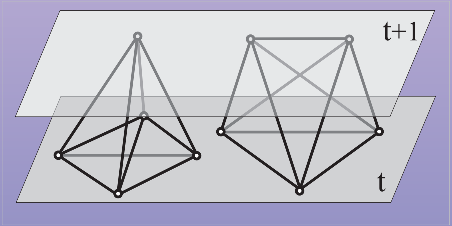
4D simplex, two types in CDT



Causal Dynamical Triangulation - properties

- By construction we have global proper-time foliation.
- Spatial links have length a_s, time links have length a_t.
- Wick rotation is well defined, $a_t \to ia_t$.
- Such formulation involves only geometric invariants like length and angles.
- We don't introduce coordinates.
- Manifestly diffeomorphism-invariant.
- Sum over triangulations (gluings).

$$Z = \int \frac{\mathcal{D}_{\mathcal{M}}[g]}{Diff_{\mathcal{M}}} e^{iS^{EH}[g]} \quad \to \quad Z = \sum_{\mathcal{T}} \frac{1}{s(\mathcal{T})} e^{-S^E[g]}$$

- Instead of the sum over the whole enormous phase-space of configurations, we probe its finite part with given probabilities.

Monte Carlo simulations - Alexander moves

- We construct a starting space-time manifold with given topology ($S^3 \times S^1$) and perform a random walk over configuration space.

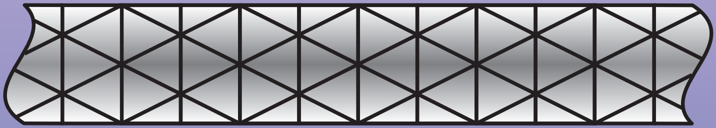
Ergodicity In the dynamical triangulation approach all possible configurations are generated by the set of Alexander moves.

Fixed topology The moves don't change the topology.

Causality Only moves that preserve the foliation are allowed.

4D CDT We have 4 types of moves.

Minimal configuration



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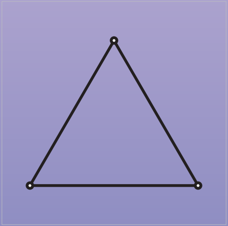
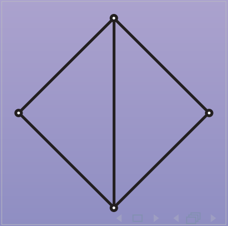
Moves in 2D





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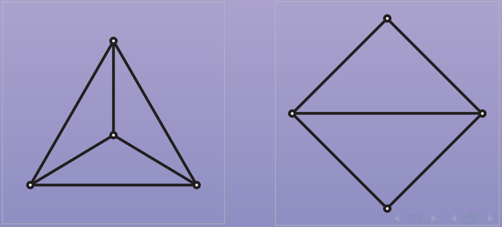

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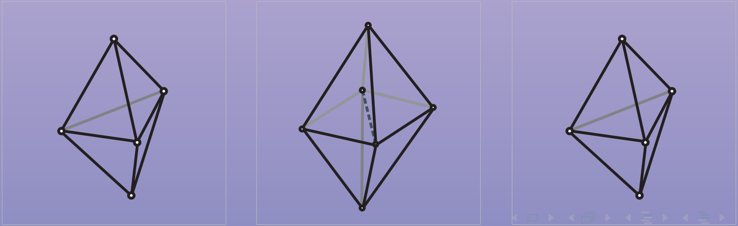

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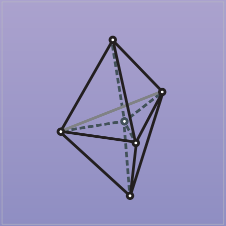
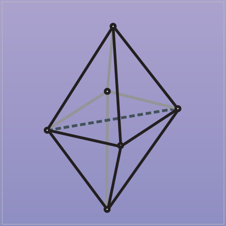
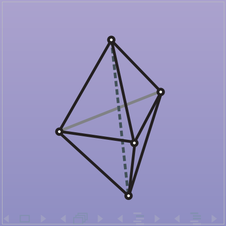






Monte Carlo Markov Chain

- We perform a random walk in the phase-space of configurations (space of piecewise linear geometries).
- Each step is one of the 4D CDT moves.
- The weight (acceptance probability) $W(\mathcal{A} \to \mathcal{B})$ of a move from configuration $\mathcal{A}$ to $\mathcal{B}$ is determined (not uniquely) by the detailed balance condition:

$$P(\mathcal{A})W(\mathcal{A} \to \mathcal{B}) = P(\mathcal{B})W(\mathcal{B} \to \mathcal{A})$$

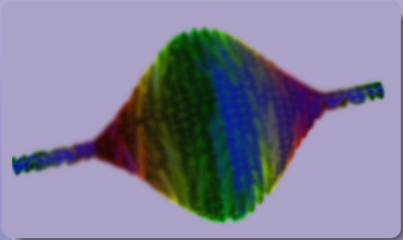
- The Monte Carlo algorithm ensures that we probe the configurations with the probability $P(\mathcal{A})$.
- After sufficiently long time, the configurations are independent.
- All we need, is the probability functional for configurations $P(\mathcal{A})$.

We generate a large number of such configurations with the probability

$$P[configuration] \propto e^{-S}$$

We use the Einstein-Hilbert action . . .

$$S = -\frac{1}{G}\int \mathrm{d}t \int \mathrm{d}\Omega\sqrt{g}(R - 6\lambda)$$



- G gravitational constant
- λ cosmological constant
- g determinant of a spacetime metric
- R scalar curvature

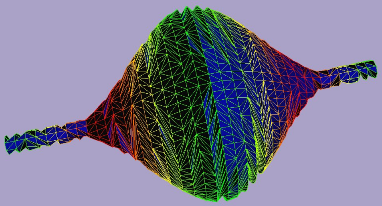
The Einstein-Hilbert and Regge action

We generate a large number of such configurations with the probability

$$P[configuration] \propto e^{-S}$$

...or the Regge action in the discrete version

$$S = -K_0 N_0 + K_4 N_4 + \Delta(N_{14} - 6N_0)$$



- N_0 number of vertices
- N_4 number of simplices
- N_{14} number of simplices of type $\{1,4\}$
- K_0 K_4 Δ bare coupling constants

Measurements

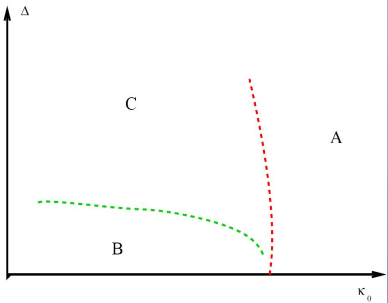
- To calculate the expectation value of an observable, we approximate the path integral by a sum over MC configurations

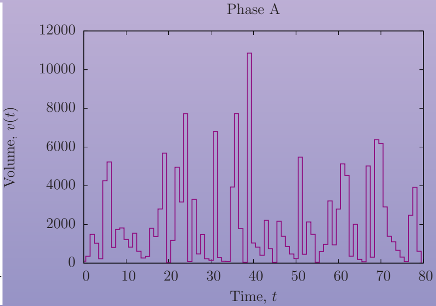
$$\langle \mathcal{O}[g] \rangle = \frac{1}{Z} \int \frac{\mathcal{D}_{\mathcal{M}}[g]}{Diff_{\mathcal{M}}} \mathcal{O}[g] e^{-S[g]} \to \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}[g^i]$$

- The Monte Carlo algorithm probes the configuration space with the probability $P[g] = \frac{1}{Z} e^{-S[g]}$.
- An simple example of an observable is the spatial volume at time t: $\mathcal{O} = v(t)$.
- For periodic boundary conditions, both the Einstein-Hilbert action and Regge action are invariant under proper time translations.
- In order to perform appropriate average of spatial volume $\bar{v}(t)$, we have to fix the position of the center of the blob.

Phase diagram of 4D CDT

- Depending on the values of coupling constants K_0 and Δ, we observe three qualitatively different behaviours of a typical configuration - phases.
- We tune the value of K_4 to its critical value, so that the total volume fluctuates around some fixed value.





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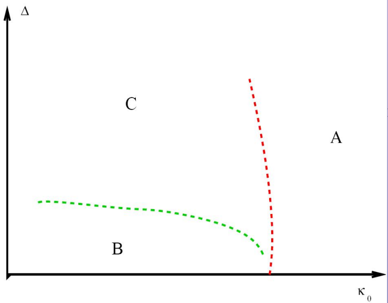

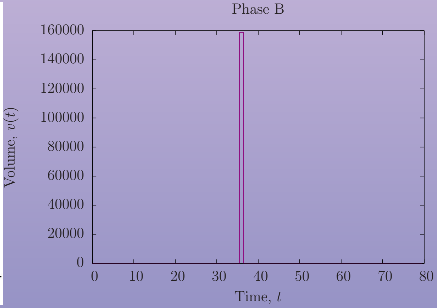

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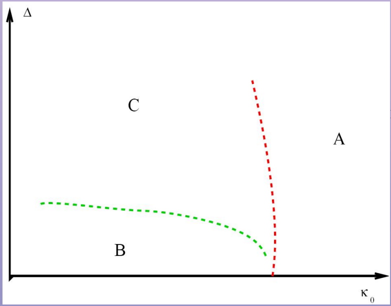

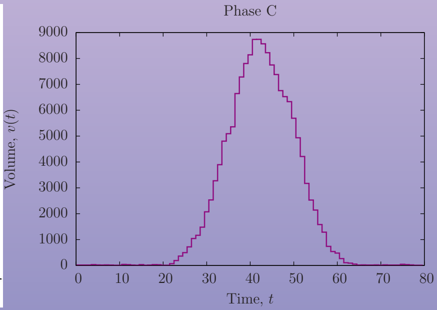

Phase diagram: 4D CDT vs Euclidean DT

- Asymmetry between a_t and a_s is important. In the Euclidean version of the model, without imposed causality, one either got
  - a "crumpled phase" with infinite Hausdorff dimension or
  - a "branched polymer phase" dominated by spacetimes where the 4-simplices form treelike structures with Hausdorff dimension two,

  even though they are built from 4D simplices. Unfortunately, the phase transition between them is of the 1^{st} order.

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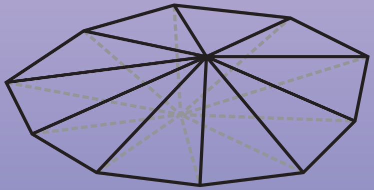

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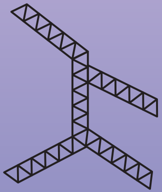
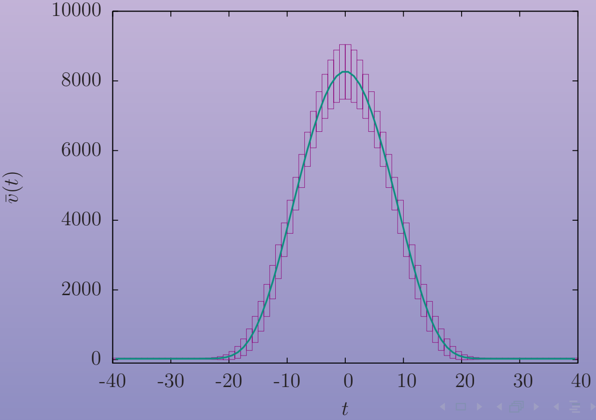


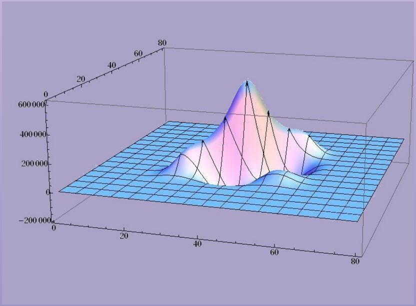
- For a certain range of bare coupling constants, a typical configuration has a "bloblike" shape and behaves as a well defined four-dimensional manifold.
- The averaged spatial volume $\bar{v}(t)$ is proportional to $\cos^3(t/B)$.



Volume fluctuations

- The next observable we can measure are the correlations of the spatial volume fluctuations around the classical solution.
- We define the two-point correlation function

$$C_{tt'} = \langle (v(t) - \bar{v}(t))(v(t') - \bar{v}(t')) \rangle$$



- Studying this matrix one can obtain the effective action for the volume fluctuations.

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Renate Loll (IS, Utrecht)

http://arxiv.org/pdf/0712.2485

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