Chaos and critical phenomena in gravitational collapse

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Obserwatorium Astronomiczne UJ

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One meteorologist remarked that if the theory were correct, one flap of a seagull's wings would be enough to alter the course of the weather forever.

E. N. Lorenz

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Critical phenomena

Singularity theorems (Penrose 1965, Hawking and Penrose 1970)

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Critical phenomena (Choptuik 1993)

What kind of solution correspond to the critical value of $p = p^*$?

Critical phenomena

- Choptuik's numerical experiment (massless scalar field)
  - black hole mass scaling $M \sim (p - p^*)^\gamma$ – non–generic naked singularity for $p = p^*$
  - universality
  - discrete self-similarity

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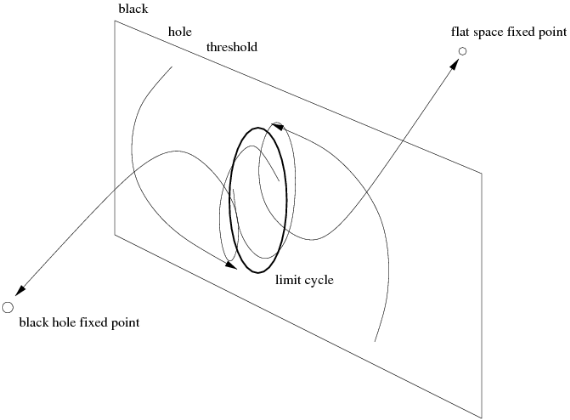
- More then 100 articles devoted to critical phenomena — different matter fields
- Study simpler PDE and search for a counterpart of the critical phenomena
- Dynamical system picture of GR

Critical phenomena



Chaos and critical phenomena in vacuum

The breakthrough — BCS ansatz (Bizoń, Chmaj, Schmidt 2005)

Evade Birkhoff's theorem for a price of going to higher dimensions

$$ds^2 = -Ae^{-2\delta}dt^2 + A^{-1}dr^2 + \frac{1}{4}r^2\left[e^{2B}\sigma_1^2 + e^{2C}\sigma_2^2 + e^{-2(B+C)}\sigma_3^2\right],$$

where A, δ, B, and C are functions of time t, radius r and

$$\sigma_1 + i\,\sigma_2 = e^{i\psi}(\cos\theta\; d\phi + i\; d\theta), \quad \sigma_3 = d\psi - \sin\theta\; d\phi$$

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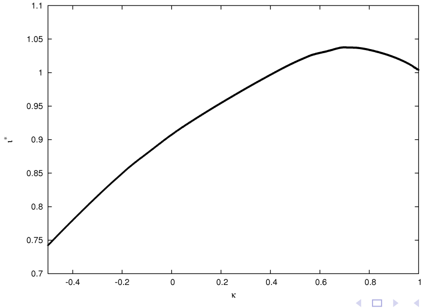
- General picture of the critical phenomena confirmed but interesting new properties
- In context of dynamical system picture of GR the critical phenomena reduce to a study of the basin boundary between two attractors
- Basin boundaries can be either smooth or fractal – so far the second property was never observed in the critical phenomena restricted to regular initial data

Fractal basin boundaries

- Chaotic scattering on three geometrically equivalent copies of the critical solution

The uncertainty dimension

Let S be one–dimensional set in one–dimensional parameter phase space (we have one free parameter κ). The probability that any two random points κ_A, κ_B separated by a distance ϵ belong to different basins $h(\kappa_A) \neq h(\kappa_B)$ scales as $P(\epsilon) \sim \epsilon^{1-dim(S \cap B)}$, where B is a basin boundary.



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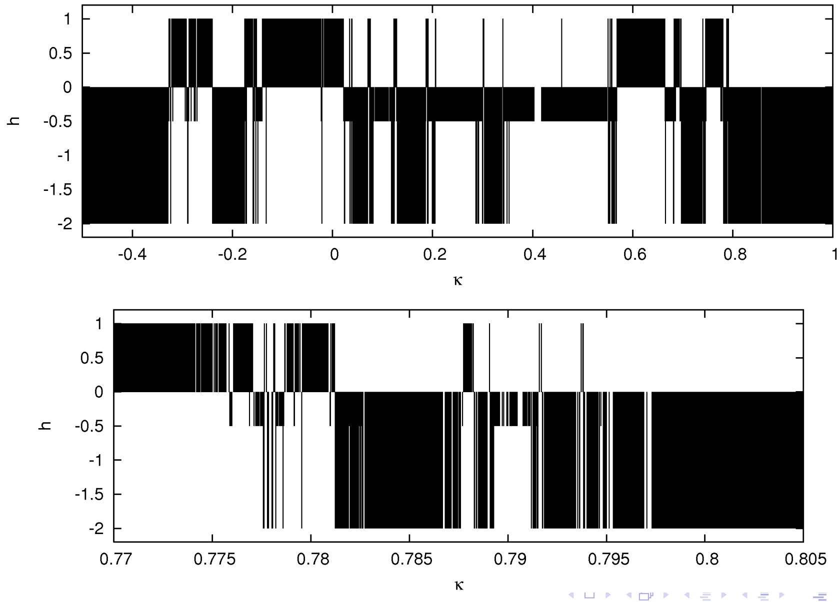

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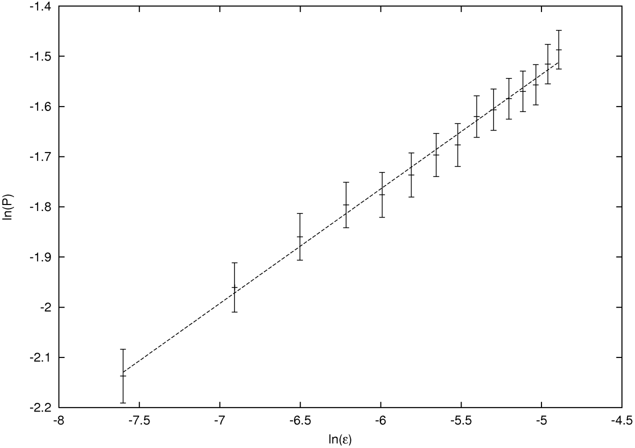

the uncertainty dimension

$$dim(S \cap B) = 0.771 \pm 0.005$$

Conclusions

A flip of the wings of the butterfly may influence the process of black hole formation (at least in this setting — is it more generic phenomenon?)

- the first example of chaos in context of gravitational collapse (regular initial data)
- rich dynamics
- a hint for a different models