# The Selforganized Universe

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what is QG ?

Dyson: Whenever I try to imagine a gedanken experiment which probes the quantum nature of gravity I end up with a black hole.

String theory offers (in my opinion) disappointingly little insight in QG (if it exists)

How does $\langle g_{\mu\nu}(x) \rangle$ emerge from nothing and how do we study the quantum fluctuations around $\langle g \rangle$?

(This will be the topic of my lectures!)

Conformal inv. (symmetry of ST) tells us around which $\langle g_{\mu\nu}(x)\rangle$ we can have a consistent ST

$$S = \int d^2\xi \, \sqrt{h} \, h^{\alpha\beta} \langle g_{\mu\nu}(x)\rangle \, \partial_\alpha X^\mu \, \partial_\beta X^\nu$$

But SFT, the "meta" theory which determines the dynamics of $\langle g_{\mu\nu}(x)\rangle$ + fluctuations is not really developed.

This is the old program of 86–89: The natural space of SFT: space of 2d field theories, $\langle g_{\mu\nu}(x)\rangle$ being the extremum and our universe the natural minimum

Many reasons this program did not succeed.

The second string revolution did not really improve the situation from p.o.f of QG : An even larger zoo of choices.

In fact it seems that more and more string people give up and appeal to the antropic principle for $\langle g_{\mu\nu}\,\alpha_i \rangle$

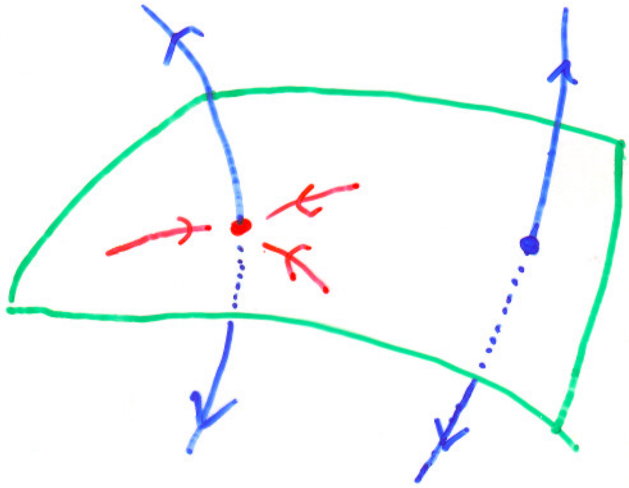
LHC is an interesting test. ( SUSY )

For me : SUSY : Take ST serious

No SUSY : ST ≡ epicycles

Awaiting these results (and anticipating that SUSY will not be obs.) we have an obligation to look elsewhere. Maybe simpler solution ?

# Asymptotic safety (Weinberg 79)



Wilsonian picture: critical surface of finite co-dimension

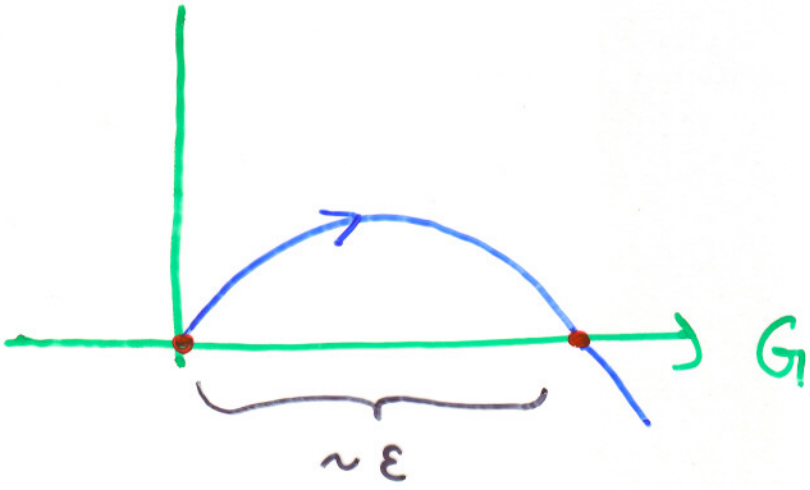
In principle it provide us with a non-perturbative def. of a QFT:

(1): Find a fixed point

(2): prove CS has finite co-dimension

Success : Fisher-Wilson fixed point in 3d

Weinberg used $2+\varepsilon$ d expansion to argue that there exists a non-trivial fixed point for QG in $d=4$ $(\varepsilon=2)$




Elaborated on by Kawai et al (early 90's)

Reuter et al, Litim, ..... last 10 years :

Exact renormalization group

Some evidence

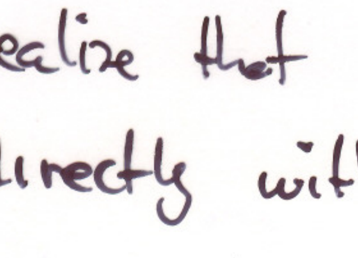
Problem: truncation of effective action.

Another approach, in the same spirit, comes from performing an explicit lattice-regularization.

Several Lattice approaches. Here only Dynamical triangulation (DT).

How to put a diff. inv. theory on lattice? realize that diff. inv. is the desire to deal directly with geometry.

Piecewise linear geometry (PLG) can be described without the use of coordinates:

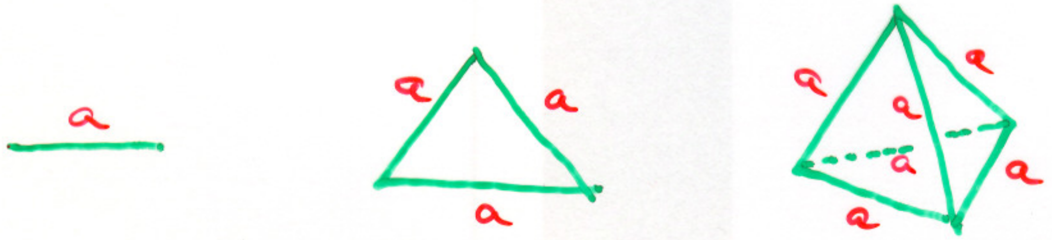


2d: Just provide length of sides and a table of neighboring triangles.

curvature lives on vertices

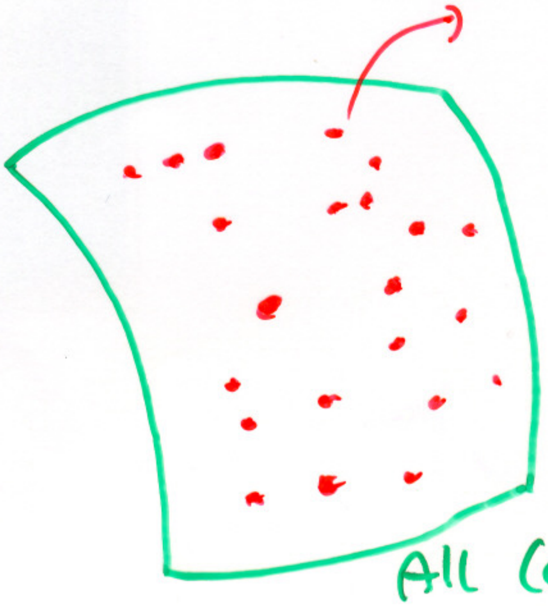
This can be generalized to higher dimensions : curvature lives on d−2 dimensional hinges.

We will make it even simpler by restricting ourselves to simple Building Blocks (BB)




a = Lattice spacing

PLG - BB



Does it work?

PLG-BB geometries a small subset of all geometries.

All (continuous) Geometries

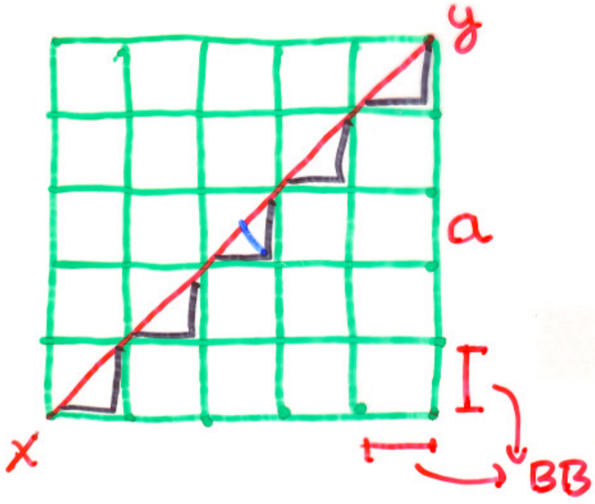
$$\sum_{PLG(a\to 0)} \longrightarrow \int \mathcal{D}[g_{\mu\nu}] \quad ?$$

Clear that we can no longer approx. a given (say) smooth geometry in an obvious way with BB

<u>Ex</u> Free particle in D Eucl. dimensions with geometric action:

$$S(P) = m \int_P d\ell = m\, L(P)$$

Hypercubic lattice : BB 




$$S(P_L) \geq \sqrt{2}\, S(P_{cont})$$

In what sense is the lattice $P_L$ close to $P_{cont}$ for $a \to 0$ ?

$$d(P_{cont}, P_L) = \max \{ d(x, P_{cont}) \mid x \in P_L \}$$

$$d(x, P_{cont}) = \min \{ d(x,y) \mid y \in P_{cont} \}.$$

This defines a distance measure on the space of path from x to y.

One can easily show that the BB-approach works for a free particle, but a more serious test is Euclidean 2d QG.

$$Z(\Lambda, G) = \int \mathcal{D}[g_{\mu\nu}]\, e^{-S[g_{\mu\nu}]}$$

$$S[g] = \Lambda \int d^2\xi \sqrt{g} - \frac{1}{G} \int d^2\xi \sqrt{g}\, R$$

As long as no topology change: $\int d^2\xi \sqrt{g}\, R = \text{const.}$

$$Z(\Lambda) = \int_0^\infty dV\, e^{-\Lambda V} \cdot \int_{\substack{V_g = V}} \mathcal{D}[g] \cdot \text{“}1\text{”}$$

$$V_g \equiv \int d^2\xi \sqrt{g}$$

$$Z(\Lambda) = \int_0^\infty dV\, e^{-\Lambda V}\, \mathcal{N}([g] \sim V)$$

$\boxed{\text{Entirely combinatorial}}$

Our BB-approach is ideal for counting $\mathcal{N}([g]=V)$

$$Z(\Lambda) = \sum_T e^{-\Lambda V} = \sum_T e^{-\mu N_T}$$

$$V = \frac{\sqrt{3}}{4} a^2 N_T \quad , \quad \Lambda a^2 = \mu$$

$$Z(\mu) = \sum_N e^{-\mu N} \sum_{T\sim N} 1$$

$$= \sum_N e^{-\mu N} \mathcal{N}(T\sim N)$$

This counting can be done. Combinatorially, (Tutte)
or using matrix models (Kazakov, David, ...)

Results agree with continuum, "standard" calculations of $\int \mathcal{D}[g_{\mu\nu}] e^{-S[g_{\mu\nu}]}$,
also for more complicated observables:

$\boxed{\text{Conclusion}}$: <u>Lattice approach works in 2d</u>

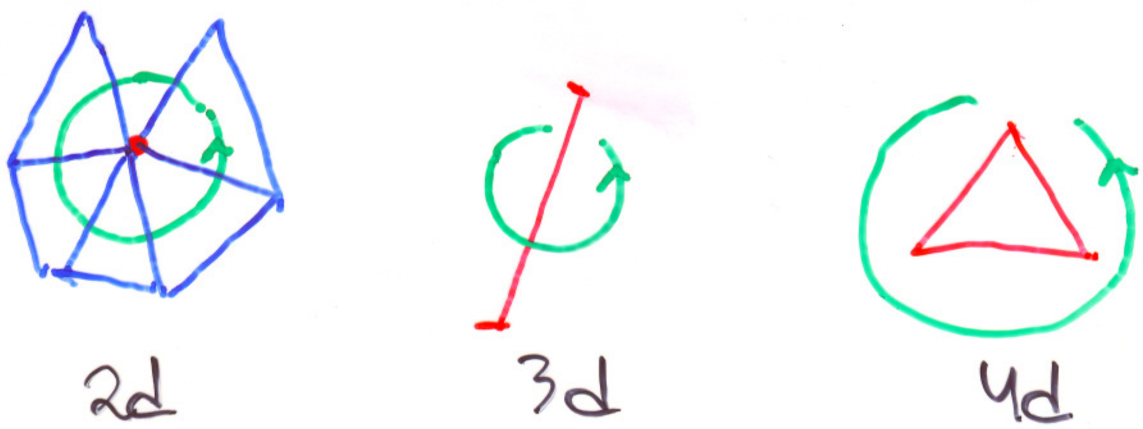
2d : No dynamical graviton

Let us jump to 4d :

BB : Four-simplices of length $a$.
glue together to form space of
fixed topology ( $S^4$, say )
in all possible ways.

what action to choose ?

There is a natural geometric action on PLGs, first discussed by Regge, expressing the curvature, living on the d-2-dim. hinges as defect angles.



Using identical BB the Regge action becomes trivial:

$$S(T) = -C_1 N_2(T) + C_2 N_4(T) = -k_0 N_0(T) + k_4 N_4(T)$$

$$Z(\Lambda, G) = \int \mathcal{D}[g]\, e^{-S[g]}$$

$$Z(k_0, k_4) = \sum_T e^{-S(T)}$$

$$= \sum_{N_0, N_4} e^{k_0 N_0 - k_4 N_4} \sum_{T \sim N_0, N_4} \text{"}1\text{"}$$

$$= \sum_{N_0, N_4} x^{N_0} y^{N_4} \mathcal{N}(T \sim N_0, N_4)$$

$x \sim e^{k_0}$, $y \sim e^{-k_4}$ :

$Z(x, y)$ is the generating function for the numbers $\mathcal{N}(N_0, N_4)$

Unfortunately no analytic solution to this counting problem.

fortunately it is possible to count approximately using MC-simulations

All the results I report about will be based on such numerical "experiments"

After dinner tonight Andrzej Görlich will give a wonderfeel talk with nice "experimental" pictures and tell you how such experiments are actually conducted.

In 2d case it is well tested that MC-simulations can produce the critical exponents of 2d QG.

In the first half of 90 the 5-6 groups carried out such simulation:

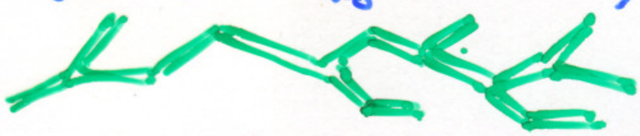
**Conclusion** It almost worked, but not quite .....

$k_0$ small ($G_0 \sim \frac{1}{k_0}$ large): **Crumpled universe**

$D(N_4) \sim$ const.

The natural entropic state of the (Euclidean) universe

$k_0$ large ($G_0 \sim \frac{1}{k_0}$ small) **Elongated universe**



A phase transition separed these two phases.

Had it only been second order ......

## Re-think:

Some general principles which could lead to a different selection of universes, with the same action.

$\boxed{\text{CDT}}$ Causal Dynamical Triangulations

Start in Minkowski space-time, insist on the requirement of causality for each space-time history (Teitelboim) and the existence of a global time coordinate.

(This causality requirement is different from the one we would impose on matter fields.!)

Only then rotate to Euclidean signature if needed.

Recall numerical setup: (Andrzej's talk)

foliation of $S^3 \times R$ ($S^3 \times S^1$)

For computer technical reasons we perform the simulations for fixed values of $N_4$:

$$Z(G,\Lambda) = \int \mathcal{D}[g_{\mu\nu}]\, e^{-\Lambda V_g + \frac{1}{G}\int \sqrt{g} R}$$

$$V_g = \int \sqrt{g}$$

$$Z(G,\Lambda) = \int dV\, e^{-\Lambda V}\, Z(G,V)$$

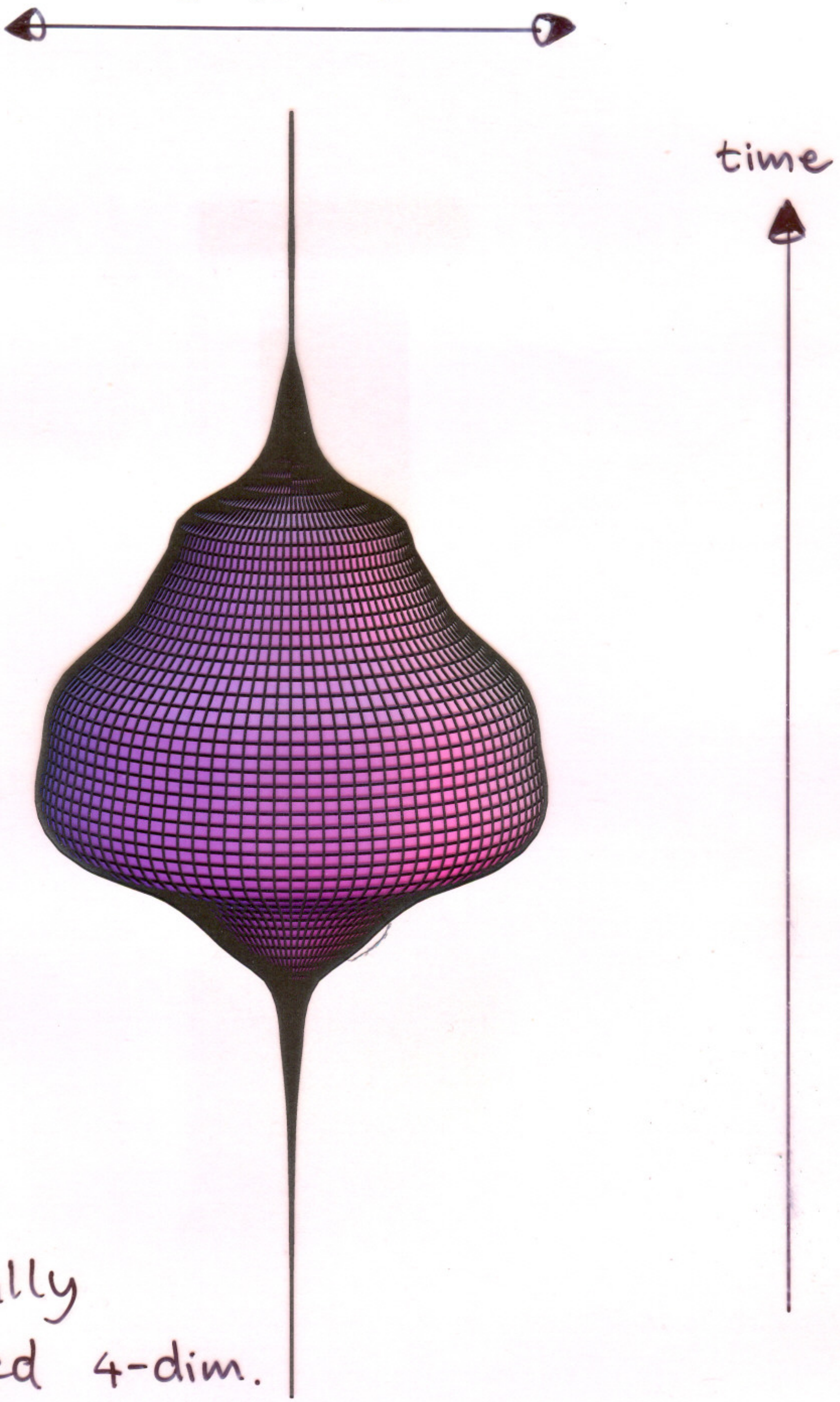
$$Z(G,V) = \int \mathcal{D}[g_{\mu\nu}]\, e^{\frac{1}{G}\int \sqrt{g} R}\, \delta\left(\int \sqrt{g} - V\right)$$

Thus:

$$\sum_{i=1}^{N_T} N_3(i) = N_4$$

What do we see?

(c)




Dynamically generated 4-dim. quantum universe, obtained from a causal path integral.

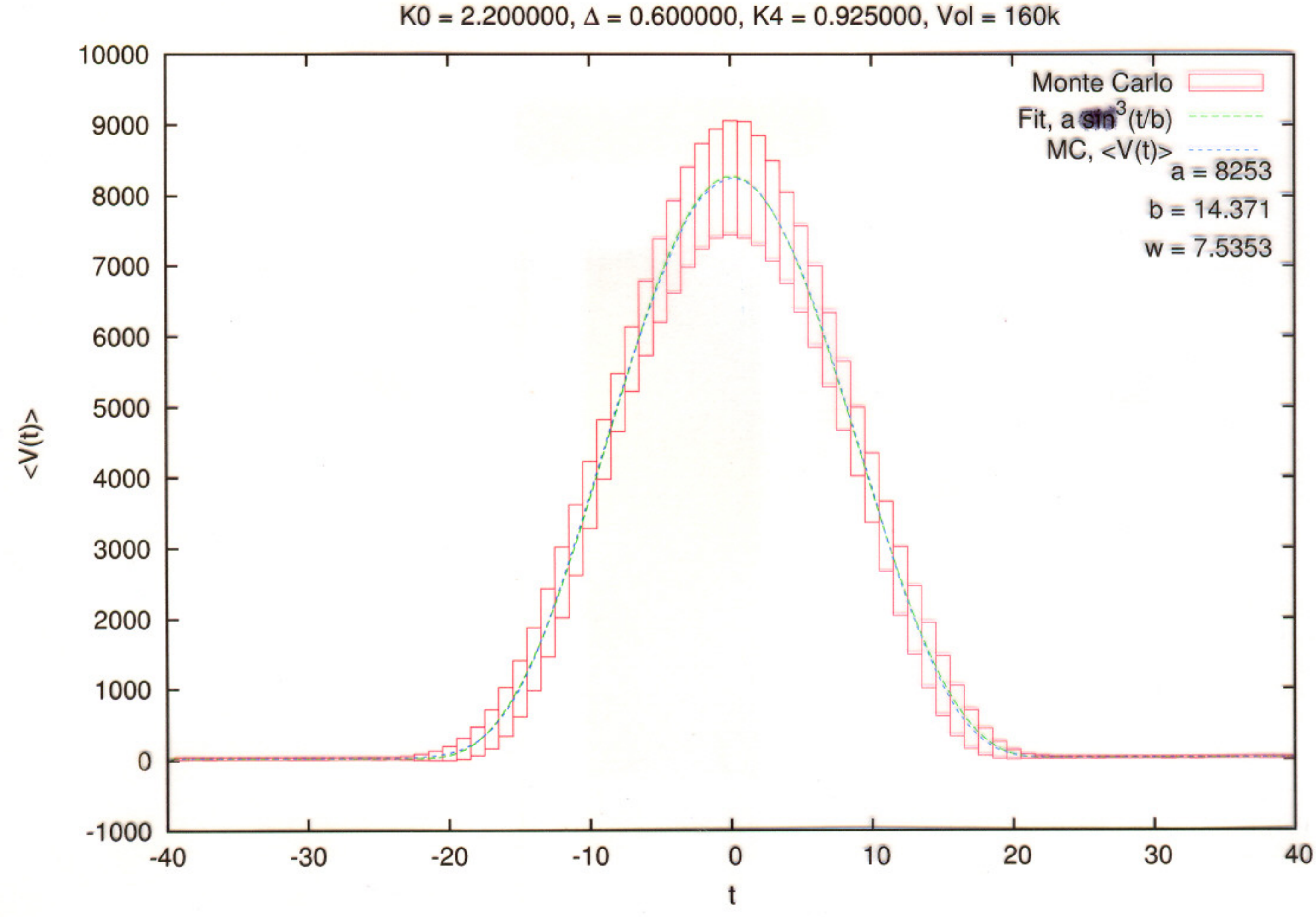
For different $N_4$ we have:

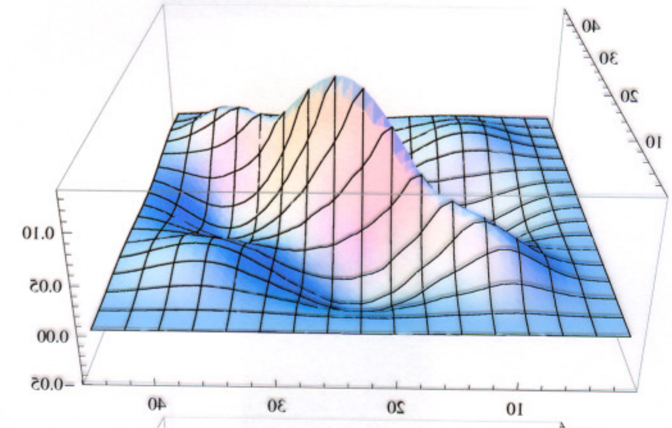
$$\boxed{N_3(i) = N_4 \ \frac{3}{4} \ \frac{1}{s_0 N_4^{1/4}} \cos^3 \frac{i}{s_0 N_4^{1/4}}}$$

$s_0 \approx 0.59$ for $(k_0, \Delta) : (2.2 , 0.6)$

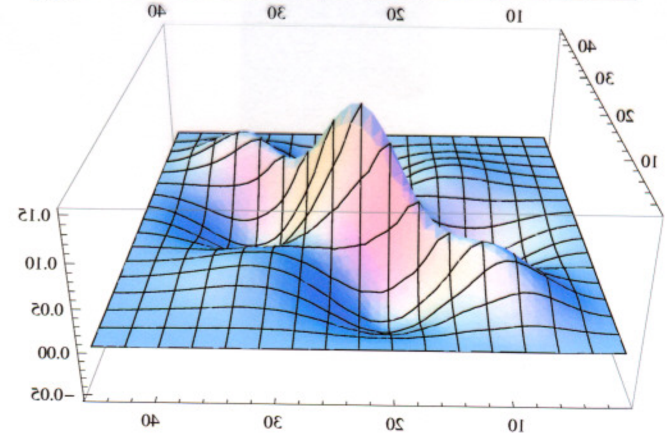
To obtain this curve we have a average over different universes and to compare different universes (comp. universes) we have to fix the center of mass.

Ambiguity up to one lattice spacing (at least)


K0 = 2.200000, Δ = 0.600000, K4 = 0.925000, Vol = 160k
Monte Carlo
Fit, a sin^3(t/b)
MC, <V(t)>
a = 8253
b = 14.371
w = 7.5353
<V(t)>
t
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
-1000
-40
-30
-20
-10
0
10
20
30
40



40
30
20
10
0.10
0.05
0.00
−0.05
40
30
20
10

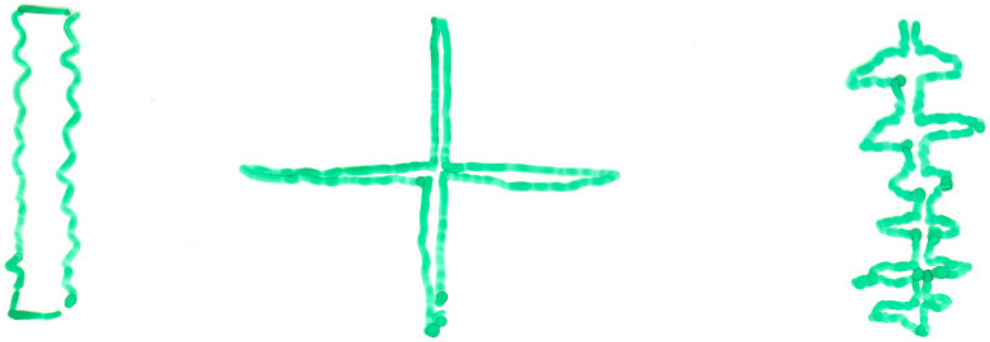
40
30
20
10
0.15
0.10
0.05
0.00
−0.05
40
30
20
10

The scaling shows:

1. Time extent of blob scales like $N_4^{1/4}$
2. spatial extent of blob scales as $N_4^{3/4}$

Thus: canonical time and space dimensions.

Nothing put in by hand: No background
No reason it should not look like:



In fact we see all these "perverse" configurations for different choices of coupling constants.

Lesson from Euclidean approach, going all the way back to DT used in non-critical strings:

$\boxed{\text{Just because you stack } d\text{-dimensional BB you do not neccesarily obtain } d\text{-dim extended objects}}$

Recall mini-superspace cosmology from the 80ties: (Hartle-Hawking .....)

$$ds^2 = g_{tt}(dt)^2 + a^2(t)\, d\Omega_3$$

$$V_3(t) = 2\pi^2 a^3(t)$$

$$S = -\frac{1}{24\pi}\frac{1}{G}\int dt \sqrt{g}\left[\frac{g^{tt}\,\dot{V}_3^2}{V_3(t)} + k_2 V_3^{1/3}\right]$$

$$\int dt \sqrt{g_{tt}}\, V_3(t) = V_4$$

$$\sqrt{g_{tt}}\, V_3(t) = V_4 \frac{3}{4B}\cos^3\frac{t}{B}, \quad \sqrt{g_{tt}} = \frac{R}{B}$$

$$V_4 = \frac{8\pi^2}{3} R^4$$

Natural discretization of this action:

(1) $$S_{Dis} = k_0 \sum_i \left[ \frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{k}_3 N_3^{1/3}(i) \right]$$

(2) $$S_{Dis} = k_0 \sum_i \left[ \frac{(N_3(i+1) - N_3(i))^2}{F(N_3(i))} - U(N_3(i)) \right]$$

While (1) by construction has a "classical" solution of the form observed, it is of interest to check to what extent the data allow us to deduce F and U

We want to derive MSA from first principles!

$$\bar{N}_3(i) = \frac{1}{K} \sum_{k=1}^{K} N_3^{(k)}(i) = \langle N_3(i) \rangle$$

$$C(i,j) = \langle (N_3(i) - \bar{N}_3(i))(N_3(j) - \bar{N}_3(j)) \rangle$$

$$= \frac{1}{K} \sum_{k=1}^{K} (N_3^{(k)}(i) - \bar{N}_3(i))(N_3^{(k)}(j) - \bar{N}_3(j))$$

$$n(i) \equiv N_3(i) - \overline{N}_3(i)$$

$$C(i,j) = \langle n(i)\, n(j) \rangle$$

$$S_{Dis}(N) = S_{Dis}(\overline{N} + n) = S_{Dis}(\overline{N}) + \frac{1}{2} \sum_{i,j} n_i \hat{P}_{ij} n_j + O(n^3)$$

$$C(i,j) \simeq \frac{\int \prod dn_i \; n_i n_j \, e^{-\frac{1}{2}\sum_{i,j} n_i \hat{P}_{ij} n_j + O(n^3)}}{\int \prod dn_i \, e^{-\frac{1}{2}\sum_{i,j} n_i \hat{P}_{ij} n_j + O(n^3)}}$$

In the quadratic approx:

$$\hat{C}(i,j) = (\hat{P}^{-1})_{ij} \quad , \quad \hat{P}_{ij} = (C^{-1})_{ij}$$