From compact objects to cold atom experiments

Isolated Neutron Star

Condensate of ^{6}Li atoms (Ketterle Group)

$T/T_F \sim 0.0001$

$T/T_F \sim 0.05$

$T/T_F \sim 0.1$

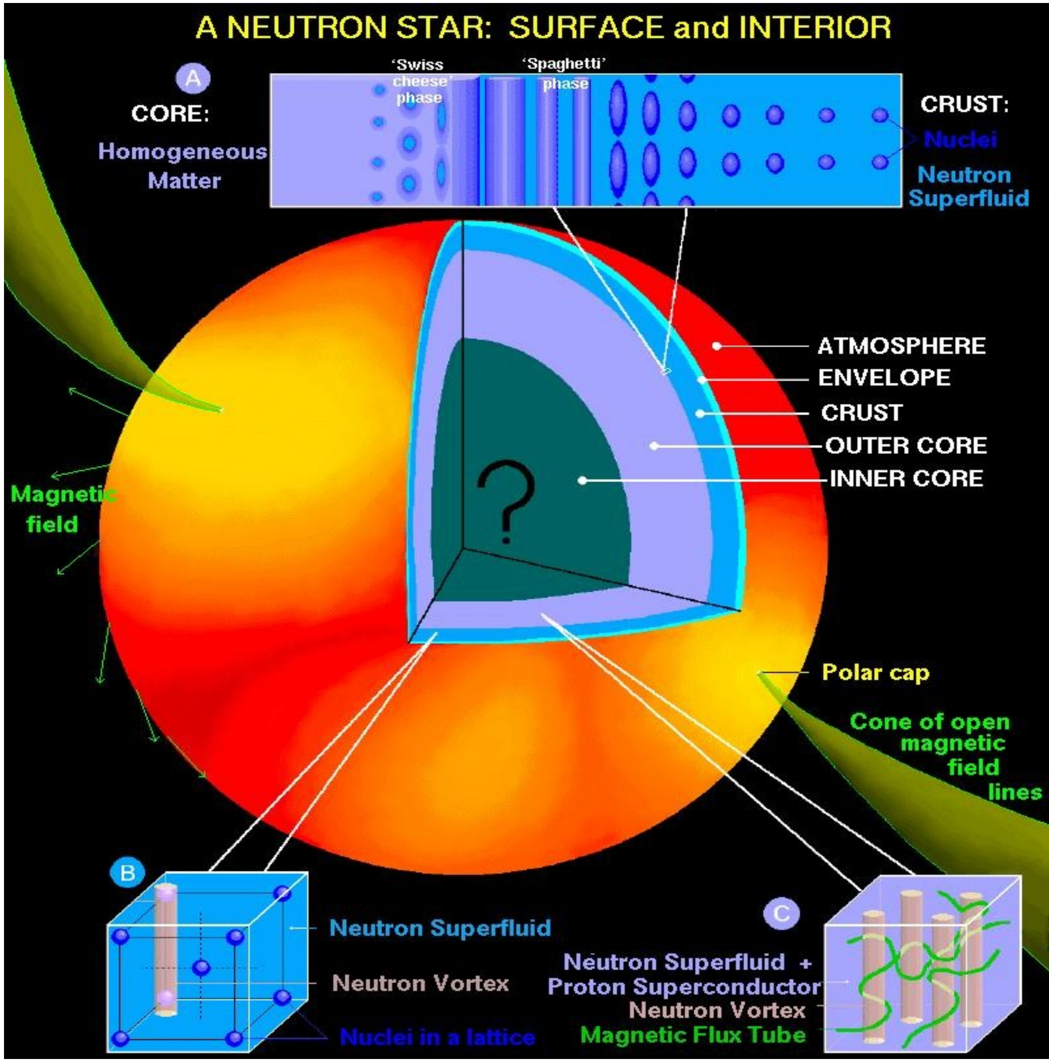
$T/T_F \sim 0.2$

How to get from here to there ?
Or alternately, how I got from there to here ?

Introduction to neutron stars: A nuclear physics perspective

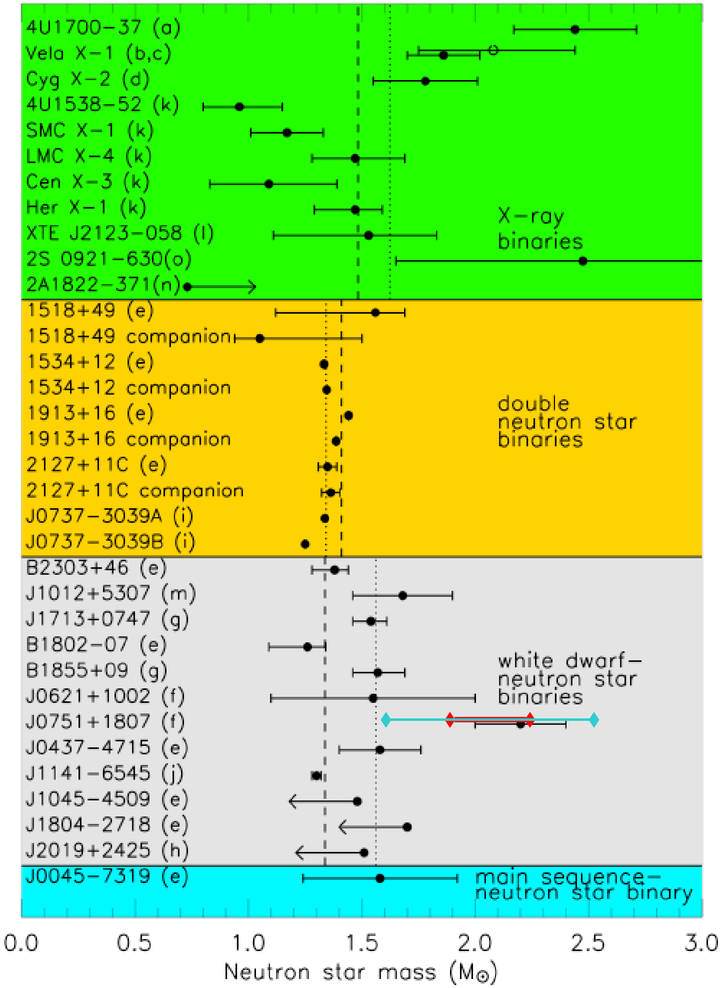
What is the nature of matter inside neutron stars ?

Difficult to probe directly, but observations + theory + simulations can help us infer:
- Mass
- Radius
- Crust Thickness
- Internal Temperature
- Dissipation rates



Page & Reddy, Ann.Rev.Nucl.Part.Sci.56:327-374, (2006)

Neutron Star Mass:



Origin of the clustering at M_{NS}~1.4 M_{solar} ?

EoS at high density: what is the heaviest neutron stars one can make ?

Difficult to make heavy NS with soft EoS.

Mass Extraction from Timing Data:

Keplerian relation:

$$\frac{(m_2 \sin i)^3}{(m_1 + m_2)^2} = \frac{x^3}{T_\odot}\left(\frac{2\pi}{P_b}\right)^2$$

General Relativistic Orbital Decay:

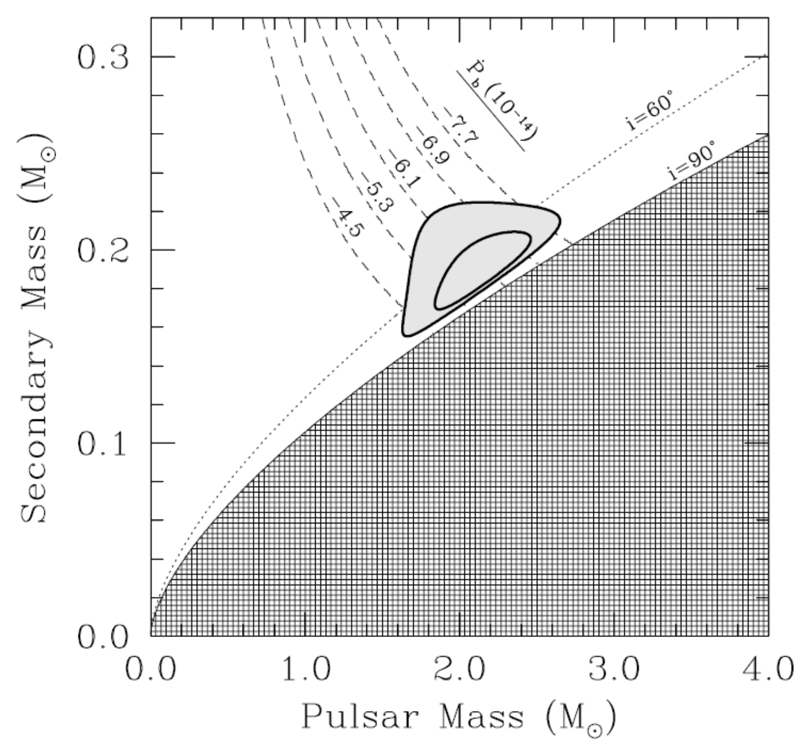
$$(\dot{P}_b)_{\rm GR} = -\left(\frac{192\pi}{5}\right)\left(\frac{2\pi}{P_b}\right)^{5/3}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) \times \frac{1}{(1-e^2)^{7/2}} T_\odot^{5/3} \frac{m_1 m_2}{(m_1+m_2)^{1/3}},$$

Heaviest neutron star (known) in a NS-WD binary (P_b~ 6 hrs): PSR J0751+1807

$$\dot{P}_b = -(6.2 \pm 0.8) \times 10^{-14}.$$

M_{NS}=2.1±0.2 $M_\odot$ (68%)

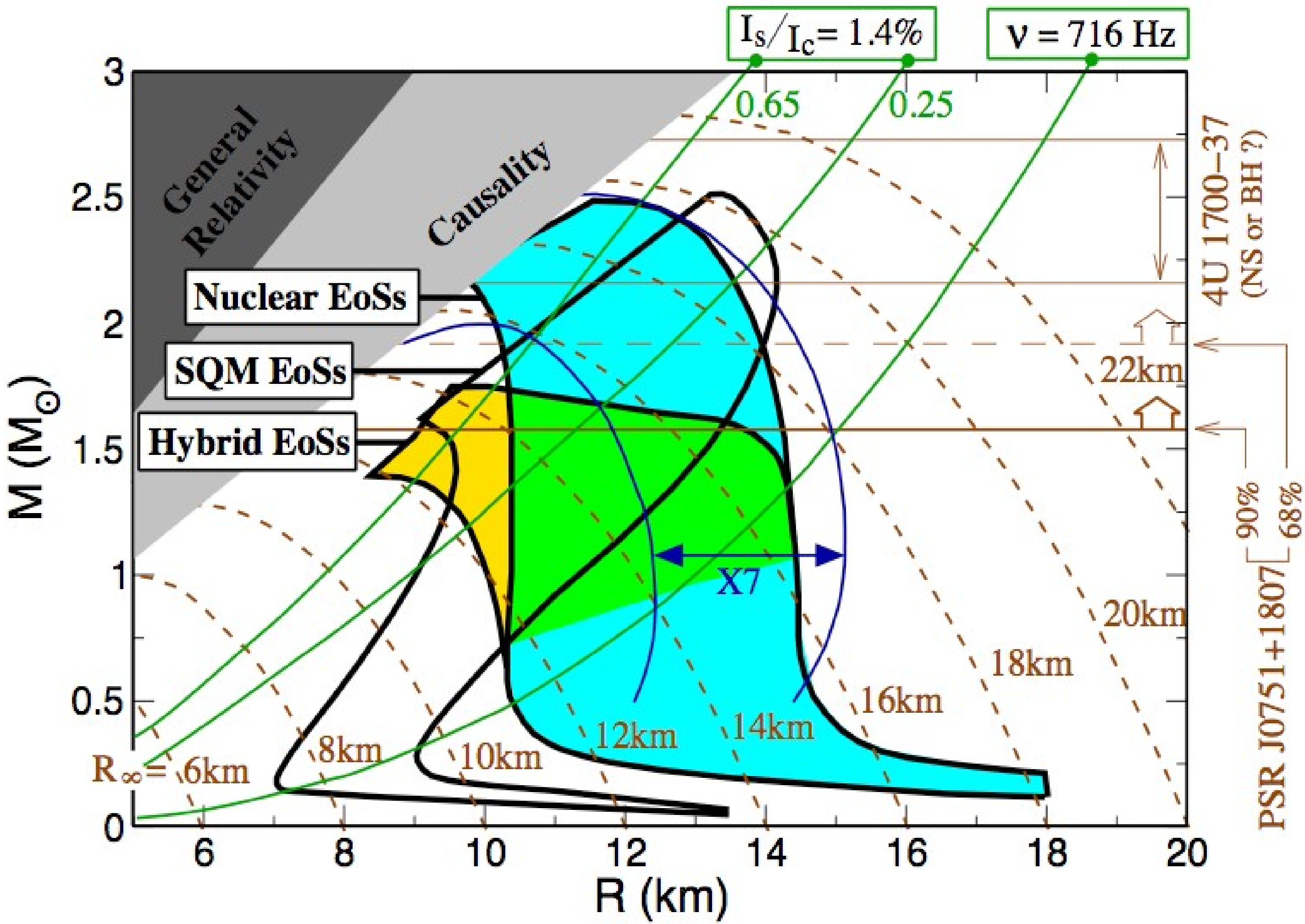
≈2.1 ± 0.5 $M_\odot$ (95%)



Mass-Radius: Model Predictions & Observational constraints

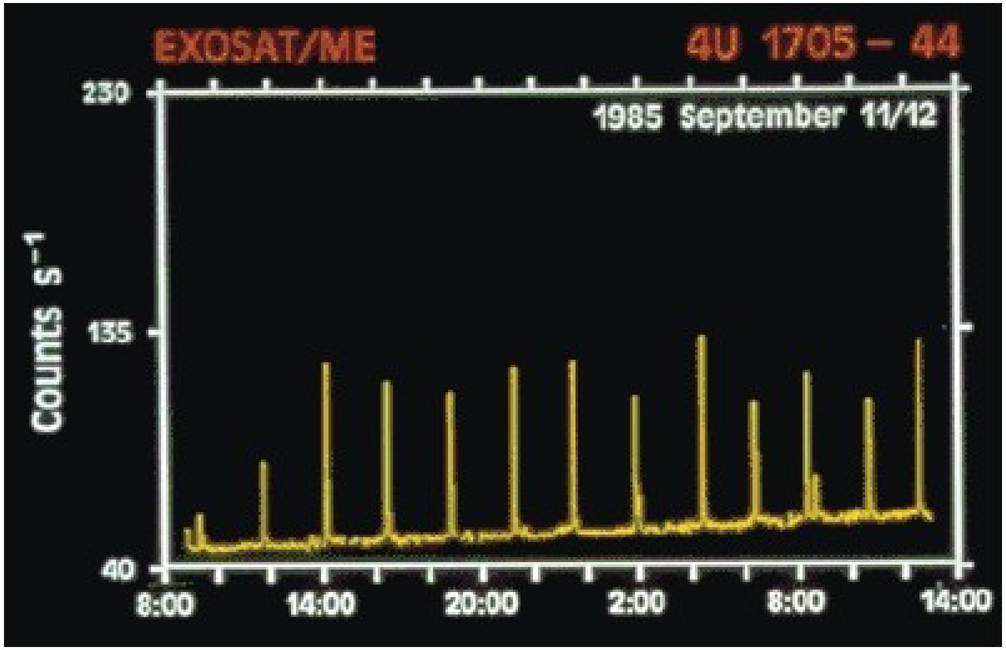
•Heavy stars would disfavor a strong 1st-order transition*

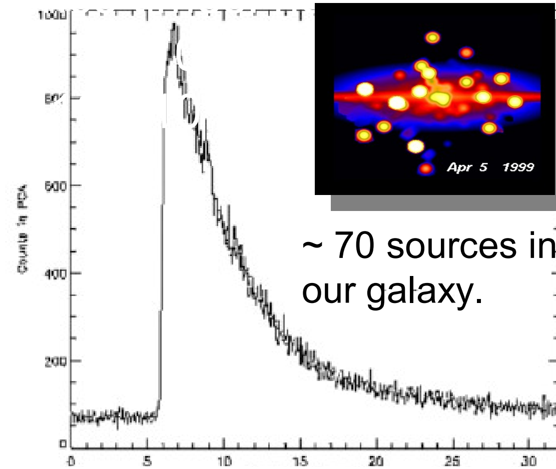
•Radius not particularly sensitive to the high density behavior



X-Ray Bursts

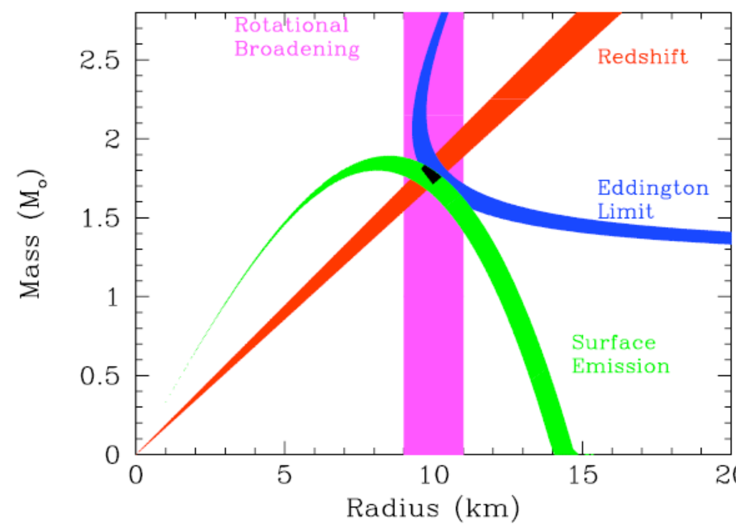
Woosley & Taam (1976),Woosley, Heger et al. (2004)
Review: Strohmayer & Bildsten (2006)





Features in light curve are sensitive to mass and radius. Eg. Secure identification of Eddington luminosity & thermal cooling in the light curve can simultaneously infer both mass & radius.

Potentially many other features exist to provide cross-checks.



Ozel, Nature 441:1115 (2006)

Quiescent Luminosity of Soft X-Ray Transients.

In some binaries accretion is intermittent.
Large outbursts (L_{burst}~10^{37}-10^{38} ergs/s) due to disk instabilities are followed by a quiescent phase with L_q ~10^{33} ergs/s.

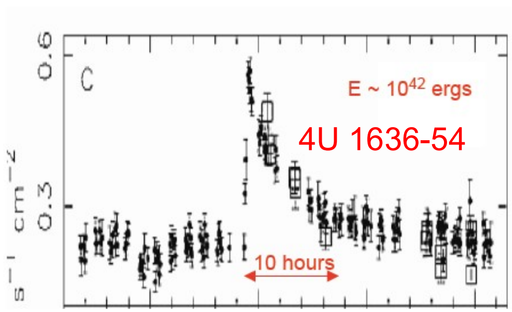
Quiescent Luminosity is powered by nuclear reactions in the inner crust.

Haensel, Zdunik, A&A 227, 321 (1990)

Brown, Bildsten, Rutledge, ApJ L95, 504 (1998)

Some sources indicate rapid neutrino cooling.

Superbursts:



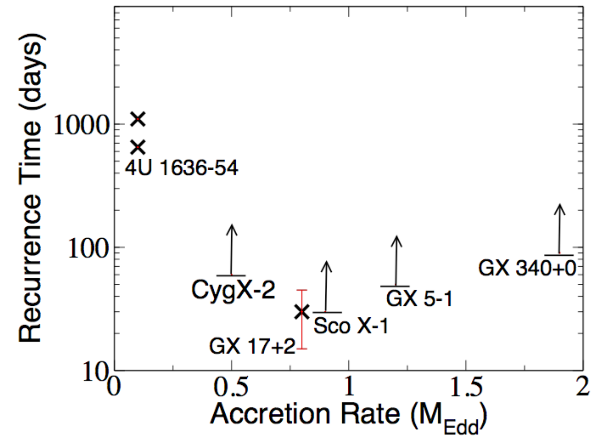
Superbursts are longer duration (hours) bursts with *recurrence times days-years*.
Likely to be ignition of carbon poor ashes produced during XRB activity.

Neutron Star Thermometer:
Ignition (recurrence times) very sensitive to the *thermal profile of the neutron star crust*.

Woosley & Taam (1976), Cumming & Bildsten (2001)

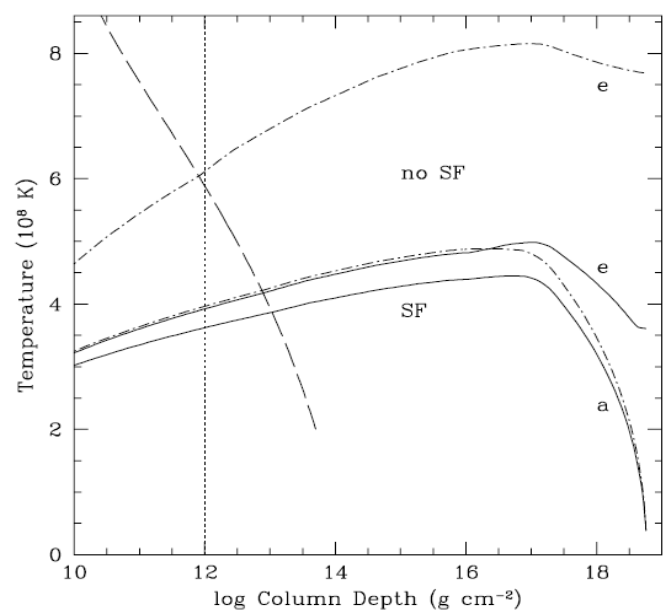
Strohmayer & Brown (2002)

Superburst Recurrence Time



Keek, in 't Zand, Cumming, astro-ph/0605689.

Thermal Profile of the Crust



Neutron Star Cooling

Minimal Cooling Model

Cooling is due to core neutrino emission for the first 10^5 -10^6 yrs.

Slow or standard cooling

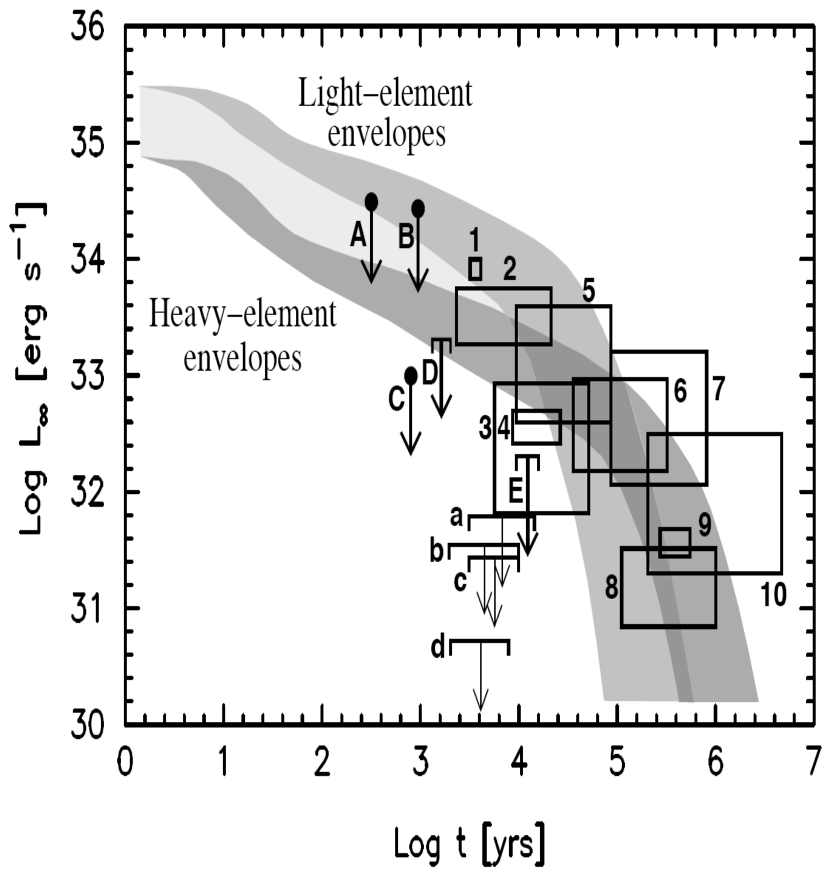
[redacted]

$L_\nu \sim 10^{21}\ T_9^{\,8}$ erg/cm^{3}s

Fast Cooling

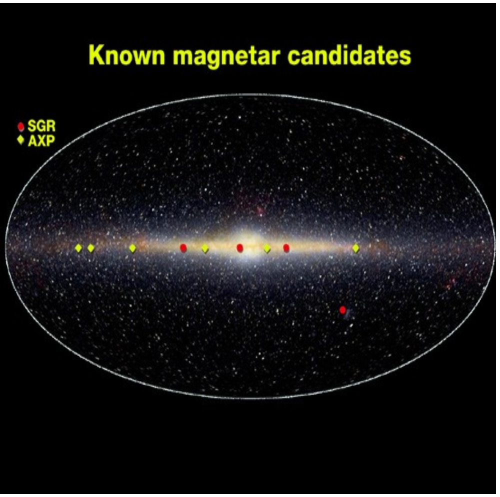
"single particle reactions"

$L_\nu \sim 10^{24\text{-}26}\ T_9^{\,6}$ erg/cm^{3}s

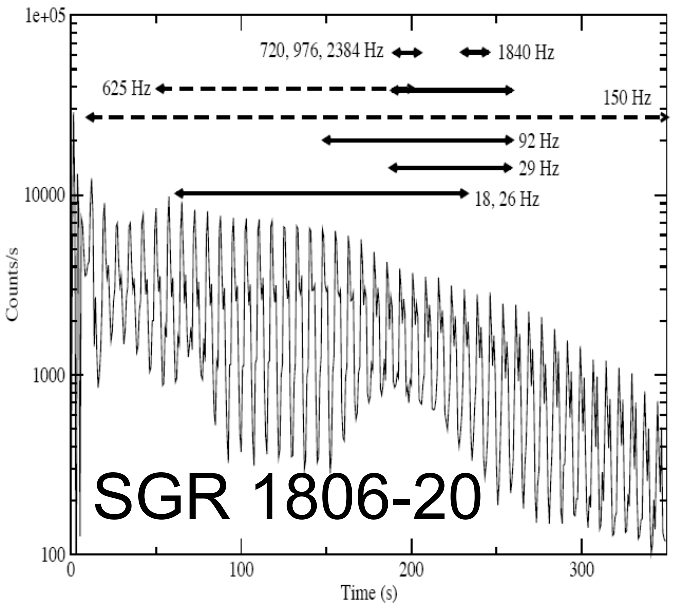


Page, Lattimer, Prakash, Steiner, ApJS.155:623 (2004)

Giant Flares & Crustal Shear Modes



SGR 0525-66
(1979)
SGR 1806-20
(1979/1986/2004*)
SGR 1900+14
(1979/1986/1998*)
SGR 1627-41
(1998)



Catastrophic outbursts from highly magnetized neutron stars.

1) How are they triggered ?
2) Are observed QPOs seismic in origin ?

QPOs likely to be crustal shear modes:
Neutron star seismology ?

Frequency (Hz)	Width (Hz)	RMS amplitude (%)	Duration (s)	Phase	Satellite
17.9 ± 0.1	1.9 ± 0.2	4.0 ± 0.3	60-230	P2/I	RHESSI
25.7 ± 0.1	3.0 ± 0.2	5.0 ± 0.3	60-230	P2/I	RHESSI
29.0 ± 0.4	4.1 ± 0.5	20.5 ± 3.0	190-260	P2/I	RXTE
92.5 ± 0.2	$1.7^{+0.7}_{-0.4}$	10.7 ± 1.2	150-260	P2/I	RXTE
92.7 ± 0.1	2.3 ± 0.2	10.3 ± 0.8	150-260	P2/I	RHESSI
92.9 ± 0.2	2.4 ± 0.3	19.2 ± 2.0	190-260	P2/I	RXTE
150.3 ± 1.6	17 ± 5	6.8 ± 1.3	10-350	P1	RXTE
626.46 ± 0.02	0.8 ± 0.1	20 ± 3	50-200	P1	RHESSI
625.5 ± 0.2	1.8 ± 0.4	8.5 ± 1.8	190-260	P2/I	RXTE
1837 ± 0.8	4.7 ± 1.2	18.0 ± 3.6	230-245	P2/I	RXTE

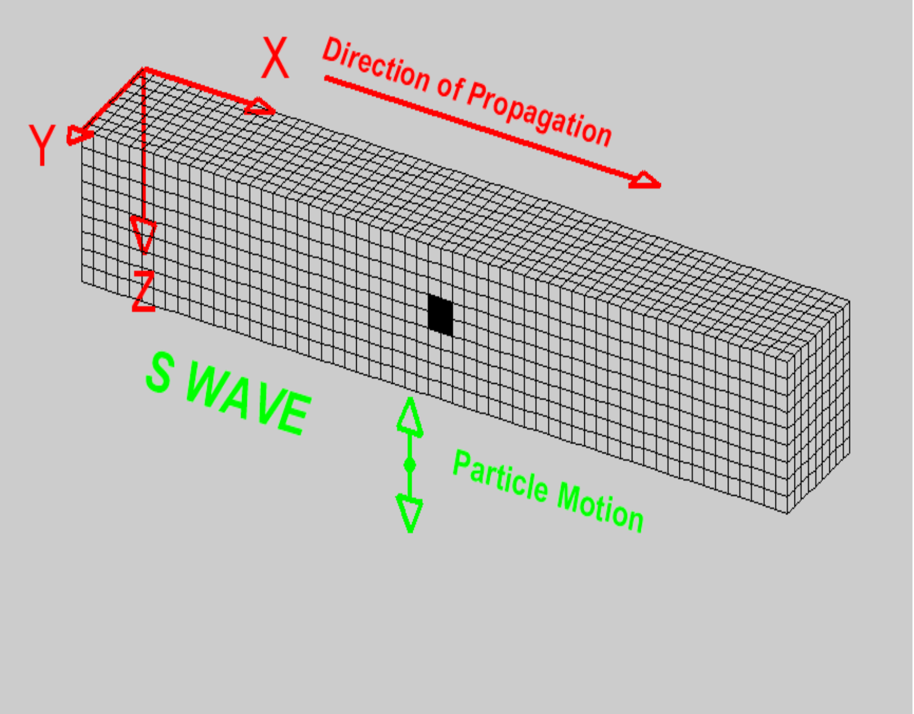
Strohmayer & Watts, astro-ph/0608463

Shear Waves in Solids

Deformation propagates. Low order modes have crust deformations ie shear motions that are tangential (toroidal modes). The Y is along the radial direction.

Motion is along X & Z.

Shear Wave Speed $v_S = (\mu/\rho)^{1/2}$



Shear Modulus:

$$\mu = \frac{0.1194}{1 + 0.595\,(\Gamma_0/\Gamma)}\, n_i\, \frac{Z^2 e^2}{a}$$

$$\Gamma = \frac{Z^2 e^2}{a k_B T} \quad (\Gamma_0 = 173)$$

$$n_i = \frac{3}{4\pi a^3}$$

Shear Oscillations of the Crust

Piro, Astrophys.J. 634 L153 (2005)

Equation of Motion (elastic modes) :

Setting $W = \xi(r)\exp(i\omega t)$

$$\frac{1}{\rho}\frac{\partial}{\partial r}\left(\mu\frac{\partial\xi}{\partial r}\right) + v_A^2\frac{\partial^2\xi}{\partial r^2} + \left[\omega^2\left(1+\frac{v_A^2}{c^2}\right) - \frac{(l+2)(l-1)\mu}{\rho R^2}\right]\xi = 0$$

Lowest-order radial modes ($\frac{\partial\xi}{\partial r} << \xi/R$):

$$\omega_{n=0} = \frac{v_S}{R}\sqrt{(l+2)(l-1)}$$

where the shear speed $v_S = \sqrt{\frac{\mu}{\rho}}$

Higher-order modes (with nodes in the radial direction):

$$\omega_{n>0} = \frac{v_S}{R}\left[\frac{(l+2)(l-1)}{R^2} + \left(\frac{n\pi\, R}{\Delta R}\right)^2\left(1+\frac{v_A^2}{v_S^2}\right)\right]^{1/2}$$

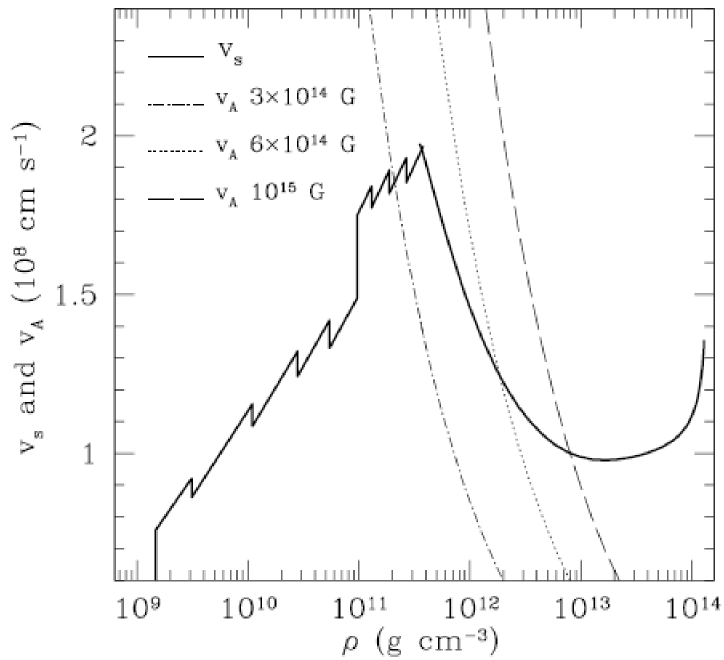
$$\simeq n\,\pi\,\frac{v_S}{R}\left(\frac{R}{\Delta R}\right)$$

Alfven Velocity: $v_A = \sqrt{\frac{B^2}{4\pi\rho}}$

Shear Speed In Compact Stars

Variation in Shear Speed inside the regular crust is small ($v_s \sim 10^8$ cm/s)

μ_e [MeV]	$\rho_{\max}$ [g/cm^3]	Element	Z	N
0.95	7.96×10^6	^{56}Fe	26	30
2.61	2.70×10^8	^{62}Ni	28	34
4.28	1.29×10^9	^{64}Ni	28	36
4.57	1.61×10^9	^{66}Ni	28	38
5.32	2.63×10^9	^{68}Ni	28	40
6.21	4.34×10^9	^{80}Ge	32	48
9.69	1.70×10^{10}	^{82}Ge	32	50
12.26	3.59×10^{10}	^{80}Zn	30	50
18.22	1.23×10^{11}	^{78}Ni	28	50
18.73	1.41×10^{11}	^{76}Fe	26	50
20.15	1.83×10^{11}	^{122}Zr	40	82
22.19	2.53×10^{11}	^{120}Sr	38	82
24.24	3.42×10^{11}	^{118}Kr	36	82
26.28	4.55×10^{11}	^{116}Se	34	82
26.82	5.05×10^{11}	^{114}Ge	32	82



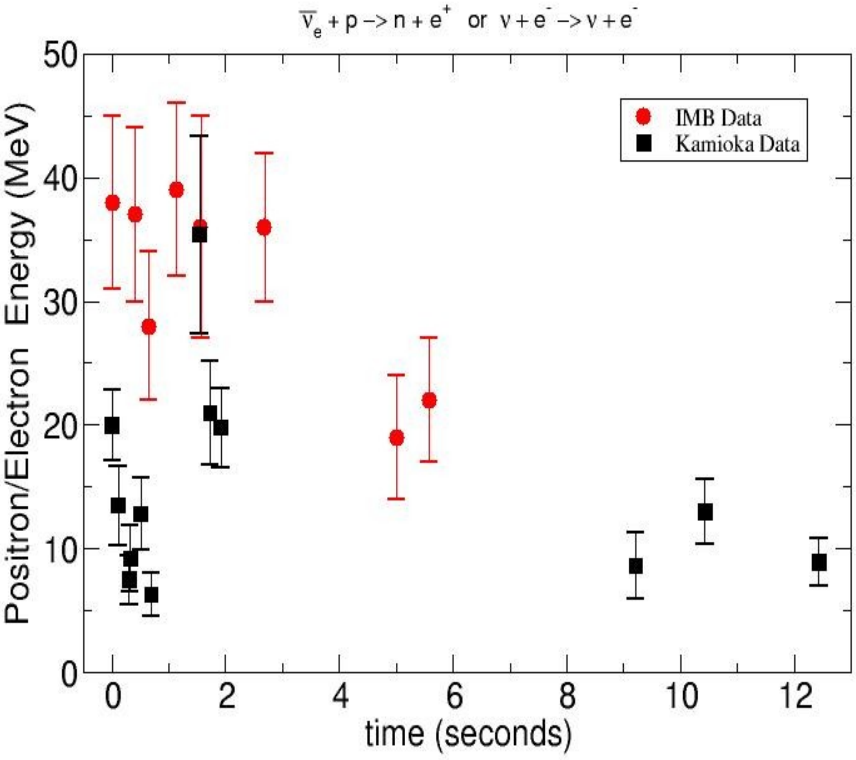
$$\omega_{n=0,l=2} \simeq 2\frac{v_S}{R}$$ ~ 30 Hz

$$\omega_{n=1} \simeq \pi \frac{v_S}{R} \frac{R}{\Delta R}$$ ~ 600 Hz

$$v_s^2 \sim \frac{\mu_e}{\tilde{M}_N} \frac{Z^{5/3}}{A} \frac{1}{1+x}$$

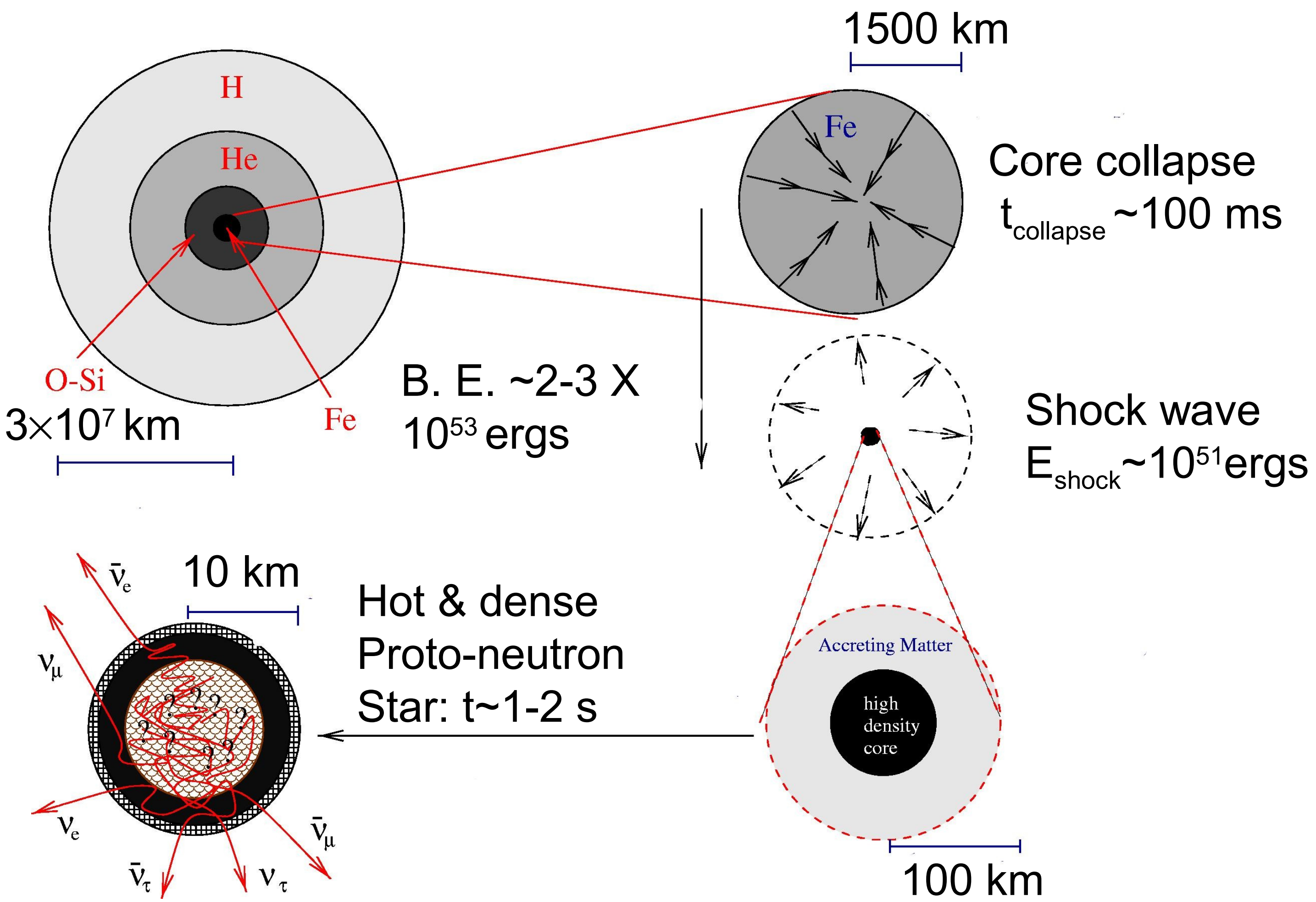
Duration of the supernova neutrino "burst"

SN 1987a: ~ 20 events ..in support of supernova theory



- 10^{57} neutrinos
- time scale ~ 10 s
- neutrino energy: $\langle E_{\nu_e} \rangle \approx 15$ MeV
- total energy emitted in neutrinos: ~ 3 X 10^{53} ergs ~ 0.2 $M_{sun}c^2$

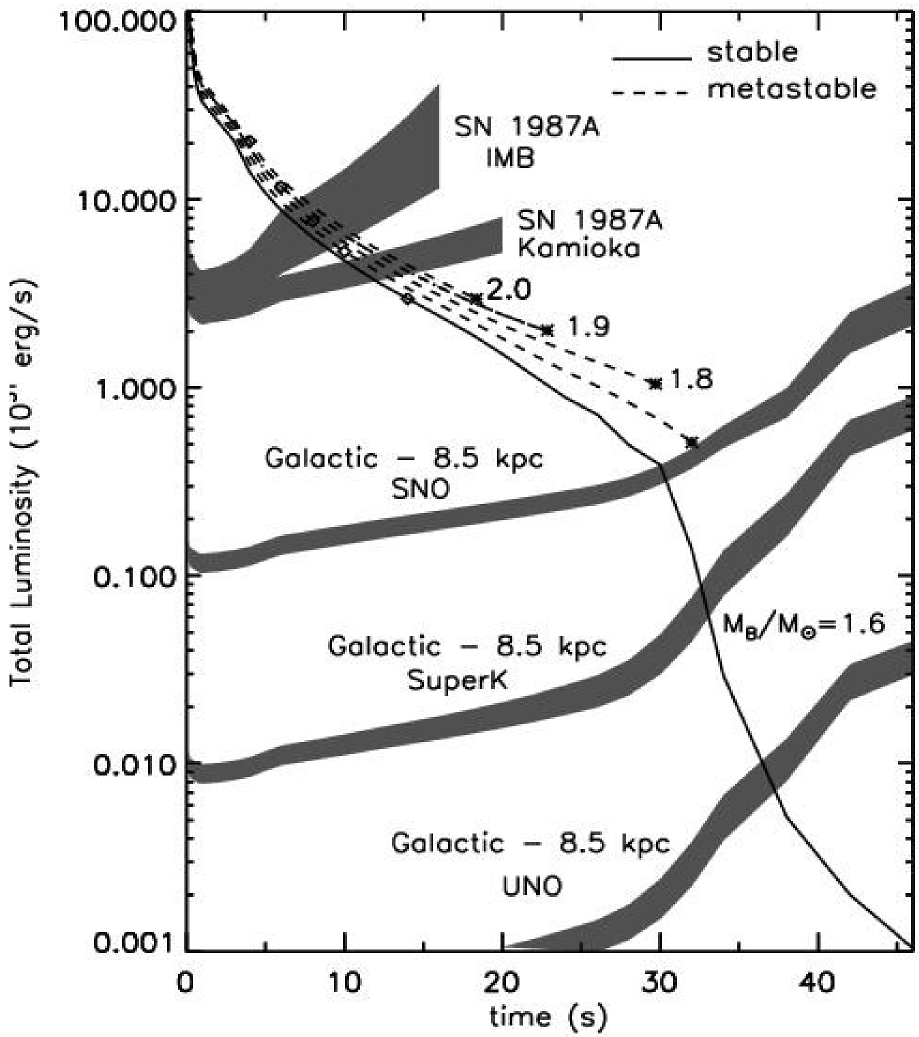
Supernova Neutrinos - a (proto) neutron star is born



Binding energy emitted in neutrinos.
Neutrino's diffuse !

$$\tau_C \approx C_V \frac{R^2}{c\langle l_n \rangle}$$

simulations with normal quark matter (delayed collapse to black-holes)



Early attempts at including quark matter in PNS simulations:

Ignores corrections to mean free paths arising due to coherent scattering & Goldstone excitations

Pons, Steiner, Prakash and Lattimer, Phys.Rev.Lett. 86, 5223 (2001)

Limiting Spin Frequency ?

Can a neutron star spin close to its Keplerian frequency ?

If r-modes are not damped NS cannot spin !

Damping due to shear, crust-core boundary layer viscosity and bulk viscosity in the core is important.

At 1 kHz, bulk viscosity is due to weak interactions

What will know about NS's in the near future ?

Answer:

- Mass
- Radius
- Temperature
- Crust thickness
- Dissipation rates

To what extent does fermion superfluidity play a role in theoretical predicitions for these quantities ?

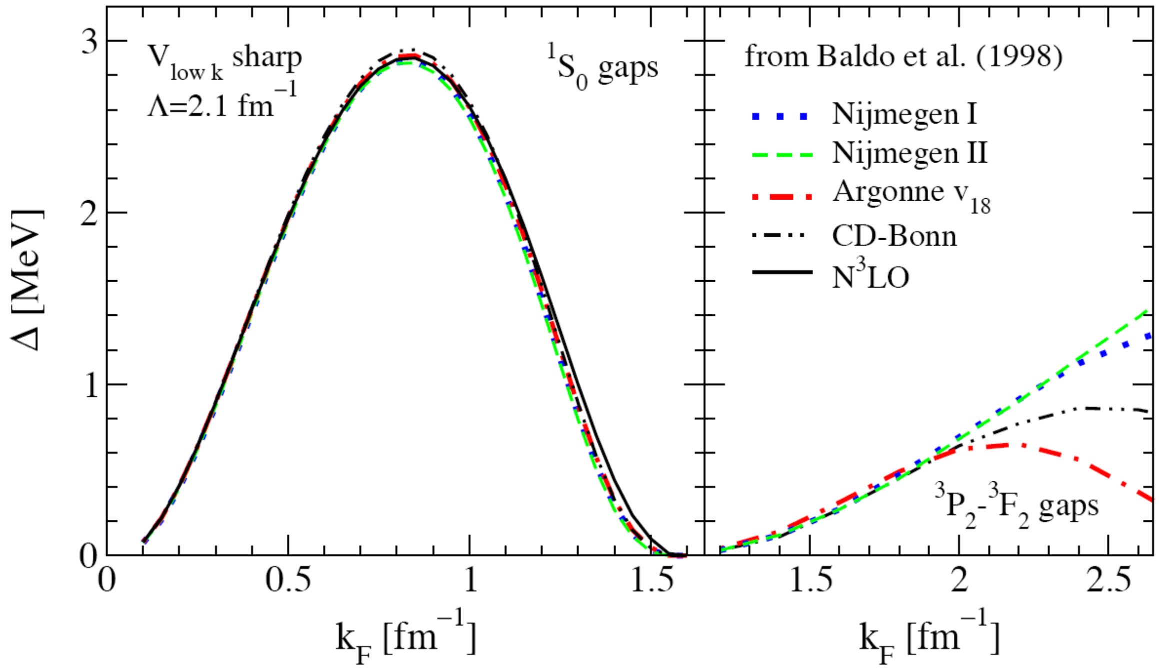
Answer:

•Known to be important for thermal evolution and dissipation.

•Could affect M and R if pairing is large.

Pairing in Nuclear Matter

- Pairing very likely.
- Gaps ~ 1 MeV.
- Role of many-body effects not well constrained yet.
- Pairing energy small compared to E_F~100 MeV.
- Unlikely to play a role in the structure (M & R).
- Very important for response properties.



BCS superfluid gaps in neutron matter

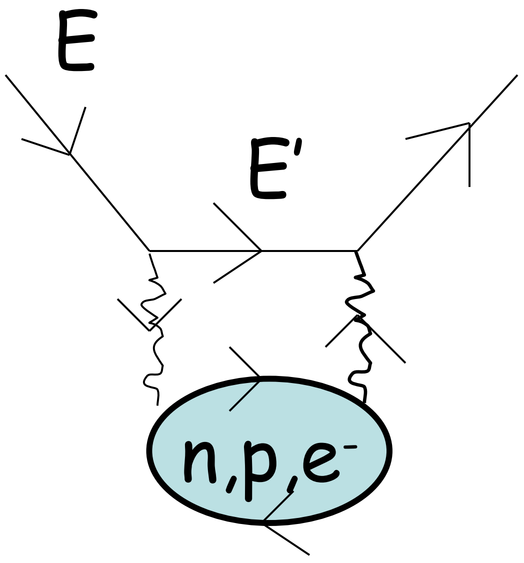
Fig. from Schwenk nucl-th/0611046

Dissipation and cooling phenomena

Neutron star cooling	→	Neutrino emissivity
Superburst recurrence time	→	Neutrino processes in inner crust
R-mode damping	→	Bulk viscosity due to weak interaction
Supernova neutrino burst duration	→	Neutrino mean free path in the dense core

Weak interactions at large density key to understanding neutron star evolution.

Neutrino interactions probe the medium



$$L = \frac{G_F}{2\sqrt{2}}\, l_\nu(x)\; j^\mu(x)$$

$$l_\nu = \bar{\nu}(x)\,\gamma_\nu\,(1-\gamma_5)\,\nu(x)$$

$$j^\mu = \bar{\psi}(x)\left(c_V\gamma^\mu - c_A\gamma^\mu\gamma_5 + iF_2\,\sigma^{\mu\nu}\frac{q_\nu}{2M}\right)\psi(x)$$

$$\frac{d^2\sigma}{V\, d\cos\theta\, dE'} \approx G_F^2\,\frac{E'}{E}\,\mathrm{Im}\left[L_{\mu\nu}(k,k+q)\,\Pi^{\mu\nu}(q)\right]$$

$$L_{\mu\nu} = \mathrm{Tr}\,[\, l_\mu(k)\; l_\nu(k+q)\,]$$

$$\Pi^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\,[\, j^\mu(p)\;\; j^\nu(p+q)\,]$$

Neutrinos probe phase structure

$$j^{\mu}(x) = \bar{\psi}(x)\,\gamma^{\mu}(c_V - c_A\gamma_5)\,\psi(x) \overset{NR}{\rightarrow} c_V\,\psi^{+}\psi\,\delta^{\mu 0} - c_A\,\psi^{+}\sigma^{i}\psi\,\delta^{\mu i}$$

Low density ($\rho < 10^{14}$g/cm^3): nucleons are non-relativistic p/M << 1

Neutrinos couple to fluctuations of density and spin

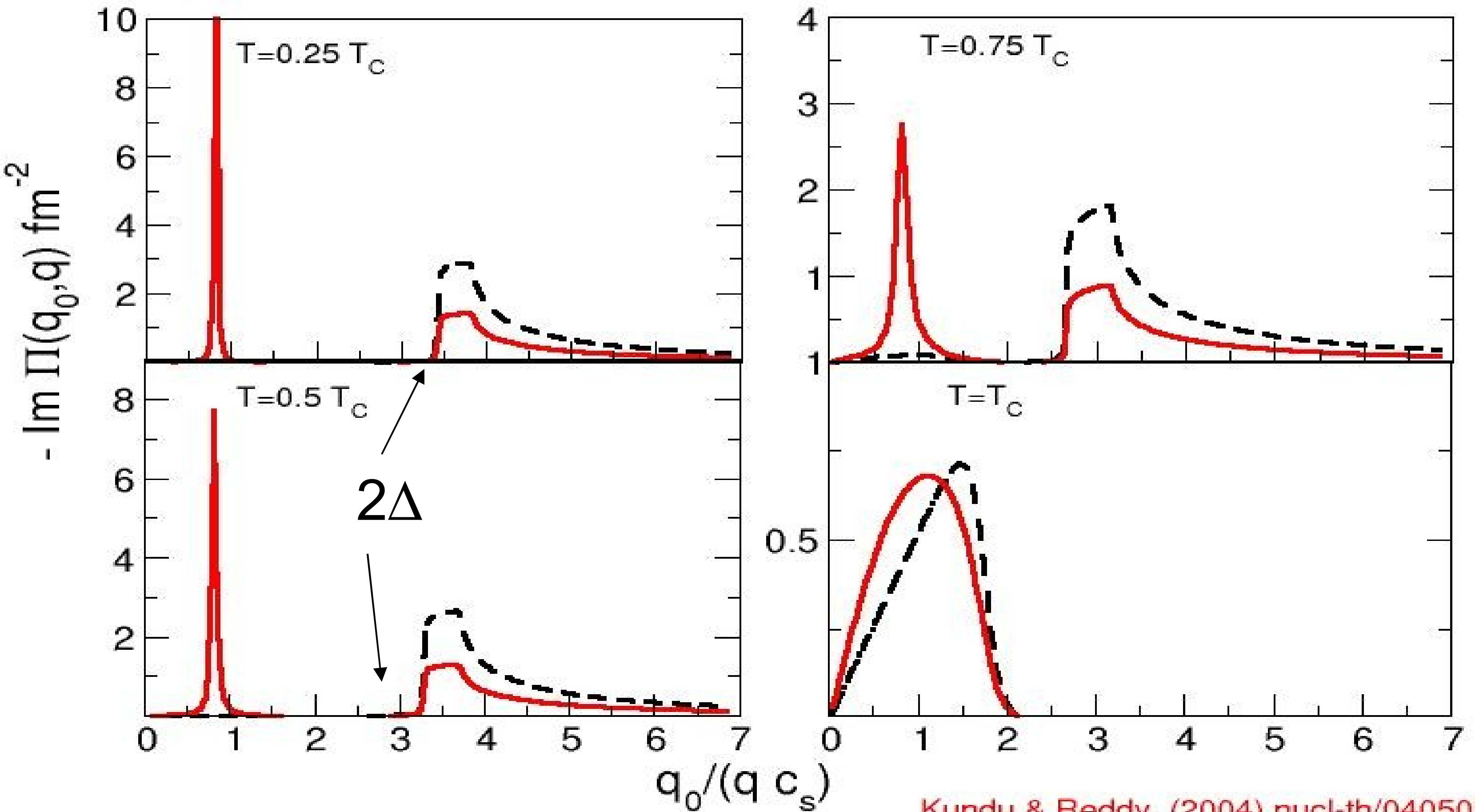
$$\frac{d^2\sigma}{V\,d\cos\theta\,dE'} \approx \frac{G_F^2 n}{8\pi^2}\,E'^2\left[c_V^2(1+\cos\theta)S_\rho(\omega,|\vec{q}|) + c_A^2(3-\cos\theta)S_\sigma(\omega,|\vec{q}|)\right]$$

$$S_\rho(\omega,|\vec{q}|) = \frac{1}{n}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\left\langle \rho(\vec{q},t)\,\rho(-\vec{q},0)\right\rangle$$

$$S_\sigma(\omega,|\vec{q}|)\,\delta_{ij} = \frac{1}{n}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\left\langle \sigma_i(\vec{q},t)\,\sigma_j(-\vec{q},0)\right\rangle$$

Spectrum of density and spin fluctuations

Spectrum of density fluctuations in Superfluids



Kundu & Reddy, (2004) nucl-th/0405055

Lecture 2: Learning about strongly coupled superfluids from cold atom experiments

Fermion Superfluids

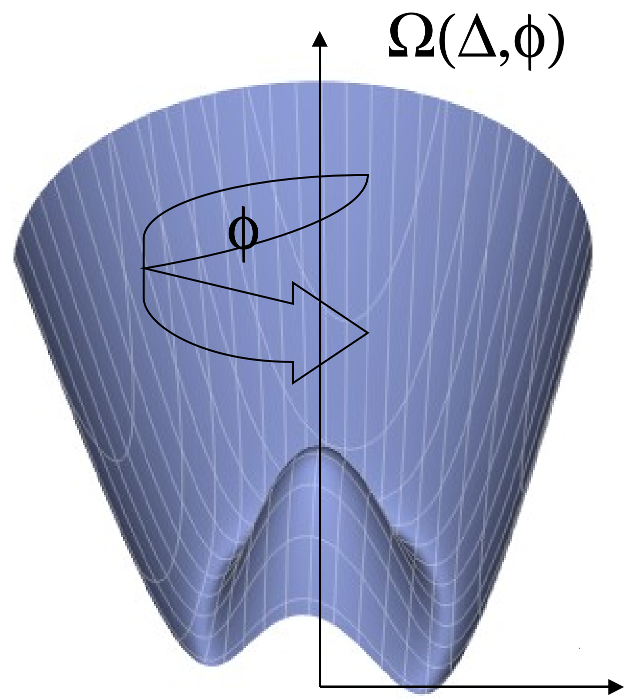
Arbitrarily weak interaction destabilizes the Fermi Gas
(Bardeen, Cooper and Schreiffer (1957))

$$H = \sum_{k,s=\uparrow,\downarrow} \left(\frac{k^2}{2m} - \mu_s \right) a_s^\dagger a_s + g \sum_{k,p,q} a^\dagger_{k+q\uparrow} a^\dagger_{p-q\downarrow} a_{k\uparrow} a_{p\downarrow}$$

$\Delta = g \langle a_{-k} a_k \rangle \quad \Delta^* = g \langle a^\dagger_{-k} a^\dagger_k \rangle$

$\Delta \rightarrow |\Delta|\, e^{i\phi}$

$$E(p) = \sqrt{\left(\frac{p^2}{2m} - m\right)^2 + D^2}$$



Pairing in Fermi Systems

- Electronic Superconductors :$(\Delta \sim 10^{-3}$ eV$)/(E_F \sim 10$ eV$) \sim 10^{-4}$

- Nuclei and Nuclear Matter : $(\Delta \sim 1$ MeV$)/(E_F \sim 10$ MeV$) \sim 10^{-1}$

- Dense Quark Matter: $(\Delta \sim 100$ MeV$)/(E_F \sim 400$ MeV$) \sim 1/4$

Cold atom experiments (^{6}Li and ^{40}K atoms) can tune the interaction through Feshback resonances. Explore BCS, BEC and the cross-over region !

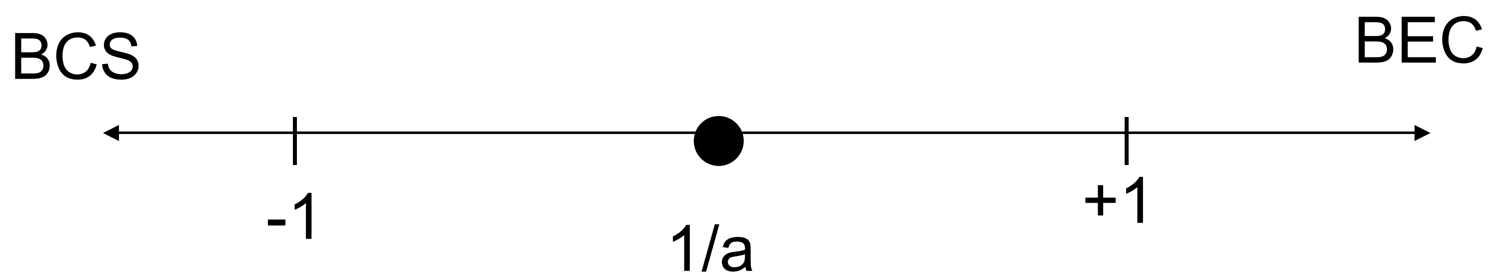
Several Groups: Hulet et al. (Rice); Ketterle et al. (MIT); Thomas et al. (Duke); D. Jin (Boulder).

Universal System: Unitary Fermi Gas

Strongly-Coupled Fermions with short-range interactions

$$\mathcal{H} = \sum_{k=1}^{A} \left(-\frac{\hbar^2}{2m_k}\nabla_k^2\right) + \sum_{i<j} v(r_{ij})$$

	Cold Fermi Atoms	Neutrons
scattering Length (a)	tunable	-18.5 fm
Effective range (r_o)	0	2.7 fm



Universal Constants at a=∞

$k_F=(3\,\pi^2\,\rho)^{1/3}$ is the only scale in the problem.

$$\mu = \xi\,\varepsilon_F = x\,\frac{k_F^2}{2m}$$

$$P = x\;P_{FG}$$

$$\mathrm{D} = h\;e_F$$

Experiment can measure ξ and η

Measuring ξ from Energy Release

Magnetic trap creates a harmonic oscillator potential to trap atoms:
10^6-10^7 atoms in ~ 100 μm^3

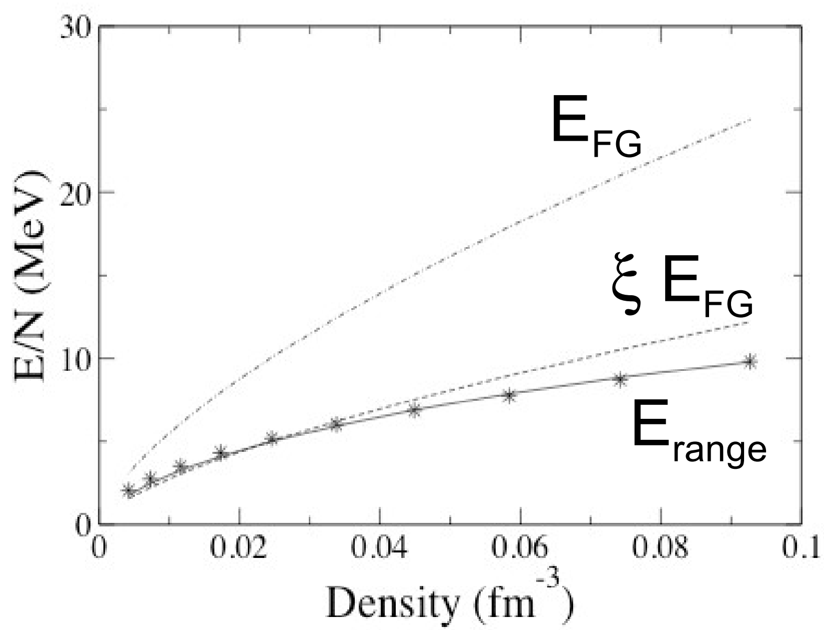
ξ	Expt
0.51 (4)	Kinast, et al., Science (2005)
0.32 (+.13,-.1)	Bartenstein, et al., PRL (2004)
0.36(15)	Bourdel, et al., PRL (2004)
0.46(5)	Partridge, et al., PRL (2004)
0.45(5)	Stewart, et al., PRL (2006)
0.41(15)	Tarruell, et al., cond-mat/0701181

Ioffe-Prichard Trap

Constraints for low-density neutron matter

In the universal regime GFMC & Lattice methods yield: $\xi = 0.4 \pm 0.2$

Neutron-Neutron interaction - dominantly s-wave (spin 0) at low energy:
Large scattering length ~ -18 fm
Modest effective range ~ 2.7 fm

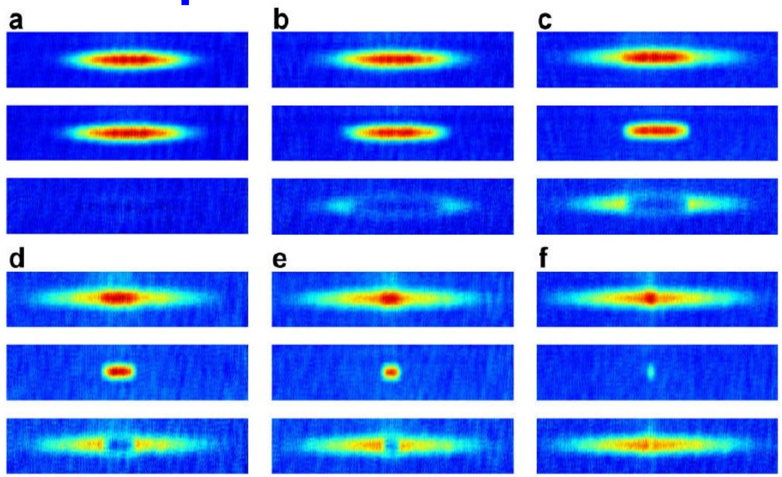


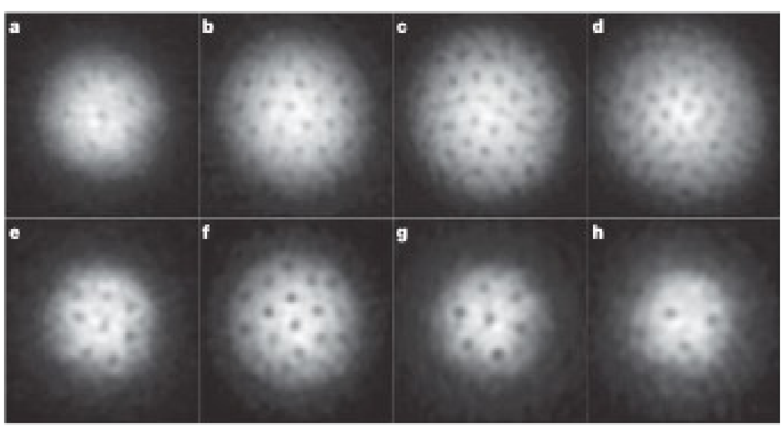
GFMC for neutron matter

Carlson (2003)

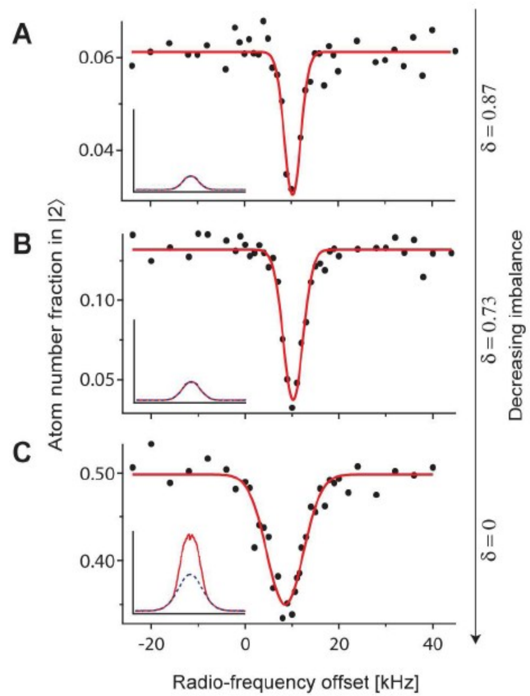
Rich Set of Experimental Results

Radial Density and polarization





Vortices



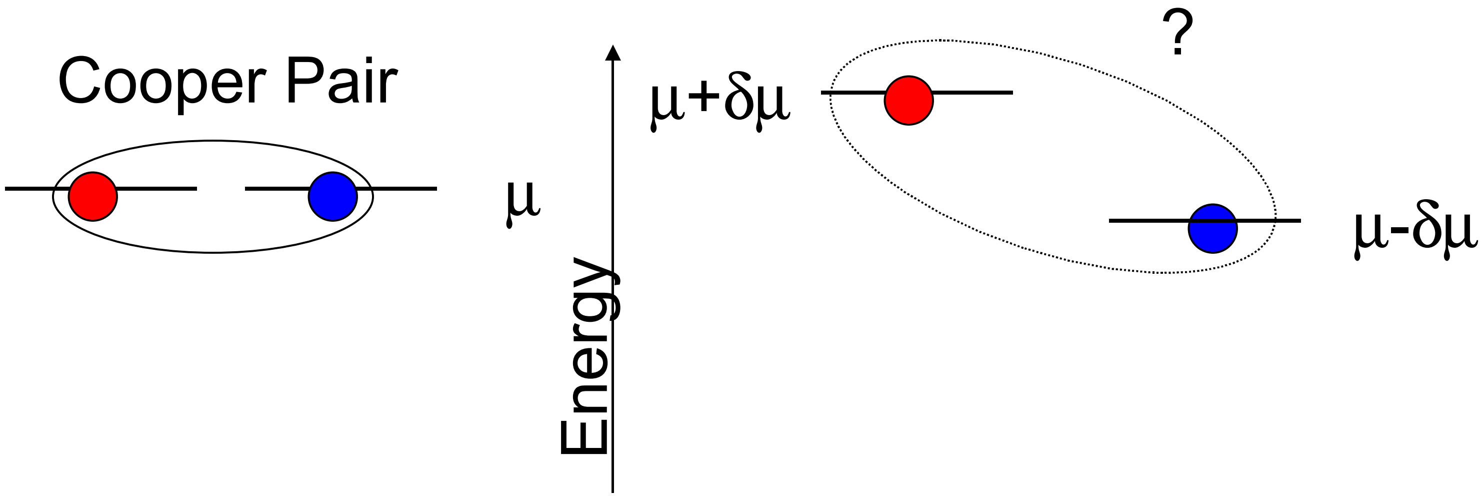
Rice

MIT

RF response

Asymmetric Fermi Systems

What happens when we split the Fermi surfaces ?

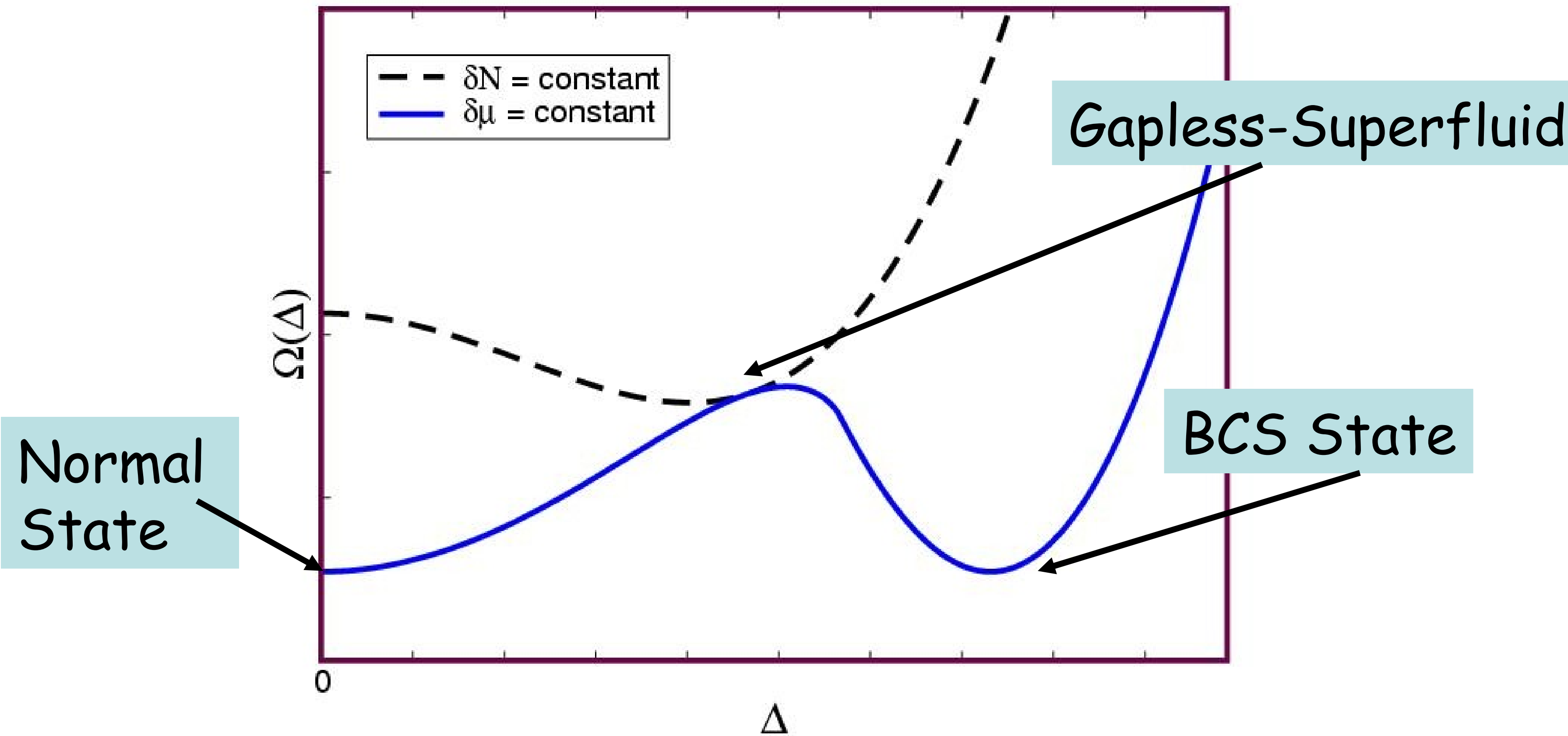


Mixed Phase | Gapless Superfluid | LOFF Phase

Asymmetric Fermion Superfluids

$\mu_\uparrow \neq \mu_\downarrow$ or $N_\uparrow \neq N_\downarrow$ $\delta\mu = \frac{(\mu_\uparrow - \mu_\downarrow)}{2}$ $\bar{\mu} = \frac{(\mu_\uparrow + \mu_\downarrow)}{2}$

Free energy in weak coupling:



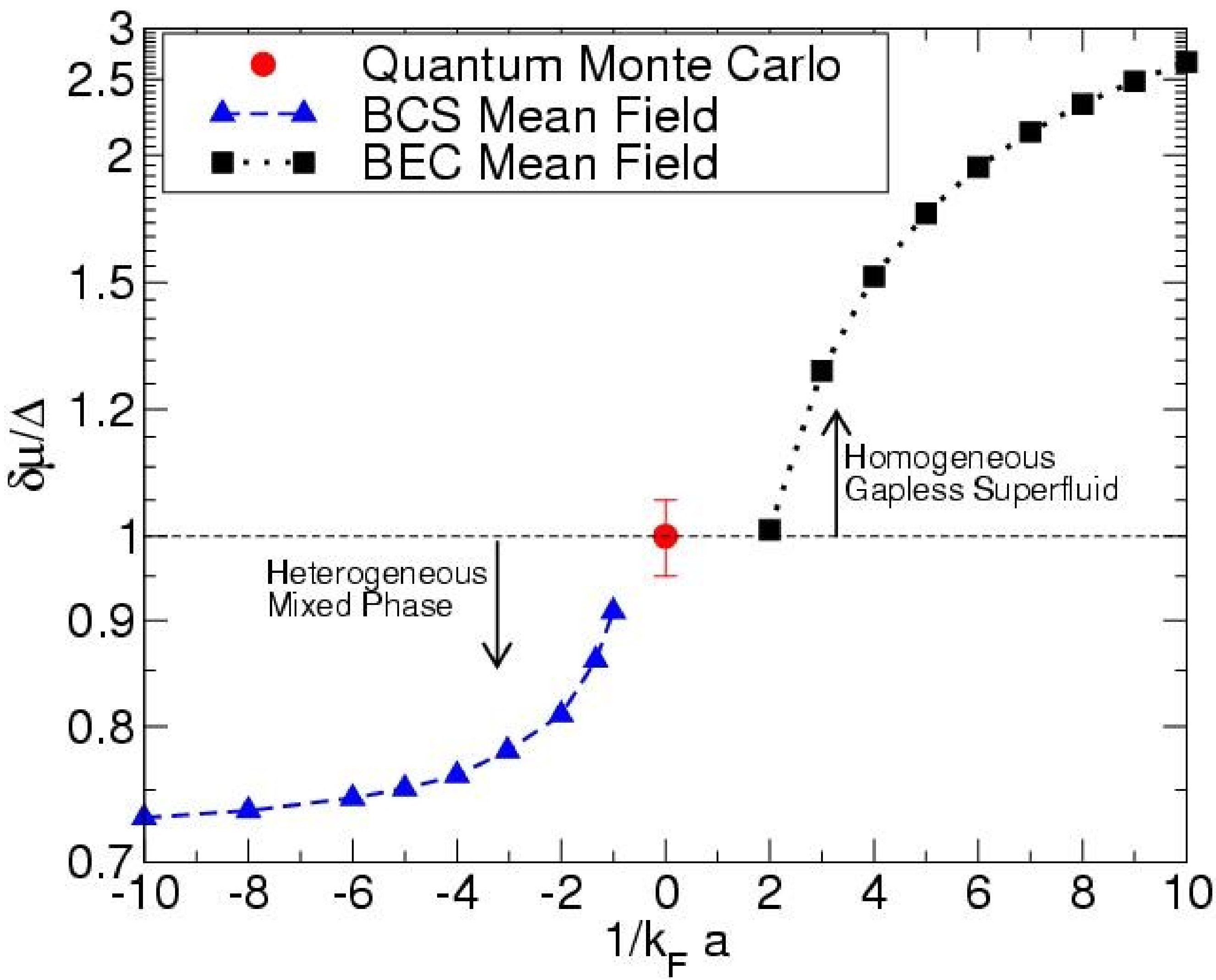
Bedaque, et al. PRL. 91:247002,2003 Liu & Wilczek PRL.90:047002,2003

Phase Separation in Strong Coupling ?

$$P_{normal}(\mu,\delta\mu)=P_{superfluid}(\mu,\delta\mu,\Delta)$$

Ratio $\delta\mu/\Delta$ increases with coupling

Carlson & Reddy PRL (2005)



Asymmetric systems in cold atoms

$\tilde{\mu}[R] = \mu - V[R]$

V[R]

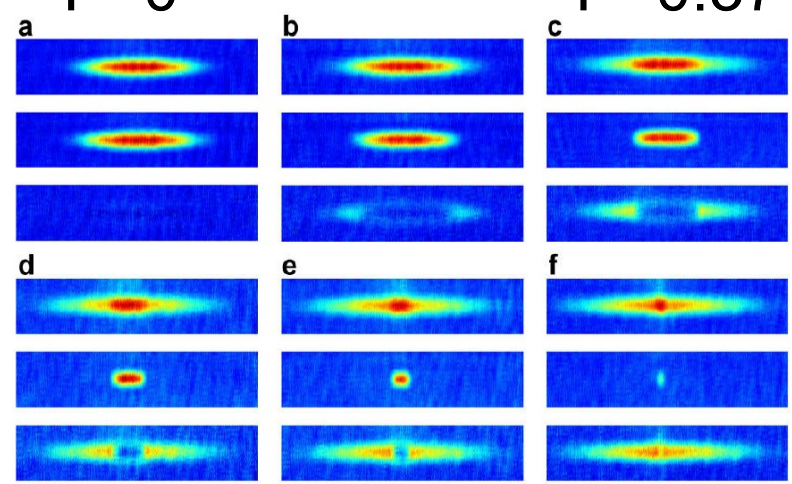
$\delta\mu$

R

Easily realized by loading different numbers.
However trapping potential induces space varying asymmetry.

Imaging by absorption:
Column density v/s R.
Tomography:
Density v/s R.

P=0 P=0.37



P=0.6 P=0.95

Hulet et al. (2006)

Testing theory

Expt. can measure:

$\delta n = n_{\uparrow} - n_{\downarrow}$

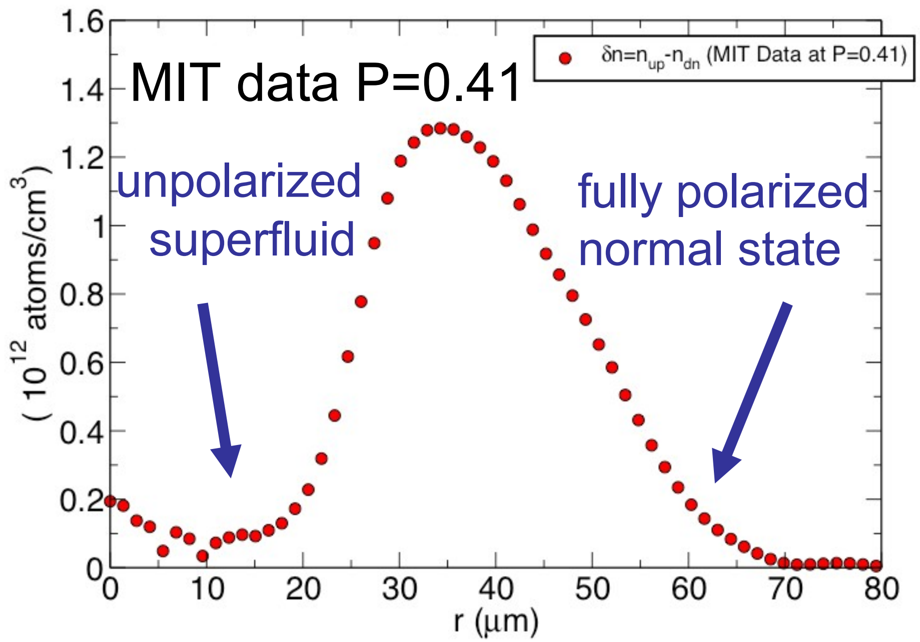
At T=0:

BCS state is unpolarized.

At finite T:

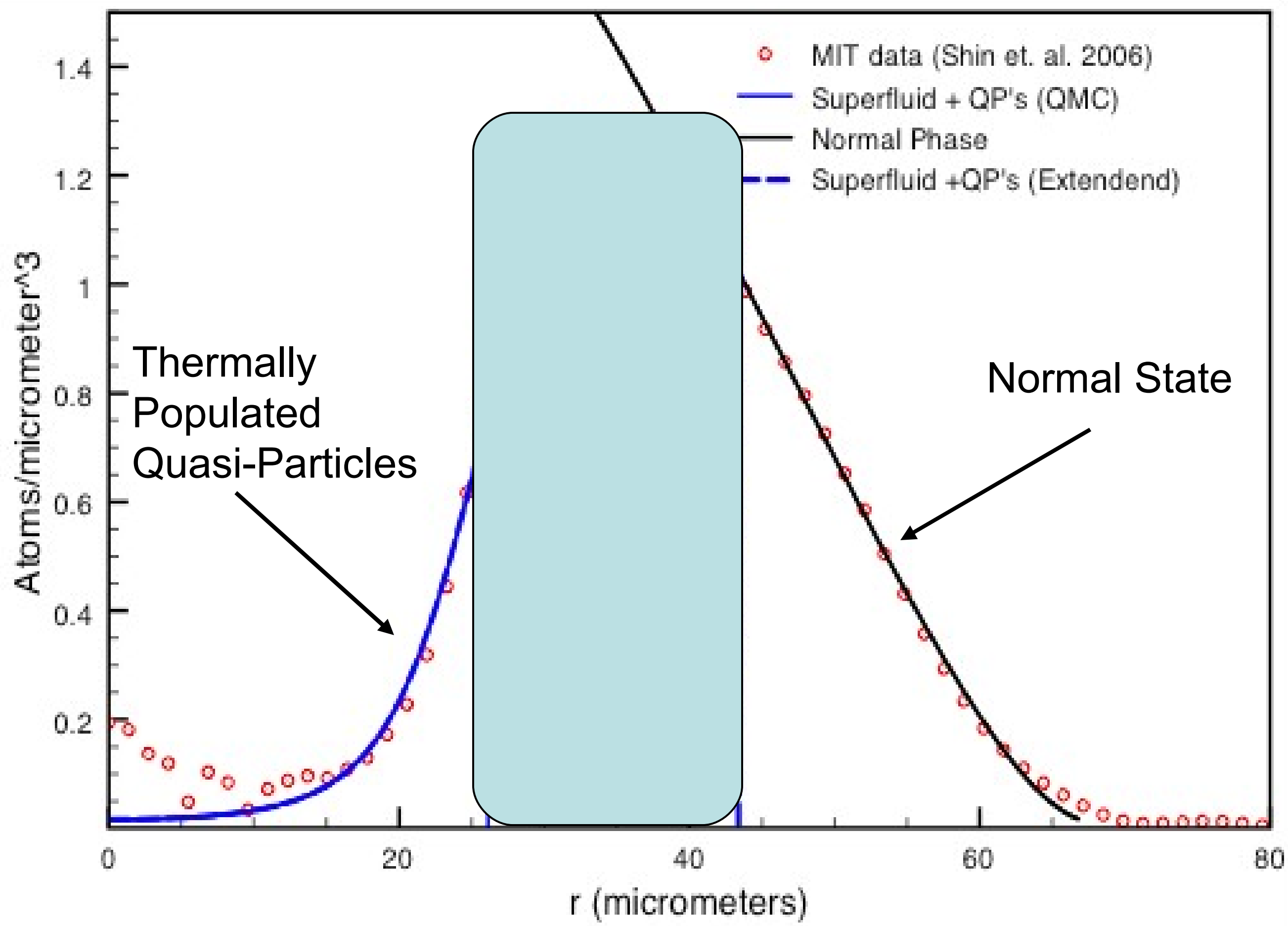
$\delta n[r] \propto \exp(-(\Delta[r]-\delta\mu)/kT)$

Polarization Density



The fully polarized normal state is a non-interacting Fermi-Gas:
Can extract: $\mu+\delta\mu$ and kT.
At the center the state is unpolarized (independent of $\delta\mu$) :
Can extract: μ

Extracting the gap from polarized systems

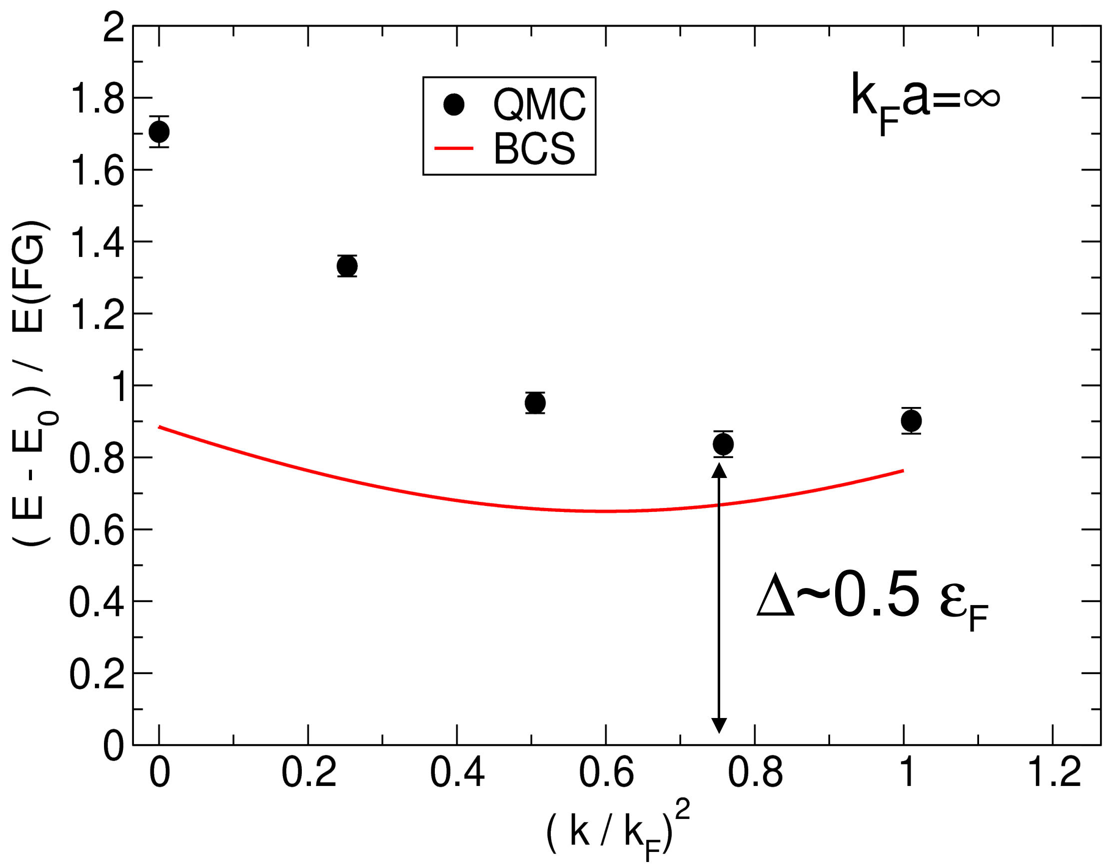


$\Delta = \eta \varepsilon_F$

$\Delta \approx 0.5\ \varepsilon_F \approx 1.2\mu$

Carlson & Reddy (2007) in prep.

Quasi-particle dispersion relation



QMC predictions for the energy of quasi-particle in the superfluid can be tested.

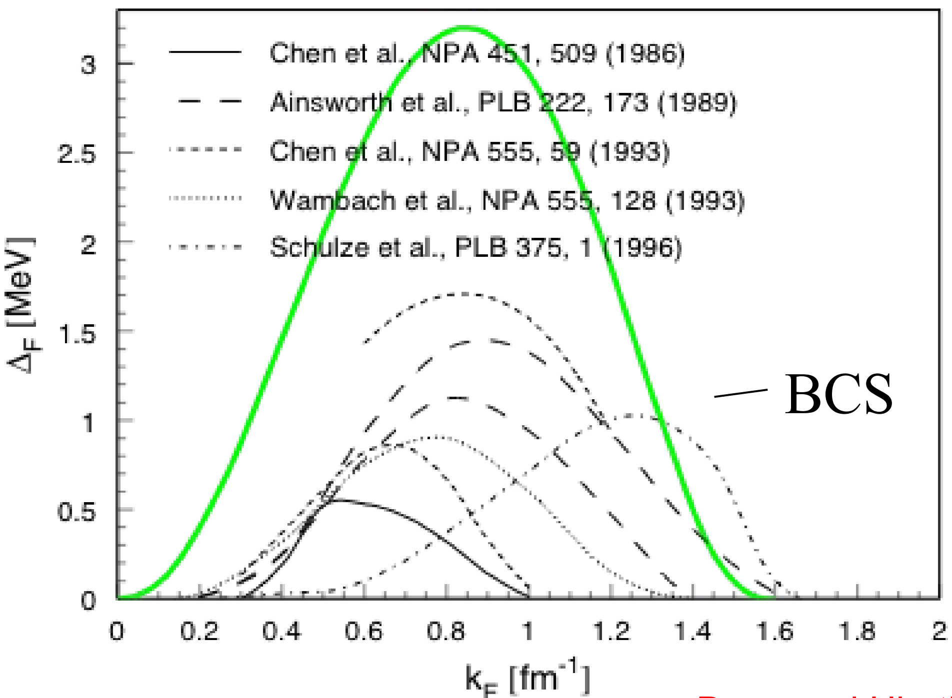
Carlson & Reddy
PRL **95**, 060401 (2005)

Neutron Matter Pairing Gap

Pairing Gap difficult to get right in approximate many-body theories.

Attempts to use GFMC with realistic neutron-neutron interactions (including range) are underway.

Carlson et al (2007) in prep.

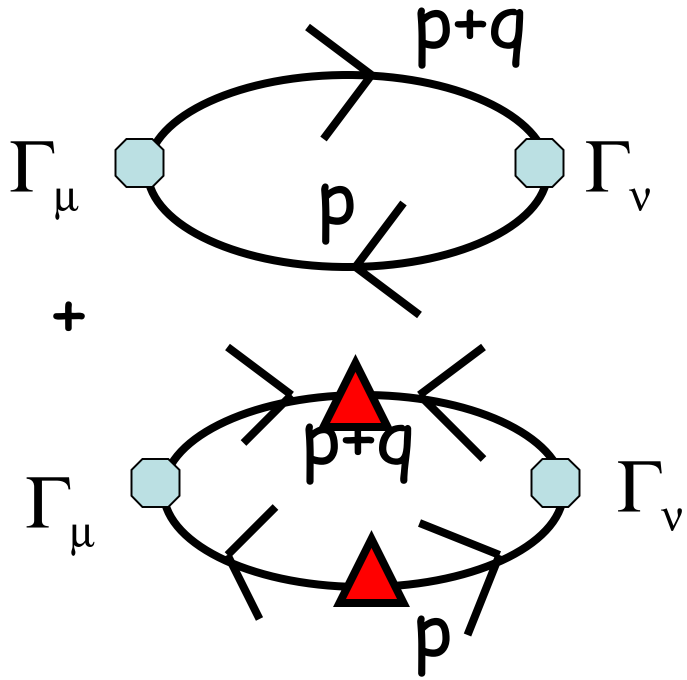


Dean and Hjorth-Jenson
RMP (2003)

Response to External Perturbations

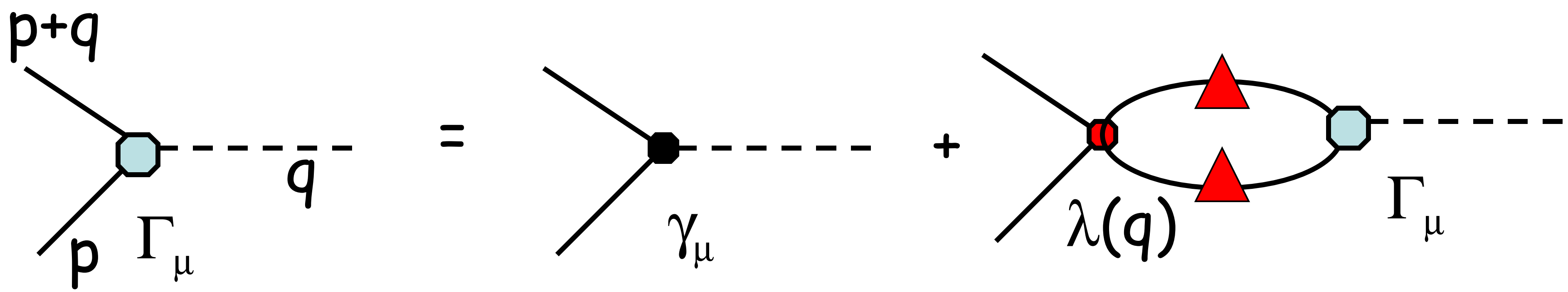
$$\Pi_{\mu\nu}(\vec{q},q_o) = \int d^4p \; Tr\,[\; G(p)\;\; \Gamma_\mu G(p+q)\Gamma_\nu \;]$$

=



Gap modifies excitation spectrum
Pairing introduces coherence effects

Collective (Goldstone) modes



Generalized Ward Identity:

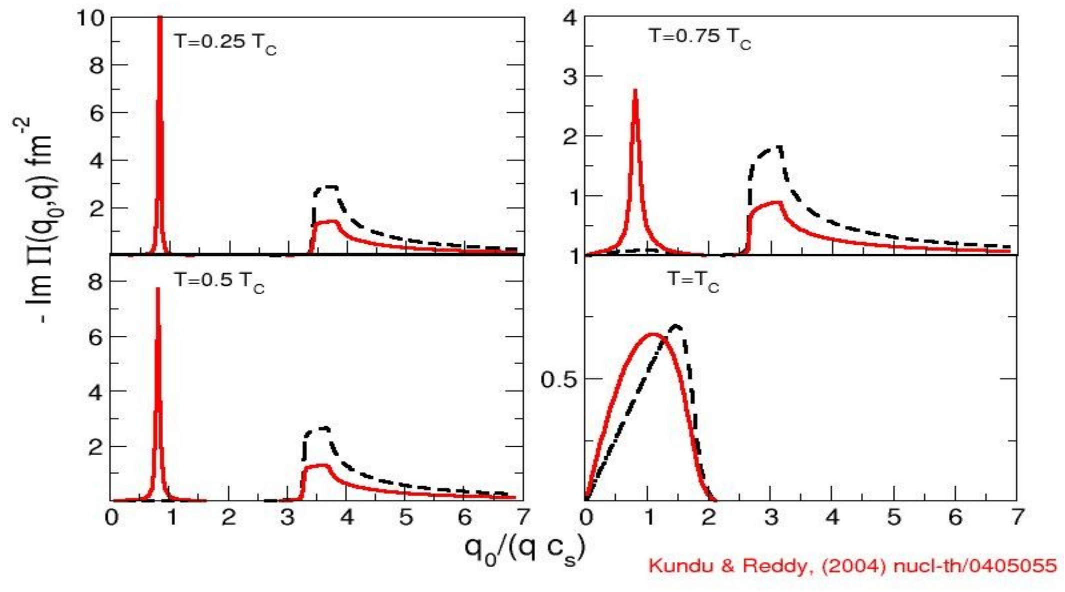
$$\sum_{m} q_{m}\Gamma_{m} = \tau_3^{NG} G^{-1}(p) - G^{-1}(p+q)\tau_3^{NG}$$

→ Goldstone mode

if $\lim_{q \to 0} \lambda(q)$ is finite

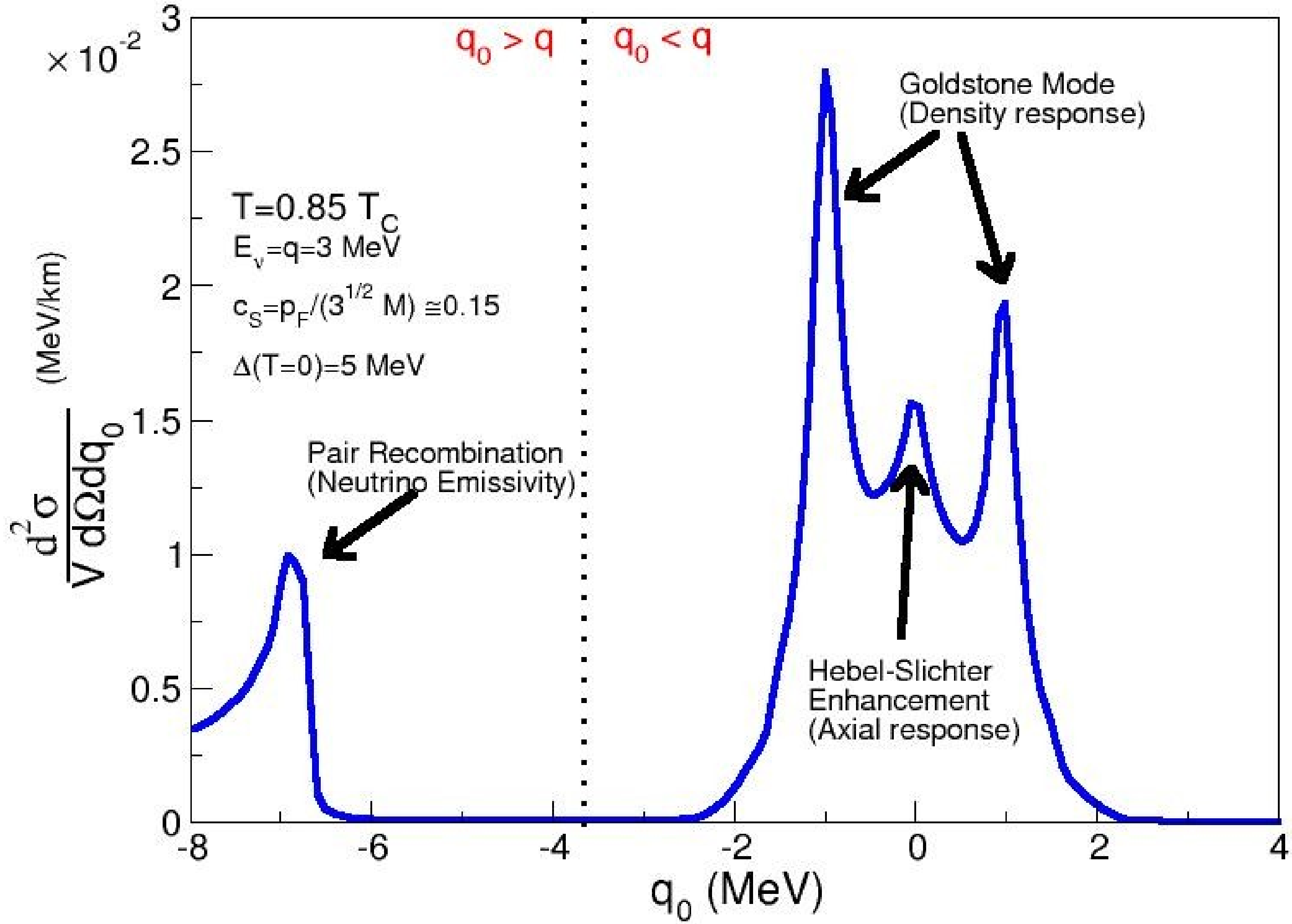
$\Gamma(q = (\vec{q}, q_o))$ is singular at

$$q_o = c_s q$$



Bogoliubov, Nuovo Cimento, 7, 6 (1958)
Anderson, Phys. Rev. **112**, 1900 (1958)
Nambu, Phys. Rev. **117**, 648 (1960)

Weak Interactions in Dense Superfluids



Carter & Reddy Phys.Rev.D62:103002,(2000)
Kundu & Reddy Phys.Rev.C70:055803,(2004)

Response Functions From Cold-Atom Expt.

Laser probe can be tuned to produce specific transitions

|3>

|2>

|1>

Example: |2>→|3> Transition

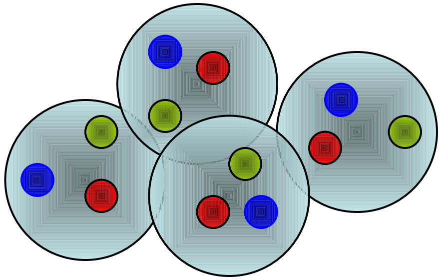
Threshold has information about the gap:

ω_{th}~0.3 ε_F Implies Δ ~ 0.5 ε_F

$$S(\omega) \propto \int d^3k\, n(k)\, \delta(\omega - \tilde{\omega}_k)$$

$$\tilde{\omega}_k = E_{QP}(k) + E_3(-k) + V_{13} - \mu$$

Quark Matter



Naïve Analysis:
Hadrons overlap-> quarks delocalize to form a Relativistic Fermi Liquid
Chiral Symmetry is restored, excitation spectrum starts at zero energy

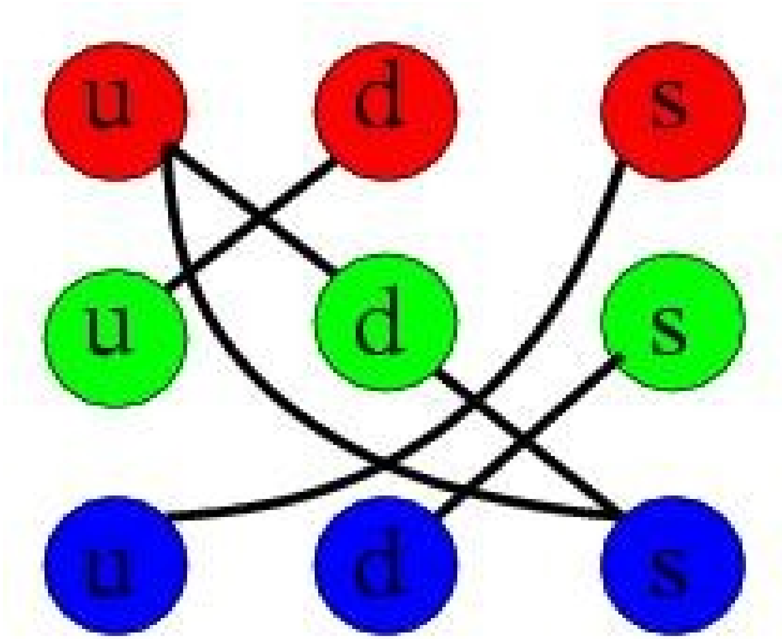
BCS Analysis:
Quark matter is a color superconductor: Gluon exchange is attractive in color anti-symmetric channel $\langle \psi^{ia}(p)\, \psi^{jb}(-p) \rangle \sim \Delta\, \varepsilon^{ijA}\, \varepsilon^{Aab}$

Color-Flavor Locked Phase

Alford, Rajagopal & Wilczek, Nucl. Phys. B **558**, 219 (1999)

BCS pairing of all 9 quarks:

$\Delta \approx 100$ MeV !



$$\Sigma\Upsilon(3)_{\chi o\lambda o\rho} \otimes \Sigma\Upsilon(3)_{\Lambda} \otimes \Sigma\Upsilon(3)_{P} \otimes \Upsilon(1)_B$$

$$\Downarrow$$

$$\Sigma\Upsilon(3)_{\chi o\lambda o\rho+\Lambda+P} \otimes Z_2$$

$E_{\gamma\lambda\upsilon o\nu} \approx gm$

$E_{\theta\upsilon\alpha\rho\kappa} \approx 2D$

Energy

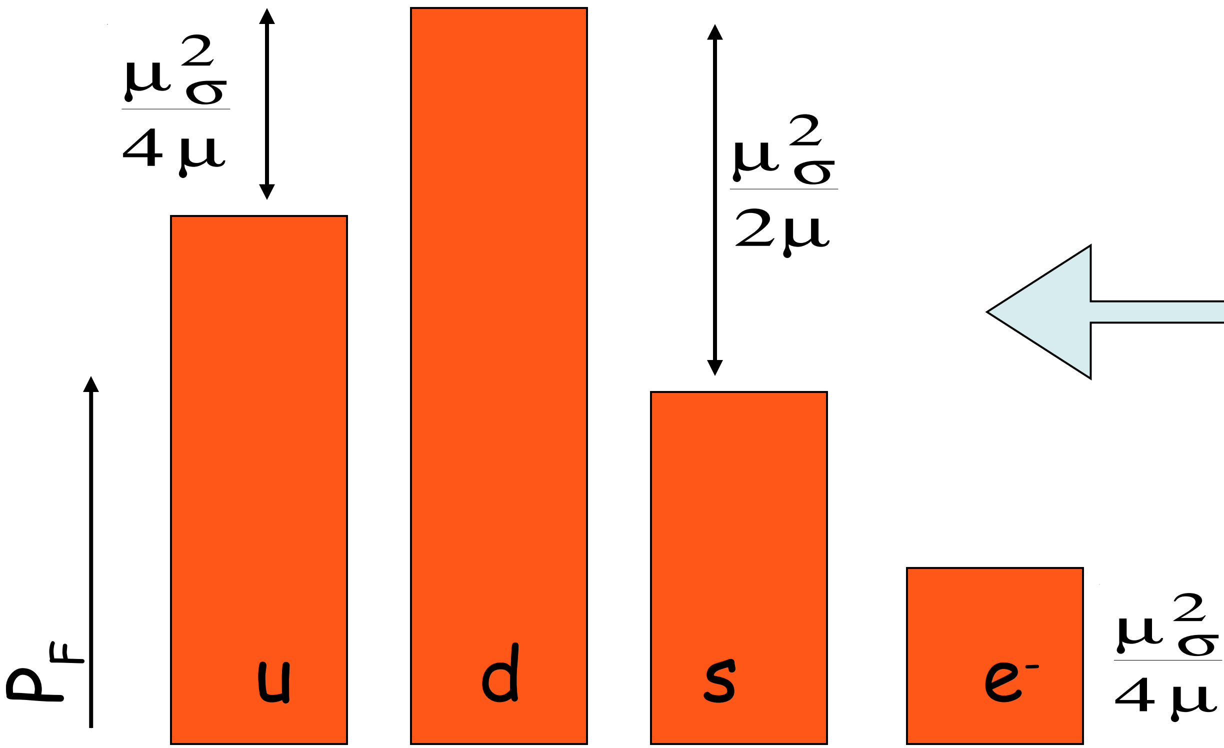
$E_{\Gamma B:\ \Sigma\Upsilon_X(3) \otimes \Sigma\Upsilon_{\Lambda}(3) \ddot{A} SU_R(3)}$

$$» \frac{D}{m}\sqrt{m_{light}\ m_s}$$

$E_{\Gamma B:\ \Upsilon_B(1)} = 0$

Excitation Spectrum

Charge Neutrality in Dense Quark Matter



Normal Quark Matter requires electrons for charge neutrality
⇓
breaks iso-spin

CFL requires $\Delta \geq m_s^2/4\mu$

Alford, Rajagopal, Reddy and Wilczek
Phys.Rev.D**64**:074017, (2001)

Outlook

Progress is being made on three fronts: (1) Observations, (2) Theory & (3) Terrestrial Experiments.

We can relate specific properties of dense matter to neutron star observables. Response functions are key to several transient phenomena.

Pairing and response of strongly interacting Fermi systems can be probed in cold-atom experiments and are useful to constrain many-body theory.

Prospects to constrain the properties of cold dense matter from astrophyics are real. But needs realistic and controlled calculations of both the astrophysics and nuclear physics.