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Renormalization group and bound states

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Question: Can $g(\lambda)$ not diverge when $\lambda \to$ "Λ_{QCD}"?

Method: Similarity Renormalization Group Procedure (SRG)

Trial study: SRG in a matrix model with AF and BS

Result: Yes: $g(\lambda)$ may behave well, but the measure of energy in the generator of SRG must include interactions

S. D. Głazek, Phys. Rev. D **75**, 025005 (2007).

S. D. Głazek, R. J. Perry, arXiv:0803.2911, to appear in Phys. Rev. D.

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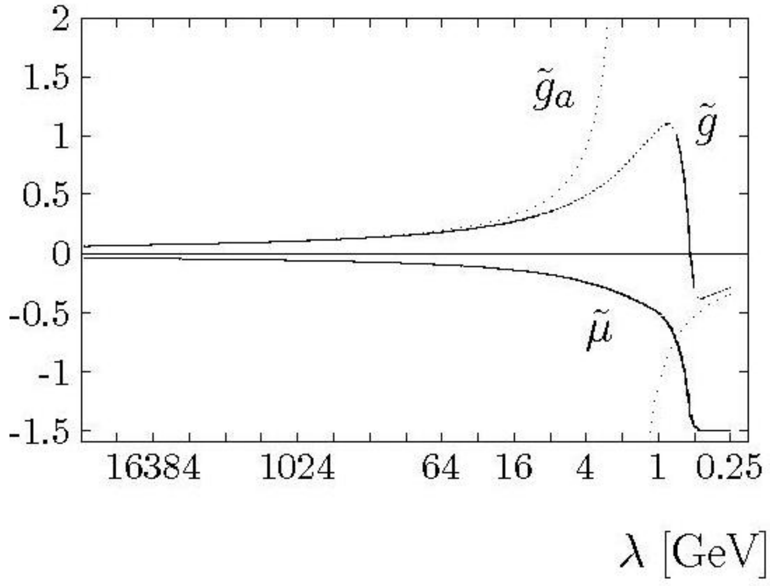
Diverging coupling constant in the model study:



$\tilde{g}_a$ - energy measured using free Hamiltonian H_0

$\tilde{g}$, $\tilde{\mu}$ - full account of interacions in the measure of energy (to be explained)

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Standard Renormalization Group Procedure:

$$\mathcal{L}_{can} = \mathcal{L}_0 + \mathcal{L}_I \quad \to \quad \mathcal{H}_{can} = \mathcal{H}_0 + \mathcal{H}_I \quad \to \quad H_{can} = \int d^3x \, \mathcal{H}_{can}(m, g)$$

$H_{can} = H_0 + H_I$ is ill-defined. It requires a cutoff Λ and $H_{CT}(\Lambda)$.

Cutoff: $H_0|n\rangle = E_n|n\rangle$, use $|n\rangle$ only such that $E_n < \Lambda$

Eigenvalues of H_0 provide the measure of energy.

$H_\Lambda = [H_{can} + H_{CT}]_\Lambda$ Schrödinger $H_\Lambda|\Psi_\Lambda\rangle = E|\Psi_\Lambda\rangle$

RG algebra: (Gaussian elimination)

$$\Lambda \to \Lambda/2 \to \Lambda/2^2 \to \ldots \to \Lambda/2^N = \lambda \ll \Lambda \quad \to \quad H_\lambda|\Psi_\lambda\rangle = E|\Psi_\lambda\rangle$$

$$H_{\lambda ij} = f_{ij}(\Lambda, CT_\Lambda, \lambda, m, g, E), \quad E_i, E_j < \lambda, \quad \frac{d}{d\Lambda} H_{\lambda ij} = 0$$

$\to$ $CT_{\Lambda mn}$ (e.g., $g \to g_\Lambda$, $m \to m_\Lambda$) $\to$ H_λ with m_λ, g_λ

K. G. Wilson, Phys. Rev. **140**, B445 (1965); Phys. Rev. D **2**, 1438 (1970).

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$$|\Psi_\lambda\rangle = \sum_{k=1}^{n} \psi_k |k\rangle \,, \quad \lambda = E_n$$

$$E_n\psi_n + \sum_{k=1}^{n} H_{Ink}\psi_k = E\psi_n \,, \quad i = n$$

$$E_i\psi_i + \sum_{k=1}^{n} H_{Iik}\psi_k = E\psi_i \,, \quad i < n$$

$$(E_n + H_{Inn} - E)\psi_n = \sum_{k=1}^{n-1} H_{Ink}\psi_k$$

$$\psi_n = (E_n + H_{Inn} - E)^{-1} \sum_{k=1}^{n-1} H_{Ink}\psi_k$$

$$E_i\psi_i + \sum_{k=1}^{n-1} H_{Iik}\psi_k + \sum_{k=1}^{n-1} \frac{H_{Iin}H_{Ink}}{E_n + H_{Inn} - E}\psi_k = E\psi_i \,, \quad i < n$$

$$H_{Iik} + \frac{H_{Iin}H_{Ink}}{E_n + H_{Inn} - E} = H'_{Iik}$$

$$\frac{1}{\lambda\,(1 - g_\lambda) - E} = problem$$

Problem: g_λ increases when λ decreases

Disaster: $g_\lambda \to \infty$ when $\lambda \to E$

In fact: $g_\lambda = g(\lambda, E) = g(g_0, \lambda/\lambda_0, E/\lambda_0) \quad \leftarrow \quad H_{\lambda_0}|\Psi_{\lambda_0}\rangle = E|\Psi_{\lambda_0}\rangle$

Simplification $E/\lambda_0 \sim 0$ does not apply when $\lambda_0 \sim E$

Solution: Gaussian **elimination** → **similarity** transformation

S. D. Głazek and K. G. Wilson, Phys. Rev. D **48**, 5863 (1993); **49**, 4214 (1994); **57**, 3558 (1998).

Flow equation: F. Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994).

SRG procedure:

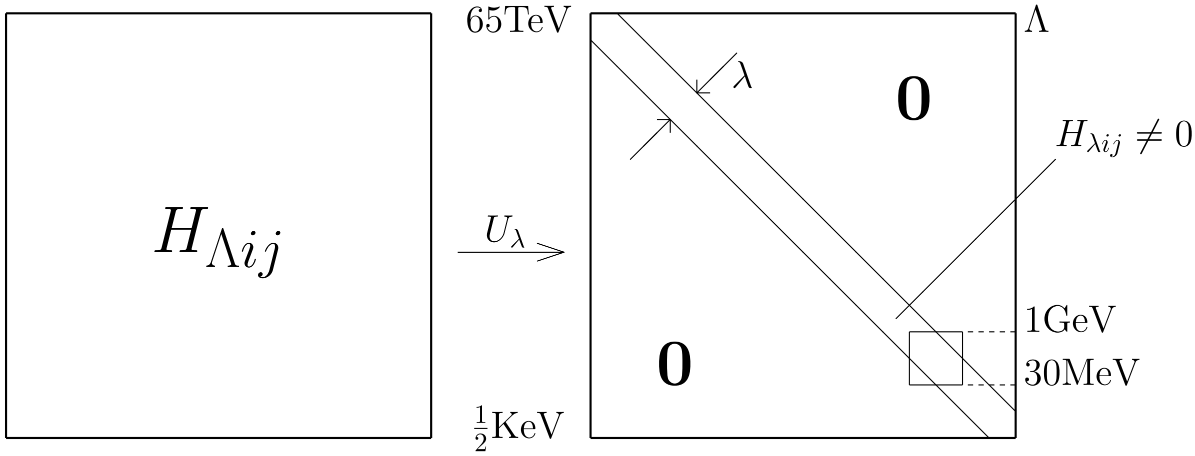
$$\frac{d}{d\lambda} H_\lambda = [\mathcal{T}_\lambda, H_\lambda]$$

$$H_\infty = [H_{can} + H_{CT}]_\Lambda$$

$$H_\lambda = H_\infty + \int_\infty^\lambda ds [\mathcal{T}_s, H_s]$$

$\mathcal{T}_\lambda = \, ?$ ← measure of energy

SRG procedure:



No small denominators in perturbation theory for H_λ

No explicit dependence of H_λ on the eigenvalues, E

Eigenvalues E appear on the diagonal when $\lambda \to 0$

$$H_\lambda = U_\lambda H_\infty U_\lambda^\dagger \qquad \frac{dH_\lambda}{d\lambda} = [\mathcal{T}_\lambda, H_\lambda] \qquad \mathcal{T}_\lambda = \frac{dU_\lambda}{d\lambda} U_\lambda^\dagger$$

$$\mathcal{T}_\lambda = [G_{f\lambda}, H_{f\lambda}]$$

$$H_\lambda = T + H_{I\lambda} = D_\lambda + V_\lambda, \qquad T_{ij} = \delta_{ij}\, T_i$$

$$D_{\lambda ij} = \delta_{ij}\ (T_i + H_{I\lambda ij}), \qquad V_{\lambda ij} = (1 - \delta_{ij})\, H_{I\lambda ij}$$

$$G_{f\lambda} = fT + (1-f) D_{f\lambda}$$

$f = 1 \leftarrow$ energy counted without interaction $(G = T)$

$f = 0 \leftarrow$ energy counted fully with interaction $(G = D)$

$$\frac{d}{d\lambda} H_\lambda = [[G_{f\lambda}, H_\lambda], H_\lambda]$$

$$H_{\lambda=\infty} = H_\Lambda$$

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Trial study: asymptotic freedom (AF)

$$H = H_0 + H_I$$

$$H_{mn} = E_m \delta_{mn} - g\sqrt{E_m E_n}$$

$$E_n = b^n \qquad b > 1 \qquad M \le n \le N \qquad \Lambda = b^N$$

Regularization $\Rightarrow$ CT $\Rightarrow$ $g \to g_N \sim 1/N = \ln b / \ln \Lambda$

Result: evolution of matrix elements of H_λ

(example with 1764 non-linear coupled integro-differential equations)

$$H_\lambda = ? \qquad H_{mn}(\lambda) = \left[E_m \delta_{mn} - g_\lambda \sqrt{E_m E_n}\right] e^{-(E_m - E_n)^2/\lambda^2}$$

$$\mathcal{V}_{mn} = \left[H(\lambda) - T\right]_{mn} / \sqrt{E_m E_n} \quad \sim \quad -g_\lambda \, , \qquad E_m, E_n \ll \lambda$$

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Example from trial study: (AF)

FLATplaneCOUPLING.pdf $f = 0$

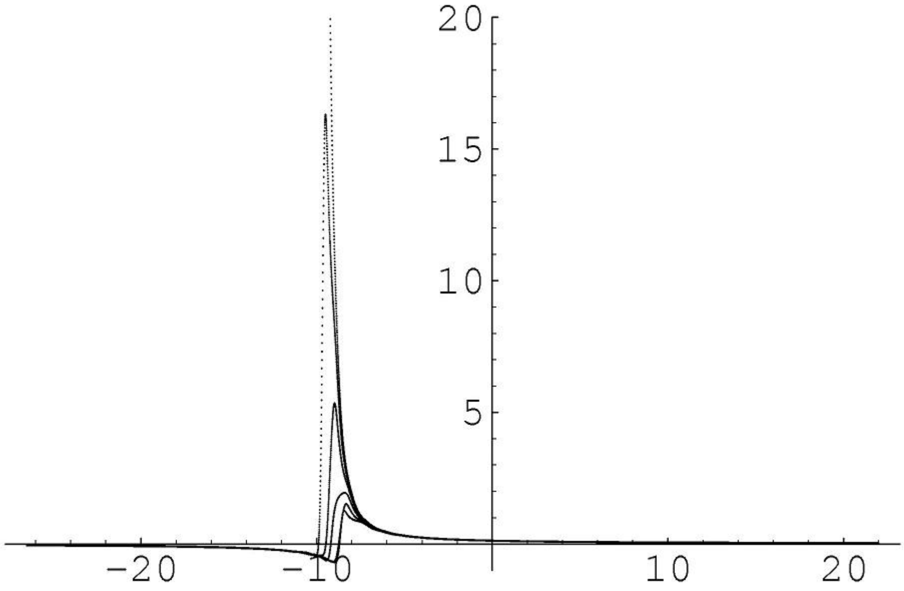

FIG. 1: The coupling constants g_f in the case of asymptotic freedom, plotted as a function of $\ln\lambda/\ln b$ (instead of $s = 1/\lambda^2$) for 6 values of f: $f = 0$ (Wegner), $f = 0.2$, 0.5, 0.75, 0.9, and 1. The correspondence between a curve and f is such that the curves for larger f reach higher and for $f = 1$ the corresponding curve apparently shoots to infinity around $\lambda \sim |E_{boundstate}|$. The ultraviolet cutoff is at b^{16}, and $b = 4$.

$\boxed{2 \times 2}$ $\quad s = 1/\lambda^2, \quad \lambda = \infty \leftrightarrow s = 0$

$$H(s) = \begin{bmatrix} E_h(s) & V(s) \\ V(s) & E_l(s) \end{bmatrix} \qquad H(0) = \begin{bmatrix} E_h(0) & V(0) \\ V(0) & E_l(0) \end{bmatrix}$$

$$D = \begin{bmatrix} E_h(s) & 0 \\ 0 & E_l(s) \end{bmatrix} \qquad G = \begin{bmatrix} fT_h + (1-f)E_h & 0 \\ 0 & fT_l + (1-f)E_l \end{bmatrix}$$

$$tr = E_h + E_l = E_1 + E_2 \quad \text{and} \quad det = E_h E_l - V^2 = E_1 E_2$$

$$H' = \frac{d}{ds} H = [[G, H], H]$$

$$\left(\frac{E_1 - E_2}{2}\right)^2 = V^2 + \left(\frac{E_h - E_l}{2}\right)^2 = x^2 + y^2 = r^2$$

$$(V^2)' = -2\left[f(T_h - T_l) + (1-f)(E_h - E_l)\right](E_h - E_l)\, V^2$$

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Mechanism of increase of g_λ for $\lambda \sim |E_B|$: $\quad g_\lambda \sim V$

g_λ is read from $E_l = T_l - g_\lambda T_l$

$f \neq 0$ is different from $f = 0$. $f = 1$ is completely different!

$$(V^2)' = -2\left[f(T_h - T_l) + (1-f)(E_h - E_l)\right](E_h - E_l)\, V^2$$

$$G = D, \quad f = 0 \quad \to \quad (V^2)' = -2(E_h - E_l)^2\, V^2$$

$$G = T, \quad f = 1 \quad \to \quad (V^2)' = -2(T_h - T_l)(E_h - E_l)\, V^2$$

and

$$\left(\frac{E_1 - E_2}{2}\right)^2 = V^2 + \left(\frac{E_h - E_l}{2}\right)^2$$

Sign of $E_h - E_l$ decides: BS $\to$ $E_B < 0$ and $E_h < E_l$ at $\lambda \sim |E_B|$.

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Case of Asymptotic Freedon: ShiftingBStoLow.nb

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Case of Limit Cycle: filmcf000.nb

AF as a part of LC

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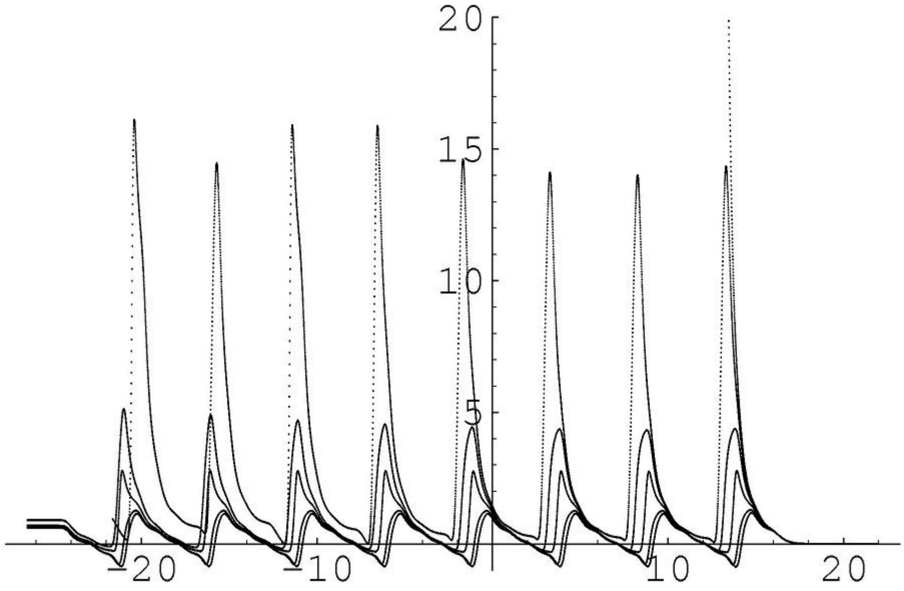

FIG. 2: The coupling constants $g_f(\lambda)$ in the case of a limit cycle, plotted as a function of $\ln\lambda/\ln b$ for 6 values of f: $f = 0$ (Wegner), $f = 0.2$, 0.5, 0.75, 0.9, and 1. The correspondence between a curve and f is such that the curves for larger f reach higher and for $f = 1$ the corresponding curve apparently shoots to infinity already around $\ln\lambda/\ln b \sim 15$. The ultraviolet cutoff is at b^{16} and $b = 4$.

Conclusion

- Generator G with $f = 1$ (energy counted without interactions) gives

  $g_{\lambda \sim mass} \to \infty$ because of BS (2 × 2 mechanism)

- Generator G with $f = 0$ (energy counted with interactions) gives

  $|g_\lambda| \lesssim 1$ including scales of binding

- Limit cycle (including periods of AF) requires interaction in G

  (no hope for understanding hierarchy of bound states using $f = 1$)

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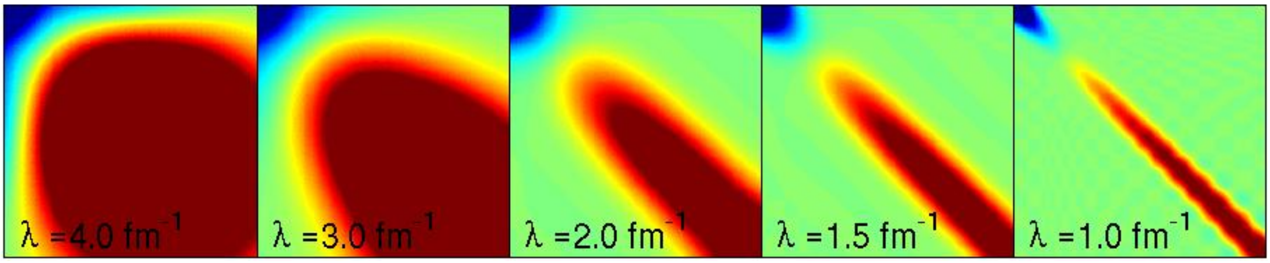
TABLE I: SRG parameters g and λ for frames shown in the earlier figure, numbered from the top to bottom. In this example, $g = 0.040002$, $h = 0$, $b = 4$, $M = -25$, $N = 16$, and all displayed numbers are rounded to 6 decimal places.

frame	$\ln(\lambda)/\ln b$	$g(\lambda)$
1 (top)	22.780321	0.040002
2	2.766096	0.092055
3	-6.864809	0.600768
4	-8.369638	1.234710
5	-8.570281	0.891475
6	-9.071891	-0.680443
7	-12.282193	-0.226083
8 (bottom)	-27.330482	-0.060769

- QCD
- Nuclear Physics
- Condensed Matter Physics F. Wegner



Picture from: http://www-physics.mps.ohio-state.edu/∼ntg/srg/

İllustraion from: E.D. Jurgenson, S.K. Bogner, R.J. Furnstahl, R.J. Perry, arXiv:0711.4252v1 [nucl-th]

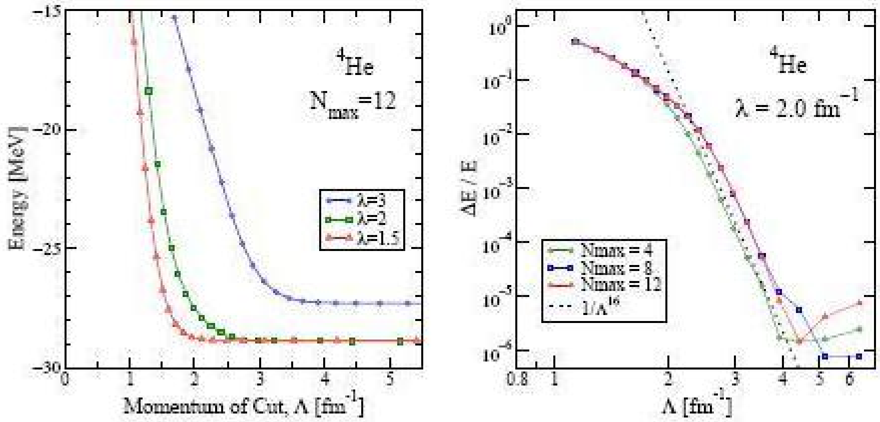


Fig. 5. Calculations of the ^{4}He ground-state energy using the NCSM. On the left is the energy obtained from the NCSM for potentials evolved to several different λ values as a function of the cut (regulator) momentum Λ with $n = 8$. On the right is the relative error of the energy for the $\lambda = 2\,\mathrm{fm}^{-1}$ case as a function of the cut momentum (with $n = 8$) for several different harmonic oscillator basis sizes. Also shown is the slope of the error in the decoupling region predicted from perturbation theory (dotted line).

Illustraion from: E.D. Jurgenson, S.K. Bogner, R.J. Furnstahl, R.J. Perry, arXiv:0711.4252v1 [nucl-th]

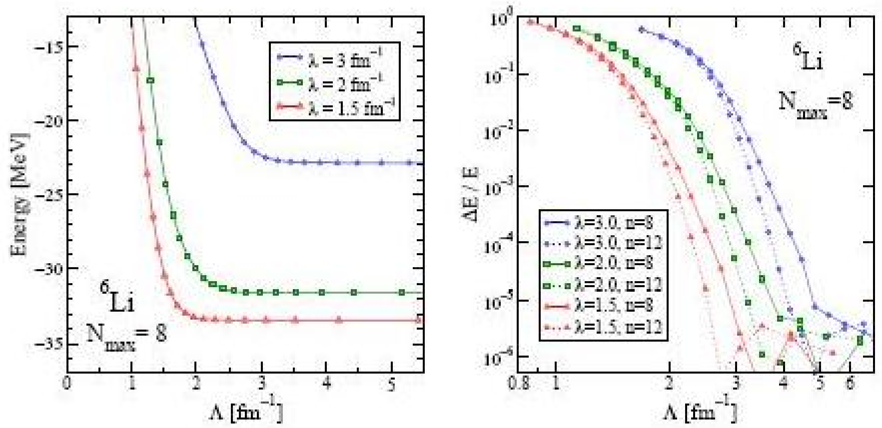


Fig. 6. Calculations of the ^{6}Li ground-state energy using the NCSM. On the left is the energy obtained from the NCSM for potentials evolved to several different λ values as a function of the cut (regulator) momentum Λ with $n = 8$. On the right is the relative error of the energy for the same λ's as a function of the cut momentum for the same λ values but with two values of n.

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