Transition of an extended object across the cosmological singularity

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Outline

Introduction

Motivation:

Finding general theoretical framework that can be used to describe possibly all available cosmological data.

Assumptions:

- evolution of the universe includes at least one quantum phase and two classical phases
- quantum phase can be described in terms of quantum p-branes propagating in higher dimensional ($d > 4$) spacetime with the cosmic singularity
- the cosmic singularity consists of pre-singularity and post-singularity epochs
- classical phase can be obtained from the quantum phase by changing topology of its spacetime, and vice versa

Remark:

We address, to some extent, the question of mathematical consistency of a cyclic universe scenario.

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Introduction (cont)

- Restriction of considerations to the neighborhood of the cosmological singularity
- Basic criterion for the choice of the model of universe in the quantum phase:
  Reasonable model should allow for propagation of quantum p-brane (i.e., particle, string, membrane,...) from pre-singularity to post-singularity epoch.
  If quantum *p*-brane cannot go through the cosmic singularity, the evolution cannot be realized.
- Model of the universe in the quantum phase:
  compactified Milne space - the simplest model of universe with the cosmic singularity that is implied by string/M theory (the simplest example of time dependent singular orbifold)

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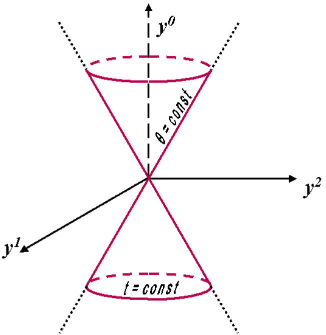
Compactified Milne space



- Isometric embedding of 2d compactified Milne space into 3d Minkowski space

$$y^0(t,\theta) = t\sqrt{1+r^2}, \quad r \in \mathbb{R}^1$$

$$y^1(t,\theta) = rt\sin(\theta/r), \quad y^2(t,\theta) = rt\cos(\theta/r)$$

$$\frac{r^2}{1+r^2}(y^0)^2 - (y^1)^2 - (y^2)^2 = 0$$

- Induced metric (for $t \neq 0$)

$$ds^2 = -dt^2 + t^2 d\theta^2, \quad (t,\theta) \in \mathbb{R}^1 \times \mathbb{S}^1$$

- Local isometry with 2d Minkowski space (for $t \neq 0$)

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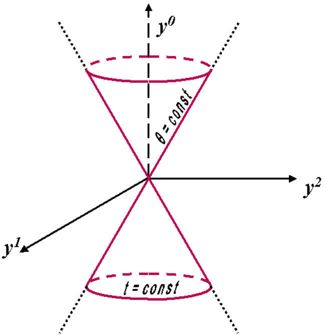

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Compactified Milne space (cont)

- Metric of the compactified Milne, CM, space

$$ds^2 = -dt^2 + dx^k dx_k + t^2 d\theta^2, \quad (t, x^k) \in \mathbb{R}^1 \times \mathbb{R}^{d-1}, \;\; \theta \in \mathbb{S}^1$$

- One term in metric disappears/appears at $t = 0$ $\Rightarrow$ CM space may be used to model big-crunch/big-bang type singularity
- Other properties of the CM space:
  - not manifold, but orbifold due to the vertex at $t = 0$
  - Riemann's tensor components equal 0 for $t \neq 0$
  - singularity at $t = 0$ of removable type: any time-like geodesic with $t < 0$ can be extended to some time-like geodesic with $t > 0$
  - extension cannot be unique due to the Cauchy problem at $t = 0$ for the geodesic equation (compact dimension shrinks away and reappears at $t = 0$)
- Orbifolding $\mathbb{S}^1$ to the segment $\mathbb{S}^1/\mathbb{Z}_2$ gives the model of two flat parallel "end of the world" branes[1] which collide and re-emerge at $t = 0$

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Classical dynamics of p-brane

The Polyakov action integral for test p-brane embedded in fixed background spacetime with metric $g_{\tilde{\mu}\tilde{\nu}}$ reads

$$S_p = -\frac{1}{2}\mu_p \int d^{p+1}\sigma \sqrt{-\gamma}\, \left[\gamma^{ab}\partial_a X^{\tilde{\mu}} \partial_b X^{\tilde{\nu}} g_{\tilde{\mu}\tilde{\nu}} - p + 1\right], \qquad (1)$$

where
μ_p is mass per unit p-volume,
$(\sigma^a) \equiv (\sigma^0, \sigma^1, \ldots, \sigma^p)$ are p-brane worldvolume coordinates,
γ_{ab} is p-brane worldvolume metric, $\gamma := det[\gamma_{ab}]$,
$(X^{\tilde{\mu}}) \equiv (X^{\mu}, \Theta) \equiv (T, X^k, \Theta) \equiv (T, X^1, \ldots, X^{d-1}, \Theta)$ are embedding functions of p-brane, i.e. $X^{\tilde{\mu}} = X^{\tilde{\mu}}(\sigma^0, \ldots, \sigma^p)$,
corresponding to $(t, x^1, \ldots, x^{d-1}, \theta)$ directions od $d + 1$ dimensional background spacetime.

Classical dynamics of p-brane (cont)

Total Hamiltonian, H_T, corresponding to the Polyakov action[2]

$$H_T = \int d^p\sigma \mathcal{H}_T, \tag{2}$$

$$\mathcal{H}_T := AC + A^i C_i, \qquad i = 1, \dots, p \tag{3}$$

where $A = A(\sigma^a)$ and $A^i = A^i(\sigma^a)$ are any 'regular' functions, and C and C_i are first-class constraints

$$C := \Pi_{\tilde{\mu}}\Pi_{\tilde{\nu}} g^{\tilde{\mu}\tilde{\nu}} + \mu_p^2\, det[\partial_a X^{\tilde{\mu}} \partial_b X^{\tilde{\nu}} g_{\tilde{\mu}\tilde{\nu}}] \approx 0, \tag{4}$$

$$C_i := \partial_i X^{\tilde{\mu}} \Pi_{\tilde{\mu}} \approx 0. \tag{5}$$

H_T does not generate time translations, but gauge transformations!

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Hamilton's equations

$$\dot{X}^{\tilde{\mu}} \equiv \frac{\partial X^{\tilde{\mu}}}{\partial \tau} = \{X^{\tilde{\mu}}, H_T\}, \qquad \dot{\Pi}_{\tilde{\mu}} \equiv \frac{\partial \Pi_{\tilde{\mu}}}{\partial \tau} = \{\Pi_{\tilde{\mu}}, H_T\}, \qquad \tau \equiv \sigma^0, \tag{6}$$

where

$$\{\cdot, \cdot\} := \int d^p\sigma \Big(\frac{\partial \cdot}{\partial X^{\tilde{\mu}}} \frac{\partial \cdot}{\partial \Pi_{\tilde{\mu}}} - \frac{\partial \cdot}{\partial \Pi_{\tilde{\mu}}} \frac{\partial \cdot}{\partial X^{\tilde{\mu}}} \Big). \tag{7}$$

Degrees of freedom

$$n_c =: 2n_p = 2(d-p),$$

where
n_c, number of independent canonical variables,
n_p, number of physical degrees of freedom,
$d+1$, dimension of spacetime,
$p+1$, number of constraints,
p, dimension of p-brane

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Propagation of a particle

Classical dynamics of a test particle in the CM space is unstable, however it can be quantized, i.e. there exists mathematically well defined quantum dynamics of a particle. For details see:

- P. Małkiewicz and WP, Class. Quantum Grav. **23** (2006) 2963, "A simple model of big-crunch / big-bang transition"
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Propagation of a string

Dynamics of a string winding around the θ-dimension in its lowest energy mode:
The string in such a state is defined by the conditions

$$\sigma^p := \theta \equiv \Theta \qquad \text{and} \qquad \partial_\theta X^\mu = 0 = \partial_\theta \Pi_\mu. \tag{8}$$

In the mode (8) the constraints read

$$C = \Pi_\mu(\tau)\,\Pi_\nu(\tau)\,\eta^{\mu\nu} + \check{\mu}_1^2\, t^2(\tau) \approx 0, \qquad C_1 = 0, \tag{9}$$

where $\check{\mu}_1 \equiv \theta_0 \mu_1$, and where $\theta_0 = 2\pi$ for $\mathbb{S}^1$ and $\theta_0 = \pi$ for $\mathbb{S}^1/\mathbb{Z}_2$ compactifications, respectively.
The equations of motion are

$$\dot{\Pi}_t(\tau) = -2A(\tau)\,\check{\mu}_1^2\, T(\tau), \qquad \dot{\Pi}_k(\tau) = 0, \tag{10}$$

$$\dot{T}(\tau) = -2A(\tau)\,\Pi_t(\tau), \qquad \dot{X}^k(\tau) = 2A(\tau)\,\Pi_k(\tau), \tag{11}$$

where $A = A(\tau)$ is any function.

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Propagation of string (cont)

In the gauge $A(\tau) = 1$, the solutions are

$$\Pi_t(\tau) = b_1 \exp(2\check{\mu}_1 \tau) + b_2 \exp(-2\check{\mu}_1 \tau), \qquad \Pi_k(\tau) = \Pi_{0k}, \tag{12}$$

$$T(\tau) = a_1 \exp(2\check{\mu}_1 \tau) + a_2 \exp(-2\check{\mu}_1 \tau), \qquad X^k(\tau) = X_0^k + 2\Pi_{0k}\,\tau, \tag{13}$$

where $b_1, b_2, \Pi_{0k}, a_1, a_2, X_0^k \in \mathbb{R}$.
Elimination of τ leads finally to

$$X^k(t) = X_0^k + \frac{\Pi_0^k}{\check{\mu}_1} \sinh^{-1}\Big(\frac{\check{\mu}_1}{\sqrt{\Pi_0^k \Pi_{0k}}}\, t\Big). \tag{14}$$

where $t(\tau) \equiv T(\tau)$ plays the role of an evolution parameter.
The solution (14) is smooth at $t = 0$, and describes stable propagation of a string across the cosmic singularity.

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Quantum string in the winding mode

In the gauge $A = 1$, the Hamiltonian of a string is

$$H_T = C = \Pi_\mu(\tau)\, \Pi_\nu(\tau)\, \eta^{\mu\nu} + \breve{\mu}_1^2\, t^2. \tag{15}$$

The quantum Hamiltonian corresponding to (15) has the form (we use the Laplace-Beltrami mapping)

$$\hat{H}_T = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial X^k \partial X_k} + \breve{\mu}_1^2 t^2, \qquad t \equiv T. \tag{16}$$

According to Dirac's quantization method physical states ψ should satisfy the equation

$$\hat{H}_T\, \psi(t, X^k) = 0. \tag{17}$$

Eq. (17) has the form of the Klein-Gordon equation. Due to this analogy we interpret t as an evolution parameter in our quantum description.

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Quantum string (cont)

To solve (17) we make the substitution

$$\psi(t, X^1, \ldots, X^{d-1}) = F(t)\, G_1(X^1)\, G_2(X^2) \cdots G_{d-1}(X^{d-1}), \tag{18}$$

which turns (17) into the following set of equations

$$\frac{d^2 G_k(q_k, X_k)}{dX_k^2} + q_k^2\, G_k(q_k, X_k) = 0, \qquad k = 1, \ldots, d-1, \tag{19}$$

$$\frac{d^2 F(q,t)}{dt^2} + (\breve{\mu}_1^2 t^2 + q^2)\, F(q,t) = 0, \qquad q^2 := q_1^2 + \ldots + q_{d-1}^2, \tag{20}$$

where $q_k^2, q^2 \in \mathbb{R}$ are the separation constants.

Quantum string (cont)

Two independent solutions to (19) have the form

$$G_{1k}(q_k, X_k) = \cos(q_k X^k), \qquad G_{2k}(q_k, X_k) = \sin(q_k X^k) \qquad (21)$$

(no summation in $q_k X^k$ with respect to k).

Two independent solutions of (20) read

$$\tilde{F}_1(q,t) = \exp\left(-i\breve{\mu}_1 t^2/2\right) H\Big(-\frac{\breve{\mu}_1 + iq^2}{2\breve{\mu}_1}, (-1)^{1/4}\sqrt{\breve{\mu}_1}\; t\Big), \qquad (22)$$

$$F_2(q,t) = \exp\left(-i\breve{\mu}_1 t^2/2\right) {}_1F_1\Big(\frac{\breve{\mu}_1 + iq^2}{4\breve{\mu}_1}, \frac{1}{2}, i\breve{\mu}_1 t^2\Big), \qquad (23)$$

where $H(a,t)$ is the Hermite function and ${}_1F_1(a,b,t)$ denotes the Kummer function.

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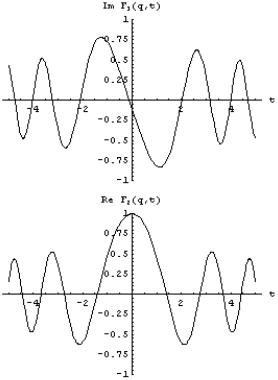
Quantum string (cont)

Construction of the Hilbert space, $\mathcal{H}$, based on the solutions (21)-(23):

Step 1 The method works if the solutions are bounded functions on $\mathbb{R} \times [-t_0, t_0]$. The function $F_2(q,t)$ is bounded, whereas $\tilde{F}_1(q,t)$ blows up as $|q| \to \infty$. Replacement:

$$F_1(q,t) := \sqrt{q}\ \exp\left(-\frac{\pi}{8\check{\mu}_1}q^2\right)\ \tilde{F}_1(q,t). \tag{24}$$



Example of two independent bounded solutions to Eq.(20), for $q = 1$, on $[-t_0, t_0]$.

Quantum string (cont)

Step 2 We introduce generalized solutions by

$$h_s(t,\vec{X}) := \int_{\mathbb{R}^{d-1}} f(q_1,\ldots,q_{d-1})\; F_s(q,t) \prod_k \exp(-iq_kX^k)\; dq_1 \ldots dq_{d-1}, \tag{25}$$

where $f \in L^2(\mathbb{R}^{d-1}), \quad s = 1,2$
and where $q^2 = q_1^2 + \ldots q_{d-1}^2, \quad (\vec{X}) \equiv (X^1,\ldots,X^{d-1})$.
Eq.(25) includes (21) due to the term $\exp(-iq_kX^k)$, with $q_k \in \mathbb{R}$.
One has $\hat{H}_T h_s = 0$.

Step 3 Eq.(25) defines the Fourier transform of the product $f\; F_s$.
Thus, due to the Fourier transform theory it defines the mapping

$$L^2(\mathbb{R}^{d-1}) \ni f \longrightarrow h_s \in L^2([-t_0,t_0]\times\mathbb{R}^{d-1}). \tag{26}$$

Replacing f by consecutive elements of a basis in $L^2(\mathbb{R}^{d-1})$ creates, roughly speaking, a basis in the Hilbert space $\mathcal{H} \subseteq L^2([-t_0,t_0]\times\mathbb{R}^{d-1})$.

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Example: $L^2(\mathbb{R}^{d-1}) := \bigotimes_{k=1}^{d-1} L_k^2(\mathbb{R})$, where $L_k^2(\mathbb{R}) \equiv L^2(\mathbb{R})$, with the basis $f_n \in L^2(\mathbb{R})$ defined as

$$f_n(q) := \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} \exp(-q^2/2)\, H_n(q), \qquad n = 0, 1, 2, \ldots, \tag{27}$$

where $H_n(q)$ is the Hermite polynomial.

The orthonormal basis (27) can be used to define a sequence of vectors $\bigotimes_{k=1}^{d-1} f_{n_k}(q^k) \in L^2(\mathbb{R}^{d-1})$, and further used to create a sequence of vectors in $\mathcal{H} = L^2([-t_0, t_0] \times \mathbb{R}^{d-1})$, owing to (26). Obtained set of vectors can be used to build another set of independent vectors by a standard method, and turned into an orthonormal basis by making use of the Gram-Schmidt procedure. Completion of the span of such an orthonormal basis defines the Hilbert space $\mathcal{H} \subseteq L^2([-t_0, t_0] \times \mathbb{R}^{d-1})$. For more details: P. Małkiewicz and W. P., Class. Quantum Grav. **24** (2007) 915, 'Propagation of a string across the cosmological singularity'

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Classical dynamics of a membrane

The physical phase space of a membrane (in the zero-mode, winding around the θ-dimension) is defined by the constraints

$$C = \Pi_\mu(\tau,\sigma)\,\Pi_\nu(\tau,\sigma)\,\eta^{\mu\nu} + \kappa^2\, t^2(\tau,\sigma)\acute{X}^\mu(\tau,\sigma)\acute{X}^\nu(\tau,\sigma)\,\eta_{\mu\nu} \approx 0, \quad (28)$$

$$C_1 = \acute{X}^\mu(\tau,\sigma)\,\Pi_\mu(\tau,\sigma) \approx 0, \qquad C_2 = 0, \quad (29)$$

where $\acute{X}^\mu := \partial X^\mu/\partial\sigma$, $\sigma \equiv \sigma^1$, and where $\kappa \equiv \pi\mu_2$. For some states of a membrane the expressions for C and C_1 are well defined[3].

To examine the algebra of constraints we 'smear' the constraints as follows

$$\check{A} := \int_0^\pi d\sigma\, f(\sigma)A(\tau,\sigma), \quad f \in C_0^\infty[0,\pi]. \quad (30)$$

The Lie bracket is defined as

$$\{\check{A},\check{B}\} := \int_0^\pi d\sigma\, \left(\frac{\partial\check{A}}{\partial X^\mu}\frac{\partial\check{B}}{\partial\Pi_\mu} - \frac{\partial\check{A}}{\partial\Pi_\mu}\frac{\partial\check{B}}{\partial X^\mu}\right) \quad (31)$$

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Classical dynamics of a membrane (cont)

Constraints in an integral form satisfy the algebra

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Quantization of the dynamics of a membrane means finding an essentially self-adjoint representation of this algebra on a dense subspace of a Hilbert space.

However, the 'structure constant', t^2, is not a constant, but a function on the phase space.

Little is known about representations of such type of an algebra!

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Summary

- classical dynamics of a particle can be quantized despite the fact that it is unstable
- dynamics of a string in the zero-mode of winding string is well defined both at classical and quantum levels
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Next steps

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  - non-zero modes of the winding string
  - possible modification of the singularity by a string
- quantization of dynamics of a membrane
- obtaining classical phase from quantum phase and vice versa
- quantization of CM space (by making use of LQG methods): big-crunch / big-bang (change of spacetime dimension) or big-bounce (no change of dimensionality of spacetime), or Big-Crunch (destruction of spacetime)
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