Pomeron-Graviton duality and resummation at high energies

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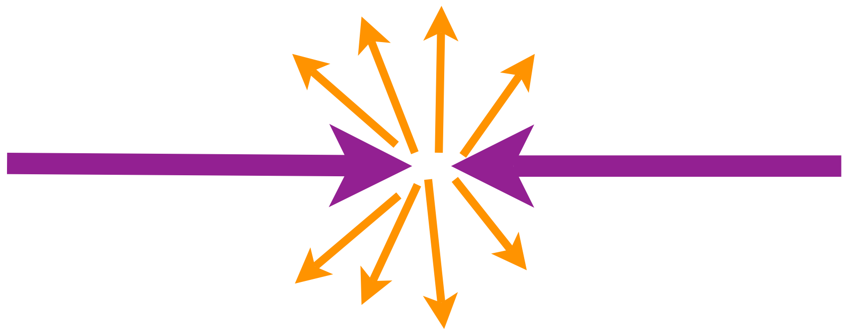
Cracow School of Theoretical Physics, XLVII Course, 2007

High - energy scattering of hadrons



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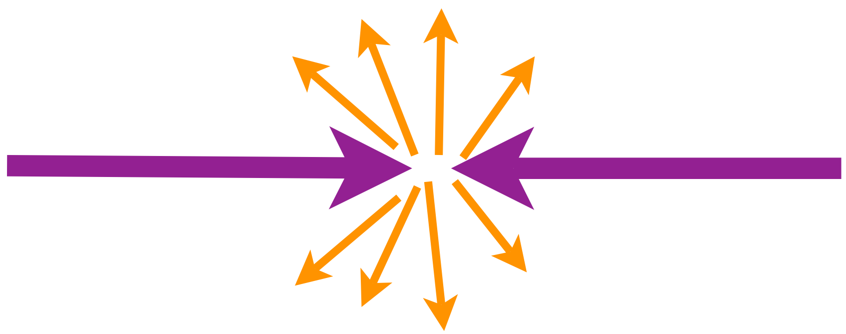
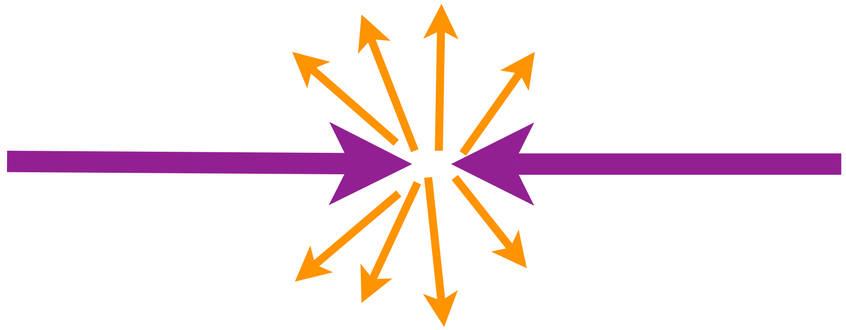


What is the dependence of the cross section at high energies?

High - energy scattering of hadrons



What is the dependence of the cross section at high energies?

Gauge theory

weak gauge coupling

Pomeron:
collective state of gluons

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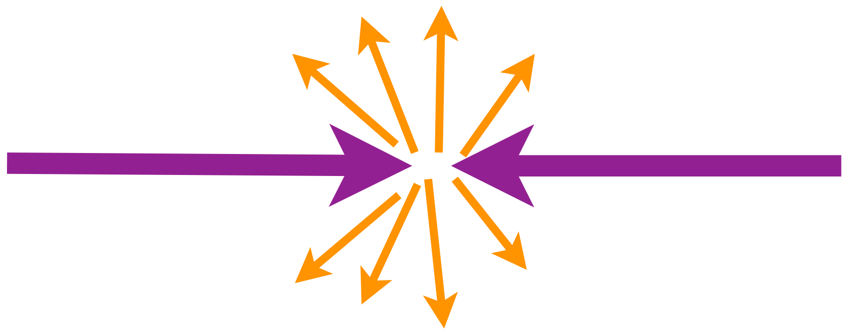

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String theory

strong gauge coupling

5-dimensional graviton
in anti - de Sitter space

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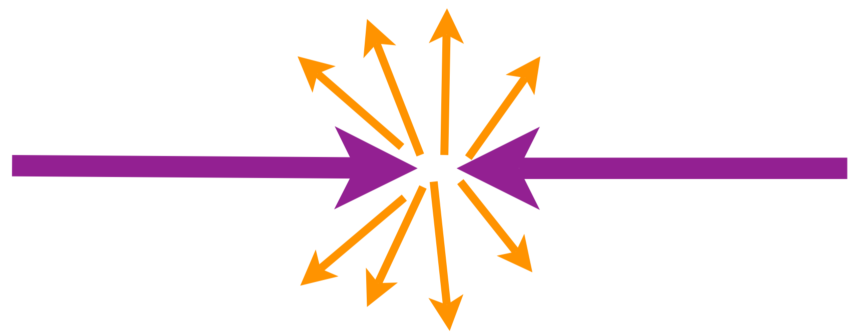

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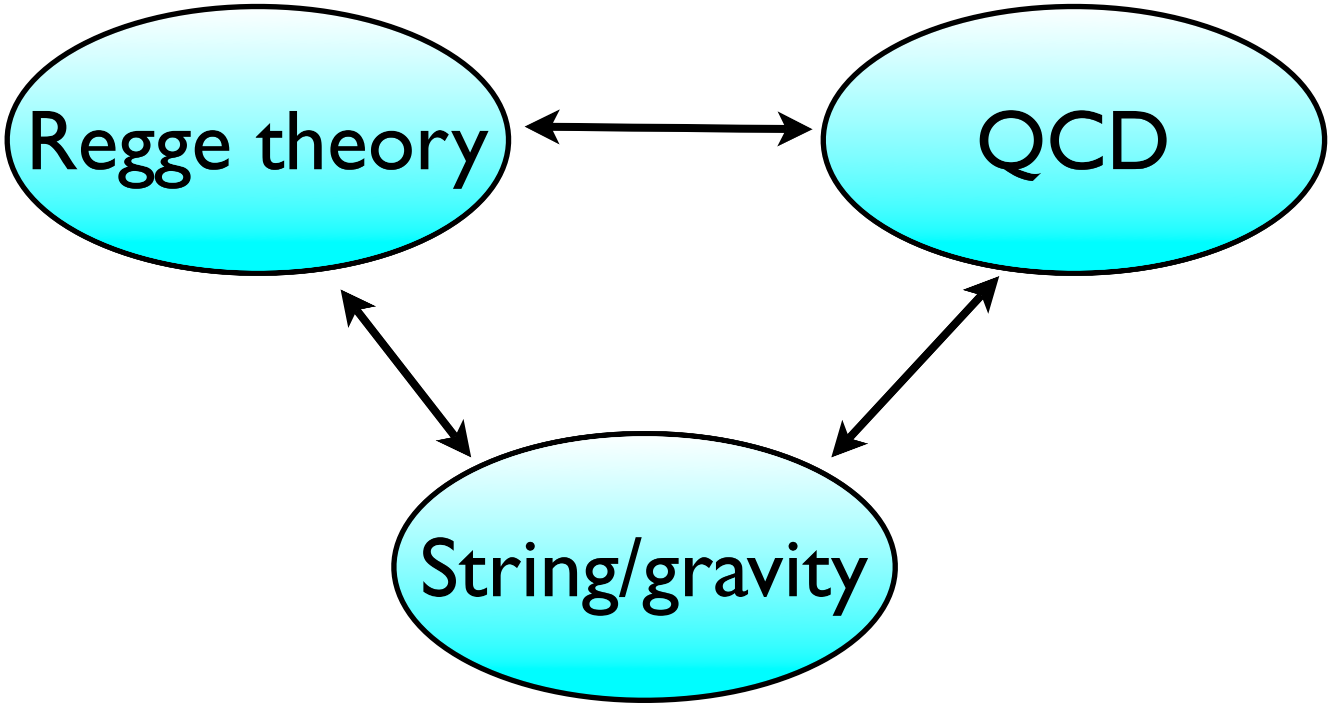
Resummation



Regge theory
QCD
String/gravity

Regge theory

Regge trajectories

- Linear dependence of the spin J on the square mass for mesons.

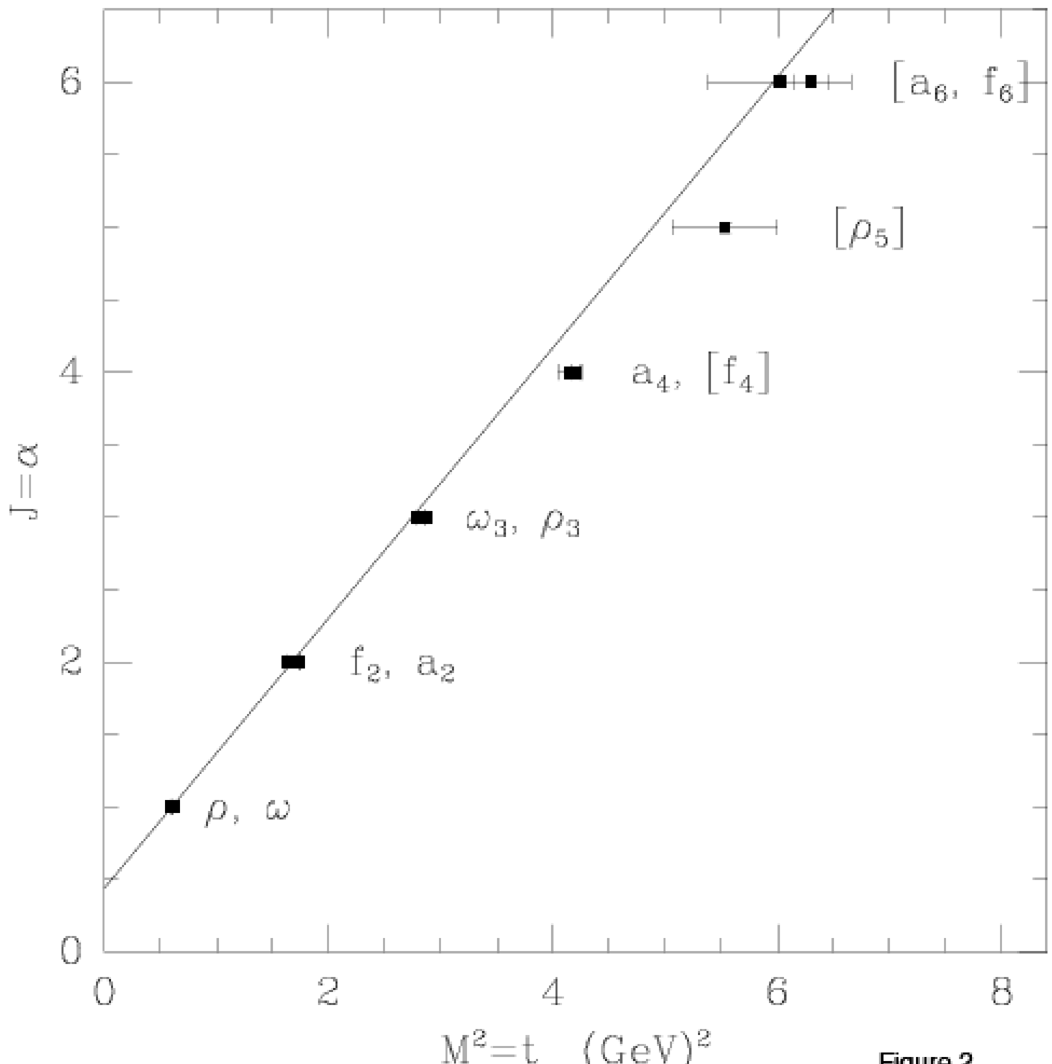


Figure 2

$$\alpha(t) = \alpha(0) + \alpha' t$$

$\alpha(0)_\rho = 0.45$ **intercept**

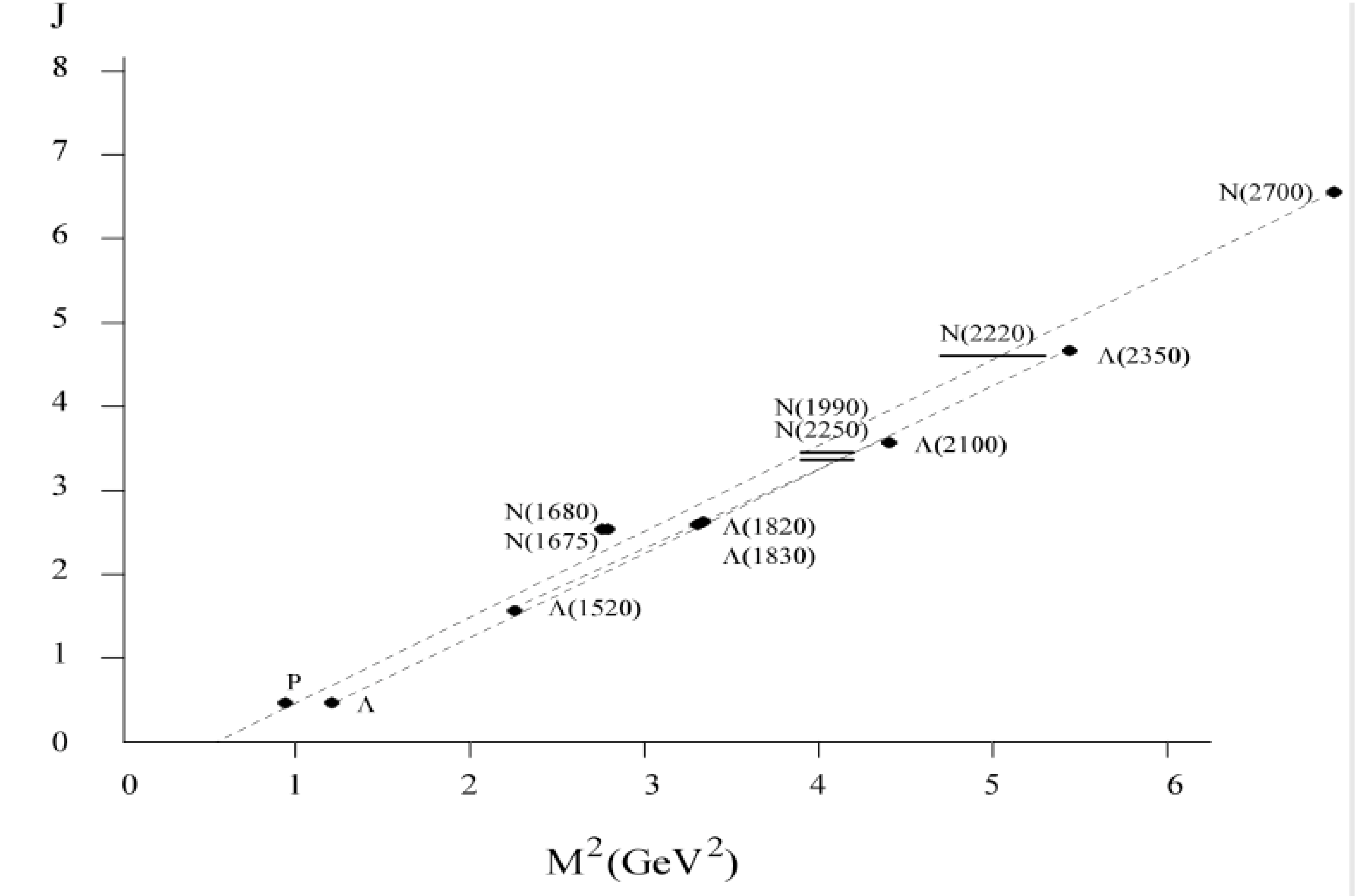
$\alpha'_\rho = 0.93\ \mathrm{GeV}^{-2}$ **slope**

$t = M^2 > 0$

for bound states

Regge trajectories

Similar regularity for baryons



S-matrix and Regge theory

S-matrix and Regge theory

General assumption about the S matrix:

- Lorentz invariance $\mathcal{A}(s,t)$
- Unitarity $S^\dagger S = SS^\dagger = 1$
- Analyticity
- Crossing $\mathcal{A}_{ab\to cd}(t,s,u) = \mathcal{A}_{a\bar{c}\to\bar{b}d}(s,t,u)$

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Decomposition into partial waves

$$\mathcal{A}(s,t) = \frac{1}{2i}\oint_C dl(2l+1)\frac{a(l,t)}{\sin\pi l}P(l,1+2s/t)$$

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Angular momentum

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Partial wave amplitude

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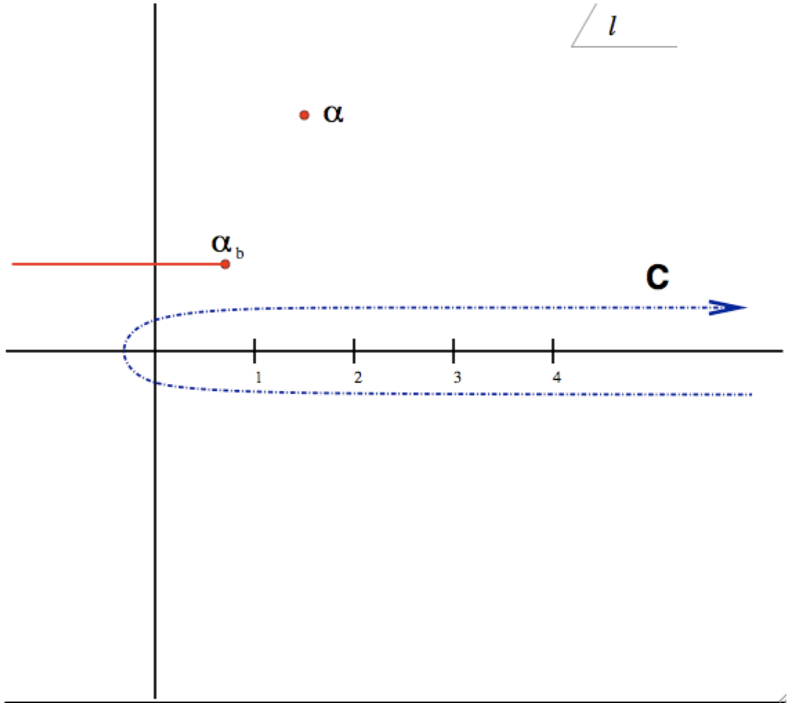
Angular momentum → dl

Partial wave amplitude → $a(l,t)$

Legendre polynomial → $P(l, 1+2s/t)$

Complex angular momentum plane

Complex angular momentum plane



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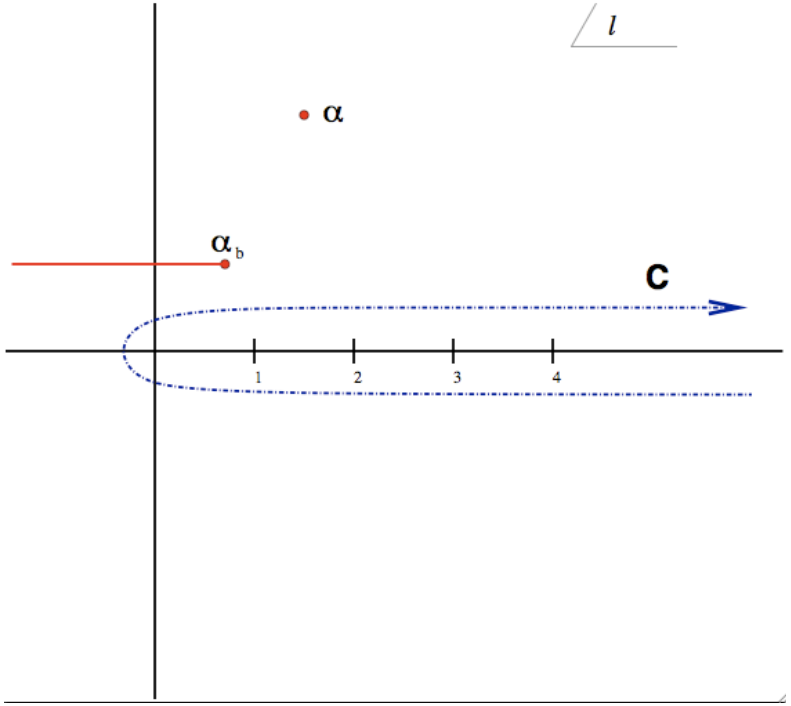
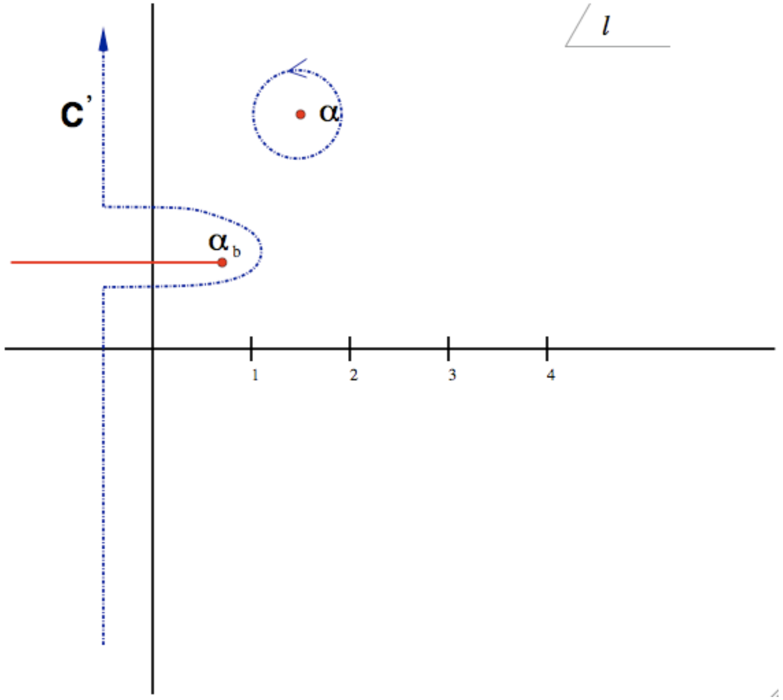


Deformation of the contour



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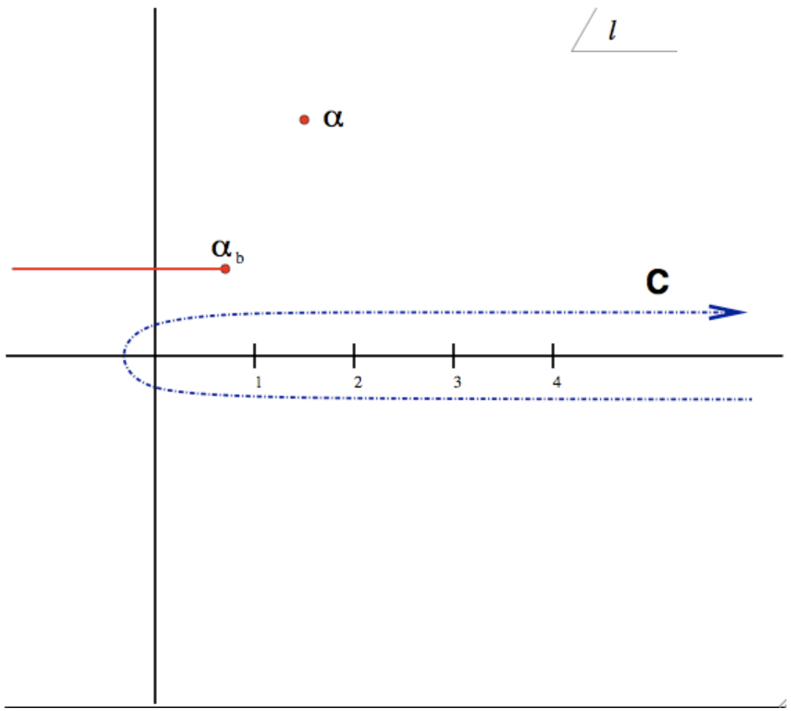

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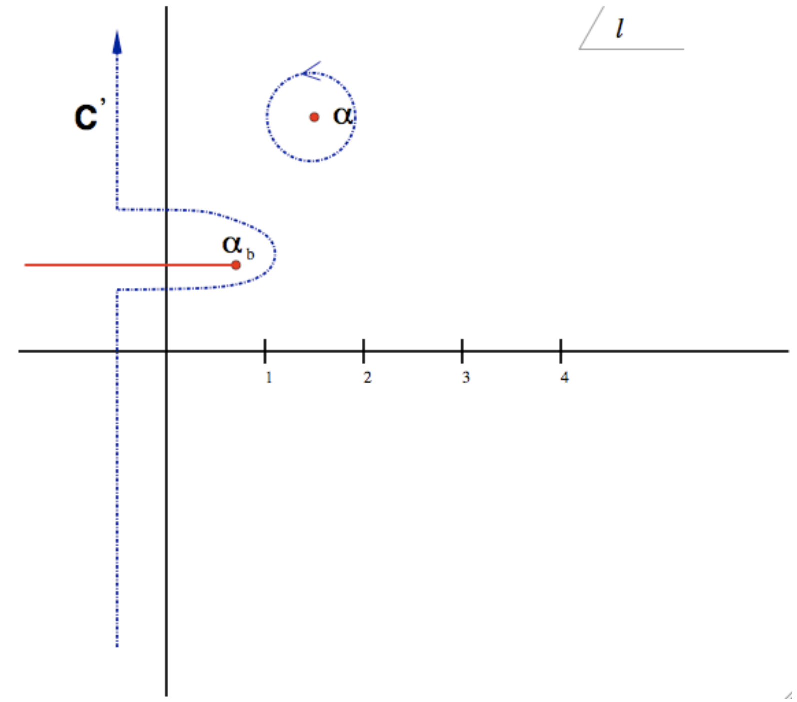
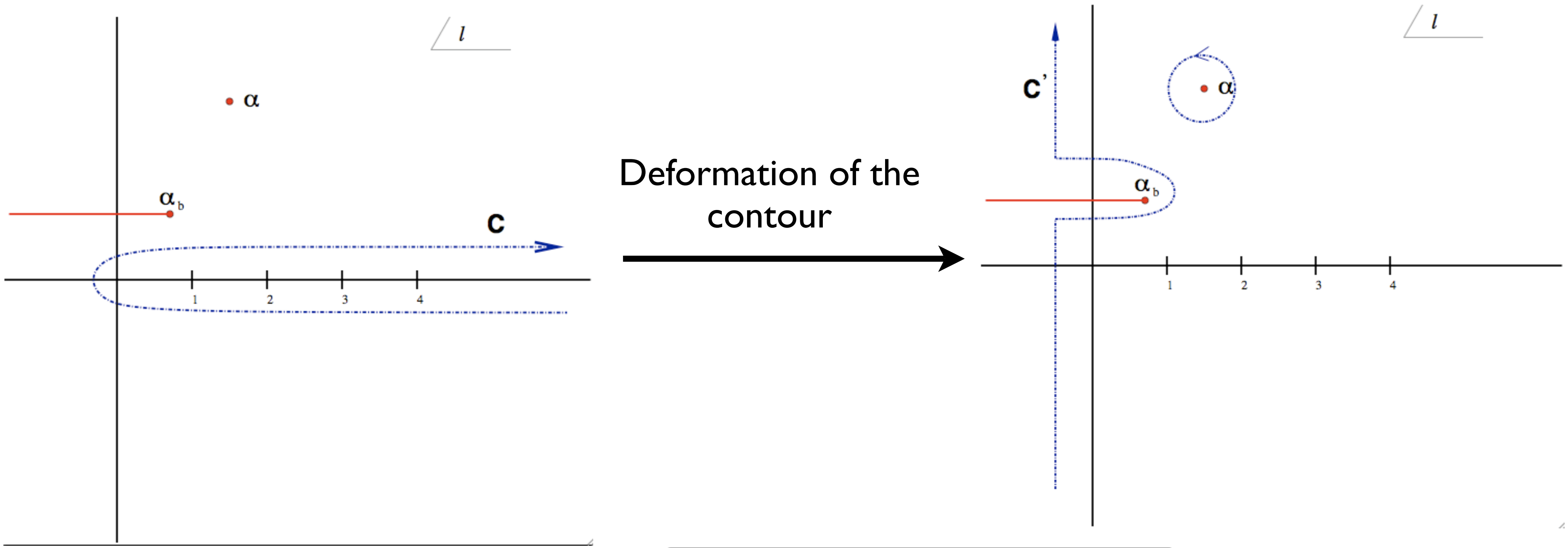


In the Regge limit: $s \gg |t| \; (s \to \infty, t \text{ fixed})$

Complex angular momentum plane



In the Regge limit: $s \gg |t| \; (s \to \infty, t \text{ fixed})$

$$\mathcal{A}(s,t) \to \frac{\eta + e^{-i\pi\alpha(t)}}{2} \beta(t)\, s^{\alpha(t)}$$

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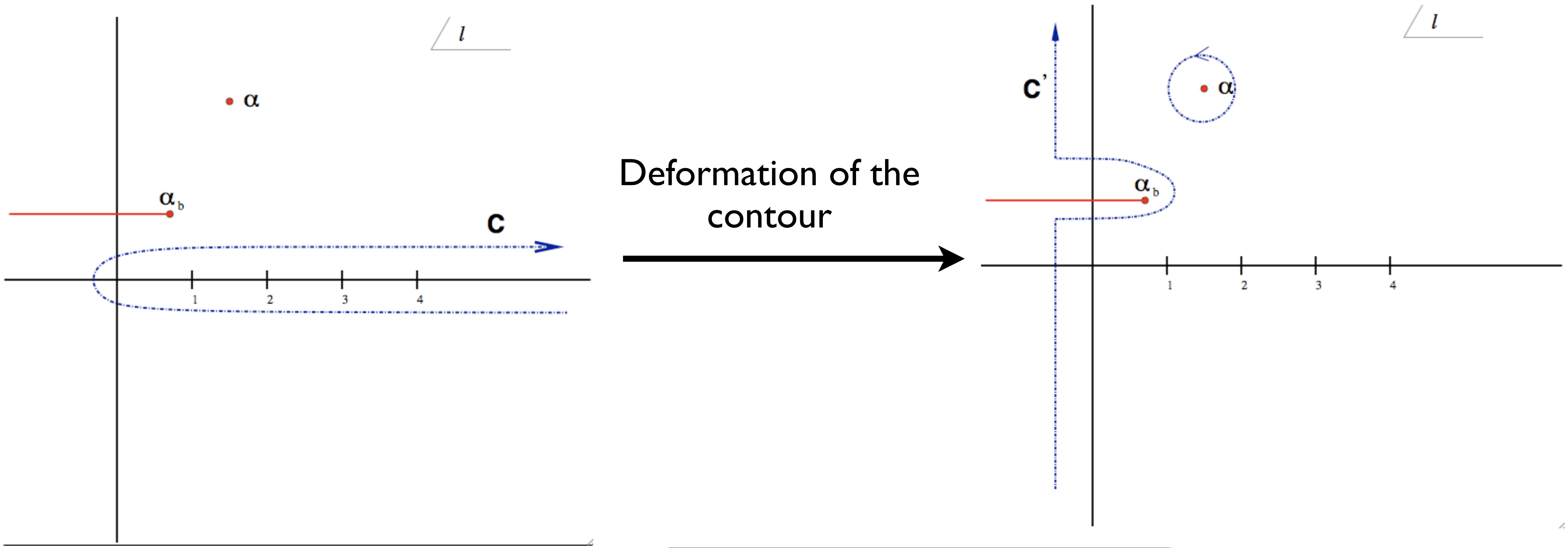

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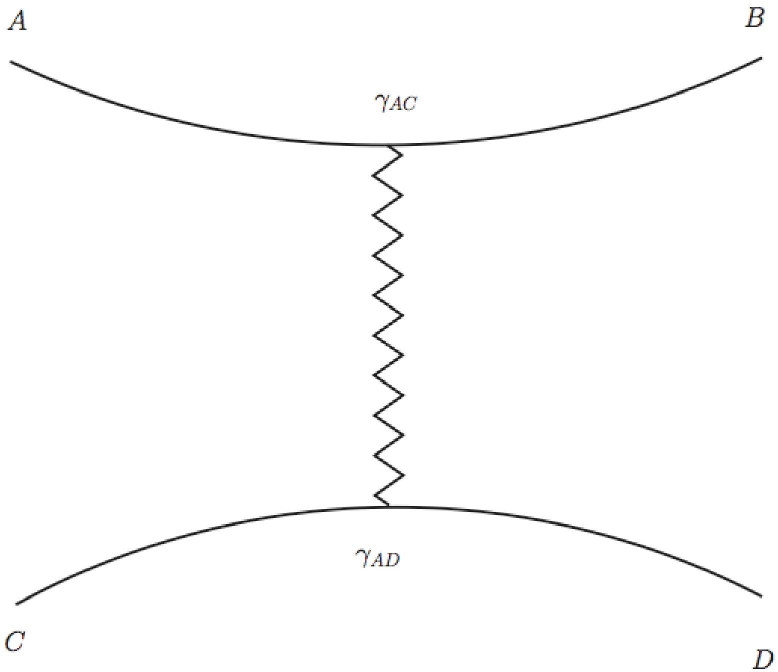
Amplitude dominated by the Regge pole with largest $\mathrm{Re}\,\alpha(t)$

Reggeon exchange

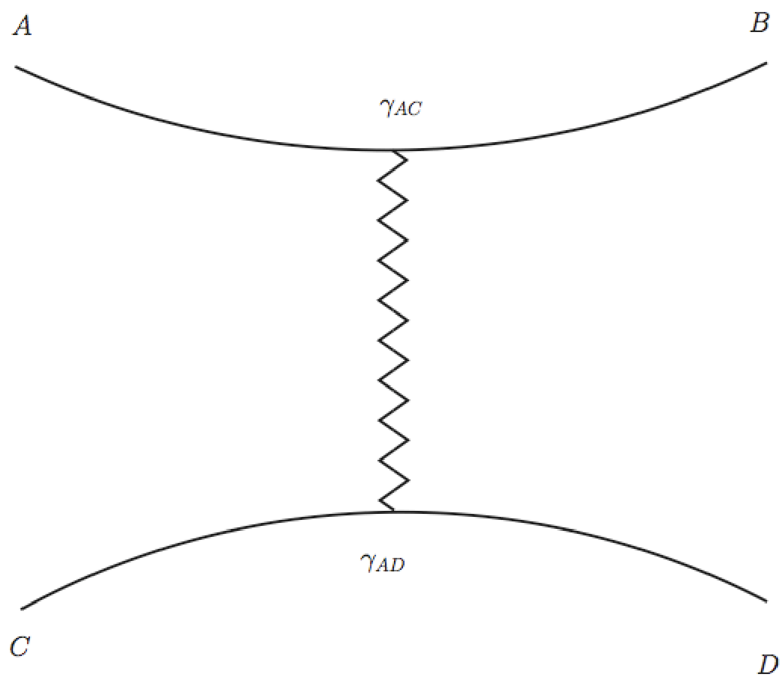
Reggeon exchange



Factorization of the couplings
and the Reggeon exchange

$$\mathcal{A}(s,t) \to \frac{\eta + e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)} \frac{\gamma_{ac}(t)\gamma_{bd}(t)}{\Gamma(\alpha(t))} s^{\alpha(t)}$$

Reggeon exchange



Factorization of the couplings and the Reggeon exchange

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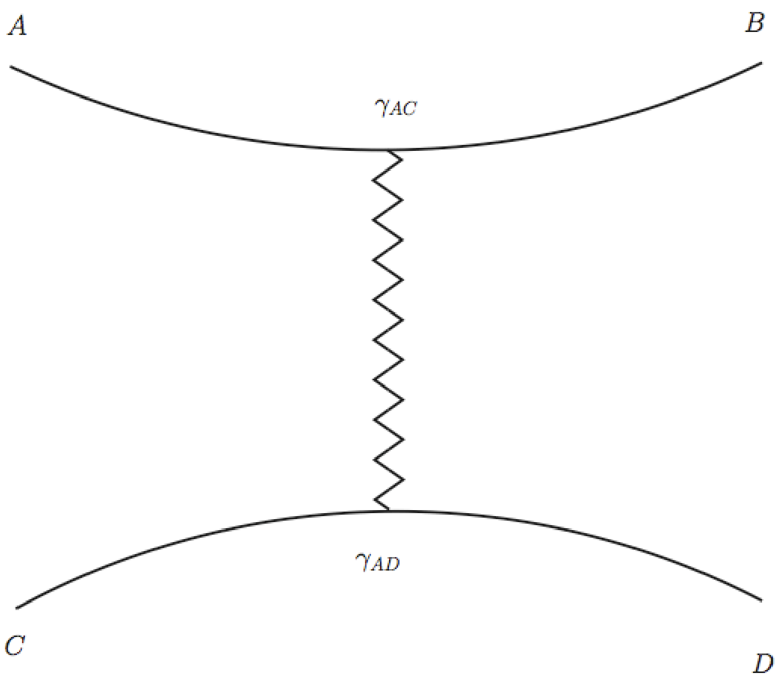
Signature

Reggeon exchange



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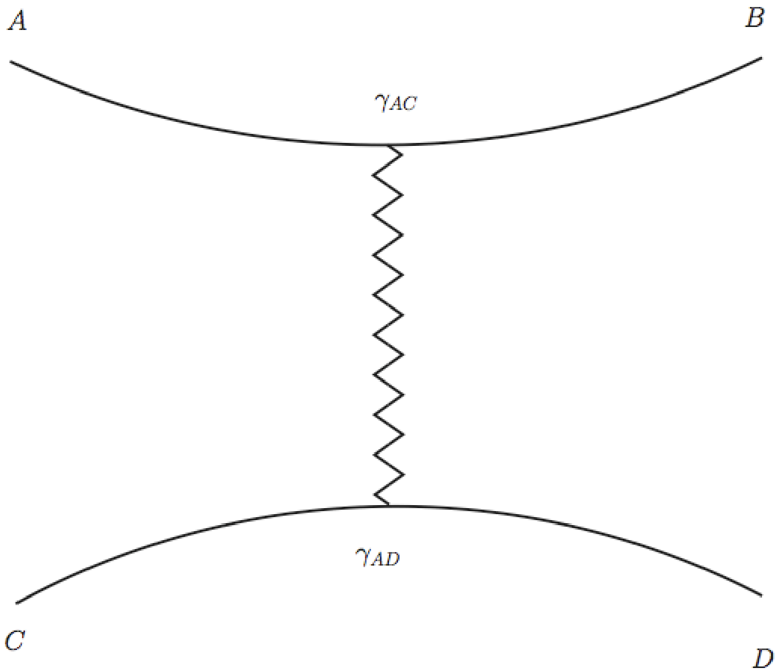
Signature

Couplings

Reggeon exchange



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Signature

Couplings

The energy behavior of the amplitude is determined by the exchange of the quasi-particle: Reggeon

Pomeron

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Vacuum exchange dominates cross sections at high energies

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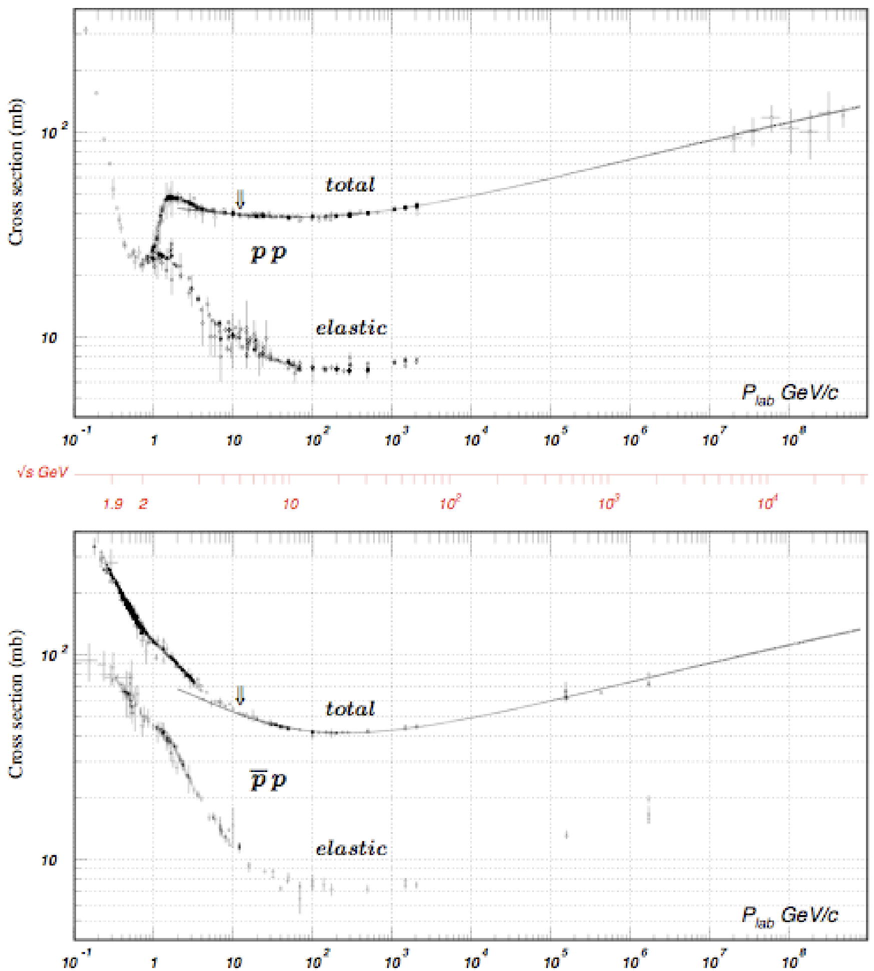
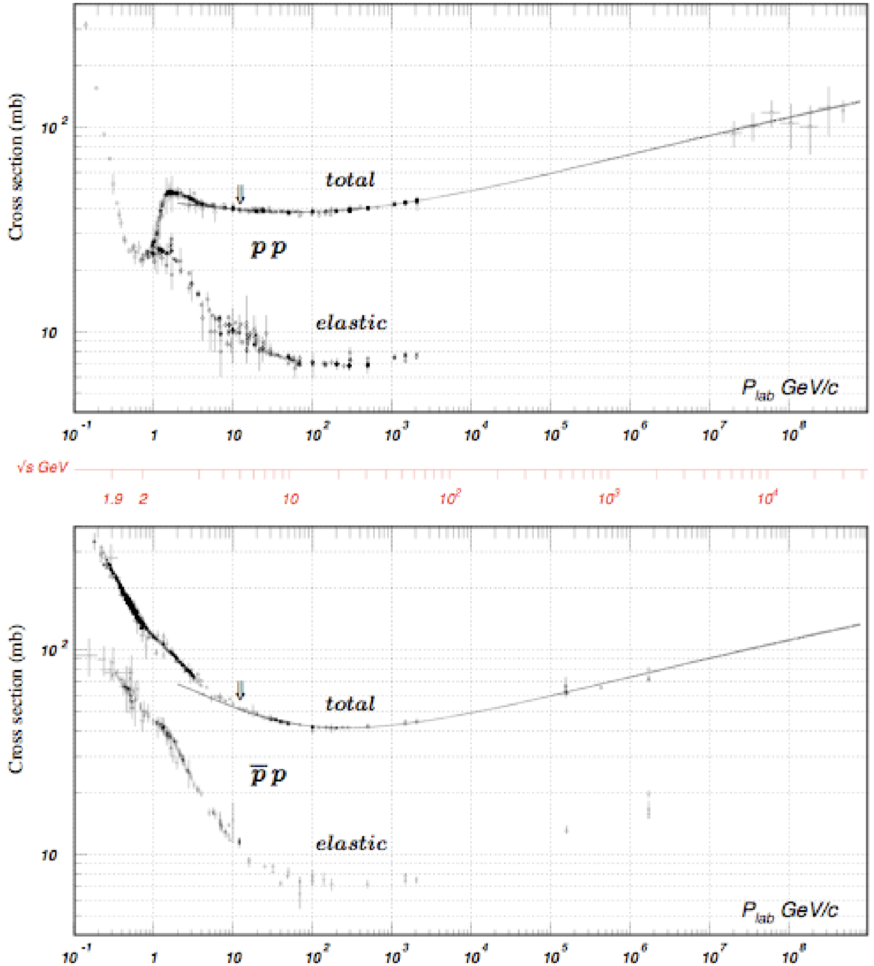


Figure 40.11: Total and elastic cross sections for pp and $\overline{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html. (Courtesy of the COMPAS group, IHEP, Protvino, August 2005.)

$$\sigma_{\mathrm{TOT}}(p\bar{p}) \sim \sigma_{\mathrm{TOT}}(pp)$$

Pomeron

Vacuum exchange dominates cross sections at high energies



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Vacuum exchange

$$\alpha(0)_P \geq 1$$

experimentally: $\alpha_P(0) \simeq 1.08,\ \sigma_{TOT} \sim s^{(\alpha(0)_P - 1)}$

Pomeron

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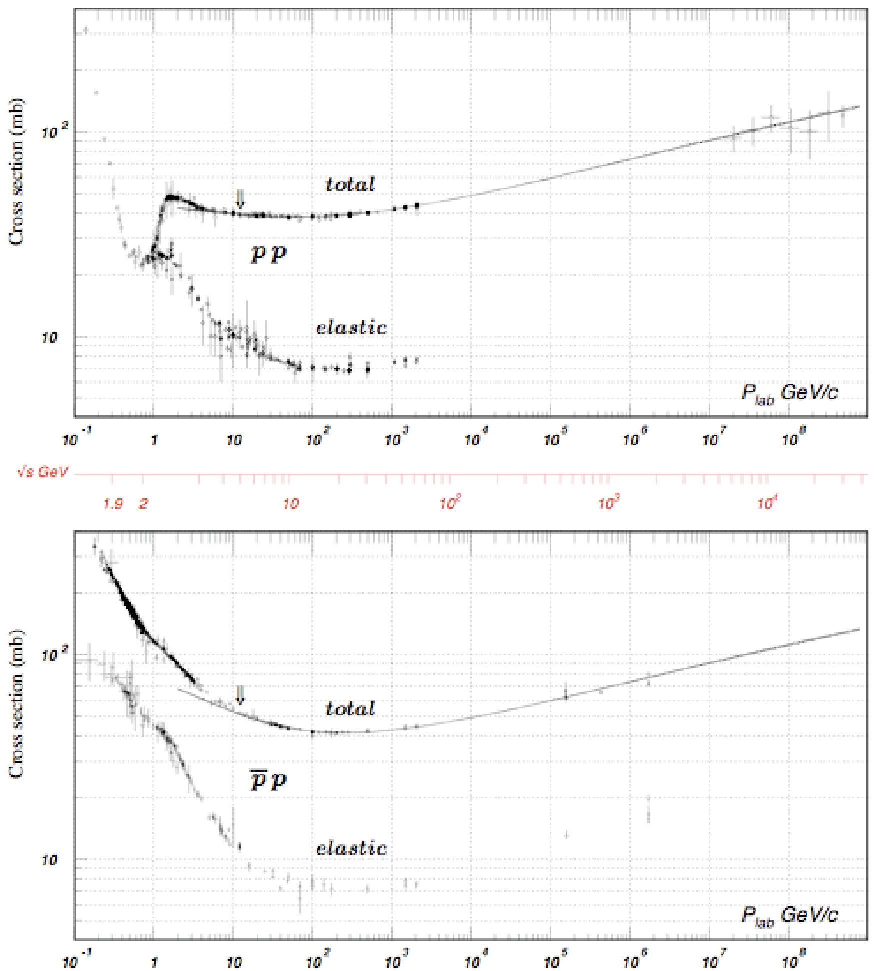

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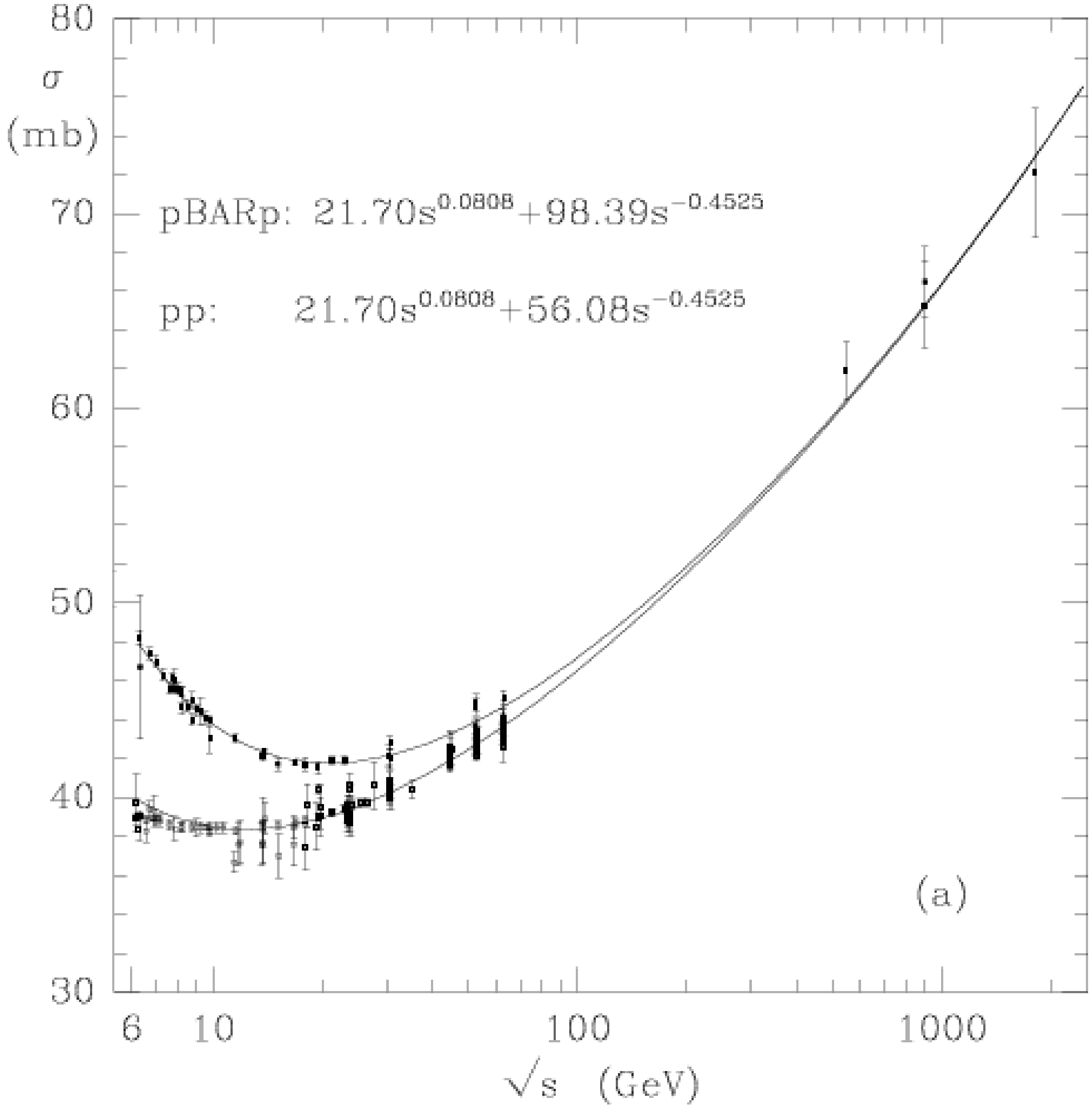
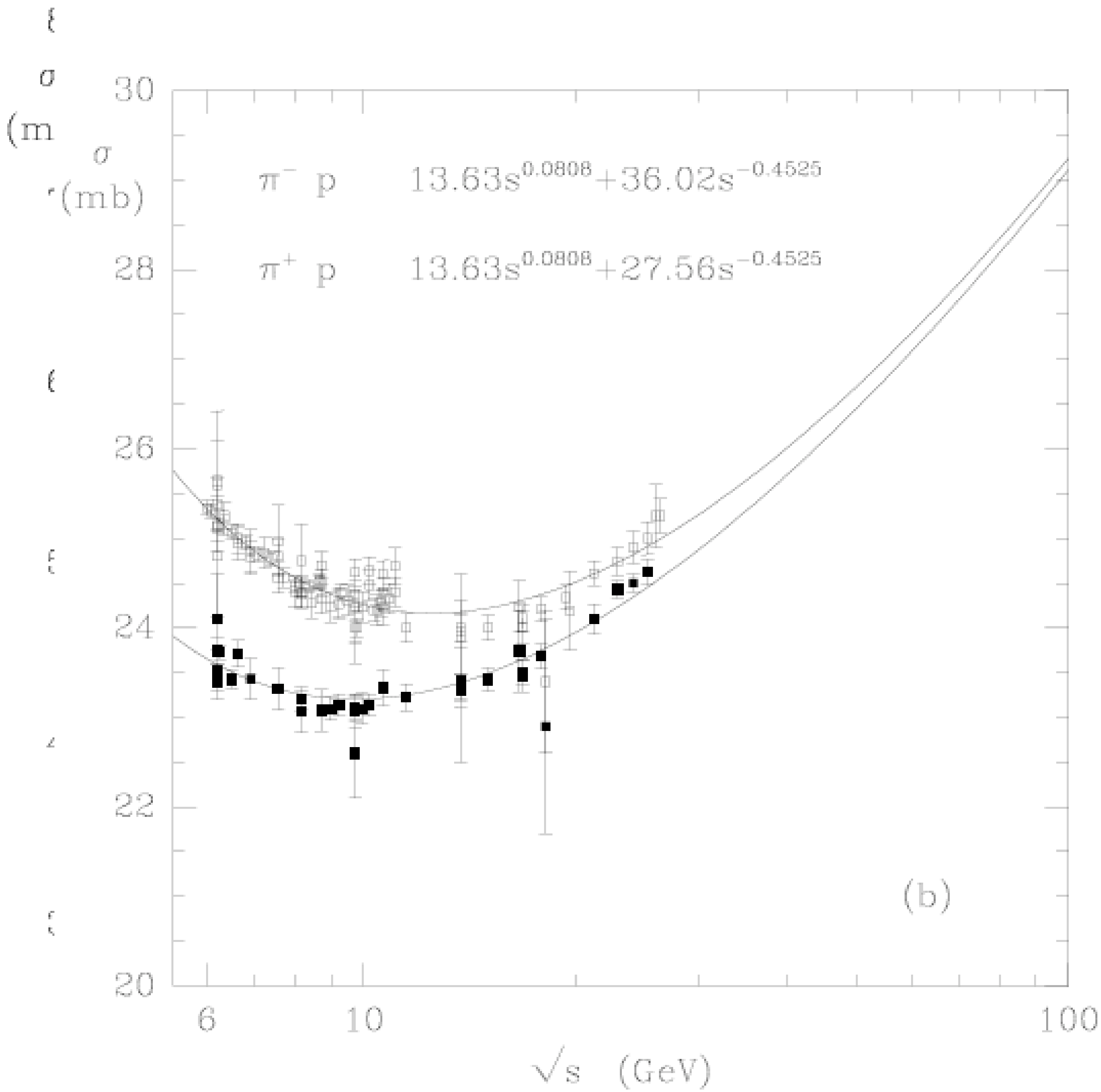
Non-vacuum exchange

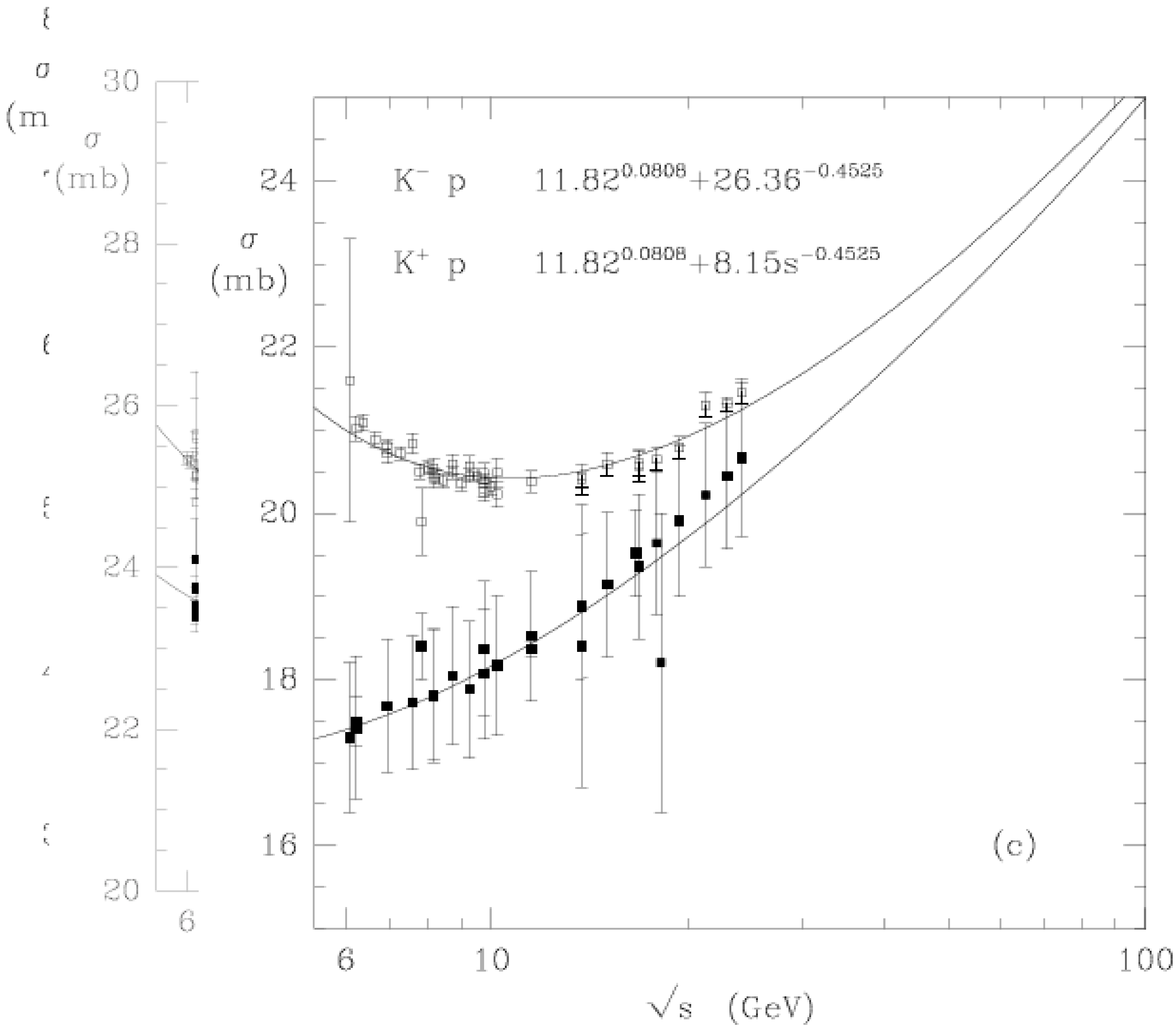
$$\alpha(0)_R < 1$$

Note: odderon $\alpha_O(0) \leq 1$

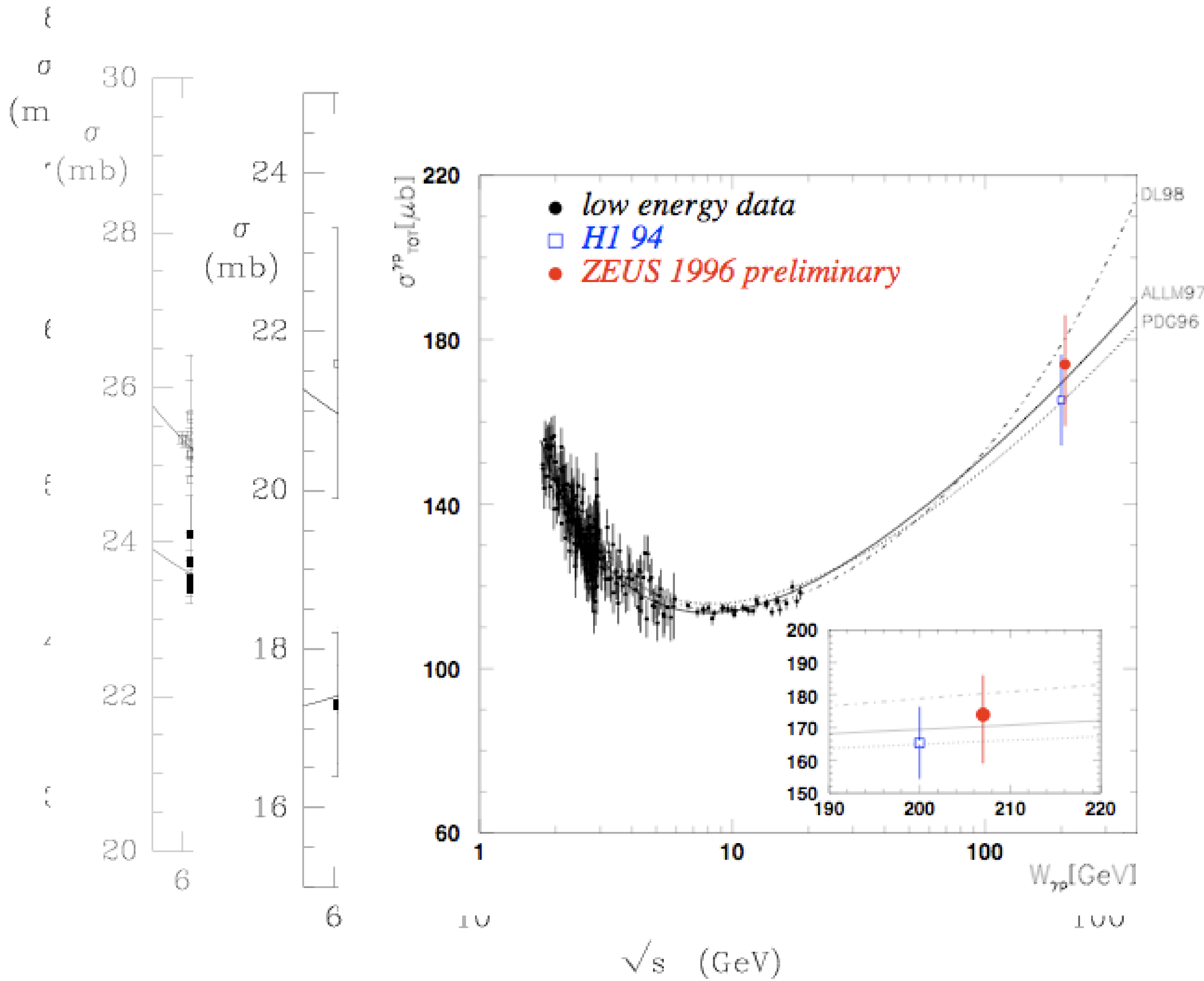

80
σ
(mb)
70
pBARp: 21.70s^0.0808+98.39s^-0.4525
pp: 21.70s^0.0808+56.08s^-0.4525
60
50
40
30
(a)
6
10
100
1000
√s (GeV)



σ (mb)
π⁻ p 13.63s^0.0808+36.02s^-0.4525
π⁺ p 13.63s^0.0808+27.56s^-0.4525
30
28
26
24
22
20
6
10
100
√s (GeV)
(b)

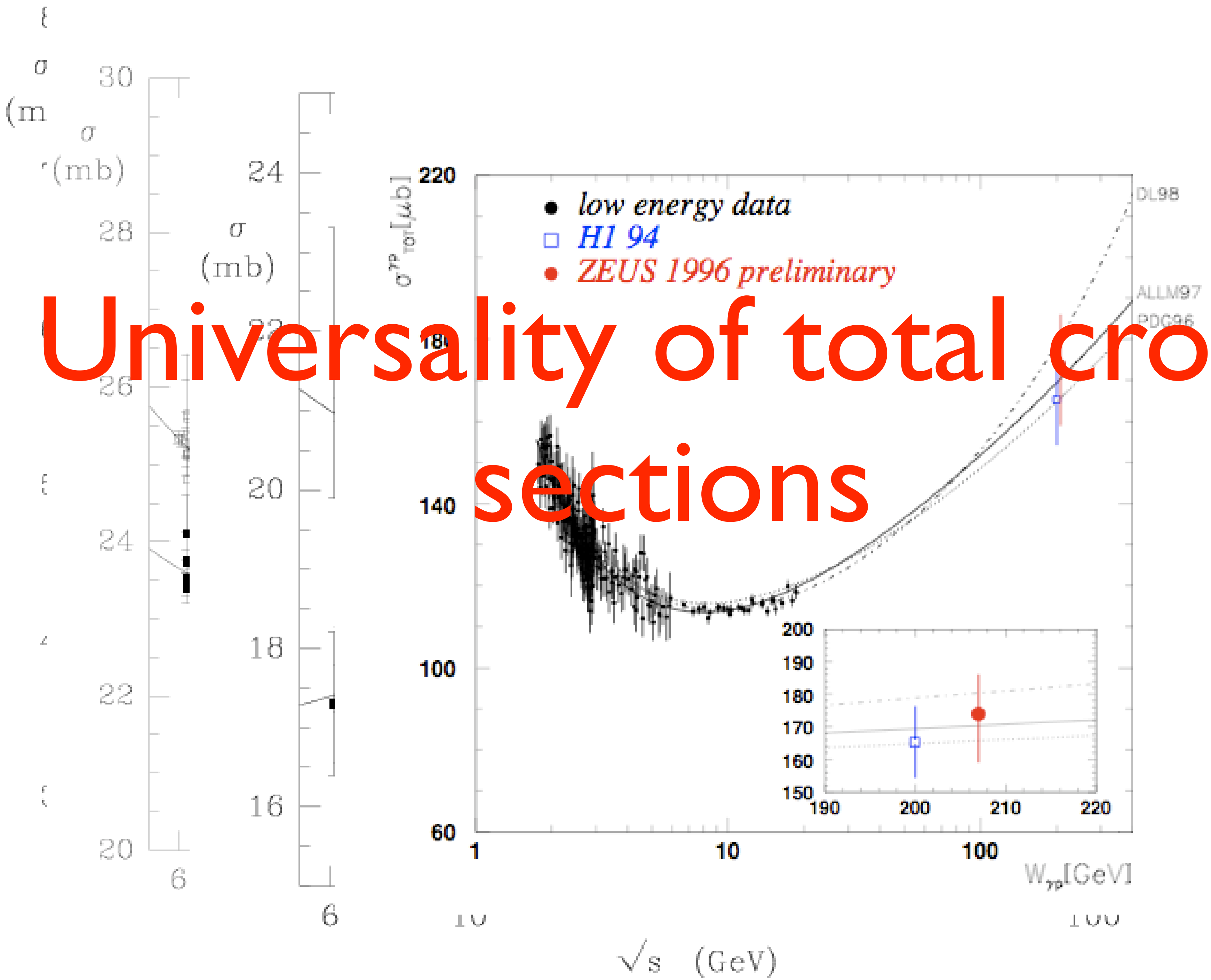


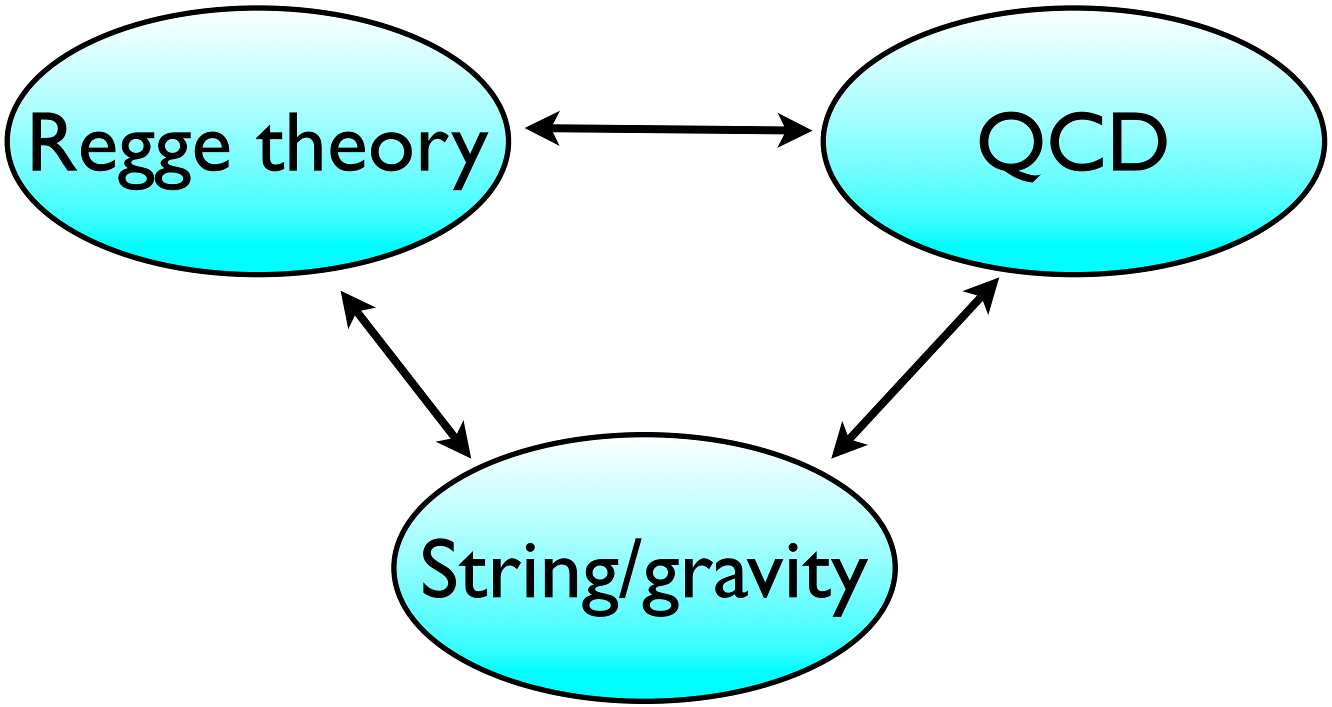
K⁻ p 11.82^0.0808+26.36^-0.4525
K⁺ p 11.82^0.0808+8.15s^-0.4525
σ (mb)
√s (GeV)
(c)

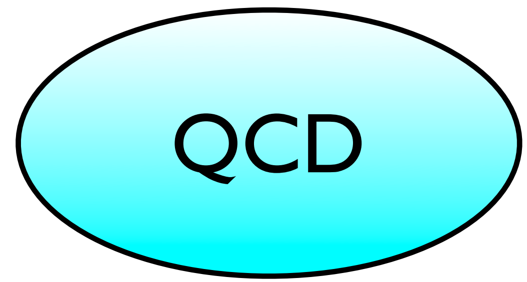


σ
(mb)
30
28
26
24
22
20
6
24
22
20
18
16
6
220
180
140
100
60
1
10
100
$\sigma^{\gamma p}_{TOT}[\mu b]$
low energy data
H1 94
ZEUS 1996 preliminary
DL98
ALLM97
PDG96
$W_{\gamma p}$[GeV]
200
190
180
170
160
150
190
200
210
220
√s (GeV)

Universality of total cross sections



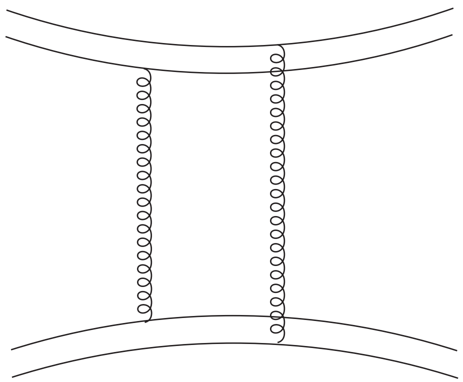

Regge theory
QCD
String/gravity


QCD

Pomeron in gauge theory

Low-Nussinov model

2-gluon exchange

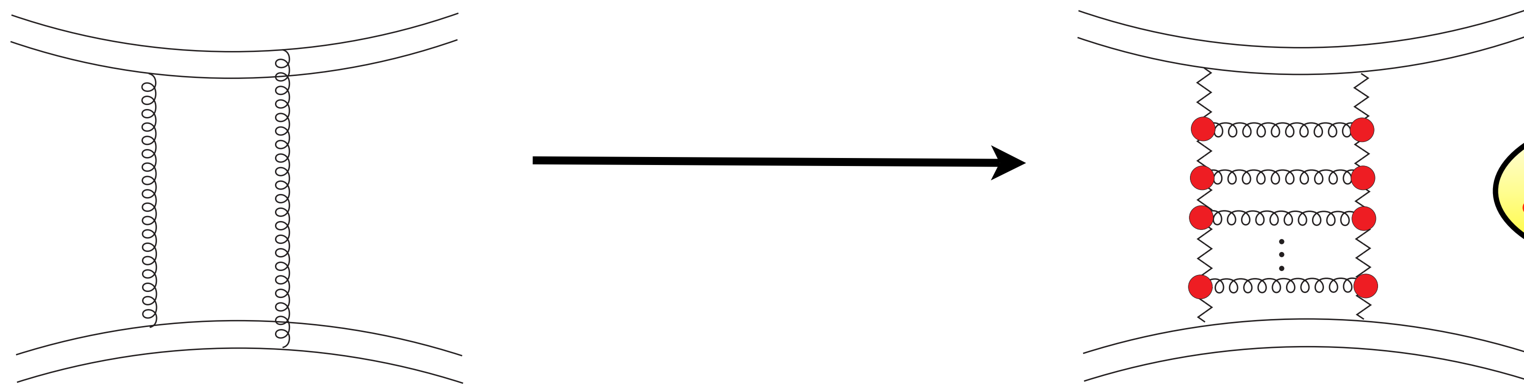


Pomeron in gauge theory

Low-Nussinov model
2-gluon exchange

BFKL resummation
color singlet

Balitskii
Fadin
Kuraev
Lipatov



$s \gg |t|$

Pomeron in gauge theory

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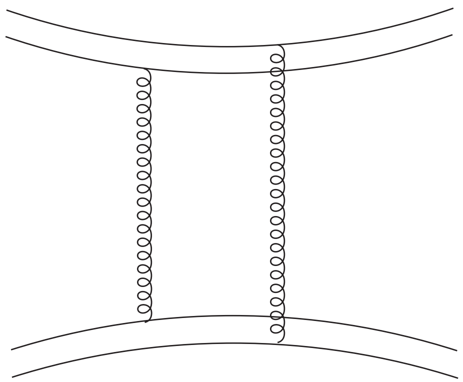

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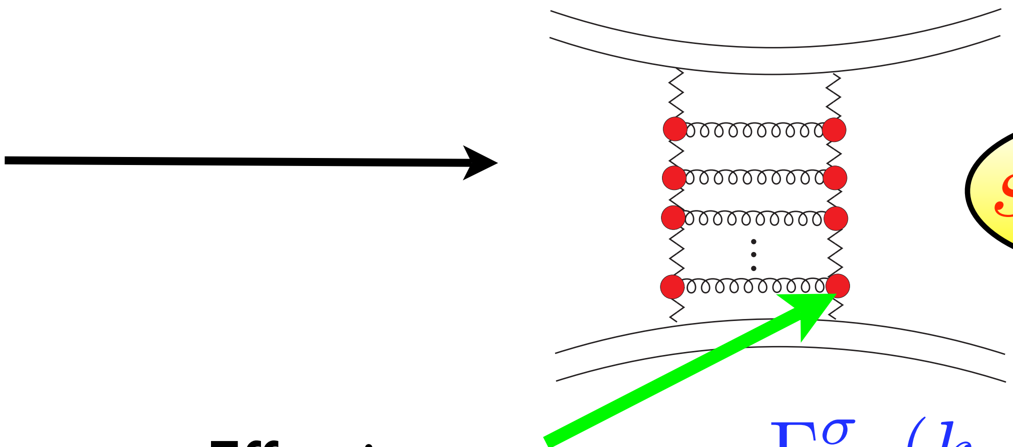
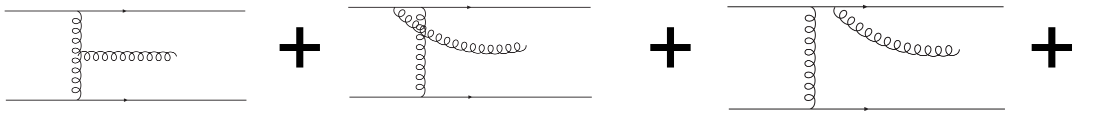


$s \gg |t|$

Effective vertex $\Gamma^{\sigma}_{\mu\nu}(k_i, k_{i+1})$



$+$ ...

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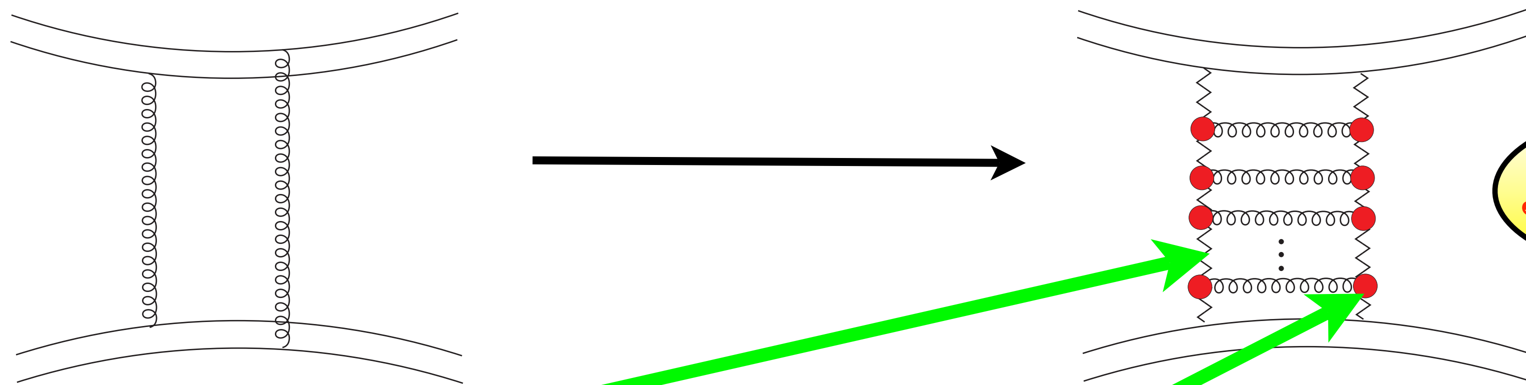
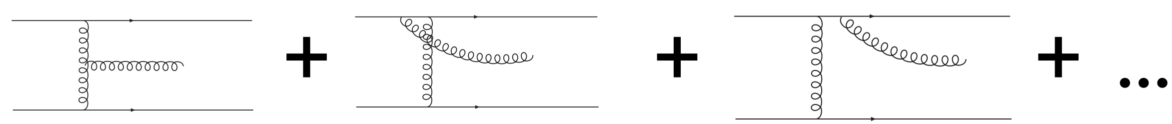


$s \gg |t|$

Reggeized gluon:

$$D_{\mu\nu}(\hat{s}, k_T^2) = \frac{i g_{\mu\nu}}{k_T^2} \left(\frac{\hat{s}}{k_T^2} \right)^{\epsilon_G(k_T^2)}$$

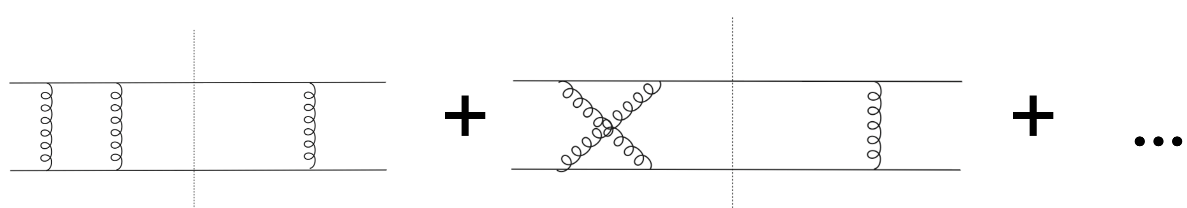
Effective vertex $\Gamma^{\sigma}_{\mu\nu}(k_i, k_{i+1})$



$$\hat{s}_i = (k_{i-1} - k_{i+1})^2$$

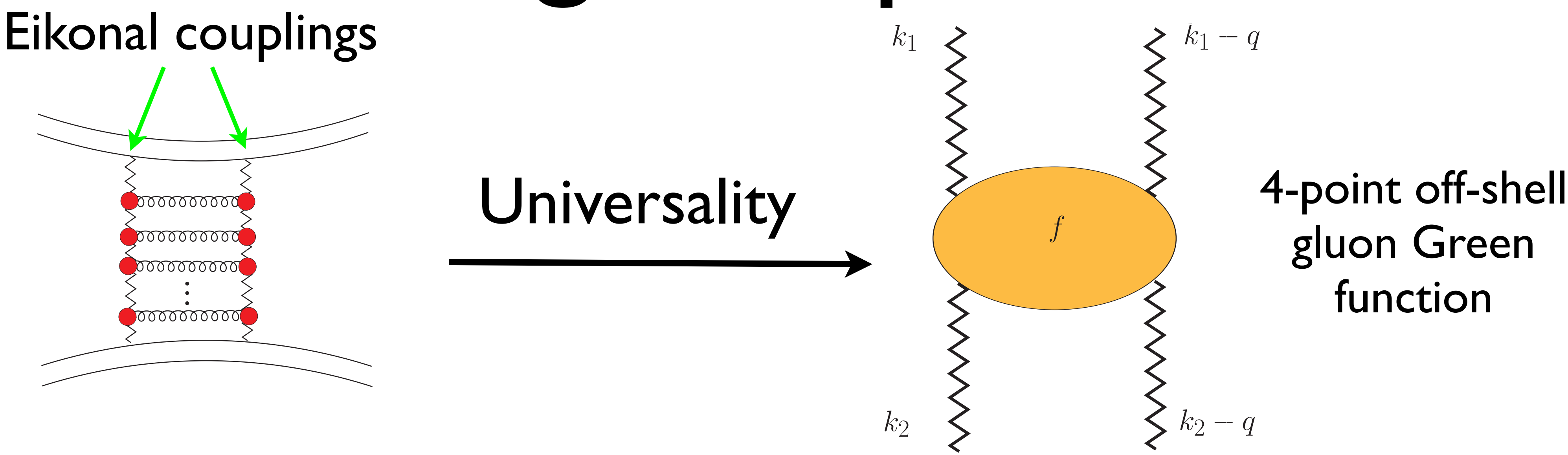
Regge trajectory: virtual diagrams

$$\epsilon_G(q_T^2) = \frac{N_c \alpha_s}{4\pi^2} \int_\Lambda d^2 k_T \frac{-q_T^2}{k_T^2 (k_T - q_T)^2}$$



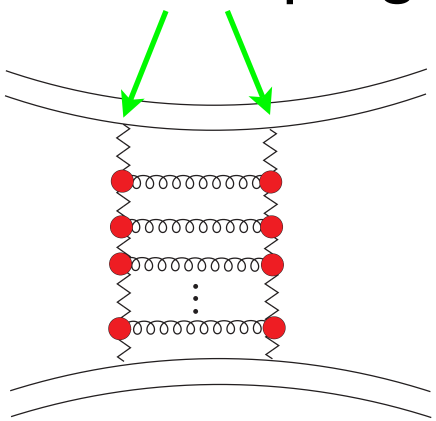
Infrared divergent!

Integral equation

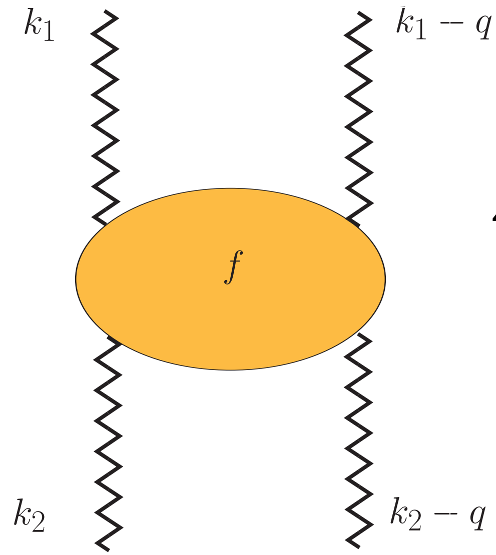


Integral equation

Eikonal couplings

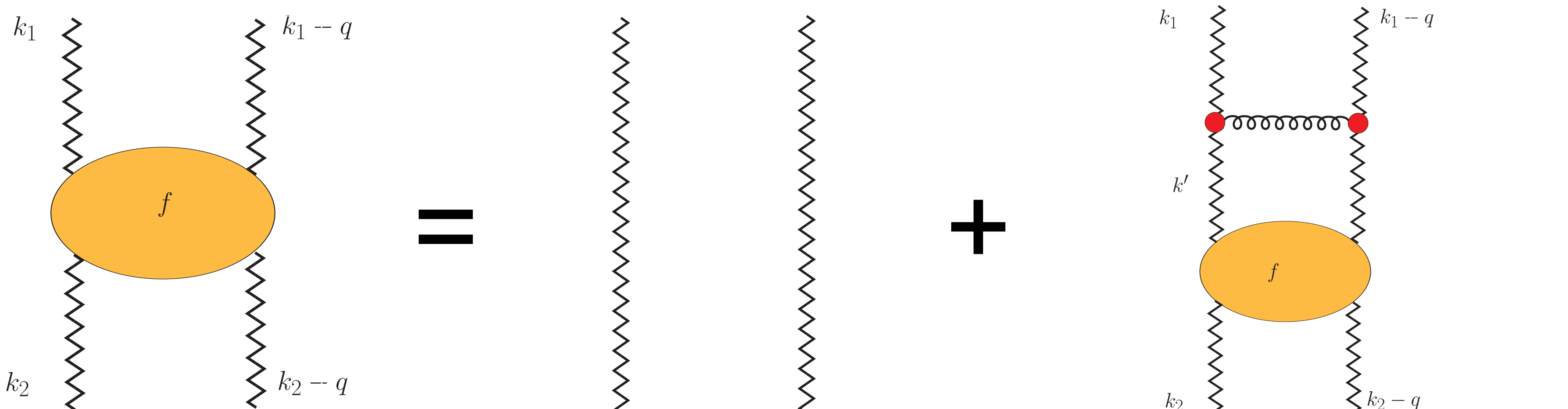


Universality



4-point off-shell gluon Green function

Integral equation for the Pomeron



Born term

Addition of one rung

Integral equation

$$f(Y;k_{1T},k_{2T},q_T) = f^{(0)}(k_{1T},k_{2T},q_T)+\int_0^Y dy\, K(k_{1T},k_{2T},q_T)\otimes f(y;k_{1T},k_{2T},q_T)$$

Convolution in transverse momenta

Rapidity: $Y = \ln 1/x = \ln s/s_0$

! Scale choice (irrelevant at lowest order) !

Integral equation

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Convolution in transverse momenta

Rapidity: $Y = \ln 1/x = \ln s/s_0$

! Scale choice (irrelevant at lowest order) !

! Factorization of longitudinal and transverse components of momenta !

Mellin transform: $\int dY e^{(-\omega-1)Y} f(Y) dY = f(\omega)$

$$\omega f(\omega;k_{1T},k_{2T},q_T) = \delta^{(2)}(k_{1T}-k_{2T}) + K(k_{1T},k_{2T},q_T) \otimes f(\omega;k_{1T},k_{2T},q_T)$$

Integral kernel has Mobius invariance.

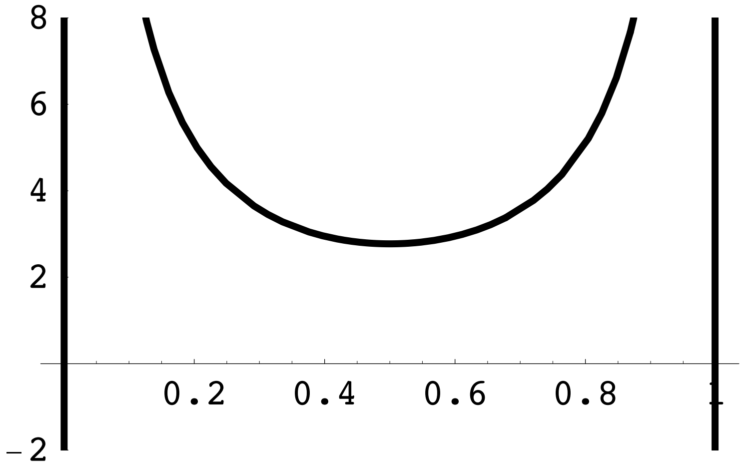
Solution of the BFKL equation

At zero momentum transfer: $q_T = 0$

Eigenfunctions: $\phi_\nu^n(k_T) = \frac{1}{\pi\sqrt{2}}(k_T^2)^{1/2+i\nu} e^{in\theta}$

Diagonalize equation: $K \otimes \phi_\nu^n = \frac{\alpha_s N_c}{\pi}\chi(\nu,n)\,\phi_\nu^n$

Eigenvalue (take n=0): $\chi(\nu,0) = 2\psi(1) - \psi(1/2+i\nu) - \psi(1/2-i\nu)$



$\gamma = 1/2 + i\nu$

Simple poles:

$$\gamma = \ldots, -2, -1, 0, 1, 2, \ldots$$

Hard Pomeron

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Saddle point solution: around $\gamma = 1/2$

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Power-like growth with energy

Diffusion pattern in transverse momenta

Regge behavior from Feynman diagrams: $\alpha_P(0) = 1 + \frac{N_c \alpha_s}{\pi} 4\ln 2$

Note: it is possible to compute Pomeron in electroweak theory also.

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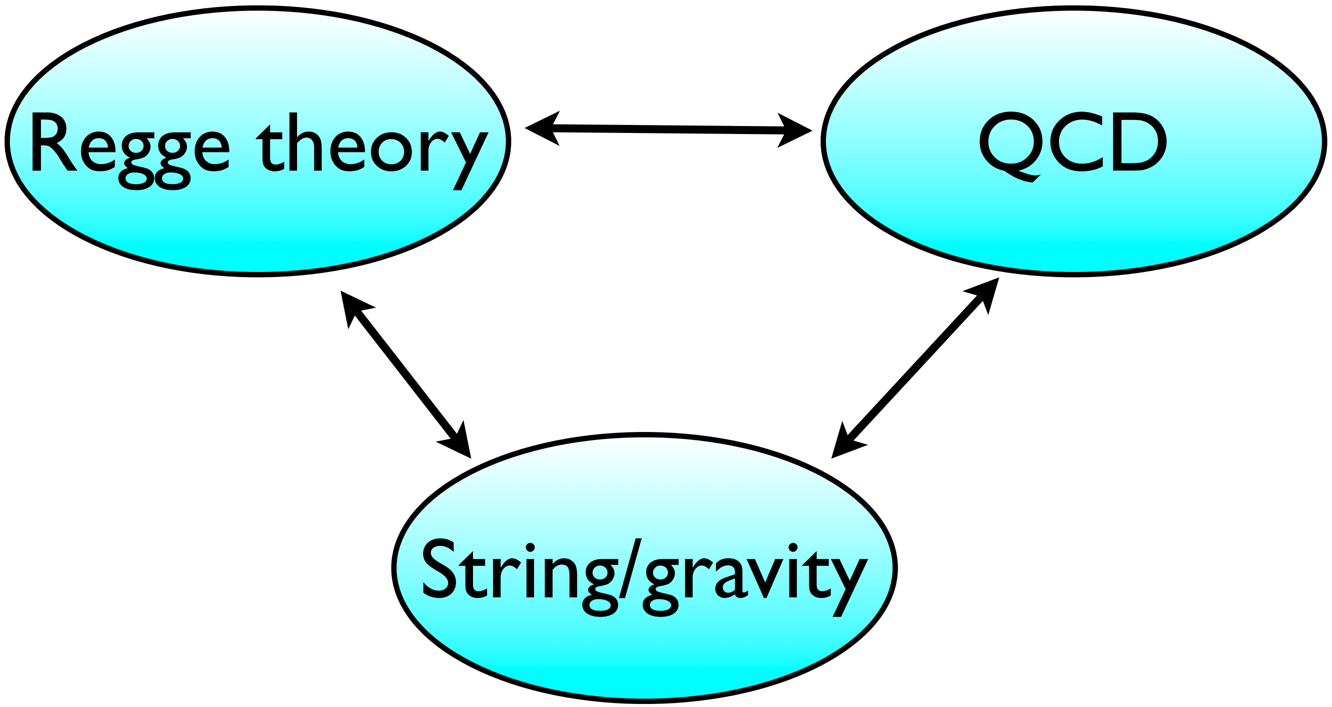

Regge theory
QCD
String/gravity

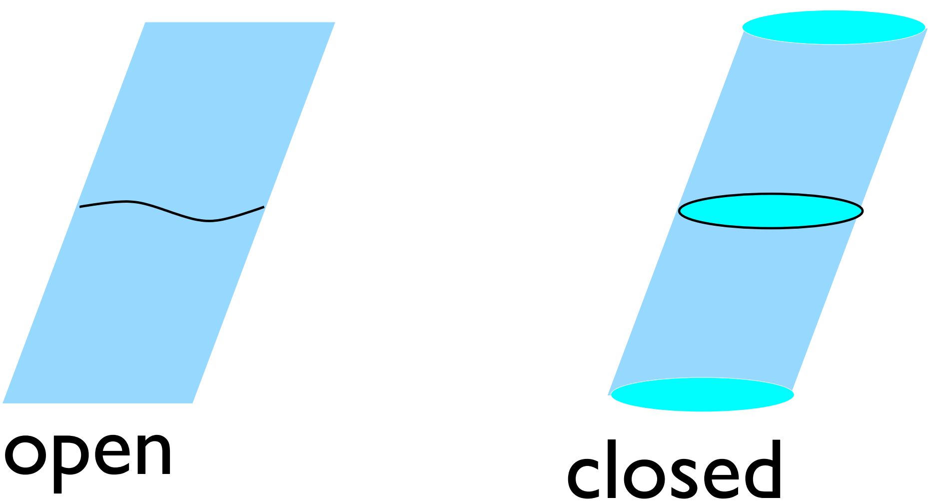

String/gravity

Graviton

Graviton

Spin 2 massless (2 polarizations) particle:
symmetric rank 2 tensor.

In string theory: closed string state.



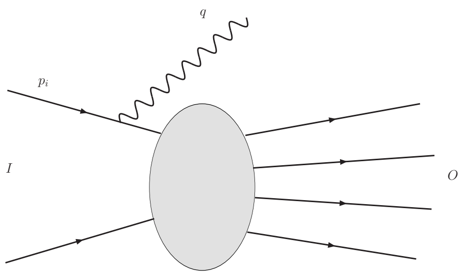
String theory includes gravity

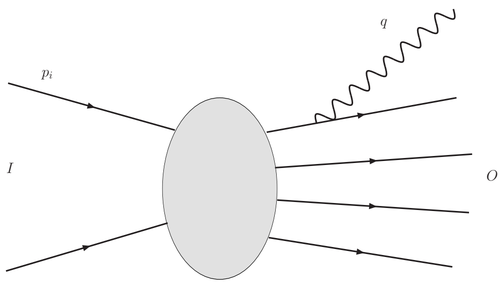
Universality of couplings

Universality of couplings

Example: *(Weinberg)*

Amplitude for emission of soft foton:





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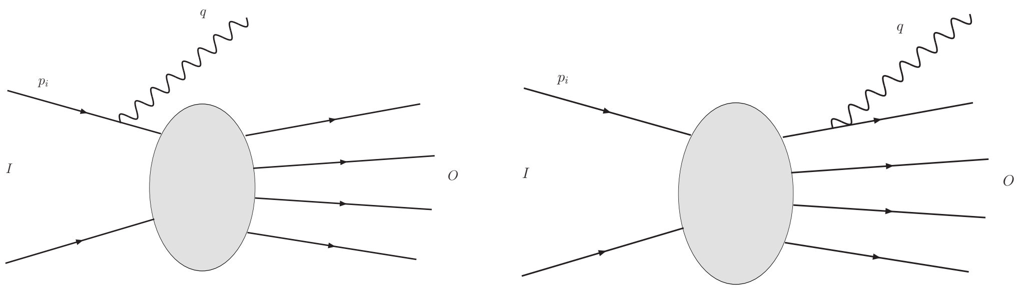

$\mathcal{M}_{IO}$ Amplitude without fotons

$$\mathcal{M}^{\mu}_{IO}(q,\gamma) = \mathcal{M}_{IO}\sum_n \frac{\sigma_n e_n p_n^{\mu}}{p_n \cdot q - i\sigma_n \epsilon}$$

e_n charge p_n momentum

σ_n ± 1 for incoming and outgoing particles

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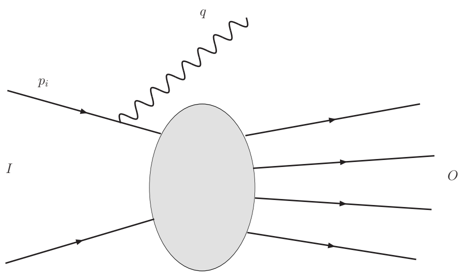

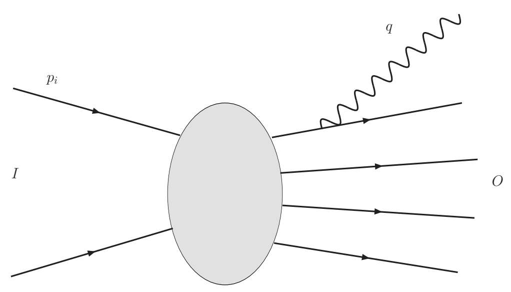

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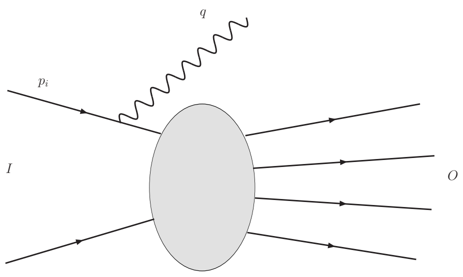

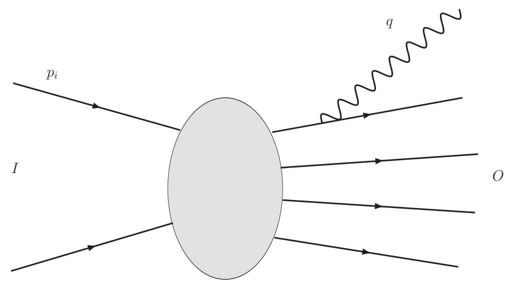

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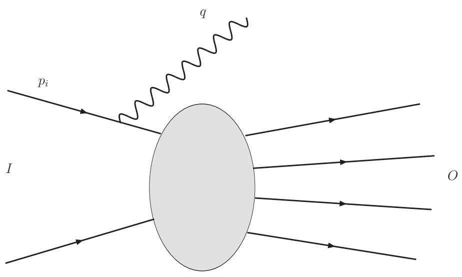

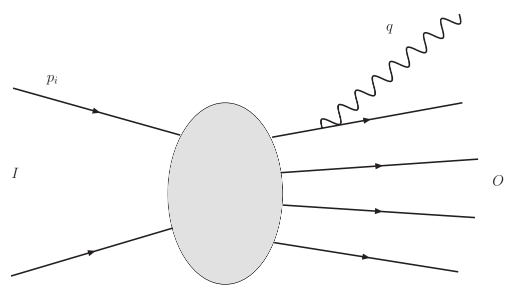

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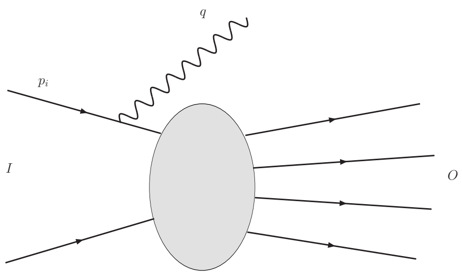

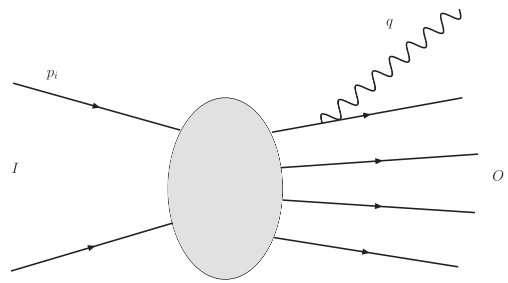

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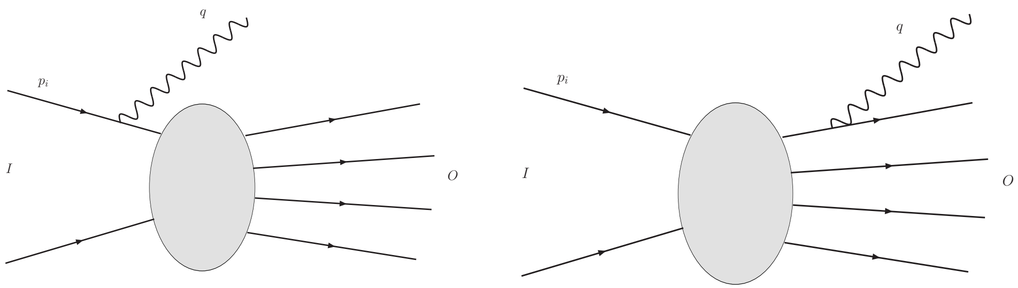

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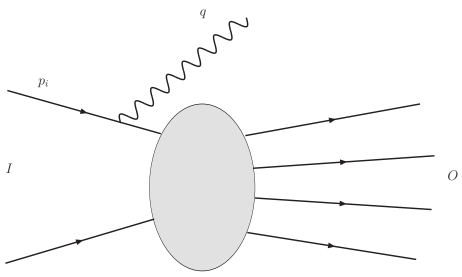

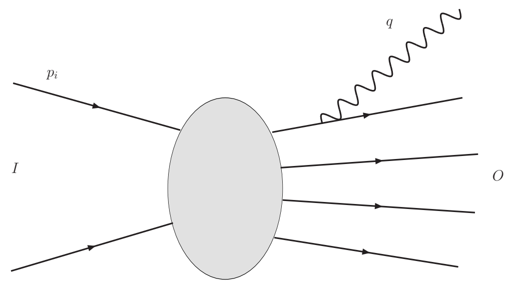

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Lorentz invariance for spin 2 particles gives principle of equivalence

Gauge/Gravity duality

Maldacena

strings in AdS(D) ⟷ CFT(d=D-1)

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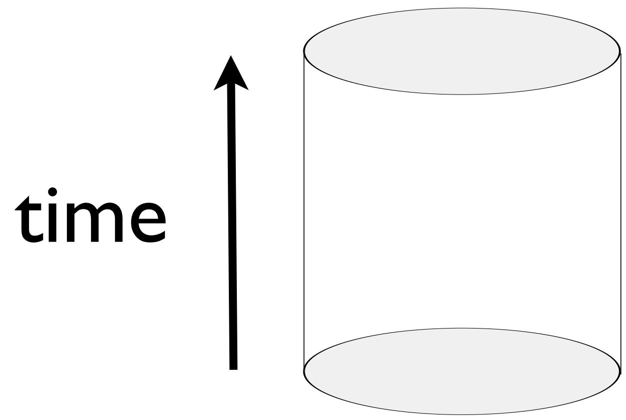

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(T,X): Minkowski coordinates

R: radius of curvature

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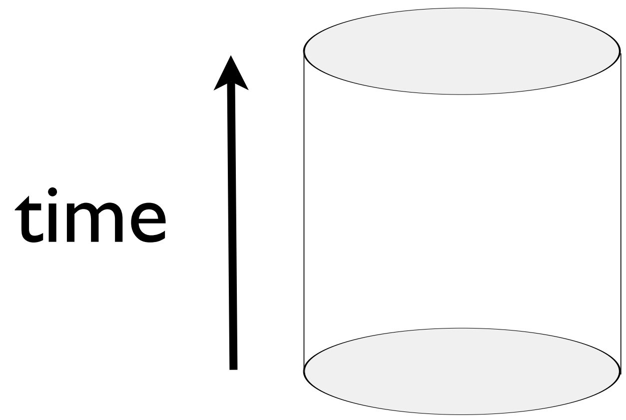

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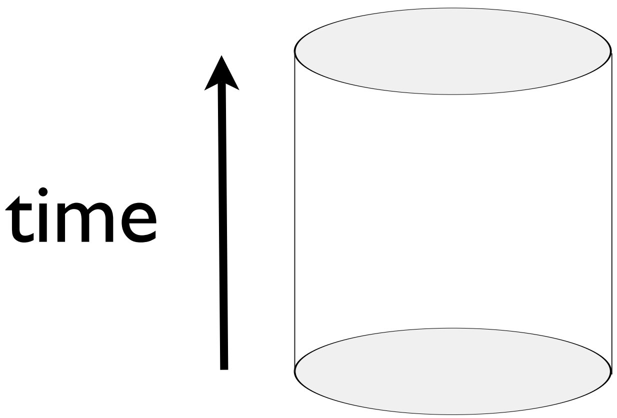

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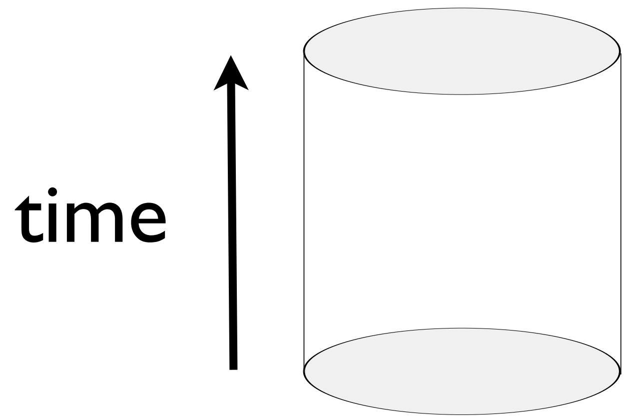

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Duality: different degrees of freedom in two different limits of the coupling $g^2 N_c$

$g^2 N_c \gg 1$

- Strongly coupled SYM
- Weakly coupled gravity

$g^2 N_c \ll 1$

- Weakly coupled SYM
- Strongly coupled gravity

String/N=4 SYM duality

Note that correspondence is expected to be valid for N=4 SYM:

- One gauge field A_μ
- Six scalars $\phi_i, \; i = 1, \ldots, 6$
- Four fermions $\chi_k, \; k = 1, \ldots, 4$
- Fields transform in the adjoint representation
- Conformal invariant $\beta \sim N - 4 = 0$

SYM N=4 very different from QCD. Nevertheless a very good “laboratory” .

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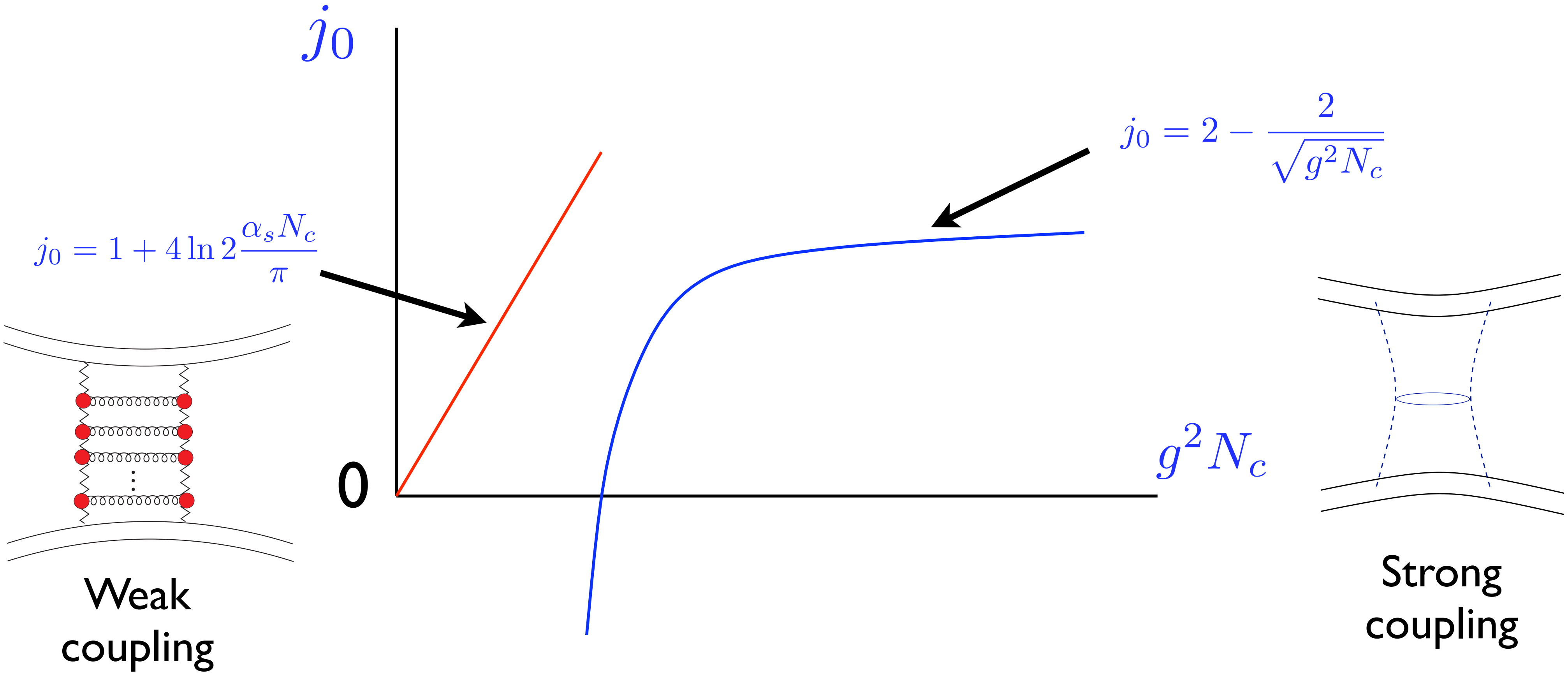
Diffusion in transverse (virtual) momenta ⟷ *Diffusion in the fifth (radial) dimension of AdS space*

Pomeron/Graviton



Pomeron: made out of many (reggeized) gluons. Growth of the cross section caused by dynamical effect: emission of many gluons.

Graviton: single object (closed string state). Growth of the cross section corresponds to the exchange of spin 2.

Resummation at high energies (small x)

- Next-to-leading order very large: $j_0 = 1 + 4\ln 2 \frac{\alpha_s N_c}{\pi}(1 - 6.45\frac{\alpha_s N_c}{\pi})$
- Sources of large corrections:

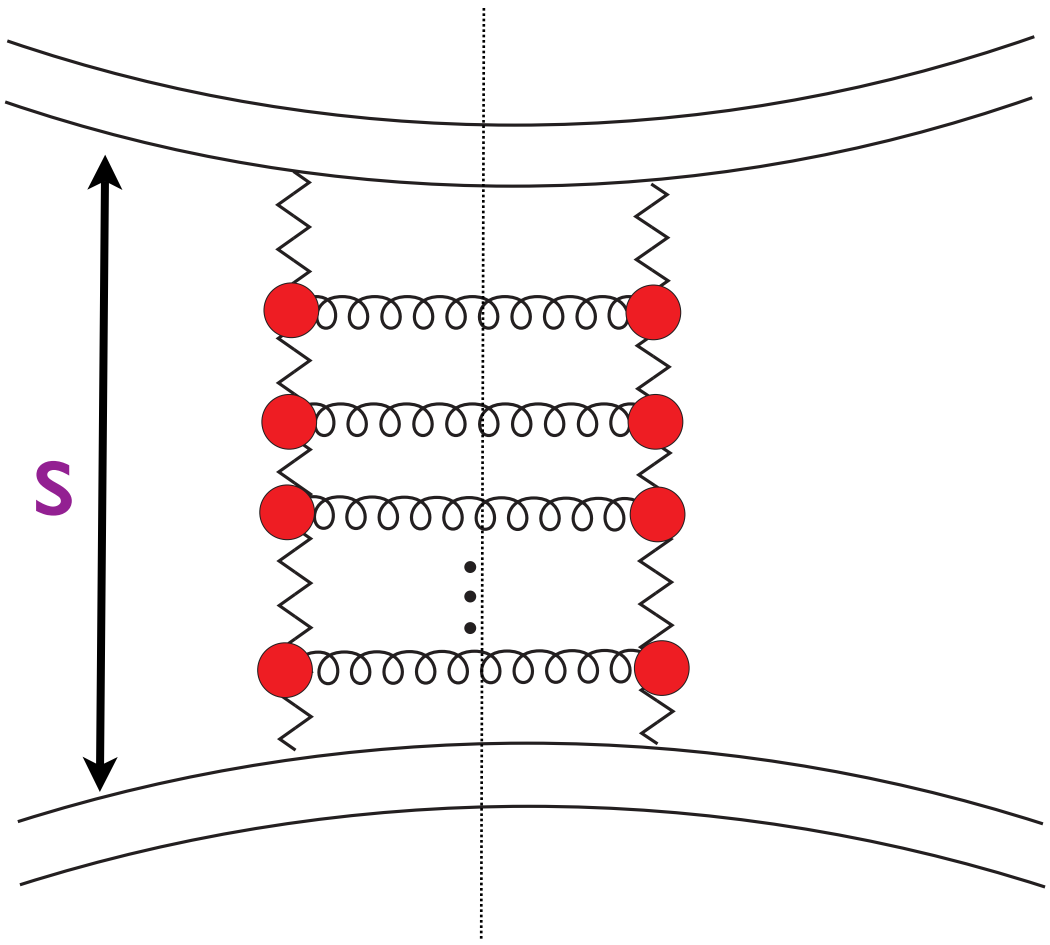
  Common to QCD and SYM

  - Kinematical effects, energy momentum conservation.
  - Running of the coupling.

  QCD only

- Other corrections: quarks in the evolution.
- Need to take more than next-to-leading order: *all orders.*

Kinematics



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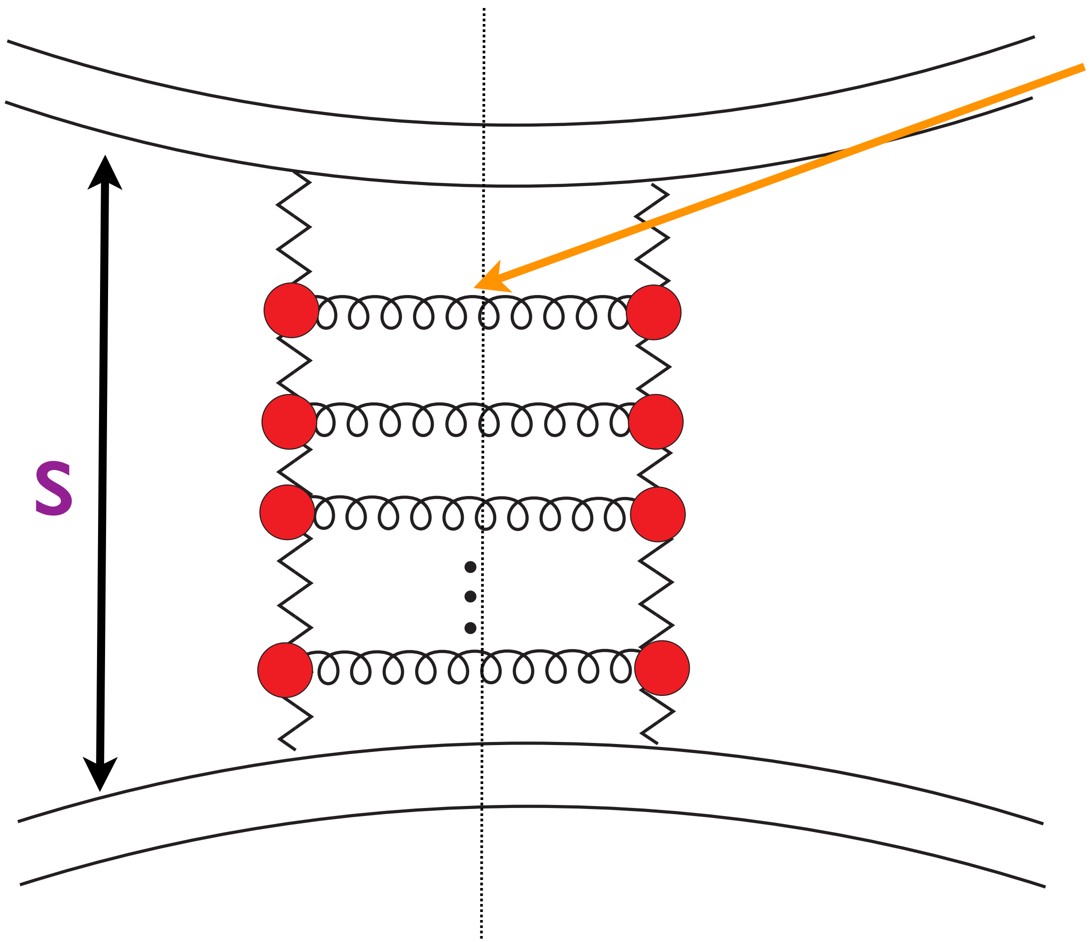

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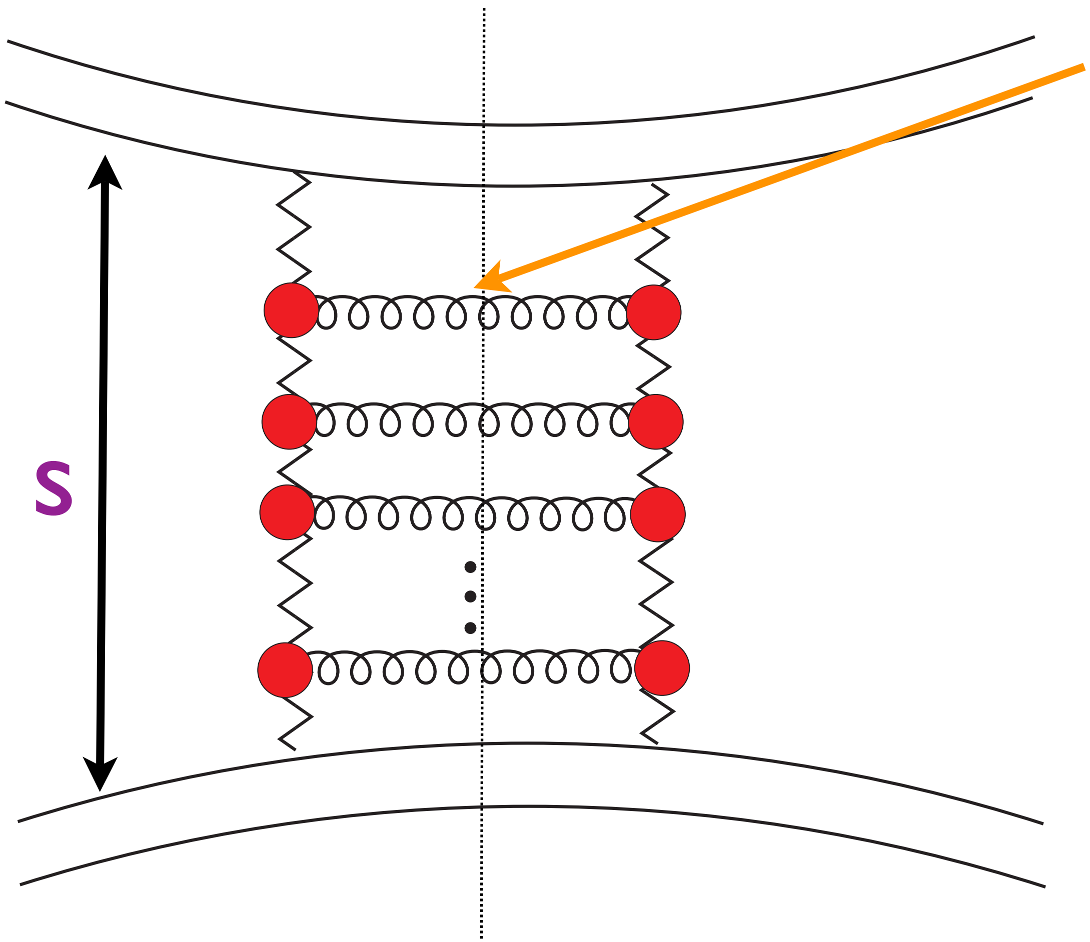

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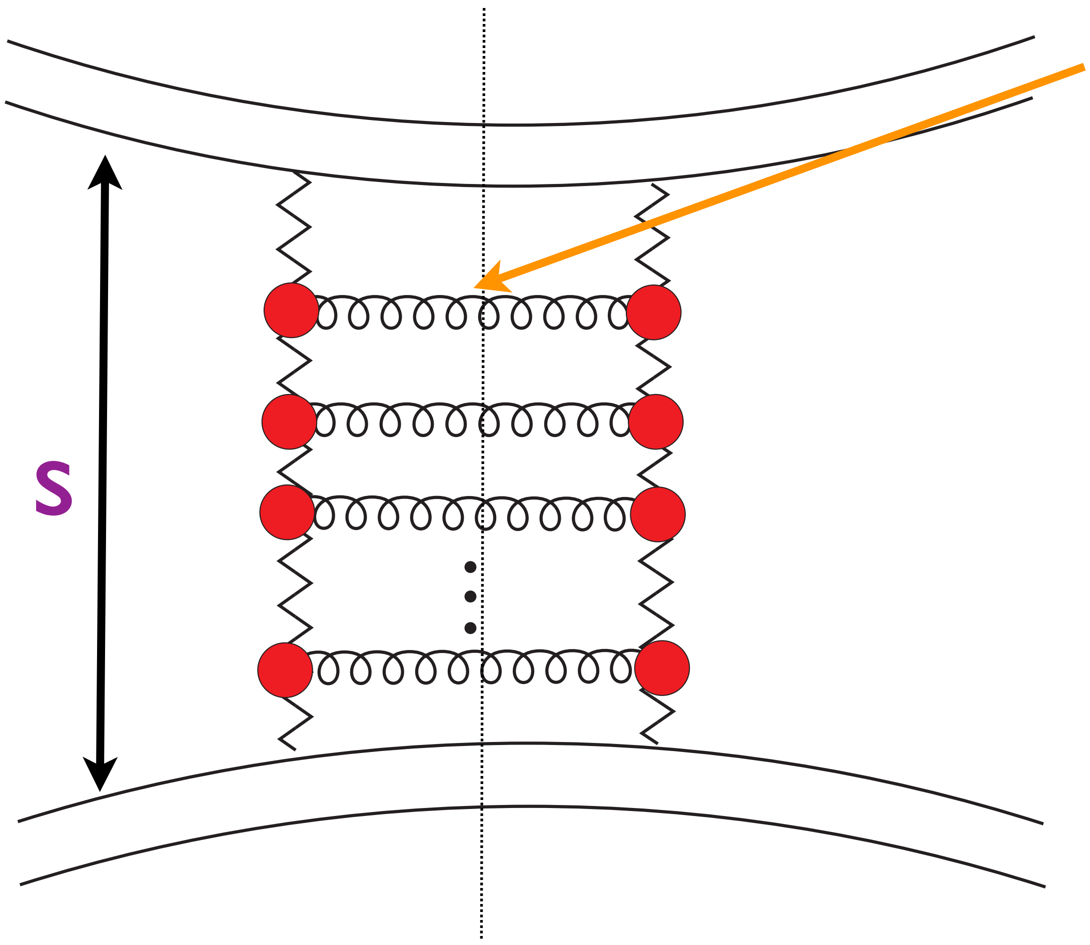

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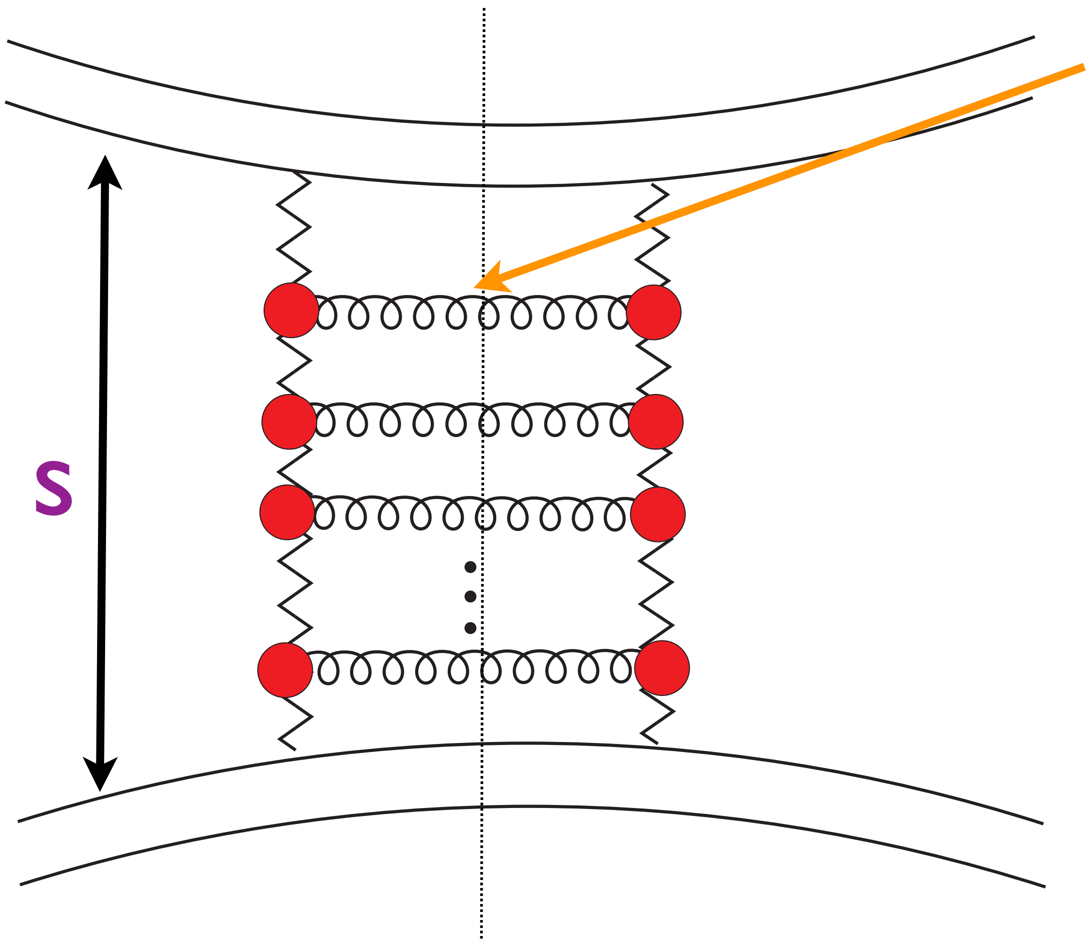
- Put the constraint to correct this.

Kinematics



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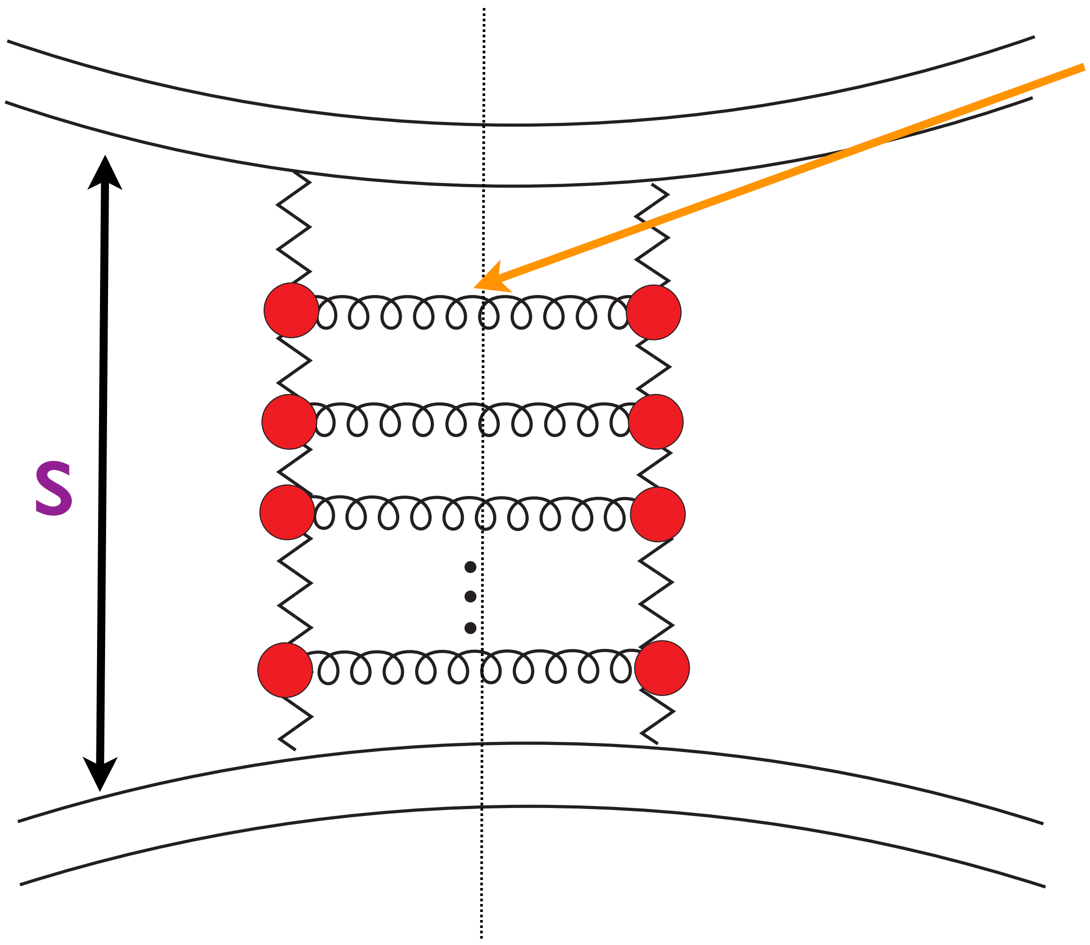

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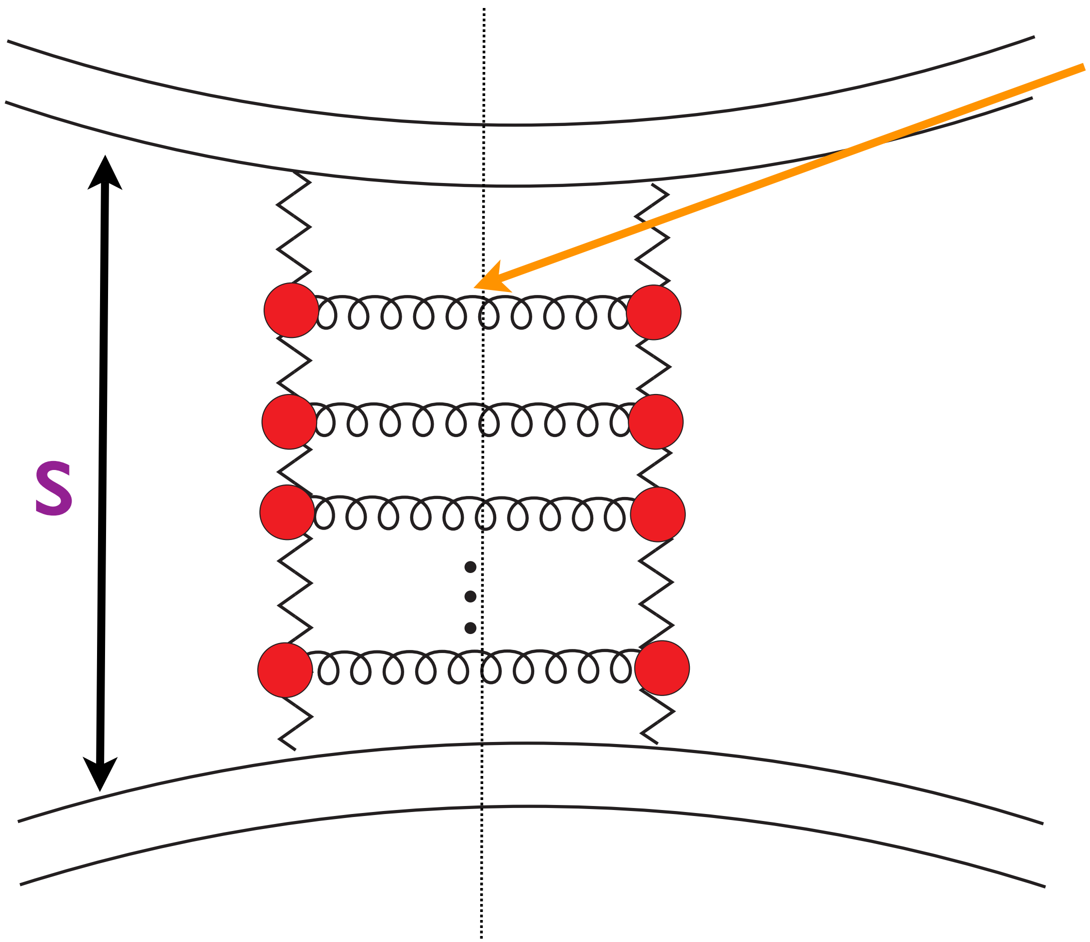

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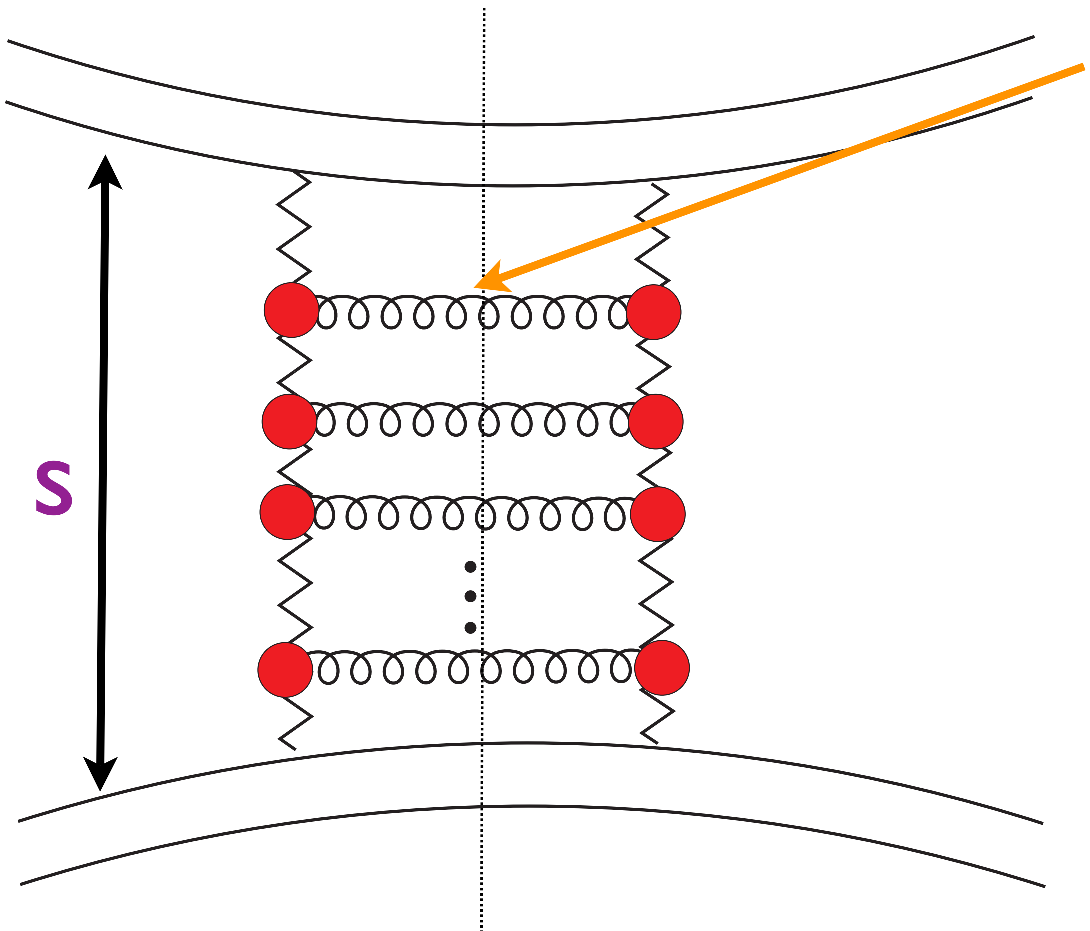

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Impose constraints to satisfy energy-momentum sum rule

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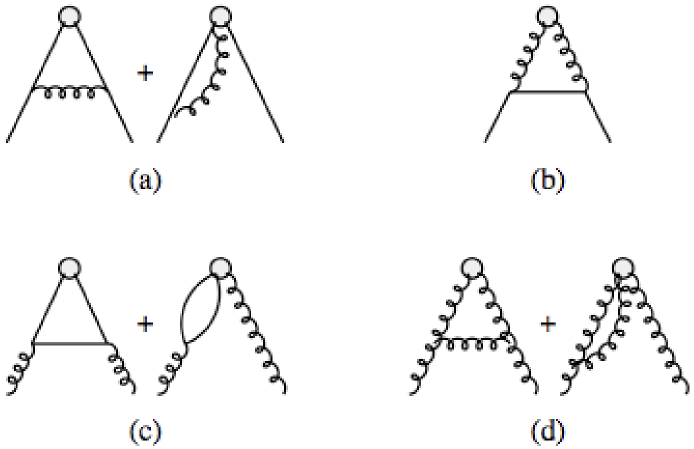
A note on anomalous dimensions in QCD

In standard operator product expansion
approach to DIS
evaluate anomalous dimensions



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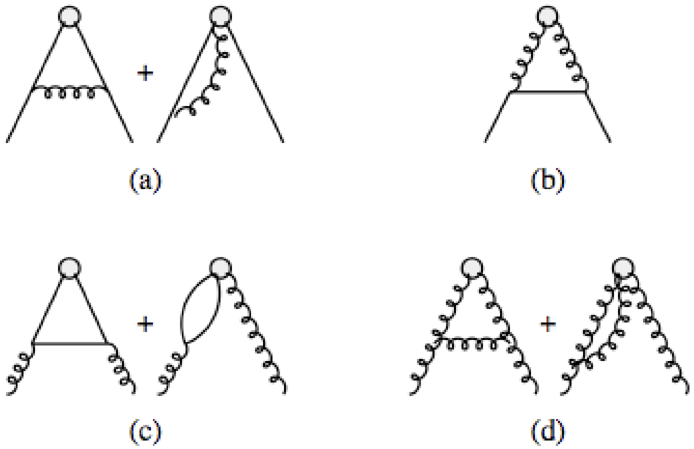

RGE:

$$[\mathcal{D}\delta_{ab} - \gamma_{ab}^{(j)}]C_b^j(g, \mu, -q^2) = 0$$

$$\mathcal{D} = \mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}$$

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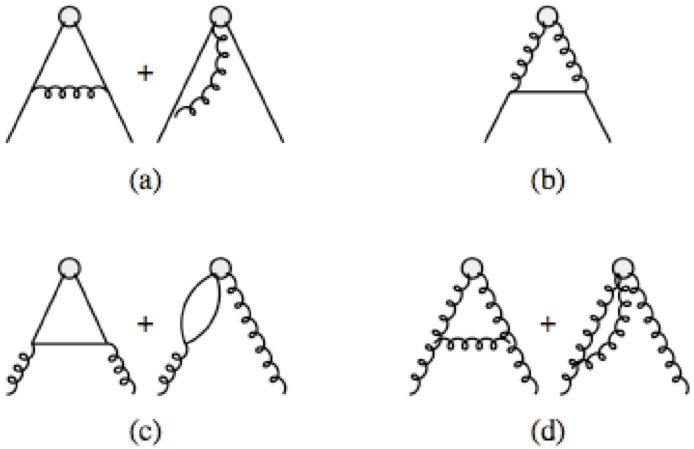

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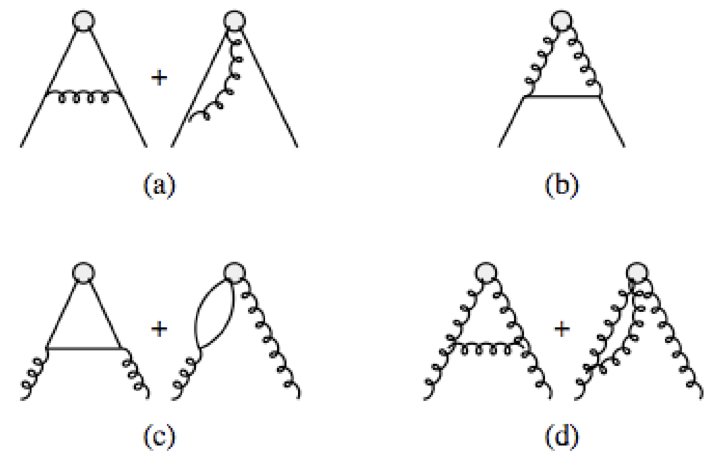

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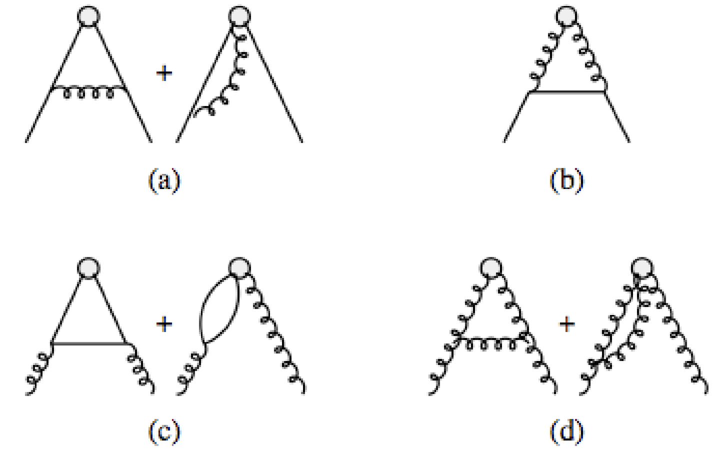

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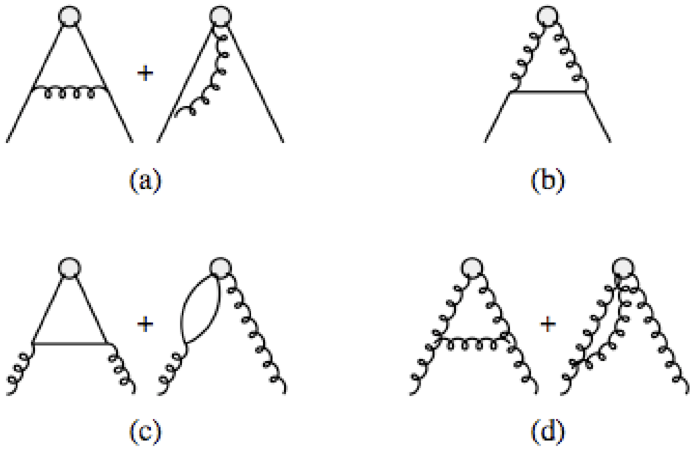

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Satisfied at each order of the perturbation theory

)

BFKL kernel eigenvalue:

$$1 = \mathcal{K}(\alpha_s, \gamma, j)$$

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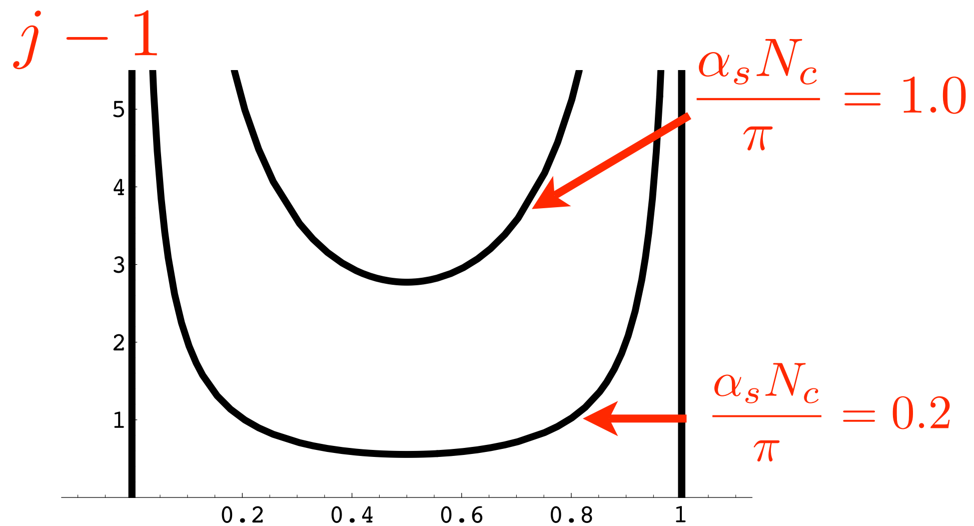

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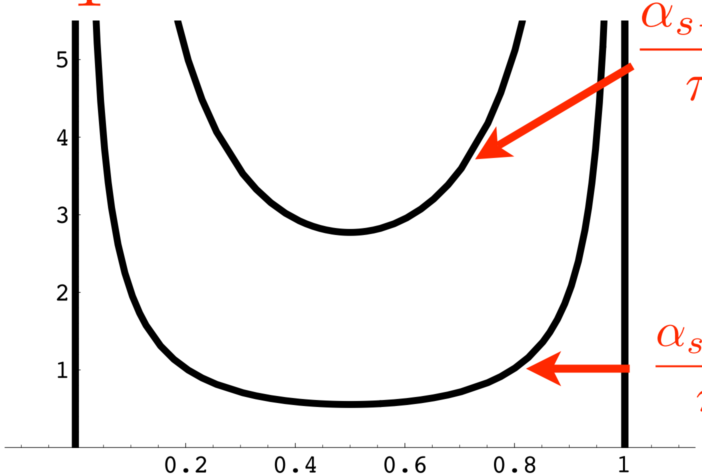
$j - 1$



$$\frac{\alpha_s N_c}{\pi} = 1.0$$

$$\frac{\alpha_s N_c}{\pi} = 0.2$$

After:

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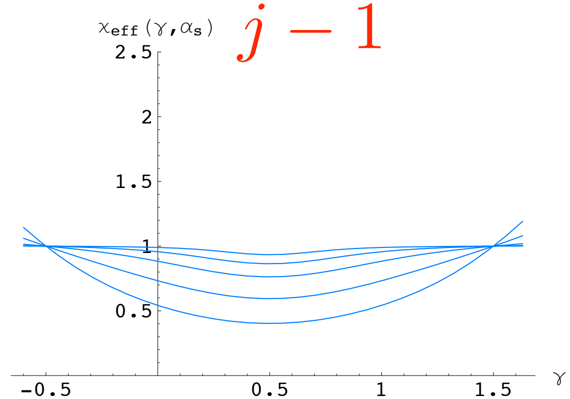

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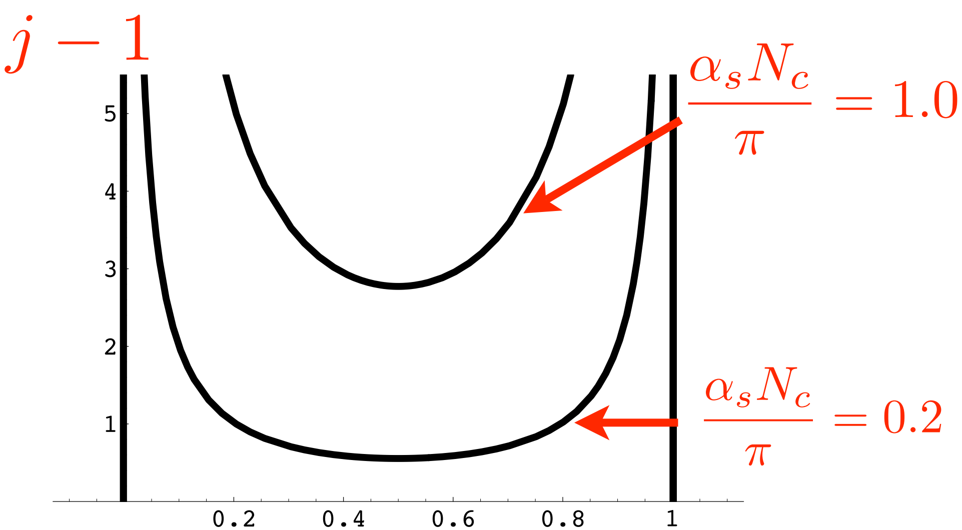

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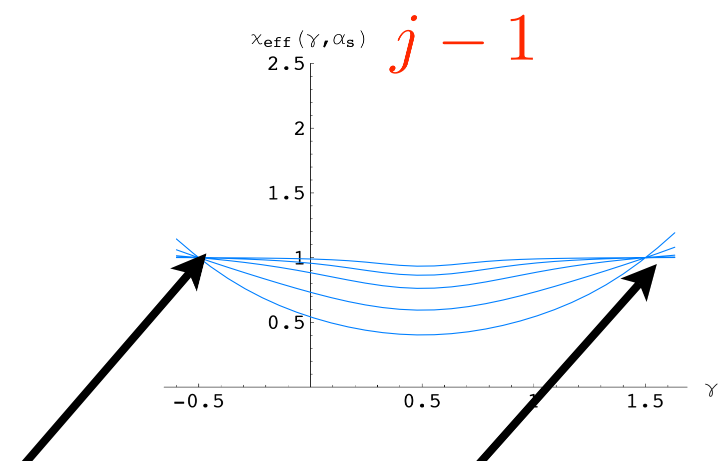
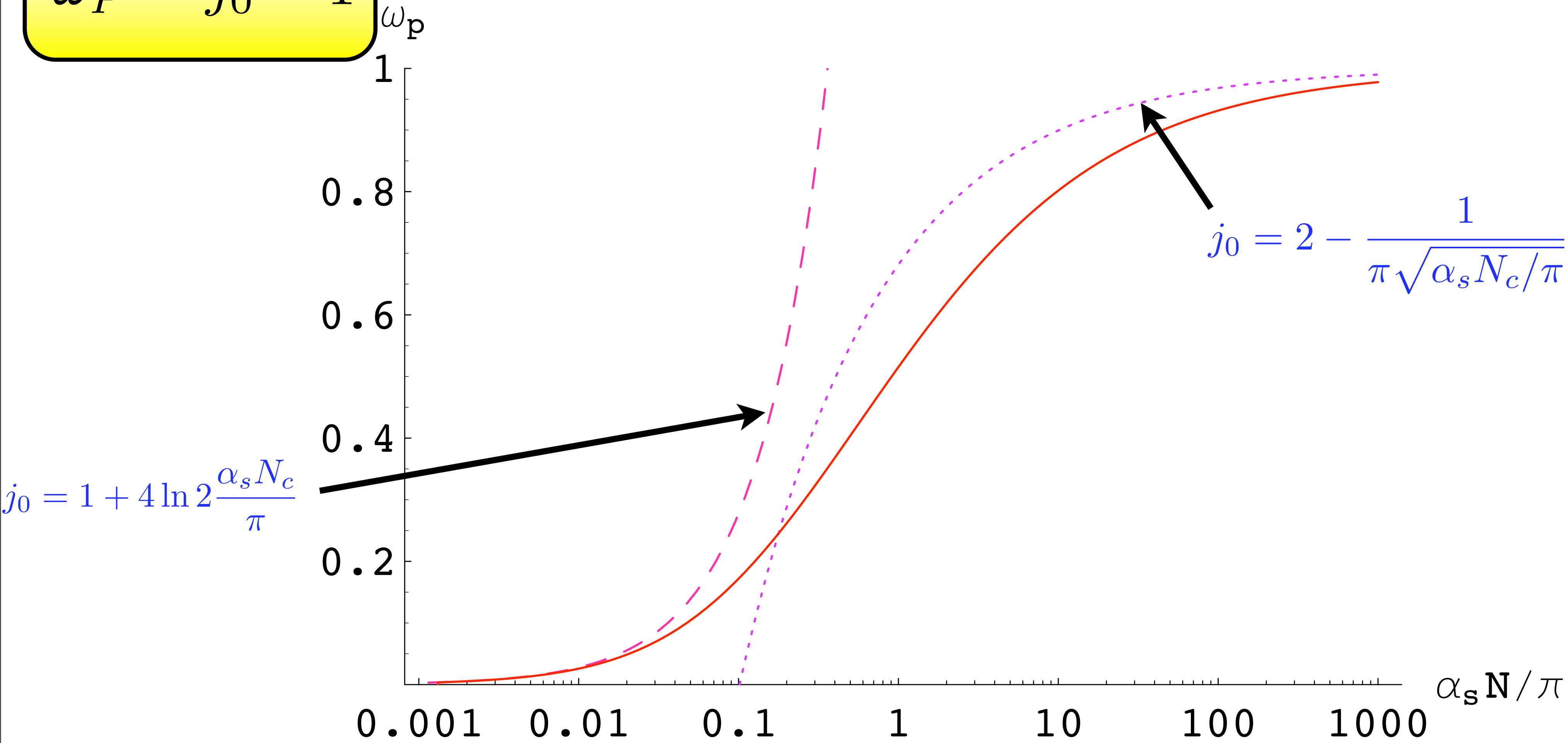


Fixed points: energy conservation

Intercept in the resummed model

$$\omega_P = j_0 - 1$$



Note the logarithmic horizontal axis

Cross section: $\sigma \sim s^{j_0 - 1}$

Vanishing diffusion and soft gluons

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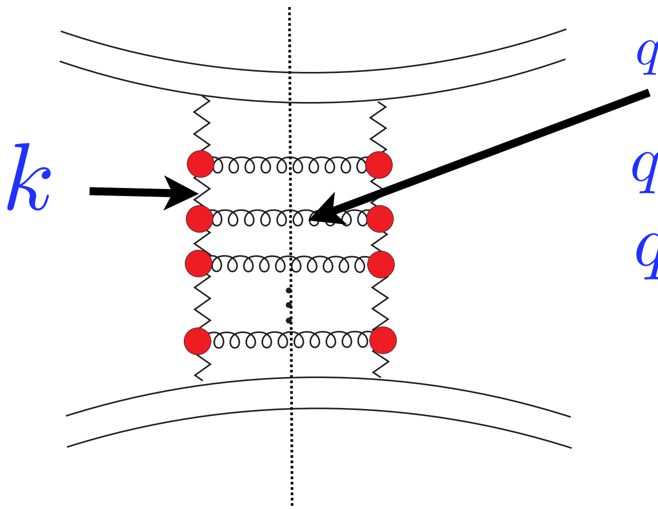
Small coupling



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$q^2 = 0$

q_T can be large

Vanishing diffusion and soft gluons

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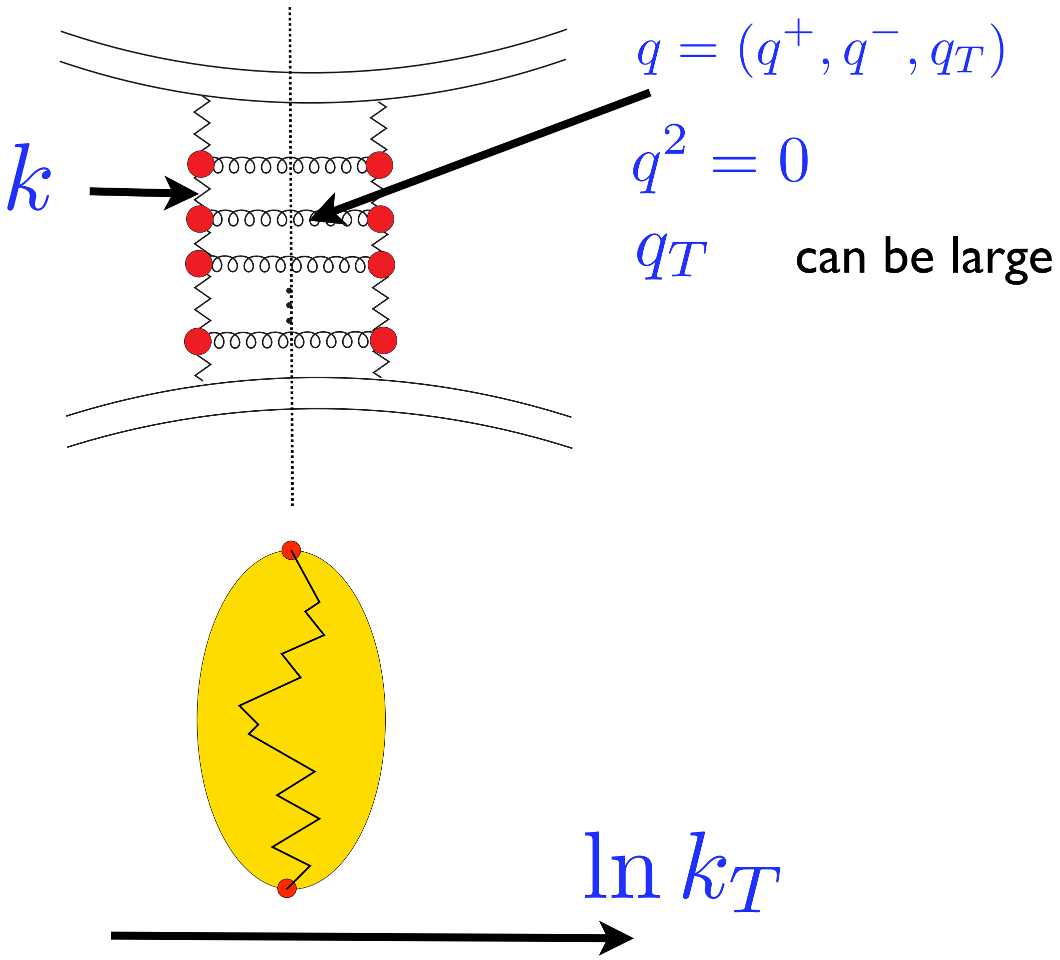

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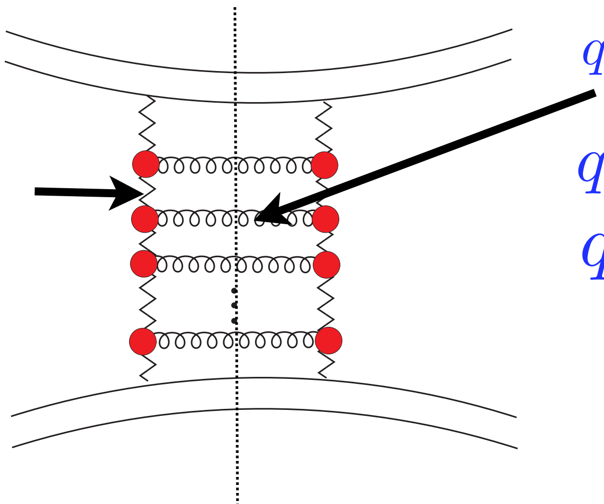

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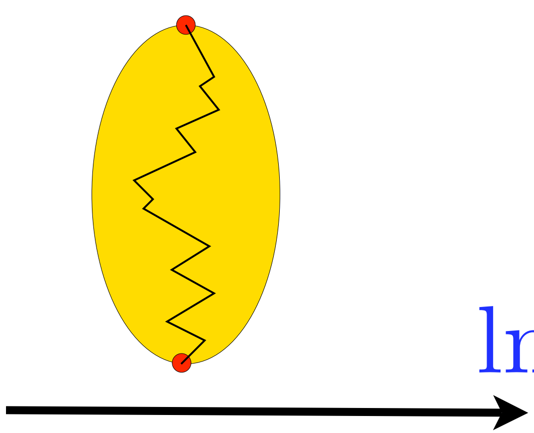

$\ln k_T$

$\chi''(\frac{\alpha_s N_C}{\pi} \ll 1)$ large

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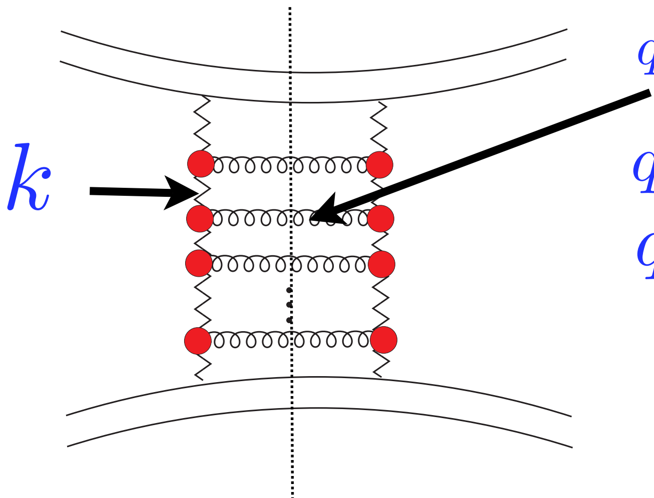

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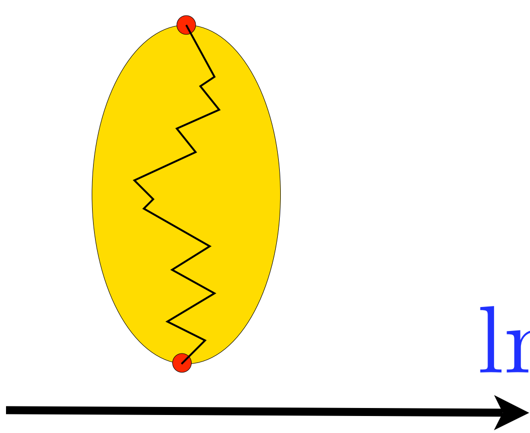

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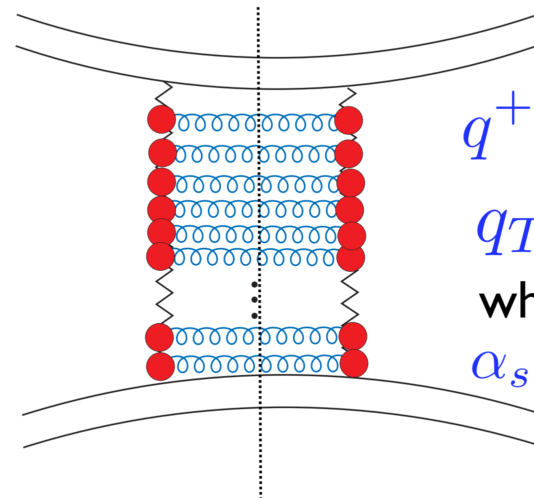

$q^+, q^- \rightarrow 0$

$q_T \rightarrow 0$

when

$\alpha_s N_c \rightarrow \infty$

Vanishing diffusion and soft gluons

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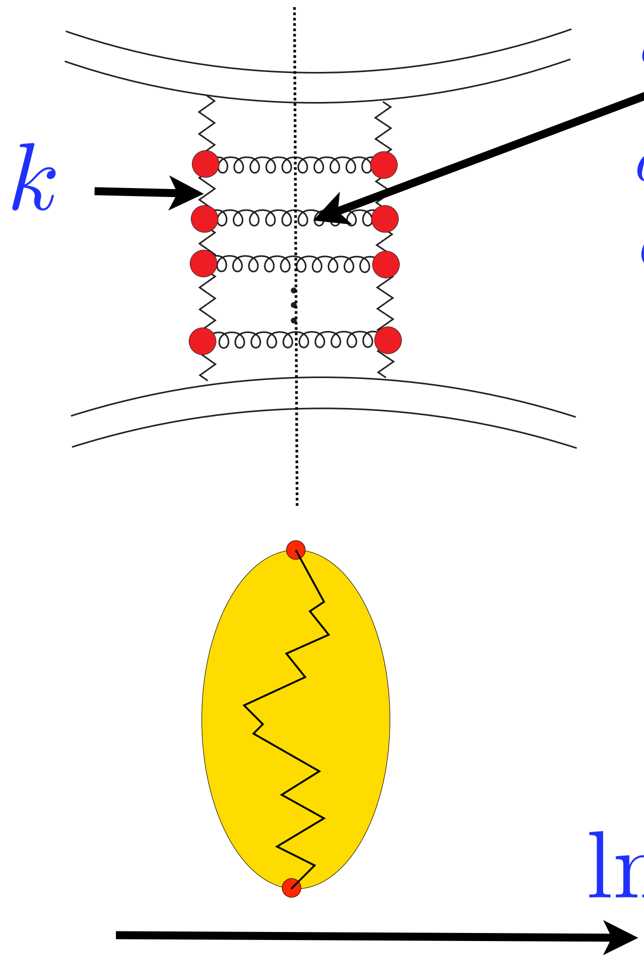

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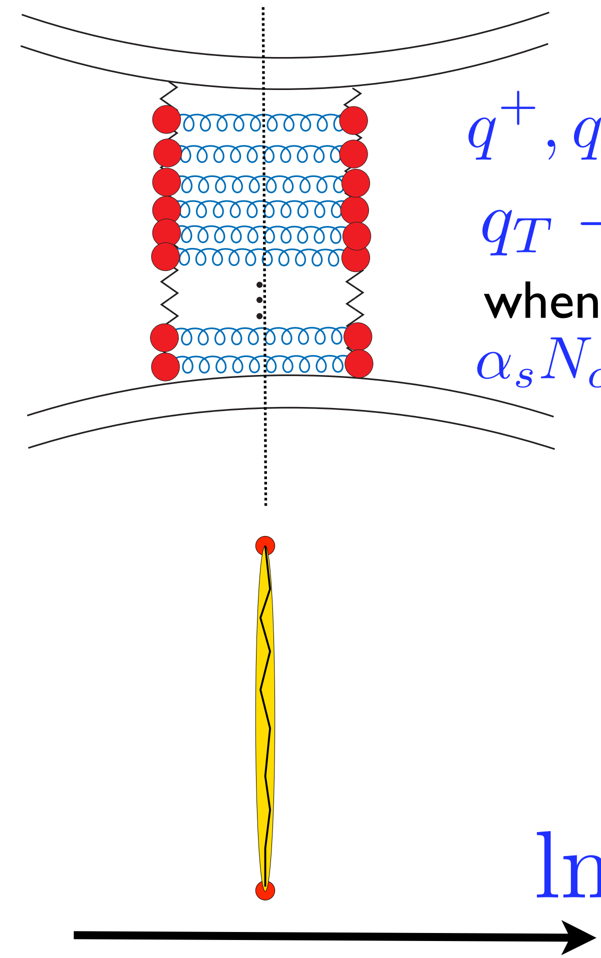

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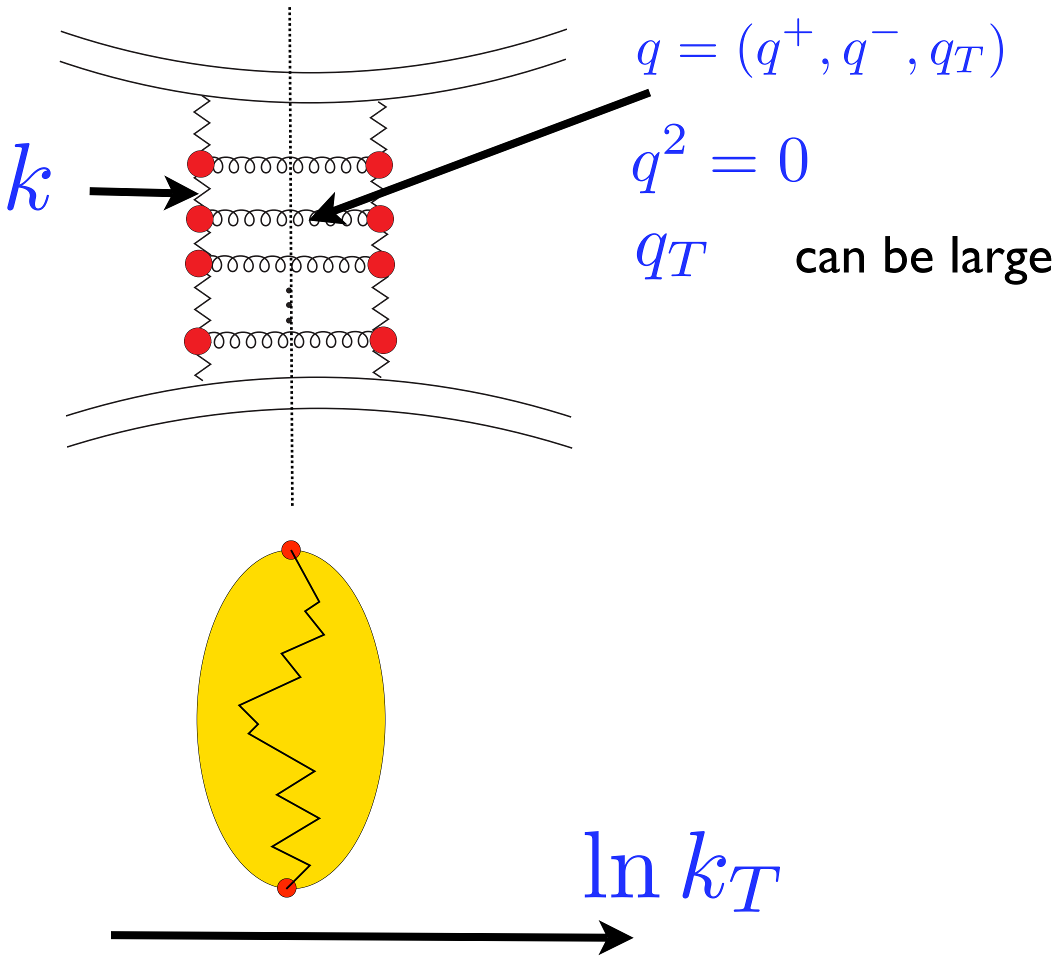

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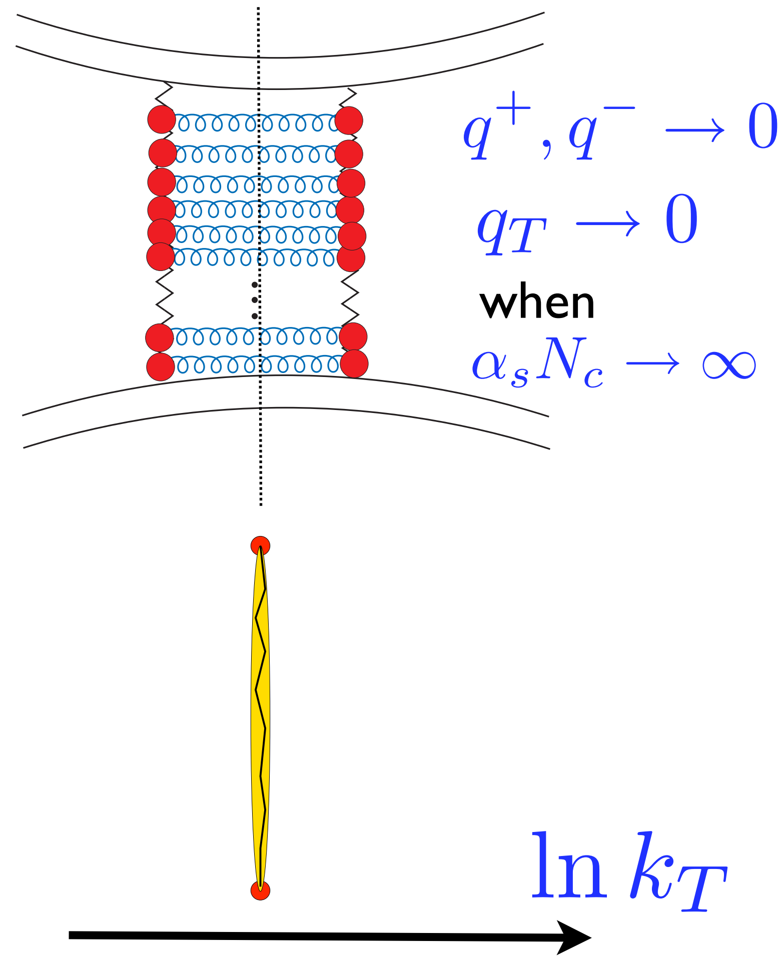

$\chi''(\frac{\alpha_s N_C}{\pi} \gg 1)$ small

Summary

- Universal growth of hadronic cross sections.
- In QCD Pomeron: compound state of gluons, dominates the high energy behavior of cross sections.
- In string(gravity) theory: graviton dominates at high energies.
- Simple kinematic constraints lead to resummation: weak to strong coupling interpolation.
- In real QCD situation more complicated: running coupling, multi-Pomeron/graviton exchanges(interactions).