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New Developments in Astrophysics and Astroparticle Physics
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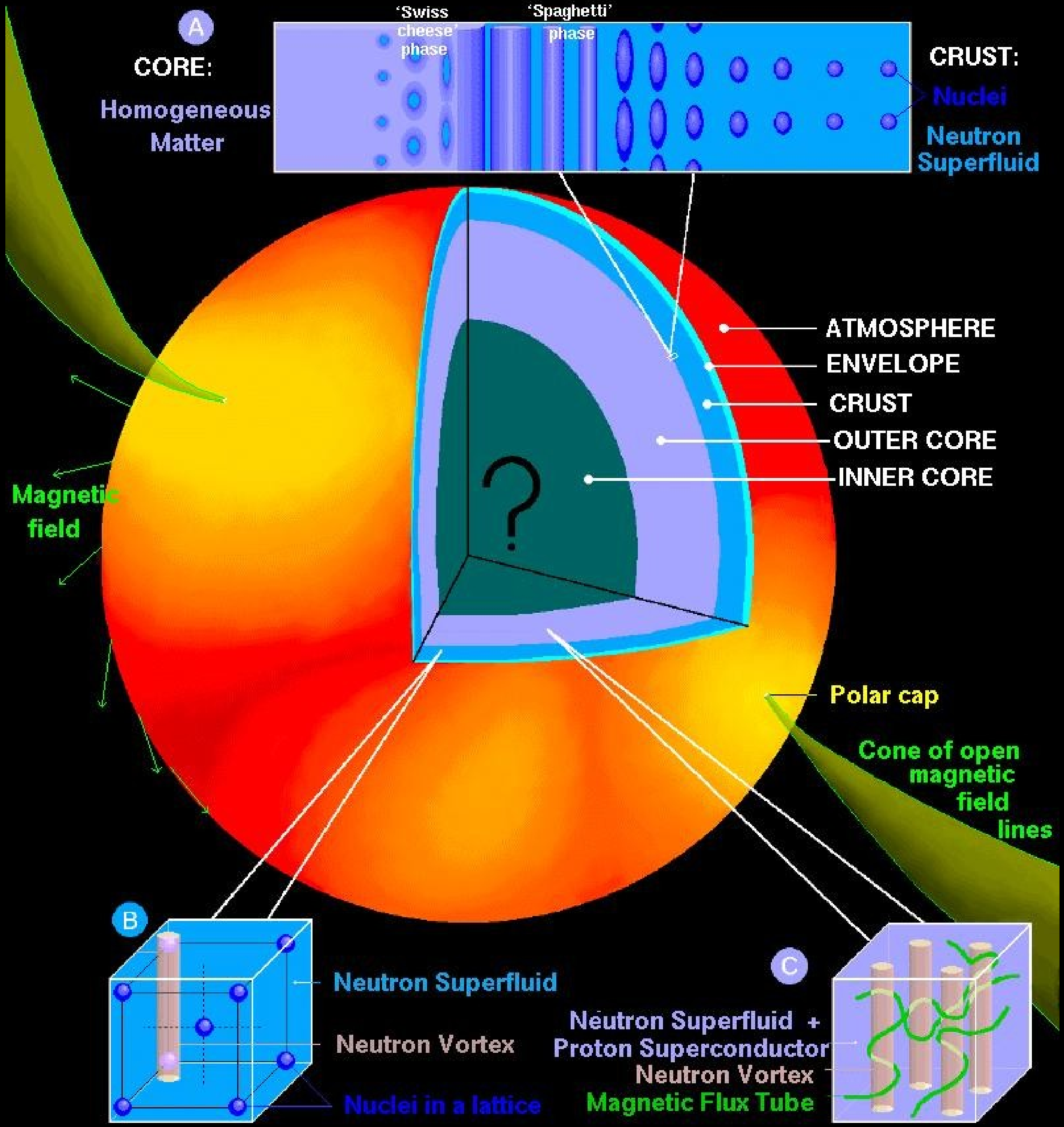
Proton Localization and Magnetic Field Evolution in Dense Neutron Star Matter

Neutron Star Structure

- Atmosphere $(\sim 1\,cm)$ $\rho \le 10\,g\,cm^{-3}$
- Outer crust $10 \le \rho \le 4.3 \cdot 10^{11}\,g\,cm^{-3}$
- Inner crust $4.3 \cdot 10^{11} \le \rho \le 2.4 \cdot 10^{14}\,g\,cm^{-3}$
- Core $2.4 \cdot 10^{14} \le \rho$

A NEUTRON STAR: SURFACE and INTERIOR



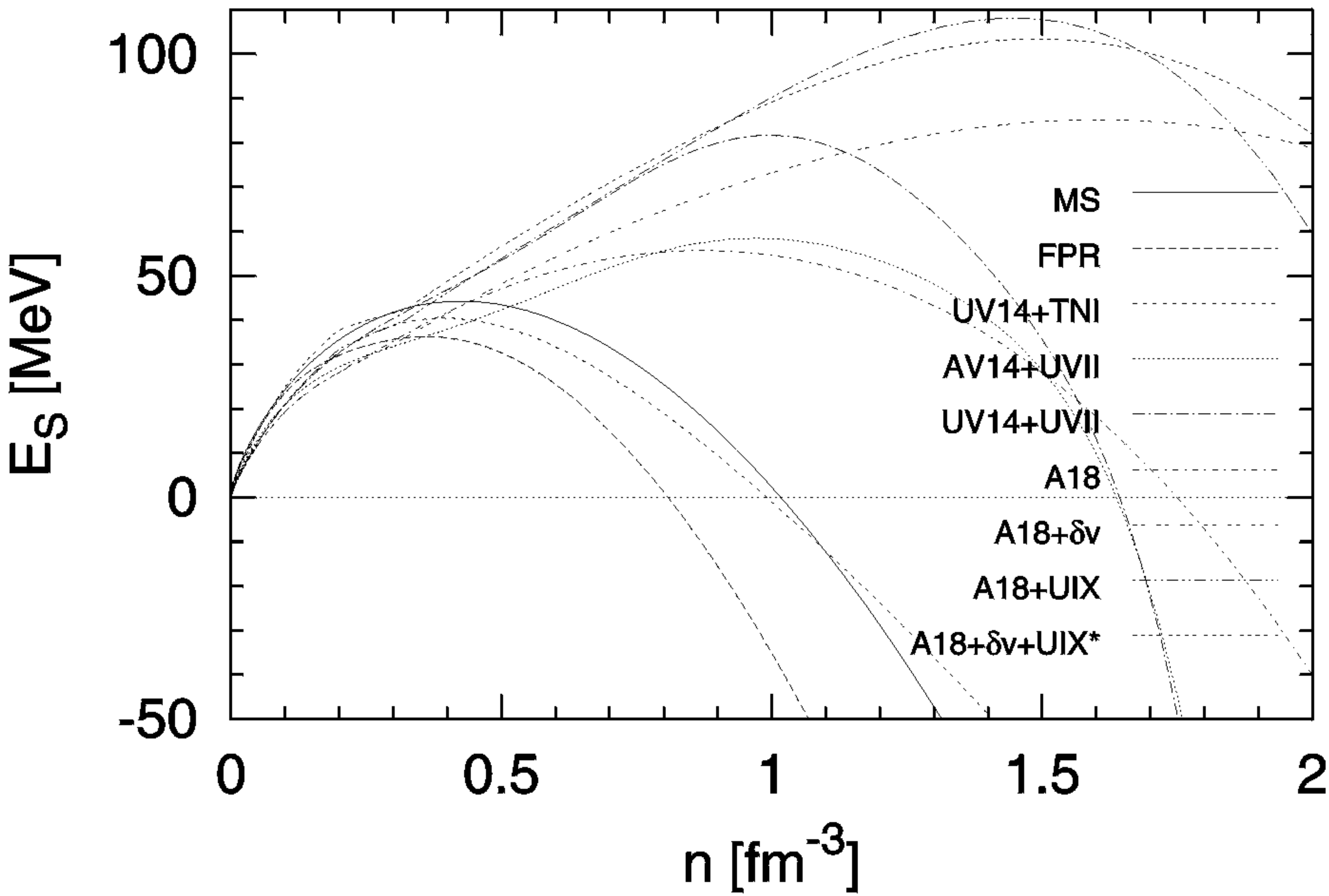
Realistic Nuclear Models:

1. Skyrme (SI', SII', SIII', SL, Ska, SKM, SGII, RATP, T6)
2. Myers-Świątecki (MS)
3. Friedman-Pandharipande-Ravenhall (FPR)
4. UV14+TNI (UV)
5. AV14+UVII (AV)
6. UV14+UVII (UVU)
7. A18
8. A18+δv
9. A18+UIX
10. A18+δv+UIX*

Symmetry Energy of Nuclear Matter

$$E(n,x) = E\left(n,\frac{1}{2}\right) + E_S(n)(2x-1)^2 \qquad x = n_P / n$$

$$E_S(n) = \frac{1}{8}\frac{\partial^2 E(n,x)}{\partial x^2}\bigg|_{x=\frac{1}{2}}$$



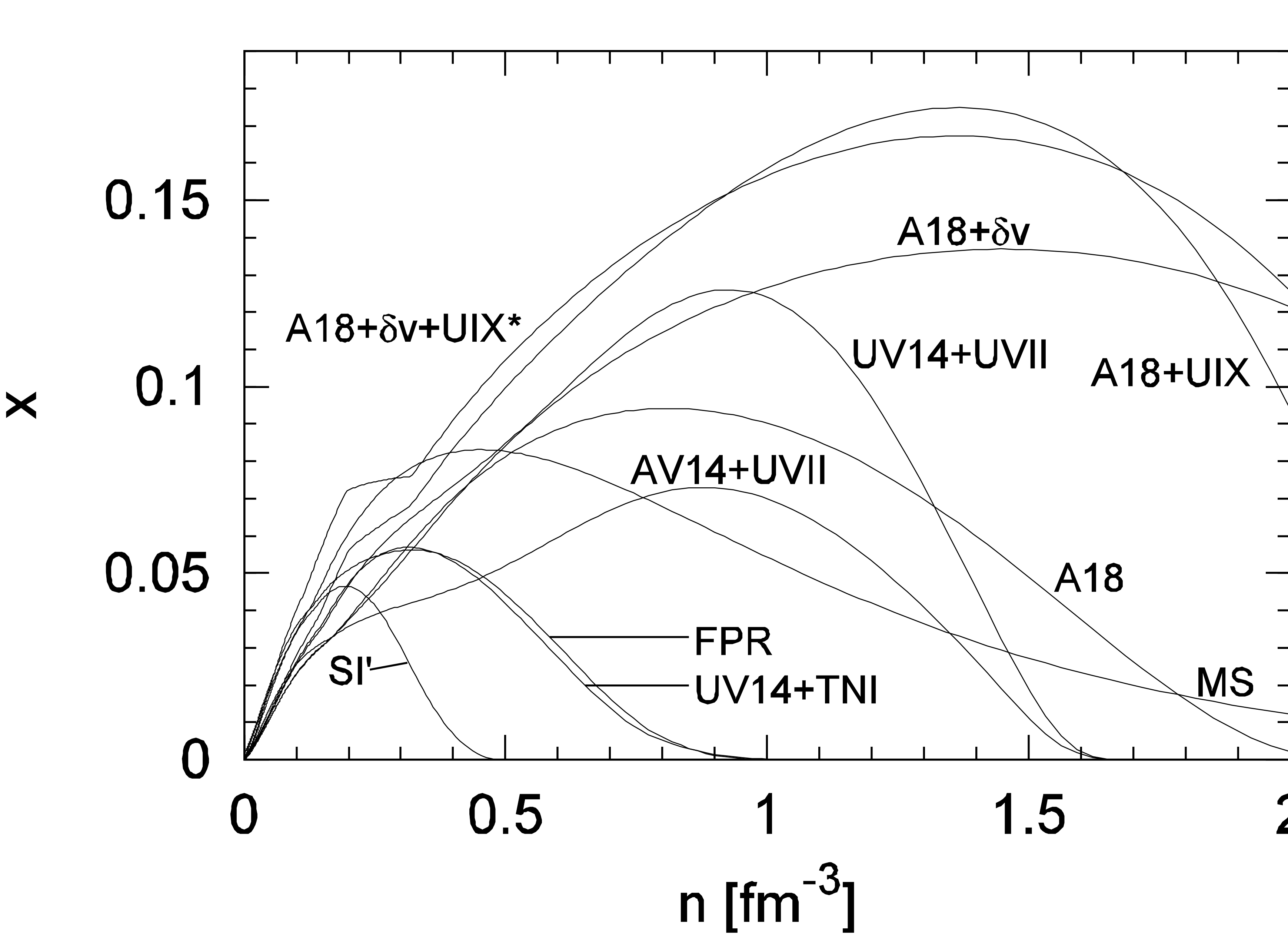
100
50
0
-50
0
0.5
1
1.5
2
E_S [MeV]
n [fm^-3]
MS
FPR
UV14+TNI
AV14+UVII
UV14+UVII
A18
A18+δv
A18+UIX
A18+δv+UIX*

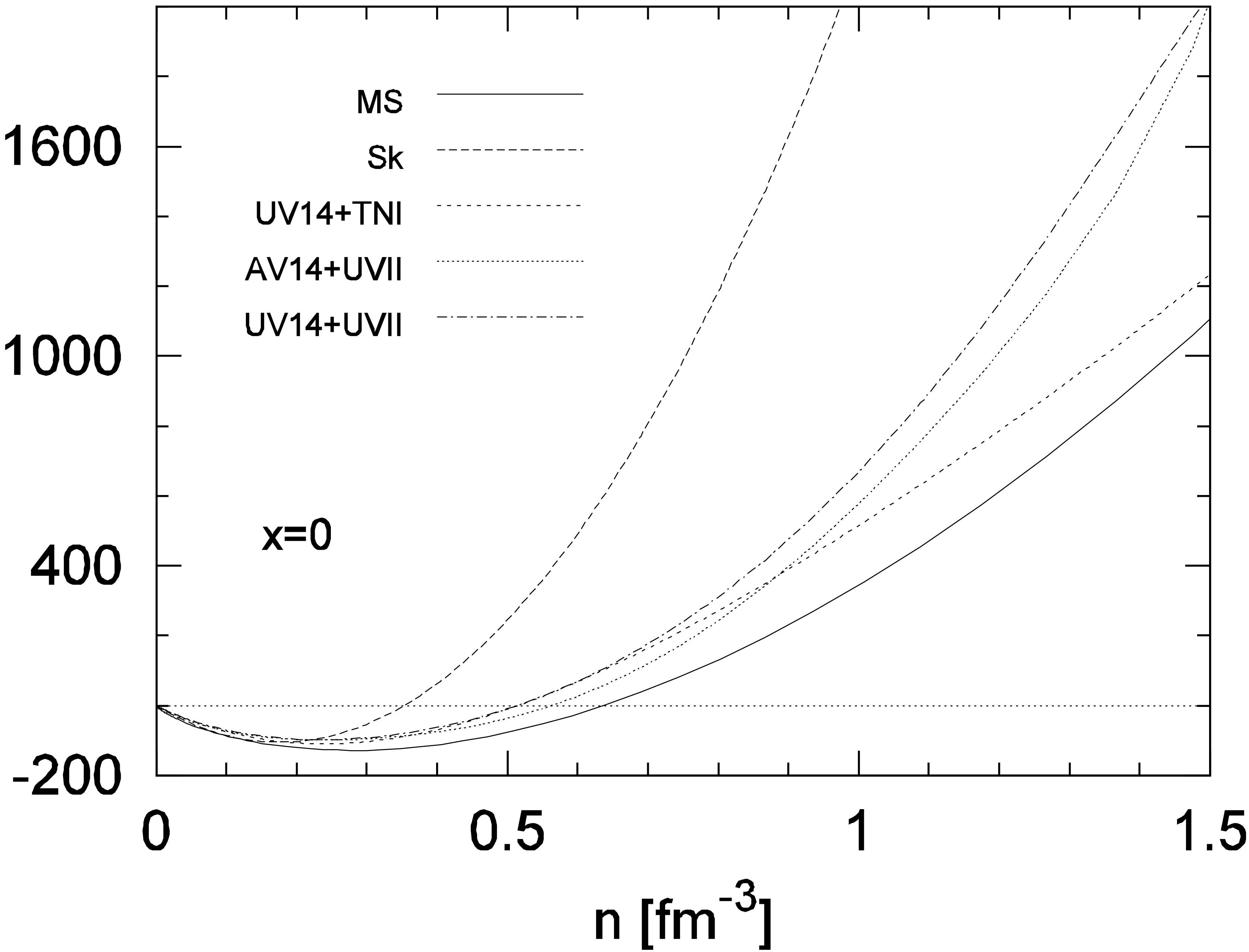
Small values of the symmetry energy:

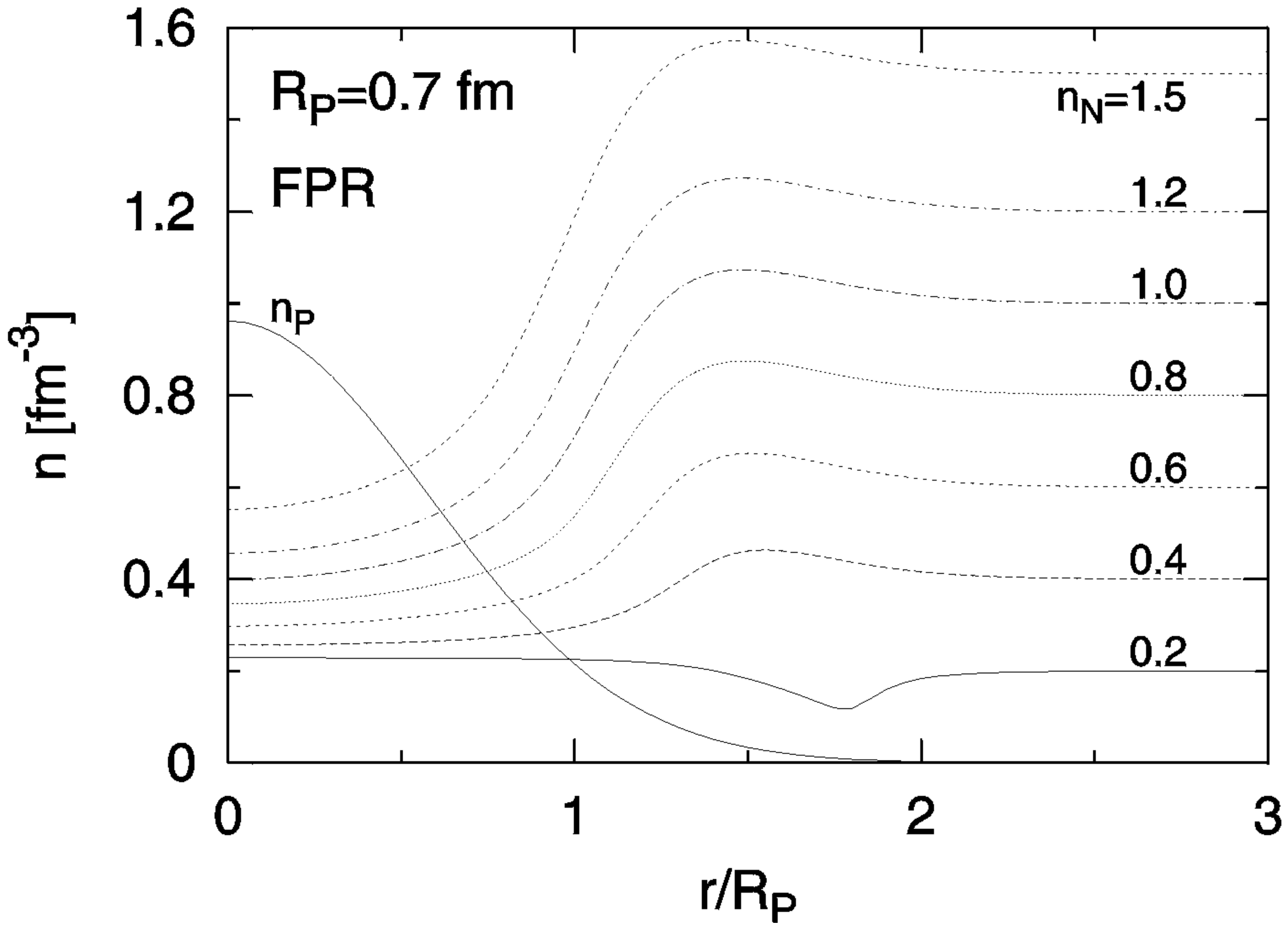
2. Low proton concentration

3. Charge separation instability

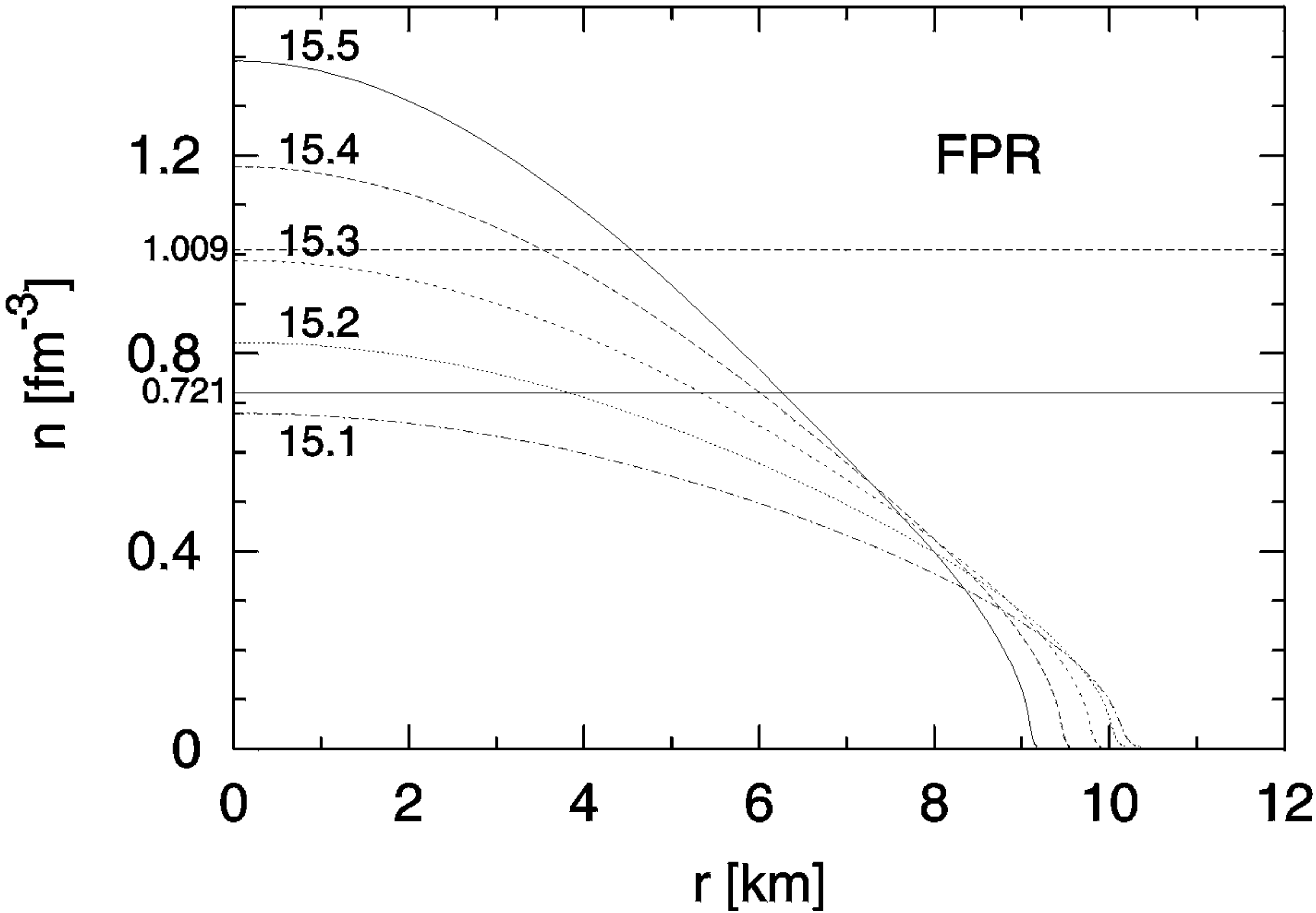
- realized eg. through proton localization


x
0.15
0.1
0.05
0
0
0.5
1
1.5
2
n [fm^{-3}]
A18+δv+UIX*
A18+δv
UV14+UVII
A18+UIX
AV14+UVII
A18
FPR
UV14+TNI
SI'
MS


MS
Sk
UV14+TNI
AV14+UVII
UV14+UVII
x=0
1600
1000
400
-200
0
0.5
1
1.5
μ_P [MeV]
n [fm^{-3}]



1.6
1.2
0.8
0.4
0
R_P=0.7 fm
FPR
n_N=1.5
1.2
1.0
0.8
0.6
0.4
0.2
n_P
n [fm^{-3}]
0
1
2
3
r/R_P



FPR
15.5
15.4
15.3
15.2
15.1
1.009
0.721
n [fm^{-3}]
r [km]
0
0.4
0.8
1.2
0
2
4
6
8
10
12

Spin instability of proton-localized phase

The energy density difference between polarized state and the normal one: $\delta\varepsilon = \delta\varepsilon_N + g^{pn}\delta s_N \delta s_P + \delta\varepsilon_P$

where the spin excess: $\delta s_N = \delta n_N\uparrow - \delta n_N\downarrow \qquad \delta s_P = \delta n_P\uparrow + \delta n_P\downarrow$

effective coupling constant: $g^{pn} = -2.5\, fm^2$

According to the Landau Fermi-liquid theory the change of the neutron energy density: $\delta\varepsilon_N = \dfrac{1+G_0^{NN}}{2N_N}\left(\delta s_N\right)^2$

where spin-dependent Landau parameter for pure neutron matter: $G_0^{NN} = 1.0$

Density of states at the Fermi level: $N_N = \pi^{-2} m_N k_F^N$

For localized protons their wave functions extend to a limited volume: $\delta\varepsilon_P = 0$

The difference of energy density: $\delta\varepsilon = \dfrac{1+G_0^{NN}}{2N_N}\left(\delta s_N\right)^2 + g^{pn}\delta s_N \delta s_P$

for $\delta s_N = -\dfrac{g^{pn} N_N}{1+G_0^{NN}}\delta s_P$ has the minimum:

$$\delta\varepsilon_{\min} = -\frac{N_N\left(g^{pn}\right)^2}{2\left(1+G_0^{NN}\right)}\left(\delta s_P\right)^2 < 0$$

The system has the minimum of the energy for fully polarized protons: $\delta s_P = n_P$

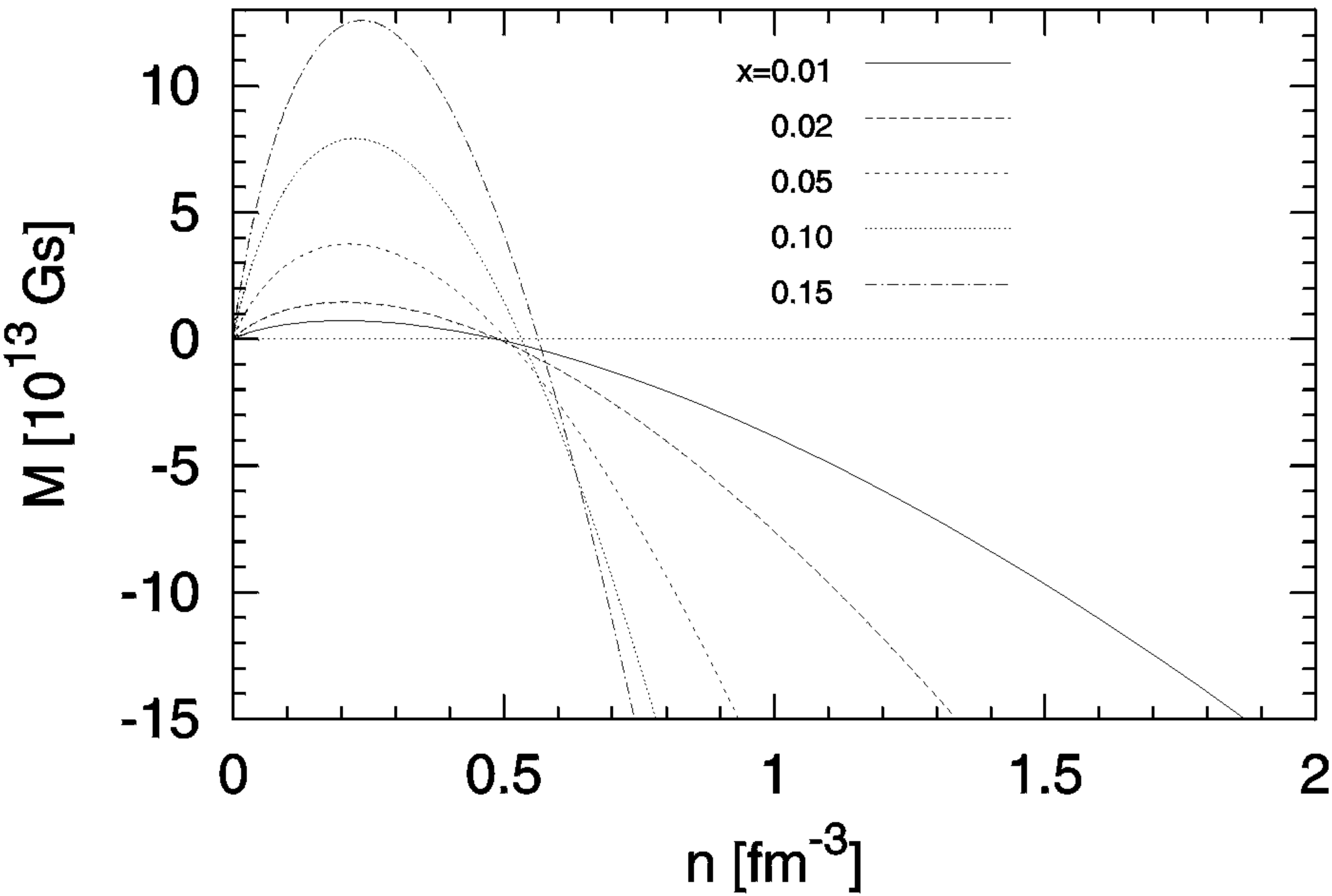
The system with polarized protons is unstable against small spin oscilations.

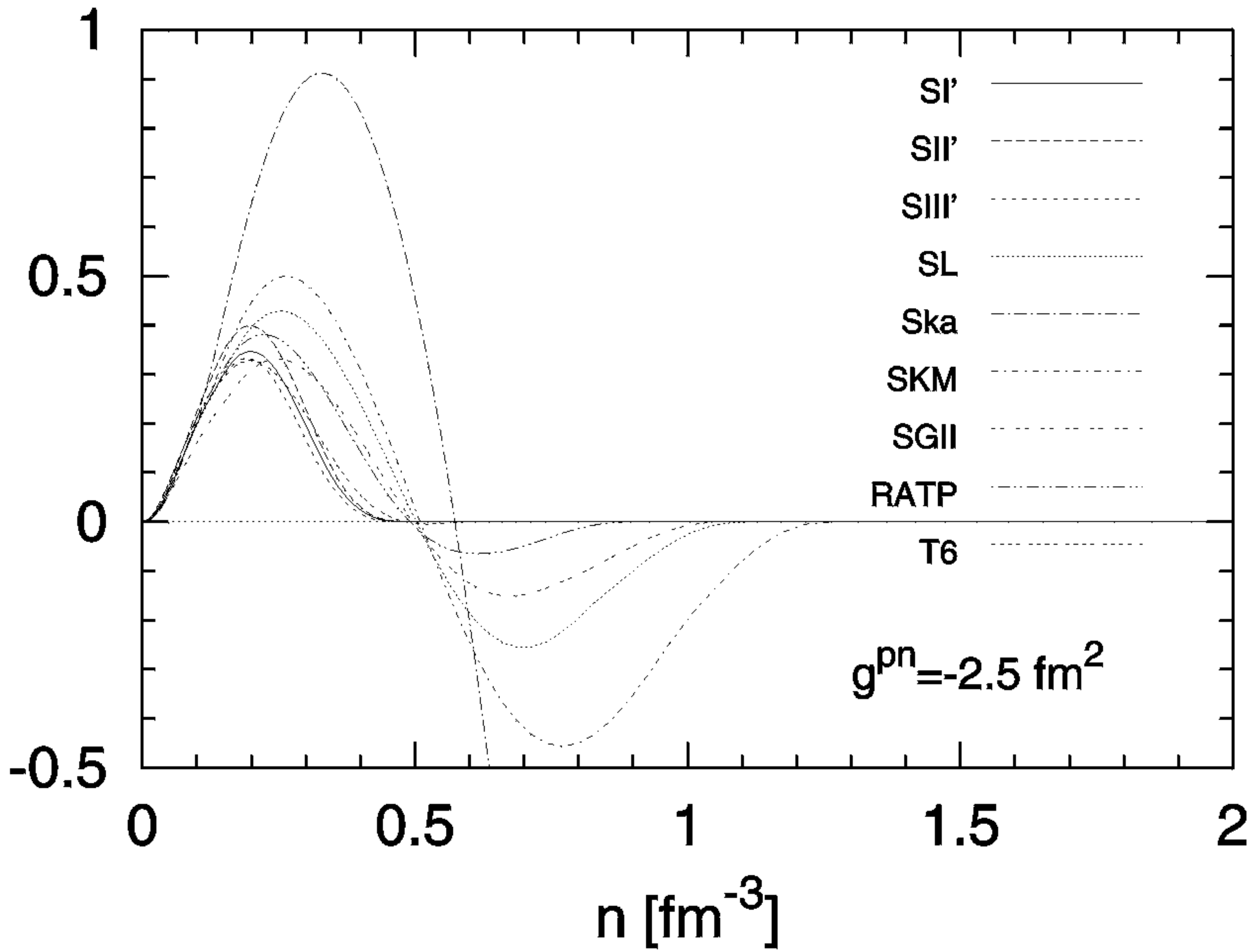
The magnetization of this phase is:

$$M = \mu_N \delta s_N + \mu_P \delta s_P = \left[- \frac{g^{pn} N_N}{1 + G_0^{NN}} \mu_N + \mu_P \right] n_P$$

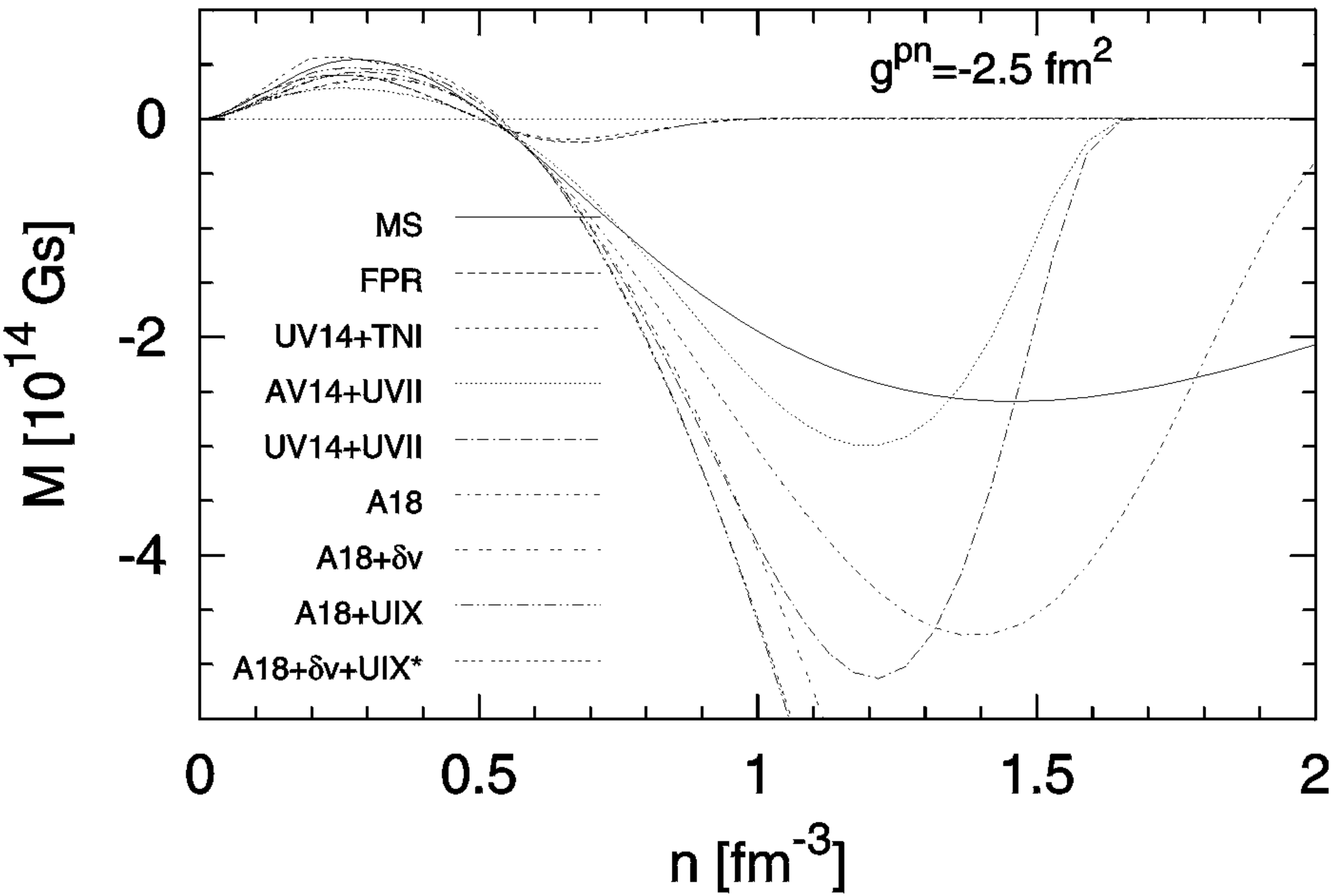
The magnetic moment of neutron and proton:

$$\mu_N = -9.66 \cdot 10^{-24}\, erg/Gs \qquad \mu_P = 1.41 \cdot 10^{-23}\, erg/Gs$$


x=0.01
0.02
0.05
0.10
0.15
M [10^13 Gs]
n [fm^-3]
10
5
0
-5
-10
-15
0
0.5
1
1.5
2



1
0.5
0
-0.5
M [10^{14} Gs]
0
0.5
1
1.5
2
n [fm^{-3}]
SI'
SII'
SIII'
SL
Ska
SKM
SGII
RATP
T6
g^{pn}=-2.5 fm^2



g^pn=-2.5 fm^2
M [10^14 Gs]
0
-2
-4
MS
FPR
UV14+TNI
AV14+UVII
UV14+UVII
A18
A18+δv
A18+UIX
A18+δv+UIX*
0
0.5
1
1.5
2
n [fm^-3]

For nonlocalized protons without any proton-proton interaction:

$$\delta \varepsilon_P = \frac{1}{2N_P}\left(\delta s_P\right)^2 \neq 0 \qquad \delta \varepsilon_{\min} = \frac{1}{2}\left[\frac{1}{N_P} - \frac{N_N}{1+G_0^{NN}}\left(g^{pn}\right)^2\right]\left(\delta s_P\right)^2$$

so the system is unstable to spin fluctuations when:

$$\delta \varepsilon_{\min} < 0 \Rightarrow \left|g^{pn}\right| > g_c^{pn}$$

$$\text{where: } g_c^{pn} = \sqrt{\frac{1+G_0^{NN}}{N_N N_P}}$$

Magnetic moment of ferromagnetic phase of volume dV is $d\boldsymbol{M} = M\, dV$

The existence of the magnetic moment implies a dipole magnetic field, which at the magnetic pole on the surface of the star of radius R has the value: $B_P = 2\boldsymbol{M} / R^3$

Emergence of magnetic field

The energy per baryon in the polarized phase could be below that for the normal phase by at least $\sim 1 MeV$, so the phase transition to magnetized matter with spin ordering is expected to occur very soon after formation of the neutron star.
This sudden switching-on of the magnetic field of the magnetized core, forming a single domain, will result in the induction of the screening field which will fully shield the ferromagnetic field:

$$B_{fer} + B_{ind}(0) = 0$$

The induced current suffers ohmic decay, and the nonzero field emerges.

Decay time of the n-th mode is: $\tau_n = 4\pi R^2 \sigma \,/ (c\pi n)^2$

The unshielded fraction of the magnetic field which emerges after time t:

$$\varepsilon(t) = \frac{\left|B_{fer} + B_{ind}(t)\right|}{\left|B_{fer}\right|} = 1 - \exp(-t/\tau_1)$$

The age and the magnetic field of the pulsar may be obtained from the period and its derivative: $t = \dfrac{P}{2\dot{P}}$

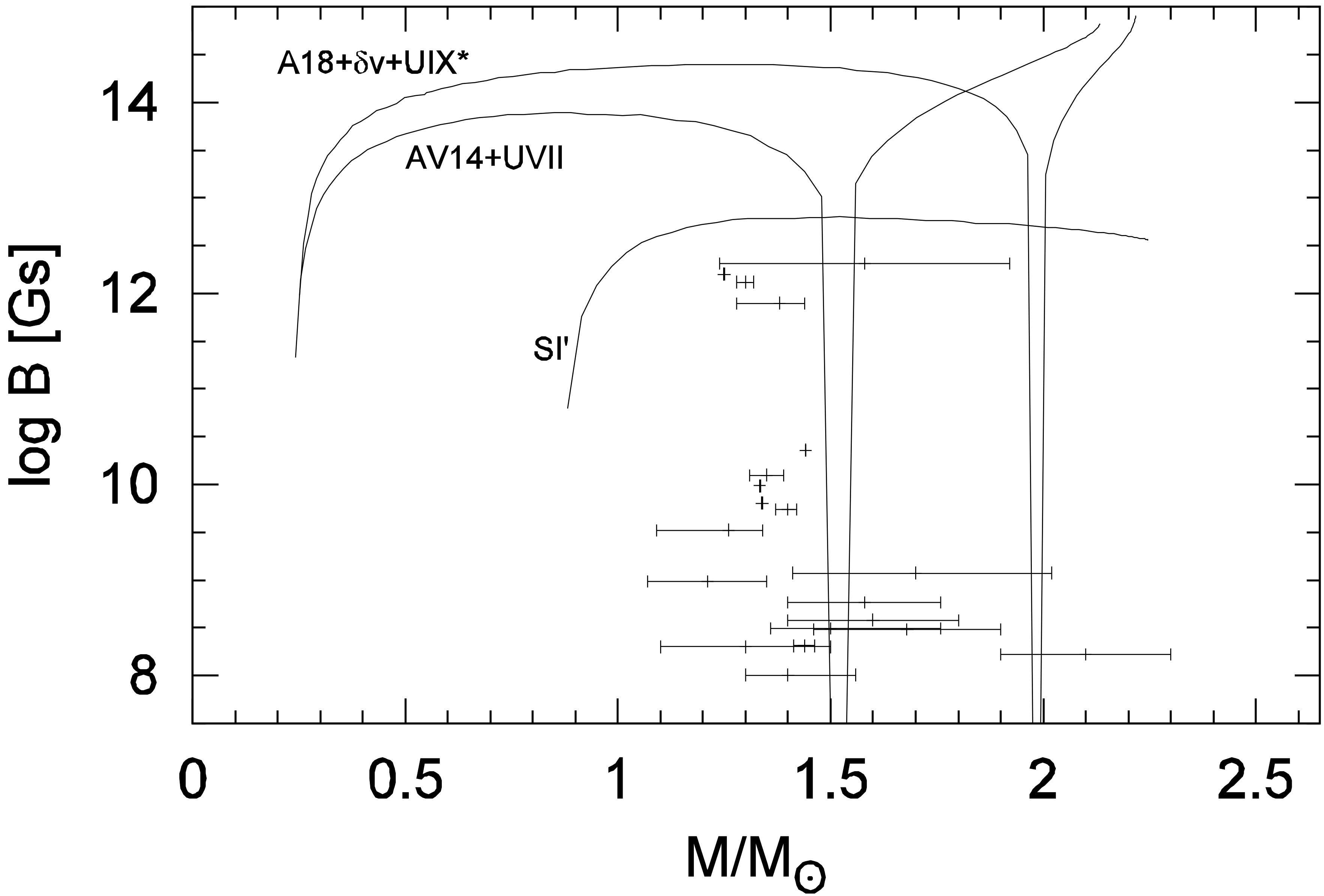
From the energy loss of the pulsar: $\dot{E} = I\Omega\dot{\Omega} = -\dfrac{B^2 R^6 \Omega^4 \sin^2\alpha}{6c^3}$

$$B_{obs} = \frac{c^3}{\pi R^3 \sin\alpha}\sqrt{\frac{3}{2} I P \dot{P}} \approx 3.2 \cdot 10^{19} \sqrt{P\dot{P}}\, Gs$$

The observed magnetic field is the sum of the residual field from the newly created neutron star and the field which emerges from the ferromagnetic core through the ohmic decay:

$$B_{obs} = B_{res} + \varepsilon(t) B_{fer}$$

Name	Mass [$M_\odot$]	P [ms]	$\dot{P}$ [ss^{-1}]	t [year]	B [Gs]	B_{res} [Gs] – AV14+UVII
J0737-3039A	1.338(1)	22.70	1.76E-18	2.04E8	6.40E9	4.64E9
J0737-3039B	1.249(1)	2773.5	8.92E-16	4.93E7	1.59E12	1.59E12
PRS1534+12	1.3332(10)	37.90	2.42E-18	2.48E8	9.69E9	7.49E9
J1756-2251	$1.40^{+0.02}_{-0.03}$	28.46	1.02E-18	4.42E8	5.45E9	2.73E9
B1913+16	1.4408(3)	59.03	8.63E-18	1.08E8	2.28E10	2.24E10
B2127+11C	1.349(40)	30.53	4.99E-18	9.69E7	1.25E10	1.17E10
J1141-6545	1.30(2)	393.9	4.29E-15	1.46E6	1.32E12	1.32E12
B2303+46	$1.38^{+0.06}_{-0.10}$	1066.4	5.69E-16	2.97E7	7.88E11	7.88E11
J0621+1002	$1.70^{+0.32}_{-0.29}$	28.85	4.73E-20	9.67E9	1.18E9	-1.47E11
J0437-4715	1.58(18)	5.757	5.73E-20	1.59E9	5.81E8	-7.05E9
J0751+1807	2.10(20)	3.479	7.79E-21	7.08E9	1.67E8	-7.66E11
J1713+0747	1.3(2)	4.570	8.53E-21	8.49E9	2.00E8	-8.74E10
PRS1855+09	$1.50^{+0.26}_{-0.14}$	5.362	1.77E-20	4.80E9	3.12E8	-3.98E9
J1909-3744	1.438(24)	2.947	1.40E-20	3.34E9	2.06E8	-1.40E10
J1012+5307	1.68(22)	5.256	1.71E-20	4.87E9	3.03E8	-6.34E10
B1802-07	$1.26^{+0.08}_{-0.17}$	23.10	4.67E-19	7.84E8	3.32E9	-5.73E9
J0045-7319	1.58(34)	926.3	4.46E-15	3.29E6	2.06E12	2.06E12
J1911-5958A	$1.40^{+0.16}_{-0.10}$	3.266	3.07E-21	1.69E10	1.01E8	-1.04E11
J1738+0333	1.6(2)	5.850	2.41E-20	3.85E9	3.80E8	-2.39E10
1802-2124	1.21(14)	12.65	7.2E-20	2.78E9	9.66E8	-3.59E10


A18+δv+UIX*
AV14+UVII
SI'
log B [Gs]
14
12
10
8
0
0.5
1
1.5
2
2.5
M/M⊙

Conclusions

1. Symmetry energy implies the inhomogeneity of dense nuclear matter in neutron stars.
2. Astrophysical consequences of proton localization:
   - spontaneous polarization of localized protons;
   - influence on neutron star cooling rate.
3. Ferromagnetic core as the source of strong magnetic field of the neutron star.

References:

[1] M. Kutschera, Phys. Lett. **B340**, 1 (1994).
[2] M. Kutschera, W. Wójcik, Phys. Lett. **B223**, 11 (1989).
[3] M. Kutschera, W. Wójcik, Phys. Rev. **C47**, 1077 (1993).
[4] M. Kutschera, S. Stachniewicz, A. Szmagliński, W. Wójcik, Acta Phys. Pol. **B33**, 743 (2002).
[5] A. Szmagliński, W. Wójcik, M. Kutschera, Acta Phys. Pol. **B37**, 277 (2006).
[6] M. Kutschera, W. Wójcik, Acta Phys. Pol. **B23**, 947 (1992).
[7] M. Kutschera, MNRAS **307(4)**, 784 (1999).