From Quark-Gluon Plasma to the Perfect Liquid (II)

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Part II

Results from RHIC
The RHIC Facility

...or what a good deal of Money and Planning Can Buy!
Main RHIC results

Important results from RHIC:

- Chemical (flavor) and thermal equilibration
- Elliptic flow = early thermalization, low viscosity
- Collective flow pattern related to valence quarks
- Jet quenching = parton energy loss, high opacity
- Strong energy loss of $c$ and $b$ quarks
- Charmonium suppression not strongly increased compared with lower (CERN) energies
- Photons unaffected by medium at high $p_T$
Collision Geometry: Elliptic Flow

Elliptic flow ($v_2$):
- Gradients of almond-shape surface will lead to preferential expansion in the reaction plane
- Anisotropy of emission is quantified by $2^{nd}$ Fourier coefficient of angular distribution: $v_2$
  - prediction of fluid dynamics

Bulk evolution described by relativistic fluid dynamics,
- F.D assumes that the medium is in local thermal equilibrium,
- but no details of how equilibrium was reached.
- Input: $\varepsilon(x,\tau_i)$, $P(\varepsilon)$, ($\eta$, etc.).
Elliptic flow is created early

Model calculations suggest that flow anisotropies are generated at the earliest stages of the expansion, on a **time scale of ~ 5 fm/c** if a QGP EoS is assumed.

$v_2(p_T)$ vs. hydrodynamics

Hydro model
- $\pi$
- $K$
- $p$
- $\Lambda$

PHENIX Data
- $\pi^+ + \pi^-$
- $K^+ + K^-$
- $p + p$

STAR Data
- $K_S^0$
- $\Lambda + \Lambda$

Failure of ideal hydrodynamics - tells us how hadrons form

Mass splitting characteristic property of hydrodynamics
In the recombination regime, meson and baryon $v_2$ can be obtained from the quark $v_2$:

$$v_2^M(p_t) = 2v_2^q \frac{\hat{c} p_t}{\hat{c}} \frac{\bar{\hat{c}}}{2}$$

$$v_2^B(p_t) = 3v_2^q \frac{\hat{c} p_t}{\hat{c}} \frac{\bar{\hat{c}}}{3}$$

Emitting medium is composed of unconfined, flowing quarks.
Photons versus hadrons

Suppression of hadrons

No suppression for photons
Radiative energy loss:

\[ \frac{dE}{dx} : \rho \Lambda \left\langle k_T^2 \right\rangle \]

Scattering centers = color charges

Density of scattering centers

\[ \tilde{\phi} = \rho \tilde{\eta} \theta^2 \frac{\delta \sigma}{\delta \theta^2} \left\langle \frac{\theta^2}{k_T^2} \right\rangle \]

Scattering power of the QCD medium:

Range of color force

\[ \frac{\mu^2}{\lambda \phi} \]
How large is q-hat?

- Data are described by a large loss parameter for central collisions:

\[ \langle \phi \rangle \approx 5 - 20 \Gamma \varepsilon^2 / \phi \mu \]

Loizides, hep-ph/0608133

\[ \langle \phi \rangle \approx 1 - 2 \Gamma \varepsilon^2 / \phi \mu \]

Zhang, Owens, Wang & Wang, nucl-th/0701045

Larger than expected from perturbation theory?
Part III

Toward the

“Perfect” Liquid
Ideal gas vs. perfect liquid

- An **ideal gas** is characterized by interactions strong enough to reach thermal equilibrium (on a reasonable time scale), but weak enough to neglect their effect on $P(n,T)$.
  - This ideal can be approached arbitrarily well by diluting the gas and waiting very patiently ($\lim t \to \infty$ first, then $n \to 0$).

- A **perfect fluid** is one that obeys the Euler equations, i.e. a fluid that has negligible viscosity and infinite thermal conductivity (relative to gradients).
  - There is no presumption with regard to the equation of state.
What is viscosity?

Shear and bulk viscosity are defined as coefficients in the expansion of the stress tensor in gradients of the velocity field:

\[ T_{ik} = e u_i u_k + P (d_{ik} + u_i u_k) - h (\dot{N}_i u_k + \dot{N}_k u_i - \frac{2}{3} d_{ik} \dot{N} \times u) + V d_{ik} \dot{N} \times u \]

Microscopically, \( \eta \) is given by the rate of momentum transport:

\[ \eta \approx \frac{1}{3} n p \lambda_f = \frac{p}{3 \sigma_{tr}} \]

Unitarity limit on cross sections suggests that \( \eta \) has a lower bound:

\[ \sigma_{tr} \ll \frac{4\pi}{p^2} \quad \text{T} \quad \eta \gg \frac{p^3}{12\pi} \]
Viscosity of materials

Temperature dependence of the shear viscosity of typical fluids:
Lower bound on $\eta/s$?

A heuristic argument for $(\eta/s)_\text{min}$ is obtained using $s \approx 4n$:

$$h \gg \frac{1}{3} n \left( \frac{pV}{cV} \right) \frac{c}{c} \frac{1}{12} s \frac{c}{c} \frac{1}{n} f$$

The uncertainty relation dictates that $\tau_f (\varepsilon/n) \geq \Box$, and thus:

$$\eta \frac{h}{12} s \approx \frac{h}{4\pi} s$$

All known materials obey this condition!

For $N=4 \ SU(N_c) \ SYM$ theory the bound is saturated at strong coupling:

$$h = \frac{s}{4p} \frac{c}{c} \left( 8g^2 N_c \right)^{3/2} + \ldots$$
Bounds on $\eta$ from $v_2$

Relativistic viscous hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with} \quad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \eta (\partial^\mu u^\nu + \partial^\nu u^\mu - \text{trace})$$

Boost invar't hydro requires $\eta/s \approx 0.1$. 

$$\Gamma_s / \tau_0 \approx \eta / s$$

$\eta/s < 0.3$ confirmed by 2-D viscous hydro calculation (Baier & Romatschke).

N=4 SUSY YM theory ($g^2 N_c \ll 1$):

$$\eta/s = 1/4\pi \quad \text{(Policastro, Son, Starinets)}.$$

Absolute lower bound on $\eta/s$?
QGP viscosity – collisions

Classical expression for shear viscosity:

\[ h \gg \frac{1}{3} n \bar{p} l f \]

Collisional mean free path in medium:

\[ l_f^{(C)} = \left( n s_{\text{tr}} \right)^{-1} \]

Transport cross section in QCD medium:

\[ s_{\text{tr}} \gg \frac{5p}{p^2} a_s^2 I(a_s) \]

\[ I(a_s) = (1 + 2a_s) \ln \frac{\xi}{\xi} + \frac{1}{a_s} \frac{\tilde{c}}{\tilde{r}} - 2 \]

Collisional shear viscosity of QGP:

\[ h_c \gg \frac{T}{s_{\text{tr}}} \gg \frac{9s}{100p a_s^2 \ln a_s^{-1}} \]
QCD matter

Low T (*Prakash et al.*)
using experimental data for 2-body interactions.

High T (*Yaffe et al.*)
using perturbative QCD

\[
\frac{\eta}{s} vs. T (\text{MeV})
\]

- --- pions
- ---- pions + kaons
- --- QGP

**RHIC data**

\[\frac{1}{4\pi}\]
Part IV

Anomalous Viscosity
Spontaneous color fields

Correlation

Length

Time

Length (z)

Non-abelian

Quasi-abelian

Noise

M. Strickland, hep-ph/0511212
QGP viscosity – anomalous

Classical expression for shear viscosity:
\[ h \gg \frac{1}{3} \eta \bar{p} l_f \]

Momentum change in one coherent domain:
\[ D_p \gg g Q^a B^a r_m \]

Anomalous mean free path in medium:
\[ l_f^{(A)} \gg r_m \left( \frac{\bar{p}^2}{(D_p)^2} \right) \gg \frac{\bar{p}^2}{g^2 Q^2 \left( B^2 \right) r_m} \]

Anomalous viscosity due to random color fields:
\[ h_A \gg \frac{\eta \bar{p}^3}{3 g^2 Q^2 \left( B^2 \right) r_m} \gg \frac{\frac{9}{4} s T^3}{g^2 Q^2 \left( B^2 \right) r_m} \]
Expansion ⇔ Anisotropy

Perturbed equilibrium distribution:

\[ f(p) = f_0(p) \hat{1} + f_1(p) (1 \pm f_0(p)) \hat{1} \]

\[ f_0(p) = \exp[-u_m p^m / T] \]

For shear flow of ultrarelat. fluid:

\[ f_1(p) = \frac{5h/s}{E_p T^2} \left( p^i p^{i'} - \frac{1}{3} d_{ij} \right) (\dot{N}u)_{ij} \]

\[ (\dot{N}u)_{ij} = \frac{1}{2} \left( \dot{N}_i u_j + \dot{N}_j u_i \right) - \frac{1}{3} d_{ij} \dot{N} \times u \]

Anisotropic momentum distributions generate instabilities of soft field modes. Growth rate \( \Gamma \sim f_1(p) \).

- Shear flow always results in the formation of soft color fields;

- Size controlled by \( f_1(p) \), i.e. \( \dot{1}u \) and \( \eta/s \).
Turbulence ⇒ Diffusion

Vlasov-Boltzmann transport of thermal partons:

\[
\frac{\partial \rho}{\partial t} + \frac{p}{E_p} \times \dot{\mathbf{N}}_r + F \times \dot{\mathbf{N}}_p \hat{\mathbf{u}} f(r, p, t) = C [f] 
\]

with Lorentz force

\[ F = gQ^a (E^a + \nu^a B^a) \]

Assuming \( E, B \) random \( \Upsilon \) Fokker-Planck eq:

\[
\frac{\partial \rho}{\partial t} + \frac{p}{E_p} \times \dot{\mathbf{N}}_r - \dot{\mathbf{N}}_p \times \mathcal{D}(p) \times \dot{\mathbf{N}}_p \hat{\mathbf{u}} f(r, p, t) = C \hat{\mathbf{g}} \hat{\mathbf{f}} \hat{\mathbf{u}} \]

with diffusion coefficient

\[
D_{ij}(p) = \tilde{\mathbf{\mathcal{N}}} dt' \langle F_i (\mathbf{r}(t'), t') F_j (r, t) \rangle.
\]

Diffusion is dominated by chromo-magnetic fields:

\[
\tilde{\mathbf{\mathcal{N}}} dt' \langle B(t')B(t) \rangle \mathfrak{S} \langle B^2 \rangle \tau_m
\]
Shear viscosity

Take moments of

\[
\frac{\partial}{\partial t} + \frac{p}{E_p} \cdot \hat{N}_r - \hat{N}_p \cdot D(p) \cdot \hat{N}_p \int f(r, p, t) = C \left[ \frac{\partial}{\partial t} \hat{N}_r - \hat{N}_p \cdot D(p) \cdot \hat{N}_p \right] f(r, p, t)
\]

with \( p_z^2 \)

\[
D_{ij}(p) = \tilde{\Pi}_{\alpha} \delta_{\alpha \beta} \Phi_{\alpha} \rho_{\alpha} (\rho, \tau) Y_{\phi \phi} (\rho, \tau) \Phi_{\phi} (\rho, \tau)
\]

\[
\frac{1}{h} = O(1) \frac{N_c}{N_c^2 - 1} \frac{\langle F^2 \rangle}{s T^3} + O(10^{-2}) \frac{g^4 \ln g^{-1}}{T^3} \frac{1}{h_A} + \frac{1}{h_C}
\]

\[
\hat{\Pi} dt' \langle F_i^+ (t') F_i^+ (t) \rangle = \langle F^2 \rangle \tau_m \frac{1}{\hat{\Phi}}
\]

= jet quenching parameter !!!
Who wins?

Smallest viscosity dominates in system with several sources of viscosity

Anomalous viscosity

$$\frac{\eta_A}{\sigma} \approx \frac{\hat{\gamma}^2 |\tilde{\nu}|}{\hat{\gamma}^2 |\tilde{\nu}|}^{3/5}$$

Collisional viscosity

$$\frac{\eta_c}{\sigma} \approx \frac{36\pi}{50 \gamma^4 \nu \gamma^{-1}}$$

$$|\tilde{\nu}| : t^{-1} \quad \bigcirc \quad \text{Anomalous viscosity wins out at small } g \text{ and } t$$

Estimate for turbulent color field intensity:

$$\langle F^2 \rangle : t^{-3.1}$$
Part IV

Exploring the "Perfect" Liquid
Connecting jets with the medium

Hard partons probe the medium via the density of colored scattering centers:

\[ \tilde{\phi} = \rho \tilde{\eta} \theta^2 \delta \theta^2 (\delta \sigma / \delta \theta^2) \]

If kinetic theory applies, thermal gluons are quasiparticles that experience the same medium. Then the shear viscosity is:

\[ \eta \approx C r \langle p l_f(p) \rangle = C \left( \frac{p}{s_{tr}(p)} \right) \]

In QCD, small angle scattering dominates:

\[ \sigma_{tr}(p) \gg \frac{4 \tilde{\phi}}{\delta \tau} \]

With \( \langle p^3 \rangle \sim T^3 \) and \( s \approx 4 \rho \) one finds:

\[ \eta_s \approx \frac{5 T^3}{4 \tilde{\phi}} \]

From RHIC

\[ T_0 \approx 335 \text{ MeV}, \quad \dot{\theta}_0 \approx 1 - 2 \Gamma \varepsilon^2 / \phi \mu \quad \rightarrow \quad \frac{T_0^3}{\dot{\theta}_0} \approx 0.12 - 0.24 \]
At strong coupling, \( \frac{\eta}{s} \) is a more faithful measure of medium "blackness".

\[
\frac{\eta}{s} \approx \frac{T^3}{\langle \Phi \rangle} \approx \frac{1}{(f/\pi)^4} (\ln T)^{-4} \approx xG_{\pi}(\xi)_{\xi \to 0}
\]

\[
\approx \frac{5T^3}{4\langle \Phi \rangle} \approx \frac{(4\pi)^{-1}}{\lambda^{-1/2}} \approx xG_{\pi}(\xi)_{\xi \to 0}
\]

\[
\approx \frac{5T^3}{4\langle \Phi \rangle} \approx \frac{(4\pi)^{-1}}{\lambda^{-1/2}} \approx xG_{\pi}(\xi)_{\xi \to 0}
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\]
Fate of the “lost” energy (I)

Perturbative QCD

Lost energy of away-side jet is redistributed to angles away from $180^\circ$ and low transverse momenta $p_T < 2$ GeV/c (Mach cone?).
Fate of the “lost” energy (II)

Near side jet phenomenon:

- Longitudinal broadening of jet cones. “Ridge” contains all additional energy: In-medium fragmentation

Possible explanation: Longitudinal diffusion of radiated gluons in random, transverse color magnetic fields (A. Majumder, ...)

Final distribution of in-medium radiated gluons
The matter created in heavy ion collisions forms a highly color opaque plasma, which has an extremely small shear viscosity. This (nearly) “perfect liquid” behaves like a strongly coupled plasma of quarks and gluons, possibly due to the presence of strong turbulent color fields, especially at early times when the expansion is most rapid.

Penetrating probes, i.e. jets, heavy quarks, photons, are the best way to further probe this medium. The extended dynamic range of RHIC II and LHC, together with detailed theoretical modeling and simulations, will allow us to quantitatively determine its essential transport properties.