Description of anisotropic plasma at strong coupling

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'Towards the description of anisotropic plasma at strong coupling',
(R. Janik, PW, arXiv:0806.2141).

Outline

- Part I: The setup and the goal
- Part II: Results
- Part III: Conclusions

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- Two approaches: weak and strong coupling.
- From the side of pure QCD the problem for now is very hard.
- AdS/CFT may help to understand some general principles that governs the QGP via weak/strong coupling duality.

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- We need to find a mechanism of fast isotropisation and termalization.
- One possibility suggested involves an appearience on instabilities in the QGP.
- Thus we focus on the problem of stability of the geometry dual to the boundary gauge field configuration.

The situation at weak coupling

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- At weak coupling one can turn to numerical calculations concerning the dynamics of the energy momentum tensor:

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_L(t) & 0 & 0 \\ 0 & 0 & p_T(t) & 0 \\ 0 & 0 & 0 & p_T(t) \end{pmatrix}$$

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- Another option is to compute the poles of the gluon propagator in an anisotropic system

The situation at weak coupling

- One then considers a momentum distribution in the medium:

$$f(p) = \sqrt{1+\xi} f_{iso}(p^2 + \xi p_L^2)$$

with ξ being an anisotropy parameter:

$$\xi = \frac{p_T}{p_L} - 1$$

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  - for $\xi > 0$ modes with transverse momentum will remain stable and those with longitudianl momentum will be unstable,
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- What is good is that this behavior may be identified with the initial stage of the numerical simulation mentioned above (for time dependent $T_{\mu\nu}$).
- It would be interestring to see this situation at the strong coupling.

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- We are interested in constructing a geometry dual to anisotropic configuarion of static QGP.
- We solve eqaution of motion:

$$R_{\alpha\beta} + 4g_{\alpha\beta} = 0$$

with the following ansatz in F-G coordinates:

$$ds^2 = \frac{1}{z^2}\left(-a(z)dt^2 + b(z)dx_L^2 + c(z)dx_T^2 + dz^2\right)$$

subject to boundary condition:

$$g_{\mu\nu}(x^\mu, z) = \eta_{\mu\nu} + z^4 \frac{2\pi}{N_c^2}\left\langle T_{\mu\nu}(x^\mu)\right\rangle .$$

The framework

- The energy momentum tensor of our plasma model is:

$$\langle T_{\mu\nu} \rangle = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_L & 0 & 0 \\ 0 & 0 & p_T & 0 \\ 0 & 0 & 0 & p_T \end{pmatrix}$$

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  with $\varepsilon = p_L + 2p_T$.
- Along with the condition for the functions from the ansatz to vanish at the boundary $z = 0$, we are able to solve Einstein equations.

The solution

- The solution reads:

$$
\begin{aligned}
a(z) &= (1+A^2z^4)^{\frac{1}{2}-\frac{1}{4}\sqrt{36-2B^2}}(1-A^2z^4)^{\frac{1}{2}+\frac{1}{4}\sqrt{36-2B^2}} \\
b(z) &= (1+A^2z^4)^{\frac{1}{2}-\frac{B}{3}+\frac{1}{12}\sqrt{36-2B^2}}(1-A^2z^4)^{\frac{1}{2}+\frac{B}{3}-\frac{1}{12}\sqrt{36-2B^2}} \\
c(z) &= (1+A^2z^4)^{\frac{1}{2}+\frac{B}{6}+\frac{1}{12}\sqrt{36-2B^2}}(1-A^2z^4)^{\frac{1}{2}-\frac{B}{6}-\frac{1}{12}\sqrt{36-2B^2}}
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\end{aligned}
$$

- Parameters A and B are related to the energy density and anisotropy of the plasma through the duality (boundary condition):

$$
\begin{aligned}
\varepsilon &= \frac{1}{2}A^2\sqrt{36-B^2} \\
p_L &= \frac{1}{6}A^2\sqrt{36-B^2} - \frac{2}{3}A^2B \\
p_T &= \frac{1}{6}A^2\sqrt{36-B^2} + \frac{1}{3}A^2B
\end{aligned}
$$

- B can also be linked with the anisotropy parameter ξ:

$$
B = \frac{6\xi}{\sqrt{18\xi^2+48\xi+36}}.
$$

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- But thus we may have problem constructing meaningful boundary condition at the singularity.
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- However it is strong indication that this solution may not be fully physical one, it may be just a snaphot of early time dependent evolution.
- Nevertheles we will try to investigate small fluctuations around this solution in aim to compare it with weak coupling result.

Small fluctuations

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- After puting $A = 1$ and seperating variables,

$$\Phi = \phi(z)e^{-i\omega t + ik_1 x^1 + ik_3 x^3}$$

and a change of variable

$$x = \frac{1}{4} arctanh z^4$$

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- We obtain:

$$\frac{d^2\phi}{dx^2} + \frac{8}{(e^{16x}-1)^{\frac{3}{2}}} \left(\omega^2 e^{2(6+\sqrt{36-2B^2})x} - k_L^2 e^{2(6+\frac{4B}{3}-\frac{1}{3}\sqrt{36-2B^2})x} + \right.$$
$$\left. - k_T^2 e^{2(6-\frac{2B}{3}-\frac{1}{3}\sqrt{36-2B^2})x} \right) \phi = 0$$

Small fluctuations

- Now the singularity is located at $x = \infty$ and asymptitically we have domination of the term accompanying the frequecy:

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- In principle this equation has as a solution combination of Hankel functions:

$$H_0^{1,2}\left(\frac{\sqrt{8}}{C}\omega e^{-Cx}\right)$$

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- It is very different from the situation of negative mass black hole, where the momentum terms would dominate and thus not allow for such a notion of boundary conditions.

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- Now we would like to see how this relation is modified by the anisotropy.
- We need to solve the Maxwell equations:

$$\partial_\alpha \left(\sqrt{-g}F^{\alpha\beta}\right) = 0$$

- Once again we will utilize translational invariance,

$$A_\mu(x) = \int \frac{d^3k d\omega}{(2\pi)^4} e^{-i\omega t + i\vec{k}\vec{x}} A_\mu(z, k)$$

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- This leads to two sets of modes:
  Longitudinal modes:

$$\text{(L-L)} \; E_y(k_l, z) = \omega A_y(k_l, z) + k_l A_t(k_l, z), \; k_l || E_y$$
$$\text{(L-T)} \; E_1(k_l, z) = \omega A_1(k_l, z), \; E_2(k_l, z) = \omega A_2(k_l, z)$$

  and transverse ones:

$$\text{(T-T)} \; E_1(k_t, z) = \omega A_1(k_t, z) + k_t A_t(k, z), \; k_t || E_1,$$
$$\text{(T-L)} \; E_y(k_t, z) = \omega A_y(k_t, z), \; E_2(k_t, z) = \omega A_2(k_t, z)$$

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$$E_1'' - \frac{3 + 4\sqrt{A}(3+B)z^4 + 9Az^8}{3z(1-Az^8)} E_1'$$
$$+ \left(\frac{\omega^2(1+\sqrt{A}z^4)}{(1-\sqrt{A}z^4)^2} - k_L^2(1-\sqrt{A}z^4)^{-\frac{B}{3}}(1+\sqrt{A}z^4)^{-1+\frac{B}{3}}\right)E_1 = 0,$$

$$E_y'' - \frac{3k_L^2(1-\sqrt{A}z^4)^2(1+\sqrt{A}z^4)^{\frac{B}{3}}(1-3\sqrt{A}z^4(4-\sqrt{A}z^4))}{3z(1-Az^8)(k_L^2(1-\sqrt{A}z^4)^2(1+\sqrt{A}z^4)^{\frac{B}{3}} - \omega^2(1-\sqrt{A}z^4)^{\frac{B}{3}}(1+\sqrt{A}}$$
$$\frac{\omega^2(1-\sqrt{A}z^4)^{\frac{B}{3}}(1+\sqrt{A}z^4)^2(3+\sqrt{A}z^4(12-8B+9\sqrt{A}z^4))}{3z(1-Az^8)(k_L^2(1-\sqrt{A}z^4)^2(1+\sqrt{A}z^4)^{\frac{B}{3}} - \omega^2(1-\sqrt{A}z^4)^{\frac{B}{3}}(1+\sqrt{A}z^4)^2)}$$
$$+ \left(\frac{\omega^2(1+\sqrt{A}z^4)}{(1-\sqrt{A}z^4)^2} - k_L^2(1-\sqrt{A}z^4)^{-\frac{B}{3}}(1+\sqrt{A}z^4)^{-1+\frac{B}{3}}\right)E_y = 0$$

$$E_2'' - \frac{3+4\sqrt{A}(3+B)z^4+9Az^8}{3z(1-Az^8)}E_2'$$
$$+(\frac{\omega^2(1+\sqrt{A}z^4)}{(1-\sqrt{A}z^4)^2} - k_T^2(1-\sqrt{A}z^4)^{-\frac{B}{6}}(1+\sqrt{A}z^4)^{-1-\frac{B}{6}})E_2 = 0,$$

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$$E_1'' + (\frac{-3k_T^2(1-\sqrt{A}z^4)^{2+\frac{B}{6}}(1-3\sqrt{A}z^4(4-\sqrt{A}z^4))}{3z(1-Az^8)(k_T^2(1-\sqrt{A}z^4)^{2+\frac{B}{6}}-\omega^2(1-\sqrt{A}z^4)^{2+\frac{B}{6}})}+$$
$$\frac{\omega^2(1+\sqrt{A}z^4)^{2+\frac{B}{6}}(3+4\sqrt{A}(3+B)z^4+9Az^8)}{3z(1-Az^8)(k_T^2(1-\sqrt{A}z^4)^{2+\frac{B}{6}}-\omega^2(1-\sqrt{A}z^4)^{2+\frac{B}{6}})})E_1'$$
$$+(\frac{\omega^2(1+\sqrt{A}z^4)}{(1-\sqrt{A}z^4)^2} - k_T^2(1-\sqrt{A}z^4)^{-\frac{B}{6}}(1+\sqrt{A}z^4)^{-1-\frac{B}{6}})E_1 = 0$$

R-charge fluctuations

- But can be solved perturbatively using:

$$g(u) = 1 + \varepsilon g_0^a(u) + \varepsilon^2 g_0^b(u) + B(g_1^a(u) + \varepsilon g_1^b(u) + \ldots) + \ldots$$

with $u = z^2$.

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- After imposing incoming boundary condition one finally gets to this order:

$$\omega = -i\frac{k_L^2 + \sqrt{k_L^4 - \frac{16}{3}A^{\frac{1}{4}}Bk_L^2}}{4\sqrt{2}A^{\frac{1}{8}}}$$

  and

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Conclusions

References