Inflation after WMAP

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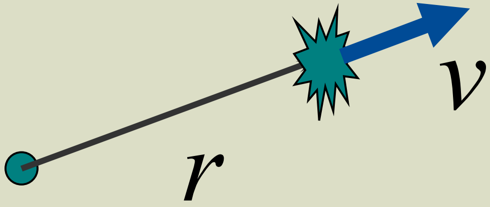
"...our mistake is not that we take our theories too seriously, but that we do not take them seriously enough. It is always hard to realize that these numbers and equations we play with at our desks have something to do with the real world..."

S. Weinberg, "The first three minutes"

- Hubble law



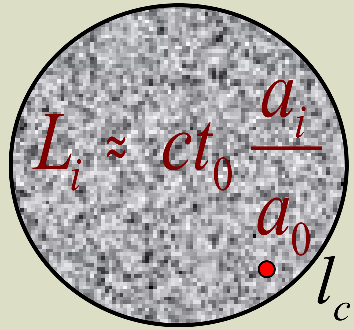
$$r = a(t)\chi_{com}$$

$$v = \dot{a}\chi_{com}$$

rate of expansion

Matter Distribution

"initial" moment of time $t_i = 10^{-43}$ sec



$l_c = c t_i \sim 10^{-33}$ cm

initial rate of expansion

$$\frac{L_i}{c t_i} \Rightarrow \approx \frac{\dot a_i}{\dot a_0}$$

current rate of expansion

In 10^{90} causally disconnected regions

Gravity is attractive force $\Rightarrow$ $\dot a_i \delta\varepsilon \dot a_0 \varepsilon$! Ł 10^{-5} !!!

In radiation dominated Universe $\dot a_i / \dot a_0 \approx 10^{30}$

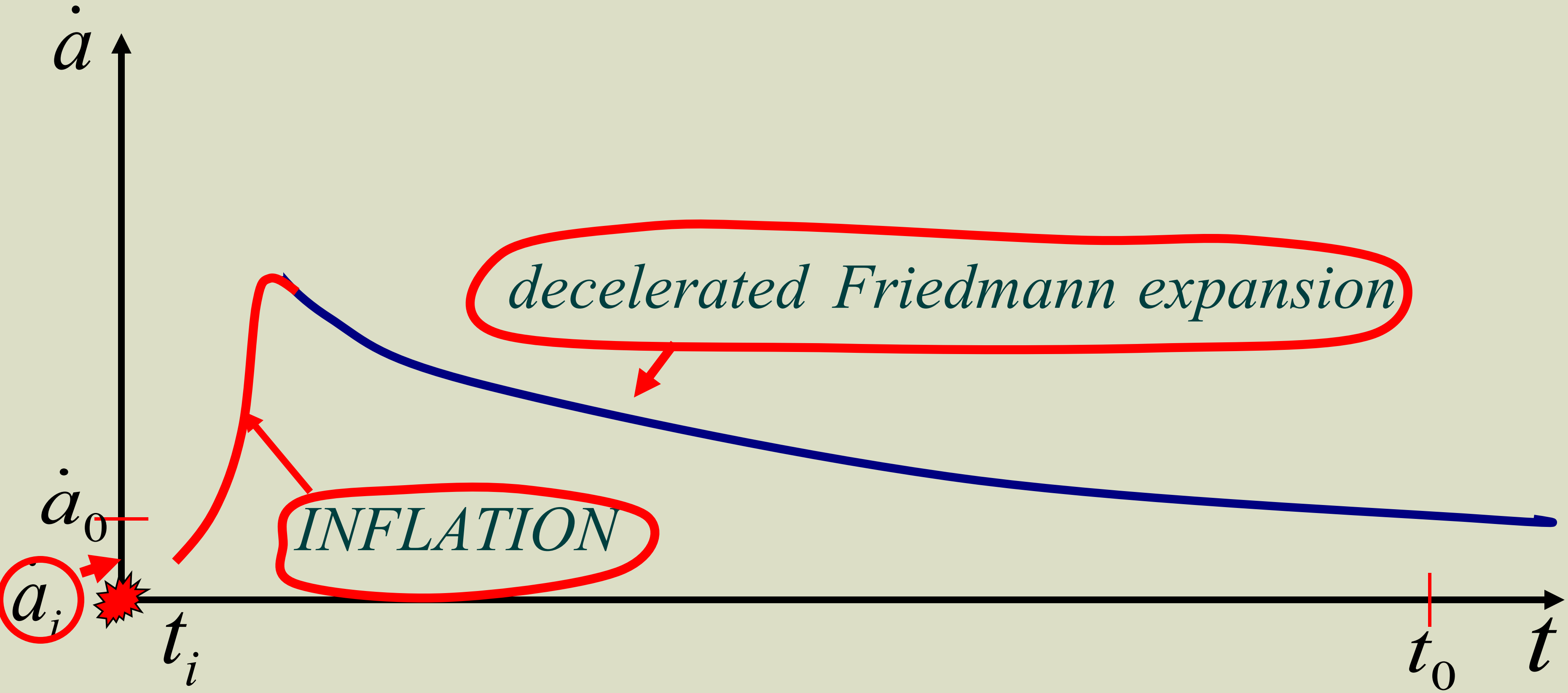
Initial velocities

- At "initial moment" ($t_i \approx 10^{-43}$ sec)

$$\frac{\left|E_i^{kin} + E_i^{pot}\right|}{E_i^{kin}} \sim \ldots \left(\frac{\dot a_0}{\dot a_i}\right)^2 \sim \ldots 10^{-60} \quad !!!$$

- For a given matter distribution error $\sim 10^{-58}\,\%$ in "initial velocities" would lead to failure in creation of "our-type" Universe

Initial conditions were VERY SPECIAL (nongeneric)!?



Necessary conditions for successful inflation:

- $\dot{a}_i << \dot{a}_0 \Rightarrow \Omega_0 \equiv \frac{|E_0^{pot}|}{E_0^{kin}} = 1$

Prediction of inflation!

- Transition from inflation to Friedmann era should be "smooth"

- How gravity can become "repulsive"?

$$\ddot{a} = -\frac{4\pi G}{3}(\varepsilon + 3p)\,a$$

acceleration

energy density

pressure

Only if $\varepsilon + 3p < 0 \Rightarrow \ddot{a} > 0 \Rightarrow$ "antigravity"

Scenarios

???

Energy density ε, pressure p

$p(\varepsilon)$ - equation of state

$p + \varepsilon = \varepsilon$ for inflation

$$p \approx -\varepsilon$$

Which concrete scenario was realized ???

- Main bonus from inflation-generation of primordial spectrum of inhomogeneities
- Inflation "washes away" all pre-inflationary inhomogeneities

However, in all scales there always remain ineveitable quantum fluctuations

Example: Quantum metric fluctuations in Minkowskii space

$$h_\lambda \approx \frac{l_{Pl}}{\lambda} \approx \frac{10^{-33} cm}{\lambda}$$

Today in galactic scales $h : 10^{-58}$

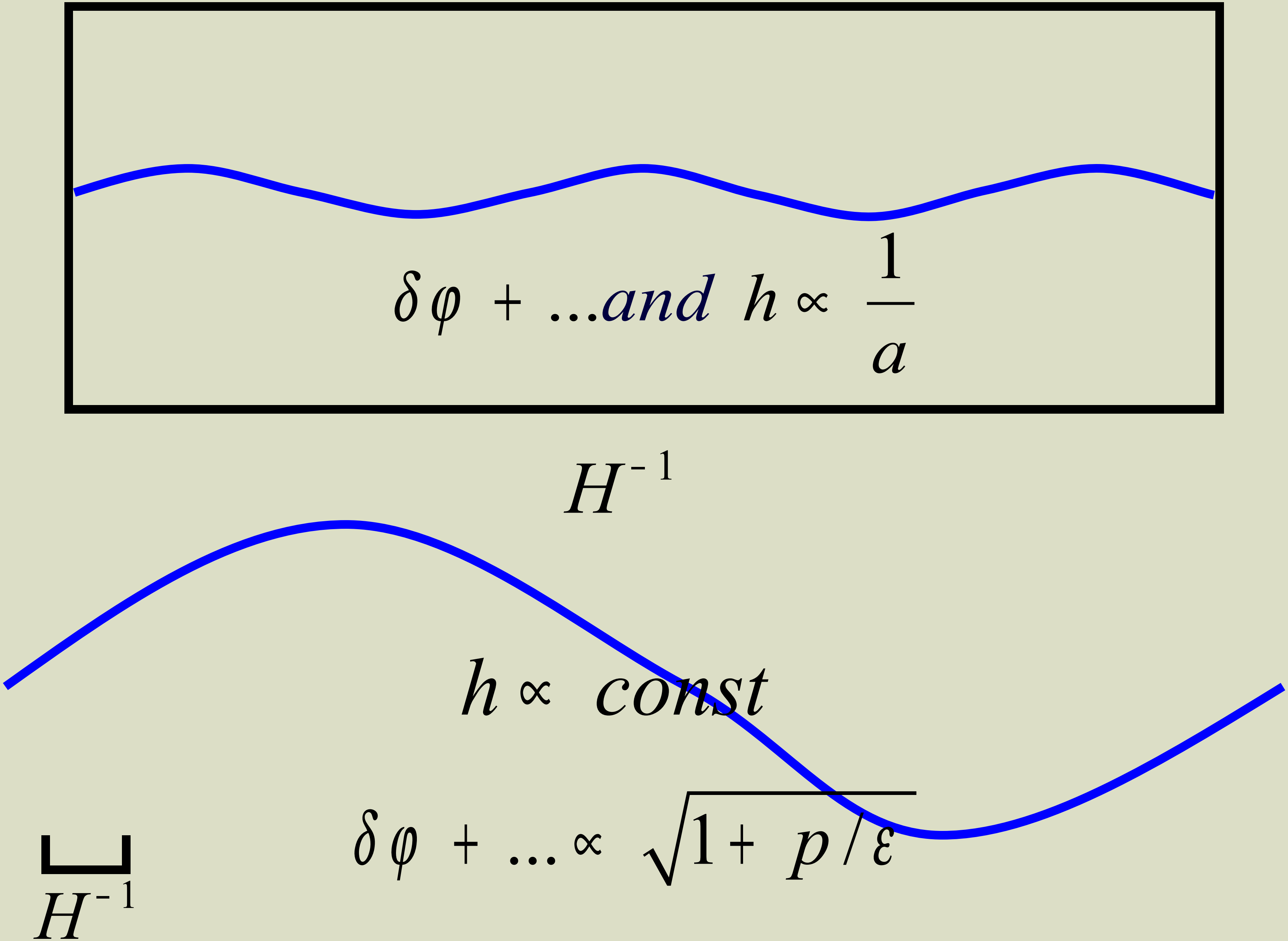
Can quantum fluctuations be amplified up to "needed" value 10^{-5} in expandng Universe???

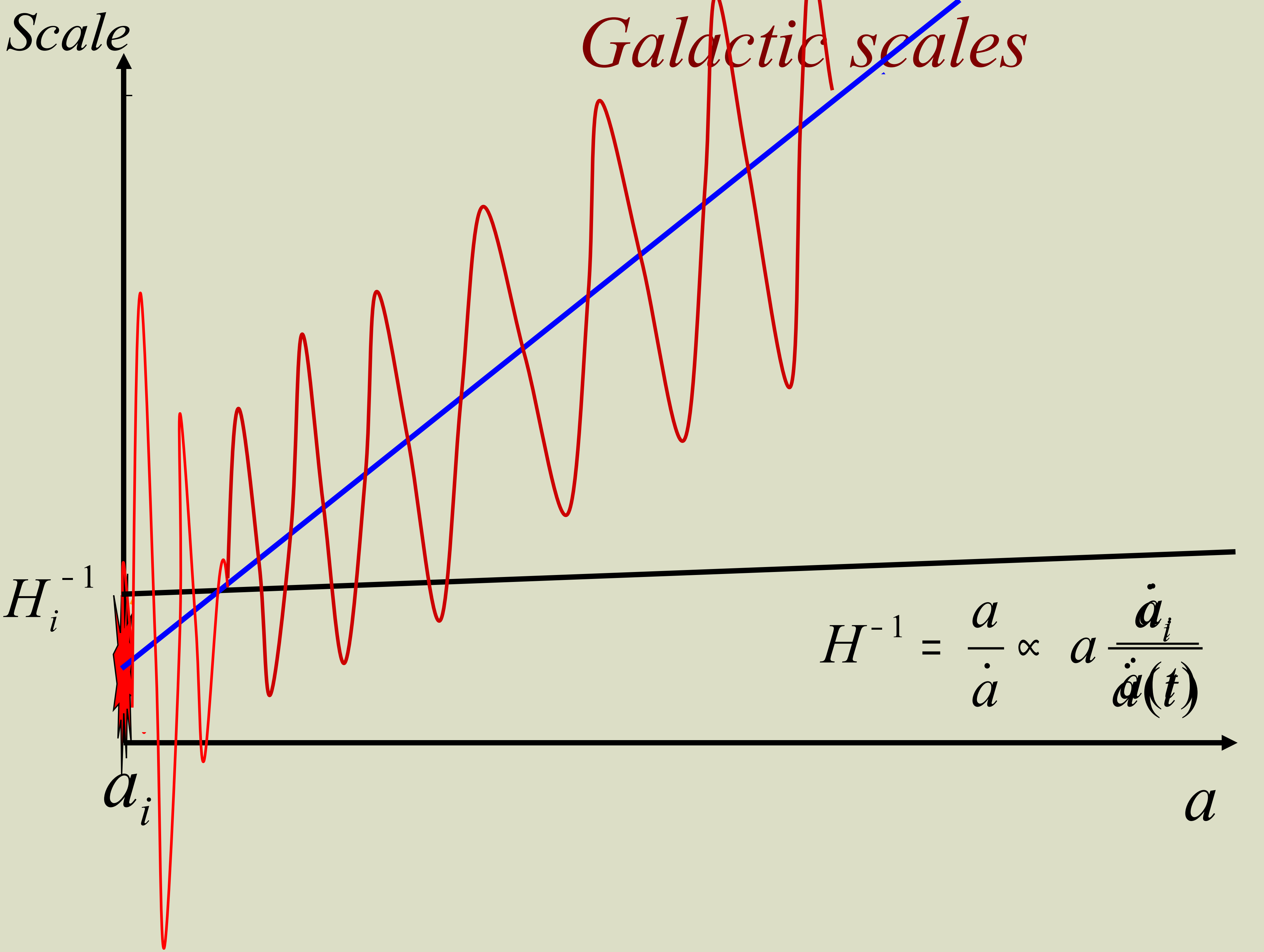
Quantum metric fluctuations are big enough (10^{-5}) only in the scales close to the Planckian scale ($10^{-33}cm$)

Purpose : Transfer these fluctuations to galactic scales ($10^{28}cm$)

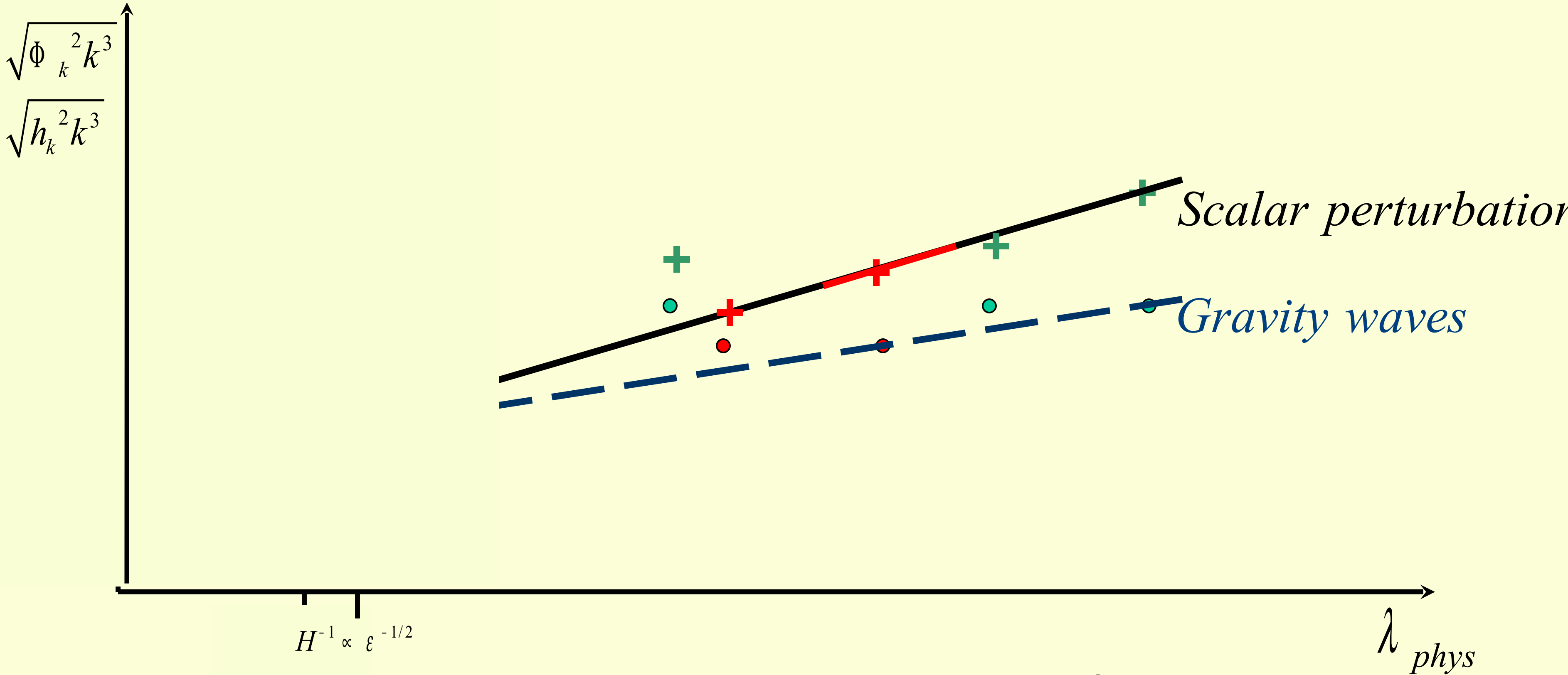
- Consider plane wave perturbation: $\delta\varphi\ ,\Phi\ \propto\ \exp\left(ik_{com}x\right)$

For given k_{com}, $\lambda_{ph}(cm) \propto\ a/k_{com} \propto\ a(t)$ and the change of the amplitude with time depends on how big is λ_{phys} compared to the curvature scale (size of Einstein lift) $H^{-1} = a/\dot{a}$


$\delta\varphi + \ldots and\ h \propto \frac{1}{a}$
H^{-1}
$h \propto const$
$\delta\varphi + \ldots \propto \sqrt{1 + p/\varepsilon}$
H^{-1}



Scale
Galactic scales
H_i^{-1}
$H^{-1} = \frac{a}{\dot{a}} \propto a \frac{\dot{a}_i}{\dot{a}(t)}$
a_i
a



$$0.92 < 0.97$$

$$n_s - 1 = O\left(\frac{p}{\varepsilon}\right)_{k\approx Ha} + \frac{1}{3}\left(\frac{d \ln(1+p/\varepsilon)}{d \ln a}\right)_{k\approx Ha} + O\left(\frac{p}{\varepsilon}\right)_{k\approx Ha}$$

$$\frac{T}{S} = O(1)\left(1+\frac{p}{\varepsilon}\right)^{1/2}_{k\approx Ha}$$

Summary

Input from HEP

???

Idea and basic properties of inflation are established:

Inflation is the stage of accelerated expansion of the universe with graceful exit to Friedmann stage

SCENARIO

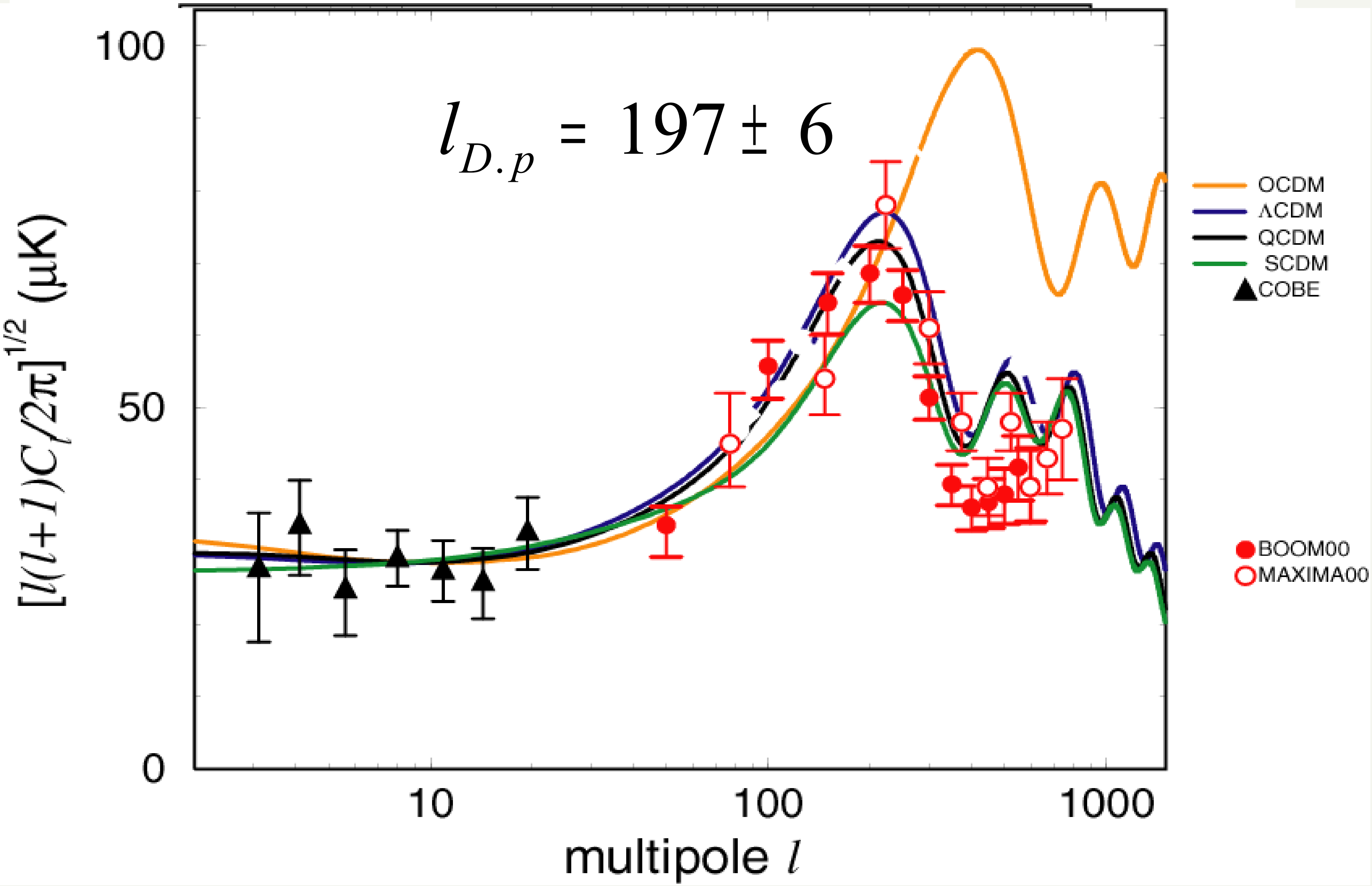
???

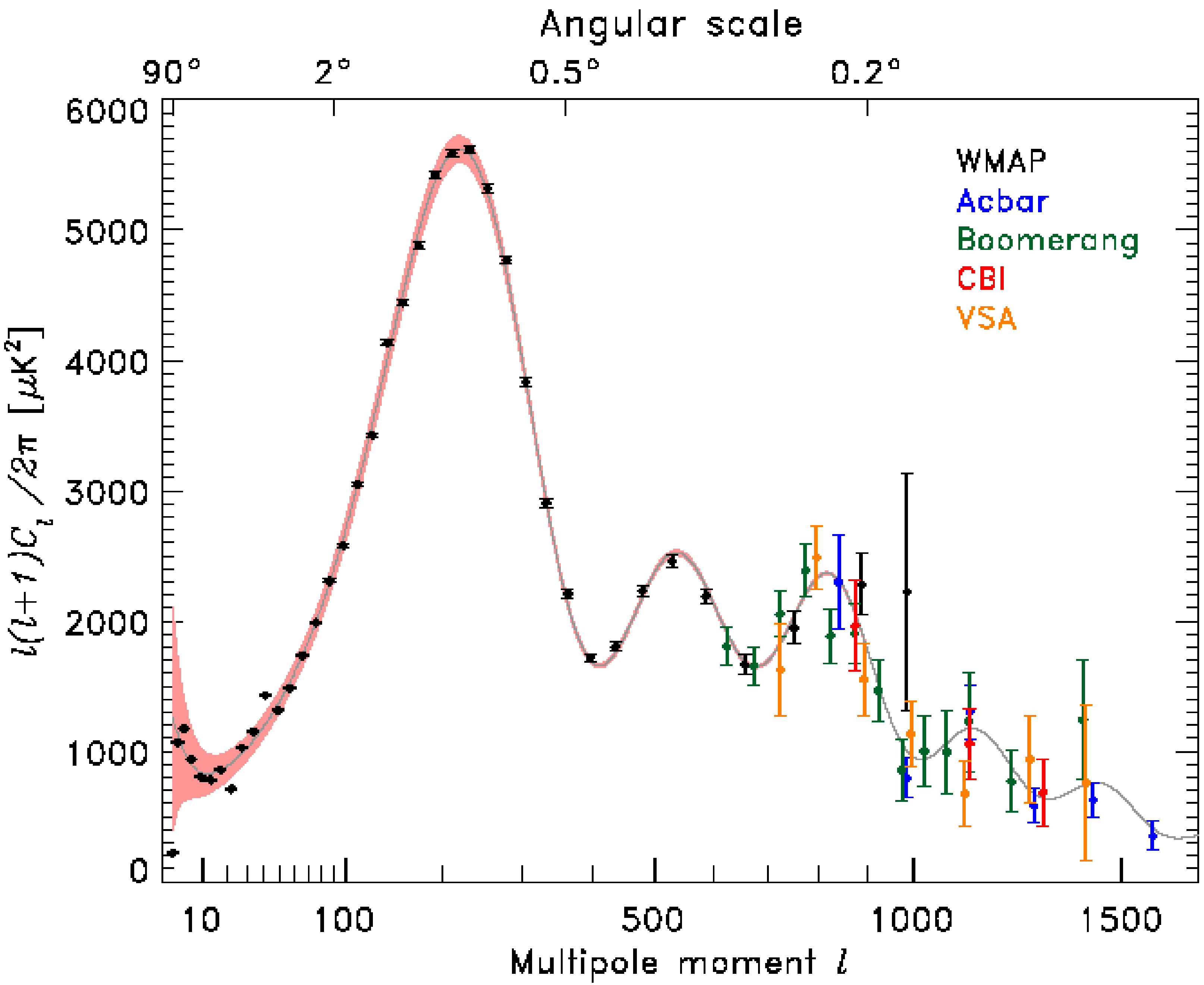
???

- Robust predictions:

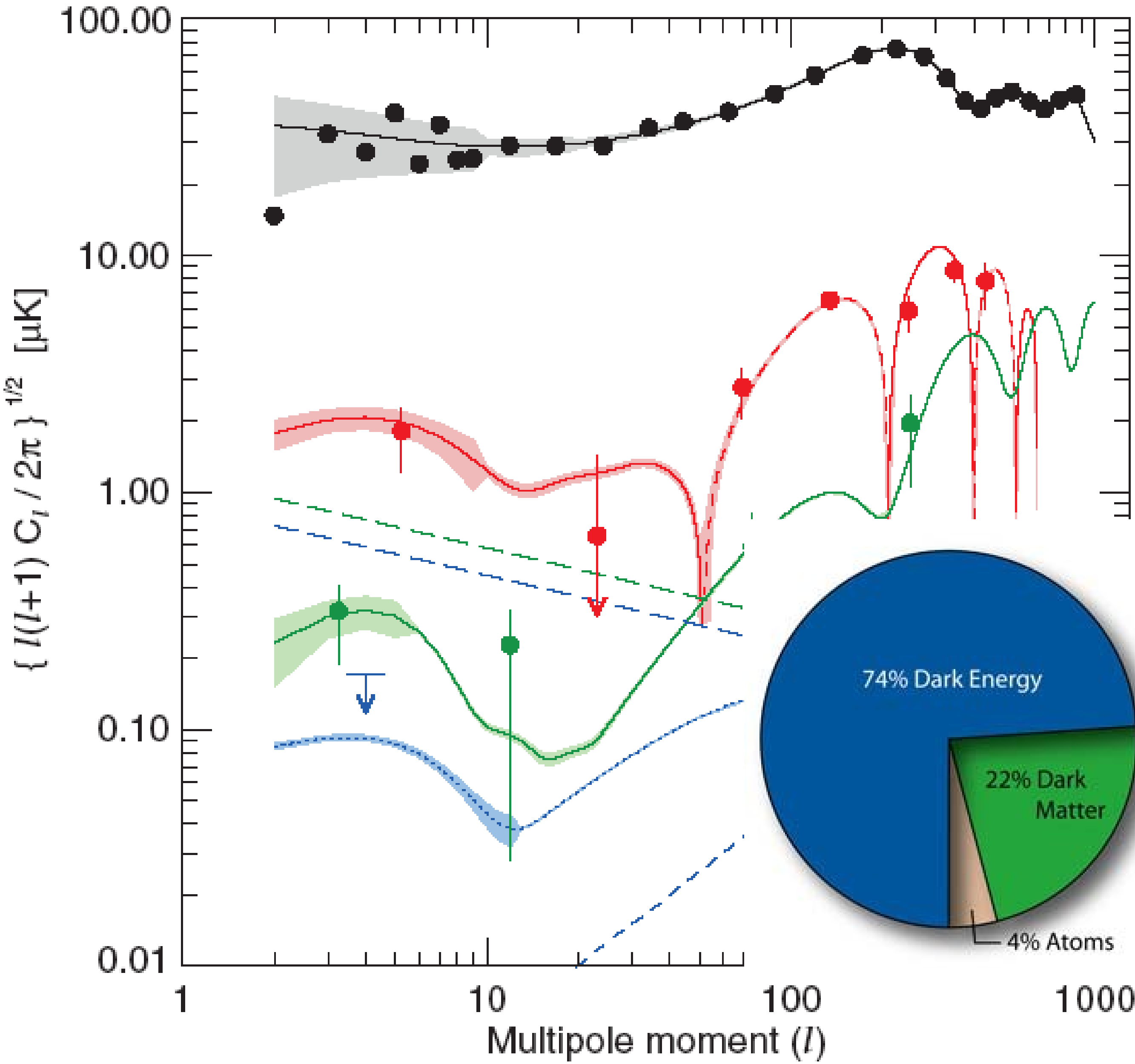
- Spatially flat Universe: $\Omega_{total} = 1 \pm 10^{-5}$
- Sligtly red-tilted spectrum of scalar perturbations $(0,92 < n_S < 0,97)$
- Perturbations are Gaussian
- Gravity waves

- Energy scale of inflation → prediction of the perturbations amplitude, concrete n_s
- Transition from inflation to Friedmann, reheating mechanism
- The origin of small number 10^{-5} characterizing perturbations


$l_{D.p} = 197 \pm 6$
100
50
0
10
100
1000
multipole l
$[l(l+1)C_l/2\pi]^{1/2}$ (μK)
OCDM
ΛCDM
QCDM
SCDM
COBE
BOOM00
MAXIMA00



Angular scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
l(l+1)C_l /2π [μK²]
WMAP
Acbar
Boomerang
CBI
VSA
10
100
500
1000
1500
Multipole moment l



100.00
10.00
1.00
0.10
0.01
{ l(l+1) C_l / 2π } 1/2 [μK]
1
10
100
1000
Multipole moment (l)
74% Dark Energy
22% Dark Matter
4% Atoms

"A finite duration of the de Sitter stage does not by itself rule out the possibility that this stage may exist as an intermediate stage in the evolution of the universe. An interesting question arises here: Might not perturbations of the metric , which would be sufficient for the formation of galaxies and galactic clusters, arise in this stage?..........

$$Q(k) \approx 3lM\left(1+\frac{1}{2}\ln\frac{H}{k}\right)$$

The fluctuation spectrum is. $n_S = 0.96$ y flat...."

(Mukhanov, Chibiov,1981)

$n_S = 0.951$

In terms of my own money, I'd bet a lot (many thousands)

"In models with the initial superdense de Sitter state ... such a large amount of relic gravitational waves is generated
...that ... the very existence of this state can be experimentally"
verified in the near future.
(Starobinsky, 1980)