Pomeron-Graviton duality and resummation at high energies

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High-energy scattering of hadrons
High - energy scattering of hadrons

What is the dependence of the cross section at high energies?
High-energy scattering of hadrons

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**Gauge theory**
weak gauge coupling

**Pomeron:**
collective state of gluons
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**Gauge theory**
- weak gauge coupling
- Pomeron: collective state of gluons

**String theory**
- strong gauge coupling
- 5-dimensional graviton in anti-de Sitter space
High-energy scattering of hadrons

What is the dependence of the cross section at high energies?

Gauge theory
weak gauge coupling

Pomeron:
collective state of gluons

String theory
strong gauge coupling

5-dimensional graviton in anti-de Sitter space

Resummation
Regge theory \rightarrow QCD \leftarrow String/gravity
Regge theory
Regge trajectories

- Linear dependence of the spin $J$ on the square mass for mesons.

\[ \alpha(t) = \alpha(0) + \alpha' t \]

$\alpha(0)_\rho = 0.45$ \hspace{1cm} \text{intercept}

$\alpha'_\rho = 0.93 \text{ GeV}^{-2}$ \hspace{1cm} \text{slope}

$\rho, \omega$

$\omega_3, \rho_3$

$a_4, [f_4]$

$[\rho_5]$

$[a_6, f_6]$

$t = M^2 > 0$

for bound states
Regge trajectories

Similar regularity for baryons
S-matrix and Regge theory
S-matrix and Regge theory

General assumption about the S matrix:

- Lorentz invariance
- Unitarity
- Analyticity
- Crossing

\[ S\dagger S = SS\dagger = 1 \]

\[ A_{ab\rightarrow cd}(t, s, u) = A_{a\bar{c}\rightarrow \bar{b}d}(s, t, u) \]
S-matrix and Regge theory

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Decomposition into partial waves

\[
A(s, t) = \frac{1}{2i} \oint_C dl (2l + 1) \frac{a(l, t)}{\sin \pi l} P(l, 1 + 2s/t)
\]
S-matrix and Regge theory

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Angular momentum
S-matrix and Regge theory

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Angular momentum Partial wave amplitude
S-matrix and Regge theory

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Angular momentum

Partial wave amplitude

Legendre polynomial
Complex angular momentum plane
Complex angular momentum plane
Complex angular momentum plane

Deformation of the contour

\[ \alpha_b \rightarrow \alpha_c \]
Complex angular momentum plane

In the Regge limit: $s \gg |t| (s \to \infty, t \text{ fixed})$
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\[
A(s, t) \to \eta + \frac{e^{-i\pi \alpha(t)}}{2} \beta(t) s^{\alpha(t)}
\]
In the Regge limit: $s \gg |t| (s \to \infty, t \text{ fixed})$

$$A(s, t) \to \frac{\eta + e^{-i\pi \alpha(t)}}{2} \beta(t) s^{\alpha(t)}$$

Amplitude dominated by the Regge pole with largest $\text{Re} \, \alpha(t)$
Reggeon exchange
Reggeon exchange

Factorization of the couplings and the Reggeon exchange

\[ \mathcal{A}(s, t) \rightarrow \frac{\eta + e^{-i\pi \alpha(t)}}{2 \sin \pi \alpha(t)} \frac{\gamma_{ac}(t)\gamma_{bd}(t)}{\Gamma(\alpha(t))} s^{\alpha(t)} \]
Reggeon exchange

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Reggeon exchange

The energy behavior of the amplitude is determined by the exchange of the quasi-particle: Reggeon
Pomeron
Pomeron

Vacuum exchange dominates cross sections at high energies
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\[ \sigma_{TOT}(p\bar{p}) \sim \sigma_{TOT}(pp) \]
Pomeron

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Vacuum exchange

\[ \alpha(0)_{P} \geq 1 \]

experimentally: \[ \alpha_{P}(0) \simeq 1.08, \sigma_{TOT} \sim s^{(\alpha(0)_{P}-1)} \]
Pomeron

Vacuum exchange dominates cross sections at high energies

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Non-vacuum exchange

\[ \alpha(0)_{R} < 1 \]

Note: odderon \( \alpha_{O}(0) \leq 1 \)
pBARp: $21.70 s^{0.0808} + 98.39 s^{-0.4525}$

pp: $21.70 s^{0.0808} + 56.08 s^{-0.4525}$
\[ \sigma^{- p} = 13.63s^{0.0806} + 36.02s^{-0.4525} \]
\[ \sigma^{+ p} = 13.63s^{0.0806} + 27.56s^{-0.4525} \]
Universality of total cross sections
QCD
Pomeron in gauge theory

Low-Nussinov model
2-gluon exchange
Pomeron in gauge theory

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BFKL resummation
color singlet

$s \gg |t|$
Pomeron in gauge theory

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Effective vertex

\[ \Gamma_{\mu\nu}(k_i, k_{i+1}) \]

\[ s \gg |t| \]
Pomeron in gauge theory

Low-Nussinov model
2-gluon exchange

BFKL resummation
color singlet

Reggeized gluon:

\[ D_{\mu\nu}(\hat{s}, k_T^2) = \frac{ig_{\mu\nu}}{k_T^2} \left( \frac{\hat{s}}{k_T^2} \right) \epsilon_G(k_T^2) \]

\[ \hat{s}_i = (k_{i-1} - k_{i+1})^2 \]

Regge trajectory: virtual diagrams

\[ \epsilon_G(q_T^2) = \frac{N_c \alpha_s}{4\pi^2} \int_\Lambda d^2 k_T \frac{-q_T^2}{k_T^2(k_T - q_T)^2} \]

Infrared divergent!

Balitskii
Fadin
Kuraev
Lipatov

\[ s \gg |t| \]
Integral equation

Eikonal couplings

Universality

4-point off-shell gluon Green function
Integral equation

Eikonal couplings

Universality

4-point off-shell gluon Green function

Integral equation for the Pomeron

Born term

Addition of one rung
Integral equation

\[ f(Y; k_{1T}, k_{2T}, q_T) = f^{(0)}(k_{1T}, k_{2T}, q_T) + \int_0^Y dy \ K(k_{1T}, k_{2T}, q_T) \otimes f(y; k_{1T}, k_{2T}, q_T) \]

Rapidity: \[ Y = \ln \frac{1}{x} = \ln \frac{s}{s_0} \]

Convolution in transverse momenta

! Scale choice (irrelevant at lowest order)!
Integral equation

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Convolution in transverse momenta

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Mellin transform:

\[ \int dY \ e^{(-\omega^{-1})Y} f(Y) dY = f(\omega) \]

\[ \omega f(\omega; k_{1T}, k_{2T}, q_T) = \delta^{(2)}(k_{1T} - k_{2T}) + K(k_{1T}, k_{2T}, q_T) \otimes f(\omega; k_{1T}, k_{2T}, q_T) \]

Integral kernel has Mobius invariance.
Solution of the BFKL equation

At zero momentum transfer: \( q_T = 0 \)

Eigenfunctions:
\[
\phi^\nu_n(k_T) = \frac{1}{\pi \sqrt{2}} (k_T^2)^{1/2 + i\nu} e^{in\theta}
\]

Diagonalize equation:
\[
K \otimes \phi^\nu_n = \frac{\alpha_s N_c}{\pi} \chi(\nu, n) \phi^\nu_n
\]

Eigenvalue (take \( n=0 \)):
\[
\chi(\nu, 0) = 2\psi(1) - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)
\]

Simple poles:
\[
\gamma = \ldots, -2, -1, 0, 1, 2, \ldots
\]

\[
\gamma = 1/2 + i\nu
\]
Hard Pomeron
Hard Pomeron

Saddle point solution: around $\gamma = 1/2$

$$\chi(\nu) \simeq 4 \ln 2 - 14 \zeta(3) \nu^2$$
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Approximate solution:

$$f(y = \ln s/s_0, k_{1T}, k_{2T}) \simeq \frac{1}{4 \sqrt{k_{1T}^2 k_{2T}^2}} \sqrt{14 \zeta(3) \alpha_s N_c \pi^2 \ln s/s_0} \left( \frac{s}{s_0} \right)^{4 \ln 2 \alpha_s N_c / \pi} \exp \left( - \frac{\pi \ln^2 \frac{k_{1T}^2}{k_{2T}^2}}{28 \zeta(3) \alpha_s N_c \ln s/s_0} \right)$$
Hard Pomeron

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Diffusion pattern in transverse momenta
Hard Pomeron

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Power-like growth with energy

Diffusion pattern in transverse momenta

Regge behavior from Feynman diagrams: $\alpha_P(0) = 1 + \frac{N_c \alpha_s}{\pi} 4 \ln 2$

Note: it is possible to compute Pomeron in electroweak theory also.
Pomeron phenomenology
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- Lowest order BFKL calculation incompatible with the experimental data.
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- The energy behavior is much too strong (0.5 vs 0.25 in DIS)
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• But in general the Pomeron as a color singlet object dominated by the gluons is well established:
  • Diffraction processes in QCD (processes with large rapidity gaps).
  • Forward jet production in DIS.
  • Universal growth of the total cross sections.
String/gravity
Graviton
Graviton
Spin 2 massless (2 polarizations) particle: symmetric rank 2 tensor.

In string theory: closed string state.

String theory includes gravity
Universality of couplings
Universality of couplings

Example: (Weinberg)

Amplitude for emission of soft photon:
Universality of couplings

Example: (Weinberg)

Amplitude for emission of soft photon:

\[ \mathcal{M}_{IO} \]  Amplitude without photons

\[ \mathcal{M}_{IO}(q, \gamma) = \mathcal{M}_{IO} \sum_n \frac{\sigma_n e_n p_n^\mu}{p_n \cdot q - i\sigma_n \epsilon} \]

- \( e_n \) charge  
- \( p_n \) momentum
- \( \sigma_n \pm 1 \) for incoming and outgoing particles
Universality of couplings

**Example:** (Weinberg)

**Amplitude for emission of soft photon:**

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\(e_n\) charge \(p_n\) momentum \(\sigma_n \pm 1\) for incoming and outgoing particles

**Ward identity:**

\[
q_\mu \mathcal{M}_{IO}^\mu(q) = 0 \quad \rightarrow \quad \sum_n \sigma_n e_n = 0
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Total charge is conserved
Universality of couplings

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Amplitude of one soft graviton:

\[ \mathcal{M}_{IO}^{\mu\nu}(q, g) = \mathcal{M}_{IO} \sum_n \frac{\sigma_n f_n p_n^\mu p_n^\nu}{p_n \cdot q - i\sigma_n \epsilon} \]
Universality of couplings

Example: (Weinberg)

Amplitude for emission of soft photon:

\[ M_{\mu I O}(q, \gamma) = M_{I O} \sum_n \frac{\sigma_n e_n p_n^{\mu}}{p_n \cdot q - i\sigma_n \epsilon} \]

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From E-M conservation:

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 \nu} = 0
\]

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From E-M conservation:

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\sum_n \sigma_n p_n = 0 \quad \Rightarrow \quad f_1 = f_2 = \ldots = f_n
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All couplings are equal

Lorentz invariance for spin 2 particles gives principle of equivalence
Gauge/Gravity duality

Maldacena

strings in AdS(D) $\leftrightarrow$ CFT($d=\text{D-1}$)
Gauge/Gravity duality

strings in AdS(D) \leftrightarrow \text{CFT}(d=D-1)

ds^2 = \frac{R^2}{Z^2}(-dT^2 + dX^2 + dZ^2)

(T,X): Minkowski coordinates
R: radius of curvature
Z: AdS radial coordinate
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States: \phi(T, X; Z = 0) \leftrightarrow Local operators: \mathcal{O}(T, X)
Gauge/Gravity duality

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Graviton in string AdS corresponds to stress energy tensor in CFT.
Gauge/Gravity duality

Maldacena

strings in AdS(D) \leftrightarrow \text{CFT}(d=D-1)

\begin{align*}
    ds^2 &= \frac{R^2}{Z^2}(-dT^2 + dX^2 + dZ^2) \\
    (T,X) : & \text{Minkowski coordinates} \\
    R : & \text{radius of curvature} \\
    Z : & \text{AdS radial coordinate}
\end{align*}

States: \( \phi(T, X; Z = 0) \) \leftrightarrow \text{Local operators: } \mathcal{O}(T, X) 

Graviton in string AdS corresponds to stress energy tensor in CFT.

Duality: different degrees of freedom in two different limits of the coupling \( g^2 N_c \)

\begin{align*}
    g^2 N_c \gg 1 & \quad \text{Strongly coupled SYM} \quad \text{Weakly coupled gravity} \\
    g^2 N_c \ll 1 & \quad \text{Weakly coupled SYM} \quad \text{Strongly coupled gravity}
\end{align*}
Note that correspondence is expected to be valid for N=4 SYM:

- One gauge field \( A_\mu \)
- Six scalars \( \phi_i, i = 1, \ldots, 6 \)
- Four fermions \( \chi_k, k = 1, \ldots, 4 \)
- Fields transform in the adjoint representation
- Conformal invariant \( \beta \sim N - 4 = 0 \)

SYM N=4 very different from QCD. Nevertheless a very good “laboratory”. 
Exchange of graviton in AdS
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Exchange of the graviton trajectory would lead to

\[ \sigma(s) \sim s^{j_0-1}, \quad j_0 = 2 \]
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Computation in string theory:

\[ f(g^2 N_c \gg 1; \ln s, r_1, r_2) \sim s^{j_0 - 1} \frac{e^{-[\ln(r_1/r_2)]^2/(4D \ln s)}}{\sqrt{4\pi D \ln s}} \]

Janik; Brower, Polchinski, Strassler, Tan
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Intercept: $j_0 = 2 - \frac{2}{\sqrt{g^2 N_c}}$

Diffusion coefficient: $D = \frac{1}{2 \sqrt{g^2 N_c}}$

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**Intercept:** \( j_0 = 2 - \frac{2}{\sqrt{g^2 N_c}} \)  
**Diffusion coefficient:** \( D = \frac{1}{2\sqrt{g^2 N_c}} \)

Compare with gauge theory result:
\[ f(g^2 N_c \ll 1; \ln s, k_1, k_2) \sim s^{j_0 - 1} \frac{e^{-\left[\ln(k_1/k_2)\right]^2/(4D \ln s)}}{\sqrt{4\pi D \ln s}} \]

**Intercept:** \( j_0 = 1 + 4 \ln 2 \frac{\alpha_s N_c}{\pi} \)  
**Diffusion coefficient:** \( D = 7\zeta(3) \frac{\alpha_s N_c}{\pi} \)
Exchange of graviton in AdS

Exchange of the graviton trajectory would lead to

\[ \sigma(s) \sim s^{j_0 - 1}, \quad j_0 = 2 \]

Computation in string theory:

\[
f(g^2N_c \gg 1; \ln s, r_1, r_2) \sim s^{j_0 - 1} e^{-[\ln(r_1/r_2)]^2/(4D \ln s)} \frac{1}{\sqrt{4\pi D \ln s}}
\]

Intercept: \( j_0 = 2 - \frac{2}{\sqrt{g^2N_c}} \)

Diffusion coefficient: \( D = \frac{1}{2 \sqrt{g^2N_c}} \)

Compare with gauge theory result:

\[
f(g^2N_c \ll 1; \ln s, k_1, k_2) \sim s^{j_0 - 1} e^{-[\ln(k_1/k_2)]^2/(4D \ln s)} \frac{1}{\sqrt{4\pi D \ln s}}
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Intercept: \( j_0 = 1 + 4 \ln 2 \frac{\alpha_s N_c}{\pi} \)

Diffusion coefficient: \( D = 7\zeta(3) \frac{\alpha_s N_c}{\pi} \)

Diffusion in transverse (virtual) momenta

Diffusion in the fifth (radial) dimension of AdS space

Janik; Brower, Polchinski, Strassler, Tan
Pomeron/Graviton

\[ j_0 = 1 + 4 \ln 2 \frac{\alpha_s N_c}{\pi} \]

Weak coupling

Pomeron: made out of many (reggeized) gluons. Growth of the cross section caused by dynamical effect: emission of many gluons.

Strong coupling

Graviton: single object (closed string state). Growth of the cross section corresponds to the exchange of spin 2.

\[ j_0 = 2 - \frac{2}{\sqrt{g^2 N_c}} \]
Resummation at high energies (small $x$)

- Next-to-leading order very large: $j_0 = 1 + 4 \ln 2 \frac{\alpha_s N_c}{\pi} \left(1 - 6.45 \frac{\alpha_s N_c}{\pi}\right)$

- Sources of large corrections:
  - Kinematical effects, energy momentum conservation.
  - Running of the coupling.
  - Other corrections: quarks in the evolution.

- Need to take more than next-to-leading order: all orders.
Kinematics
Kinematics

• These gluons must be on-shell.
Kinematics

- These gluons must be on-shell.
- The approximations used make gluons off-shell!
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Kinematics

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High energy approximation means:
These gluons must be on-shell.
The approximations used make gluons off-shell!
Put the constraint to correct this.
Energy - momentum is not conserved:

High energy approximation means:

\[ s \gg |t|, \Lambda^2, m_i^2 \]
These gluons must be on-shell.

The approximations used make gluons off-shell!

Put the constraint to correct this.

Energy - momentum is not conserved:

\[ s \gg |t|, \Lambda^2, m_i^2 \]

Impose constraints to satisfy energy-momentum sum rule.
A note on anomalous dimensions in QCD

In standard operator product expansion approach to DIS evaluate anomalous dimensions
A note on anomalous dimensions in QCD

In standard operator product expansion approach to DIS evaluate anomalous dimensions

RGE:

\[ [ \mathcal{D} \delta_{ab} - \gamma^{(j)}_{ab} ] C^j_b (g, \mu, -q^2) = 0 \]

\[ \mathcal{D} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \]
A note on anomalous dimensions in QCD

In standard operator product expansion approach to DIS evaluate anomalous dimensions

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Perturbative expansion:

\[ \gamma_{ab}^{(j)}(g) = \sum_{i} (g^2)^{i} \gamma_{ab}^{(j),i} \]
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\[ \gamma_{ab}^{(j)} (g) = \sum_{i} (g^2)^i \gamma_{ab}^{(j),i} \]

Momentum sum rule:

\[ \gamma_{gg}^{(j=2),i} + 2N_f \gamma_{qg}^{(j=2),i} = 0 \]

\[ \gamma_{gq}^{(j=2),i} + \gamma_{qq}^{(j=2),i} = 0 \]  

QCD
In standard operator product expansion approach to DIS evaluate anomalous dimensions

\[
[D \delta_{ab} - \gamma^{(j)}_{ab}] C^j_b (g, \mu, -q^2) = 0
\]

\[D = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\]

\[
\gamma^{(j)}_{ab} (g) = \sum_i (g^2)^i \gamma^{(j),i}_{ab}
\]

Momentum sum rule:

\[
\gamma^{(j=2),i}_{gg} + 2N_f \gamma^{(j=2),i}_{qg} = 0 \quad \gamma^{(j=2),i}_{gq} + \gamma^{(j=2),i}_{qq} = 0 \quad \text{QCD}
\]

\[
\gamma^{(j=2),i}_{\text{uni}} = 0 \quad \text{N=4 SYM}
\]
A note on anomalous dimensions in QCD

In standard operator product expansion approach to DIS evaluate anomalous dimensions

RGE:

\[ [D\delta_{ab} - \gamma_{ab}^{(j)}] C_{b}^{j} (g, \mu, -q^2) = 0 \]

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\[ \gamma_{gq}^{(j=2),i} + \gamma_{qq}^{(j=2),i} = 0 \quad \text{QCD} \]

\[ \gamma_{uni}^{(j=2),i} = 0 \quad \text{N=4 SYM} \]

Satisfied at each order of the perturbation theory
BFKL kernel eigenvalue:

\[ 1 = \mathcal{K}(\alpha_s, \gamma, j) \]

\[ \gamma \leftrightarrow \ln k_T \]
BFKL kernel eigenvalue:

\[ 1 = \mathcal{K}(\alpha_s, \gamma, j) \]

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Transverse momentum
BFKL kernel eigenvalue:

\[ 1 = \mathcal{K}(\alpha_s, \gamma, j) \]

\[ \gamma \leftrightarrow \ln k_T \]

Resummed model

\[ 1 = \alpha_s N_c / \pi \gamma_{gg}^{(0),j} \chi^0(\gamma, j) \]

\[ \chi^0(\gamma, j) = 2\psi(1) - \psi(\gamma + (j - 1)/2) - \psi(1 - \gamma + (j - 1)/2) \]

\[ \gamma_{gg}^{(0),j} \quad \text{LO anomalous dimension} \]
BFKL kernel eigenvalue:

\[ 1 = \mathcal{K}(\alpha_s, \gamma, j) \]

\[ \gamma \leftrightarrow \ln k_T \]

Resummed model

\[ 1 = \frac{\alpha_s N_c}{\pi} \gamma_{gg}^{(0),j} \chi^0(\gamma, j) \]

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\[ \gamma_{gg}^{(0),j} \]

LO anomalous dimension

Vanishes when \( j = 2 \)
BFKL kernel eigenvalue:

\[ 1 = \mathcal{K}(\alpha_s, \gamma, j) \]

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Solve (transcendental) equation for \( j \)

Vanishes when \( j = 2 \)

Transverse momentum
BFKL kernel eigenvalue:

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Transverse momentum

Vanishes when \( j = 2 \)

Solve (transcendental) equation for \( j \)

Before:

\[ \frac{\alpha_s N_c}{\pi} = 1.0 \]

\[ \frac{\alpha_s N_c}{\pi} = 0.2 \]
**BFKL kernel eigenvalue:**

\[ 1 = \mathcal{K}(\alpha_s, \gamma, j) \]

\[ \gamma \leftrightarrow \ln k_T \]

**Resummed model**

\[ 1 = \frac{\alpha_s N_c}{\pi} \gamma_{gg}^{(0),j} \chi^0(\gamma, j) \]

\[ \chi^0(\gamma, j) = 2\psi(1) - \psi(\gamma + (j - 1)/2) - \psi(1 - \gamma + (j - 1)/2) \]

**Vanishes when j=2**

**LO anomalous dimension**

Before:

\[ j - 1 \]

\[ \frac{\alpha_s N_c}{\pi} = 0.2 \]

After:

\[ \frac{\alpha_s N_c}{\pi} = 1.0 \]

\[ \chi_{eff}(\gamma, \alpha_s) \]

\[ j - 1 \]
BFKL kernel eigenvalue:
\[ 1 = \mathcal{K}(\alpha_s, \gamma, j) \]
\[ \gamma \leftrightarrow \ln k_T \]

Resummed model
\[ 1 = \frac{\alpha_s N_c}{\pi} \gamma_{gg}^{(0),j} \chi^0(\gamma, j) \]
\[ \chi^0(\gamma, j) = 2\psi(1) - \psi(\gamma + (j - 1)/2) - \psi(1 - \gamma + (j - 1)/2) \]
\[ \gamma_{gg}^{(0),j} \] LO anomalous dimension

Vanishes when \( j = 2 \)

Solve (transcendental) equation for \( j \)

Before:
\[ j - 1 \]
\[ \frac{\alpha_s N_c}{\pi} = 0.2 \]

After:
\[ \frac{\alpha_s N_c}{\pi} = 1.0 \]

Fixed points: energy conservation
Intercept in the resummed model

\[ \omega_P = j_0 - 1 \]

\[ j_0 = 2 - \frac{1}{\pi \sqrt{\alpha_s N_c / \pi}} \]

\[ j_0 = 1 + 4 \ln 2 \frac{\alpha_s N_c}{\pi} \]

Note the logarithmic horizontal axis

Cross section: \( \sigma \sim s^{j_0 - 1} \)
Vanishing diffusion and soft gluons
Vanishing diffusion and soft gluons

Small coupling

\[ q = (q^+, q^-, q_T) \]

\[ q^2 = 0 \]

\[ q_T \text{ can be large} \]
Vanishing diffusion and soft gluons

Small coupling

$q^2 = 0$

$q_T$ can be large

$\ln k_T$
Vanishing diffusion and soft gluons

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\[ q = (q^+, q^-, q_T) \]
\[ q^2 = 0 \]
\[ q_T \text{ can be large} \]

\[ \chi''(\frac{\alpha_s N_C}{\pi} \ll 1) \text{ large} \]
Vanishing diffusion and soft gluons

Small coupling

\[ k \]

\[ q = (q^+, q^-, q_T) \]

\[ q^2 = 0 \]

\[ q_T \text{ can be large} \]

\[ \ln k_T \]

\[ \chi'' \left( \frac{\alpha_s N_c}{\pi} \right) \ll 1 \text{ large} \]

Large coupling

\[ q^+, q^- \to 0 \]

\[ q_T \to 0 \text{ when } \alpha_s N_c \to \infty \]
Vanishing diffusion and soft gluons

Small coupling

$q = (q^+, q^-, q_T)$

$q^2 = 0$

$q_T$ can be large

Large coupling

$q^+, q^- \rightarrow 0$

$q_T \rightarrow 0$

when

$\alpha_s N_c \rightarrow \infty$

$\chi'' \left( \frac{\alpha_s N_c}{\pi} \ll 1 \right)$ large
Vanishing diffusion and soft gluons

**Small coupling**

\[ q = (q^+, q^-, q_T) \]

\[ q^2 = 0 \]

\[ q_T \text{ can be large} \]

\[ \chi''\left(\frac{\alpha_s N_C}{\pi} \ll 1\right) \text{ large} \]

**Large coupling**

\[ q^+, q^- \to 0 \]

\[ q_T \to 0 \text{ when } \alpha_s N_C \to \infty \]

\[ \chi''\left(\frac{\alpha_s N_C}{\pi} \gg 1\right) \text{ small} \]
Summary

• Universal growth of hadronic cross sections.

• In QCD Pomeron: compound state of gluons, dominates the high energy behavior of cross sections.

• In string(gravity) theory: graviton dominates at high energies.

• Simple kinematic constraints lead to resummation: weak to strong coupling interpolation.

• In real QCD situation more complicated: running coupling, multi-Pomeron/graviton exchanges(interactions).