Time-dependent D7-brane embeddings

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Outline

1. Introduction
2. The AdS/CFT correspondence
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4. Static D7-brane embeddings
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Boundary conditions

The Polyakov action:

$$S = -T \int d^2\sigma \eta^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu$$

We vary the above action plugging EOM:

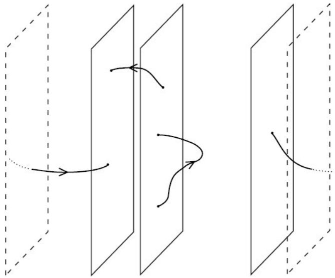
$$\delta S = -T \int d\tau [X'_\mu \delta X^\mu(0,\tau) - X'_\mu \delta X^\mu(\pi,\tau)] = 0$$

Possible boundary conditions:

- $X'_\mu = 0|_{\sigma=0,2\pi}$ von Neumann's boundary conditions
- $X^\mu(0,\tau), X^\mu(\pi,\tau) = const$ Dirichlet's boundary conditions

Brane action

$$S_{Dp} = -T_p \int d\xi^{p+1} \sqrt{-\det(P[G_{ab}])}$$



For our considerations the D3/D7-brane system is relevant.

The AdS/CFT correspondence

$\mathcal{N} = 4$ supersymmetric

Yang-Mills theory in $\mathbb{R}^4$

$\equiv$

IIB superstrings in curved

$AdS_5 \times S^5$ 10D spacetime

Strongly coupled gauge fields $\Leftrightarrow$ Weakly coupled strings

How one can relate physical observables in these two theories?

The dictionary of gauge/gravity duality

Equivalence means:

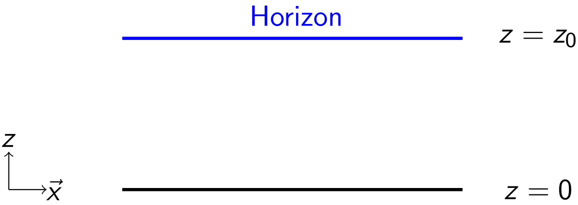
$$e^{-S_{sugra}} \approx Z_{string} = Z_{CFT} \equiv \langle e^{\int d^4x \phi_0(x) \mathcal{O}(x)} \rangle$$

Identification of some corresponding quantities:

The dictionary	
Gauge side	String side
$\mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$	dilaton
$T_{\mu\nu}$	graviton $g_{\mu\nu}$
dimension of operator	mass of the field
...	...

The AdS/CFT correspondence at $T \neq 0$

$$ds^2 = -\frac{1 - z^4/z_0^4}{z^2} dt^2 + \frac{1}{z^2} d\vec{x}^2 + \frac{1}{z^2(1 - z^4/z_0^4)} dz^2$$



Entropy:

$$S_{CFT} = S_{BH} \sim T^3$$

Geometry

$$ds^2 = \frac{1}{z^2}(-e^{a(\tau,z)}d\tau^2 + \tau^2 e^{b(\tau,z)}dy^2 + e^{c(\tau,z)}dx_\perp^2 + dz^2)$$

The three coefficients can be derived from the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 6g_{\mu\nu} = 0$$

- scaling variable $v = \frac{z}{\tau^{s/4}}$ reduces $a(\tau, z) = a(v)$
- natural criterion to determine s - geometry regularity
- $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$ has no poles nor cuts
- cancelation of 4th order pole at $v = 3^{1/4}$ requires $s = 4/3$
- this reproduces Bjorken hydrodynamics on CFT side!

Perfect fluid asymptotic geometry

Asymptotic geometry for energy density $\epsilon = \frac{e}{\tau^{4/3}}$

$$ds^2 = \frac{1}{z^2}\left(-\frac{(1 - \frac{e}{3}\frac{z^4}{\tau^{4/3}})^2}{1 + \frac{e}{3}\frac{z^4}{\tau^{4/3}}} d\tau^2 + (1 + \frac{e}{3}\frac{z^4}{\tau^{4/3}})(\tau^2 dy^2 + dx_\perp^2) + dz^2 \right)$$

The above metric is very similar to the black hole solution but with the location of the horizon moving in the bulk:

$$z_0 = \left(\frac{3}{e}\right)^{\frac{1}{4}} \tau^{\frac{1}{3}}$$

Also interesting to find subleading corrections to the metric (dependence on shear viscosity)

Fundamental matter

How one can take into account the fundamental matter?

Introduce N_f D7-branes into N_c D3-branes system ($N_c \gg N_f$)

Four different string sectors:

- close strings
- open strings
  - 3-3 strings
  - 3-7 strings
  - 7-7 strings

Gauge fields with $SU(N_c)$ symmetry

plus fundamental matter with $SU(N_f)$ symmetry

Static D7-brane embedding

The DBI action:

$$S_{D7} = -T_7 \int d^8\xi \epsilon_3 \rho^3 \sqrt{1 + \frac{g^{ab}}{\rho^2 + y_5^2 + y_6^2} (\partial_a y_5 \partial_b y_5 + \partial_a y_6 \partial_b y_6)}$$

Euler-Lagrange equation:

$$\frac{d}{d\rho} \left[\frac{\rho^3}{\sqrt{1 + \left(\frac{dy_6}{d\rho}\right)^2}} \frac{dy_6}{d\rho} \right] = 0$$

Asymptotic solution ($\rho \to \infty$):

$$y_6 = m + \frac{c}{\rho^2} + \dots$$

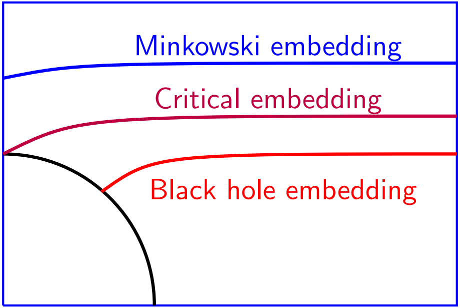
The scalars m and c are identified with the quark mass m_q and condensate $< q\bar{q} >$ respectively.

Time-dependent embeddings

An ansatz:

$$y_6 = m + \frac{c}{\rho^2 \tau^a} + \ldots$$

Should be consistent with the adiabatic picture:



Time-dependent embeddings

EOM:

$$\Box y_6 + 3\tan(y_6) - \frac{1}{2}\frac{G^{\mu\nu}\partial_\mu y_6 \partial_\nu (G^{\rho\sigma}\partial_\rho y_6 \partial_\sigma y_6)}{1 + G^{\alpha\beta}\partial_\alpha y_6 \partial_\beta y_6} = 0$$

Comparing the adiabatic and time-dependent embedding:

$$\tau^{-1} \sim T^3$$

we obtain the power of τ:

$$a = 8/3$$

The embedding up to first order:

$$y_6 = m + \frac{c}{\rho^2 \tau^{8/3}} + \dots$$

Adding viscosity

Subleading term occurs:

$$y_6 = m + \frac{c}{\rho^2 \tau^{8/3}} + \frac{c_1(\eta_0)}{\rho^2 \tau^{10/3}} + \dots$$

It is possible to determine coefficients c and c_1

$$c = -\frac{1}{6m^5} \quad c_1 = \frac{2}{9m^7}\eta_0$$

Conclusions

Summary

- The quark-gluon plasma can be described by hydrodynamics (the effective theory describing the long-distance, low-frequency behavior of interacting finite-temperature systems)
- AdS/CFT correspondence allows to compute hydrodynamical quantities (e.g. entropy)

Further work

- Calculation of the free energy (*done*)
- Analysis of the small fluctuations of the D7-brane - mesons (*work in progress*)