Proton Localization and Magnetic Field Evolution in Dense Neutron Star Matter
Neutron Star Structure

- Atmosphere ($\sim 1\, cm$) \( \rho \leq 10 \, g \, cm^{-3} \)
- Outer crust \( 10 \leq \rho \leq 4.3 \cdot 10^{11} \, g \, cm^{-3} \)
- Inner crust \( 4.3 \cdot 10^{11} \leq \rho \leq 2.4 \cdot 10^{14} \, g \, cm^{-3} \)
- Core \( 2.4 \cdot 10^{14} \leq \rho \)
Realistic Nuclear Models:

2. Myers-Świątecki (MS)
3. Friedman-Pandharipande-Ravenhall (FPR)
4. UV14+TNI (UV)
5. AV14+UVII (AV)
6. UV14+UVII (UVU)
7. A18
8. A18+δv
9. A18+UIX
10. A18+δv+UIX*
Symmetry Energy of Nuclear Matter

\[ E(n, x) = E\left(n, \frac{1}{2}\right) + E_s(n)(2x - 1)^2 \quad \text{for} \quad x = \frac{n_p}{n} \]

\[
E_s(n) = \frac{1}{8} \left. \frac{\partial^2 E(n, x)}{\partial x^2} \right|_{x=\frac{1}{2}}
\]
Small values of the symmetry energy:

2. Low proton concentration

3. Charge separation instability
   • realized eg. through proton localization
Spin instability of proton-localized phase

The energy density difference between polarized state and the normal one:

\[ \delta \varepsilon = \delta \varepsilon_N + g^{pn} \delta s_N \delta s_P + \delta \varepsilon_P \]

where the spin excess:

\[ \delta s_N = \delta n_{N\uparrow} - \delta n_{N\downarrow}; \quad \delta s_P = \delta n_{P\uparrow} + \delta n_{P\downarrow} \]

effective coupling constant:

\[ g^{pn} = -2.5 \text{ fm}^2 \]

According to the Landau Fermi-liquid theory the change of the neutron energy density:

\[ \delta \varepsilon_N = \frac{1 + G_{0}^{NN}}{2 N_N} (\delta s_N)^2 \]

where spin-dependent Landau parameter for pure neutron matter:

\[ G_{0}^{NN} = 1.0 \]

Density of states at the Fermi level:

\[ N_N = \pi^{-2} m_N k_F^N \]
For localized protons their wave functions extend to a limited volume: \( \delta \varepsilon_P = 0 \)

The difference of energy density:

\[
\delta \varepsilon = \frac{1 + G_0^{NN}}{2N_N} (\delta s_N)^2 + g^{pn} \delta s_N \delta s_P
\]

for \( \delta s_N = -\frac{g^{pn} N_N}{1 + G_0^{NN}} \delta s_P \) has the minimum:

\[
\delta \varepsilon_{\text{min}} = - \frac{N_N (g^{pn})^2}{2(1 + G_0^{NN})} (\delta s_P)^2 < 0
\]

The system has the minimum of the energy for fully polarized protons: \( \delta s_P = n_P \)

The system with polarized protons is unstable against small spin oscillations.
The magnetization of this phase is:

\[ M = \mu_N \delta s_N + \mu_P \delta s_P = \left[ -\frac{g^{pn} N_N}{1 + G_0^{NN}} \mu_N + \mu_P \right] n_P \]

The magnetic moment of neutron and proton:

\[ \mu_N = -9.66 \cdot 10^{-24} \text{ erg} / \text{Gs} \quad \mu_P = 1.41 \cdot 10^{-23} \text{ erg} / \text{Gs} \]
$g^{pn} = -2.5 \text{ fm}^2$
For nonlocalized protons without any proton-proton interaction:

\[
\delta \varepsilon_p = \frac{1}{2N_p} (\delta s_p)^2 \neq 0 \\
\delta \varepsilon_{\text{min}} = \frac{1}{2} \left[ \frac{1}{N_p} - \frac{N_N}{1 + G_0^{NN}} (g^{pn})^2 \right] (\delta s_p)^2
\]

so the system is unstable to spin fluctuations when:

\[
\delta \varepsilon_{\text{min}} < 0 \Rightarrow |g^{pn}| > g_c^{pn}
\]

where: \[g_c^{pn} = \sqrt{\frac{1 + G_0^{NN}}{N_N N_P}}\]

Magnetic moment of ferromagnetic phase of volume \(dV\) is \(dM = M \, dV\)

The existence of the magnetic moment implies a dipole magnetic field, which at the magnetic pole on the surface of the star of radius \(R\) has the value: \[B_P = 2M / R^3\]
Emergence of magnetic field

The energy per baryon in the polarized phase could be below that for the normal phase by at least $\sim 1\,\text{MeV}$, so the phase transition to magnetized matter with spin ordering is expected to occur very soon after formation of the neutron star. This sudden switching-on of the magnetic field of the magnetized core, forming a single domain, will result in the induction of the screening field which will fully shield the ferromagnetic field:

$$B_{\text{fer}} + B_{\text{ind}}(0) = 0$$

The induced current suffers ohmic decay, and the nonzero field emerges.

Decay time of the n-th mode is:

$$\tau_n = \frac{4\pi R^2 \sigma}{(c\pi n)^2}$$
The unshielded fraction of the magnetic field which emerges after time $t$: 

$$\varepsilon(t) = \frac{|B_{fer} + B_{ind}(t)|}{|B_{fer}|} = 1 - \exp(-t/\tau_1)$$

The age and the magnetic field of the pulsar may be obtained from the period and its derivative: 

$$t = \frac{P}{2\dot{P}}$$

From the energy loss of the pulsar: 

$$\dot{E} = I\Omega \dot{\Omega} = -\frac{B^2 R^6 \Omega^4 \sin^2 \alpha}{6c^3}$$

$$B_{obs} = \frac{c^3}{\pi R^3 \sin \alpha} \sqrt{\frac{3}{2}} IPP \approx 3.2 \cdot 10^{19} \sqrt{PP} \text{ Gs}$$

The observed magnetic field is the sum of the residual field from the newly created neutron star and the field which emerges from the ferromagnetic core through the ohmic decay:

$$B_{obs} = B_{res} + \varepsilon(t) B_{fer}$$
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Conclusions

1. Symmetry energy implies the inhomogeneity of dense nuclear matter in neutron stars.

2. Astrophysical consequences of proton localization:
   - spontaneous polarization of localized protons;
   - influence on neutron star cooling rate.

3. Ferromagnetic core as the source of strong magnetic field of the neutron star.
References: