From Quark-Gluon Plasma to the Perfect Liquid (II)

Berndt Mueller – *Duke University*

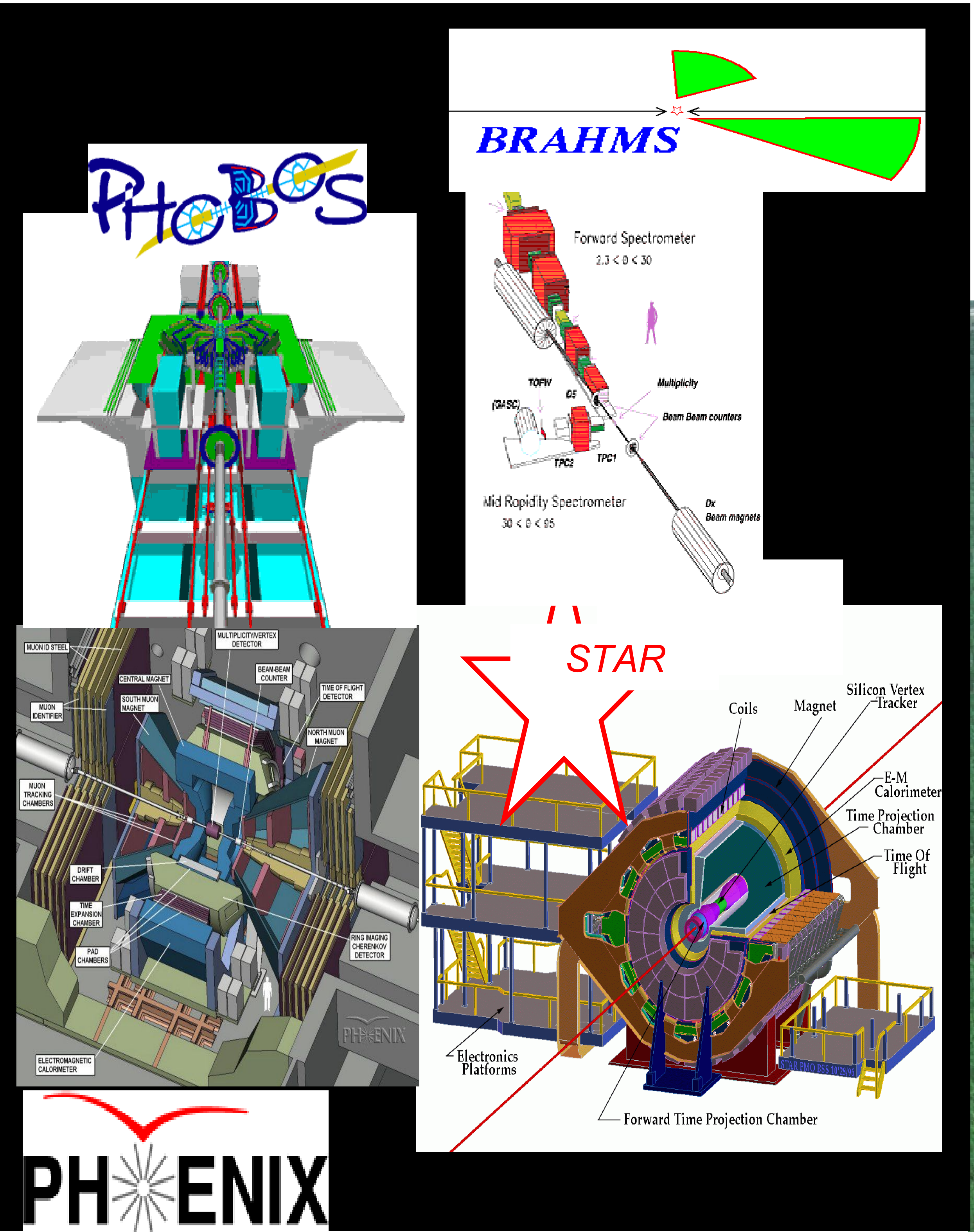
Krakow School of Theoretical Physics

Zakopane, 14-22 June 2007

Part II

Results from RHIC

The RHIC Facility



…or what a good deal of

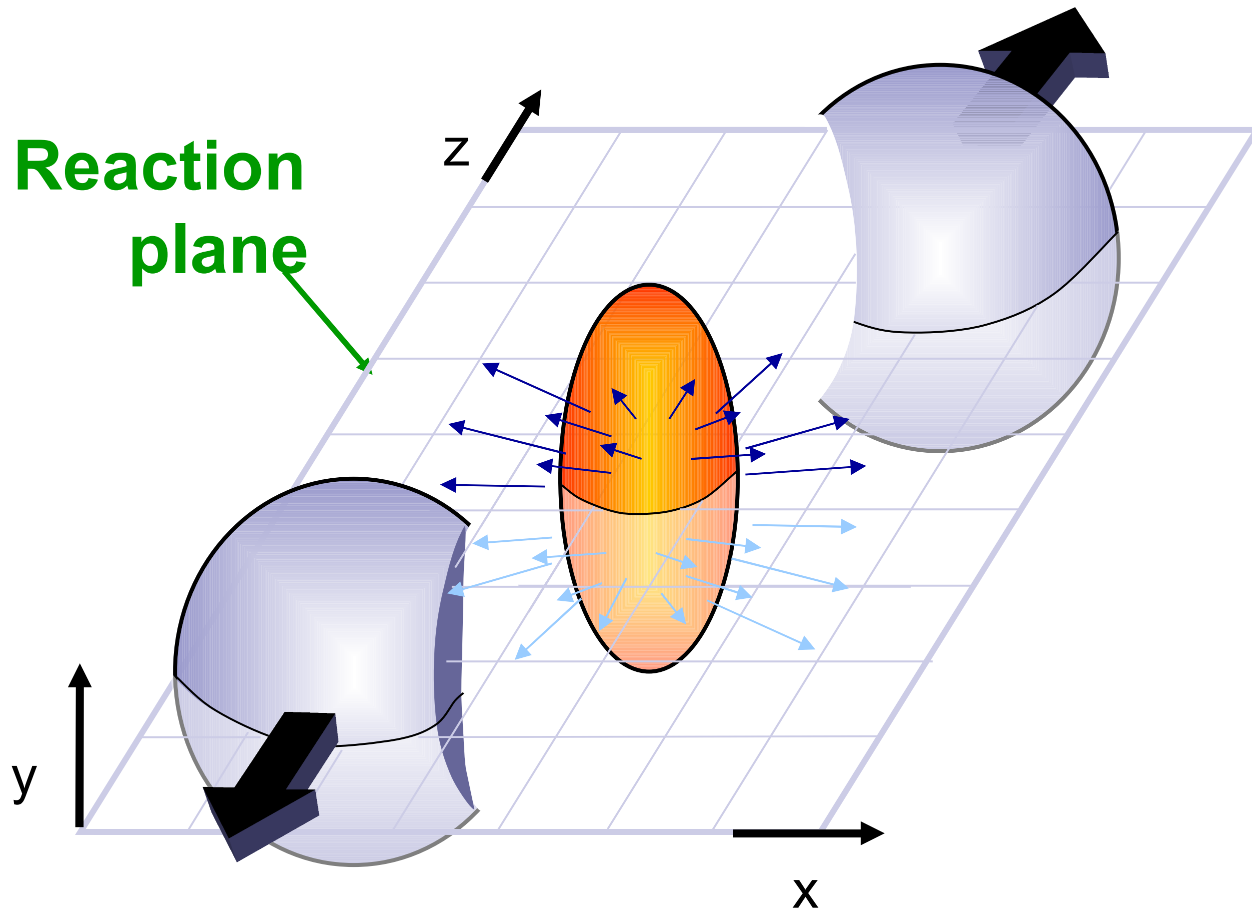
Money and Planning Can Buy!

Main RHIC results

Important results from RHIC:

- Chemical (flavor) and thermal equilibration
- Elliptic flow = early thermalization, low viscosity
- Collective flow pattern related to valence quarks
- Jet quenching = parton energy loss, high opacity
- Strong energy loss of *c* and *b* quarks
- Charmonium suppression not strongly increased compared with lower (CERN) energies
- Photons unaffected by medium at high p_T

Collision Geometry: Elliptic Flow

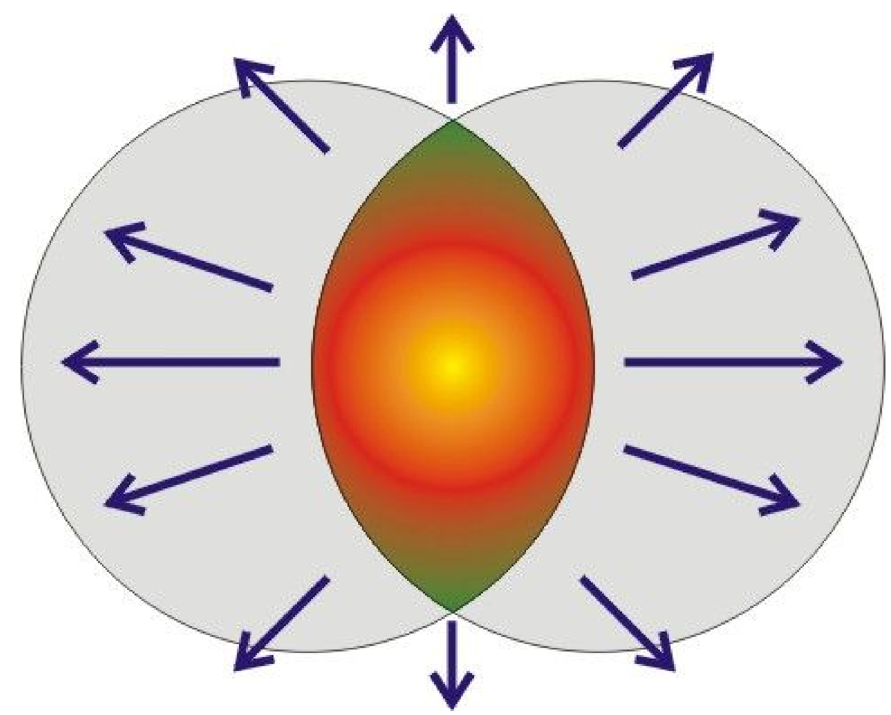


- Bulk evolution described by relativistic fluid dynamics,
- F.D assumes that the medium is in local thermal equilibrium,
- but no details of how equilibrium was reached.
- **Input: $\varepsilon(x,\tau_i)$, $P(\varepsilon)$, (η,etc.).**

Elliptic flow (v_2):

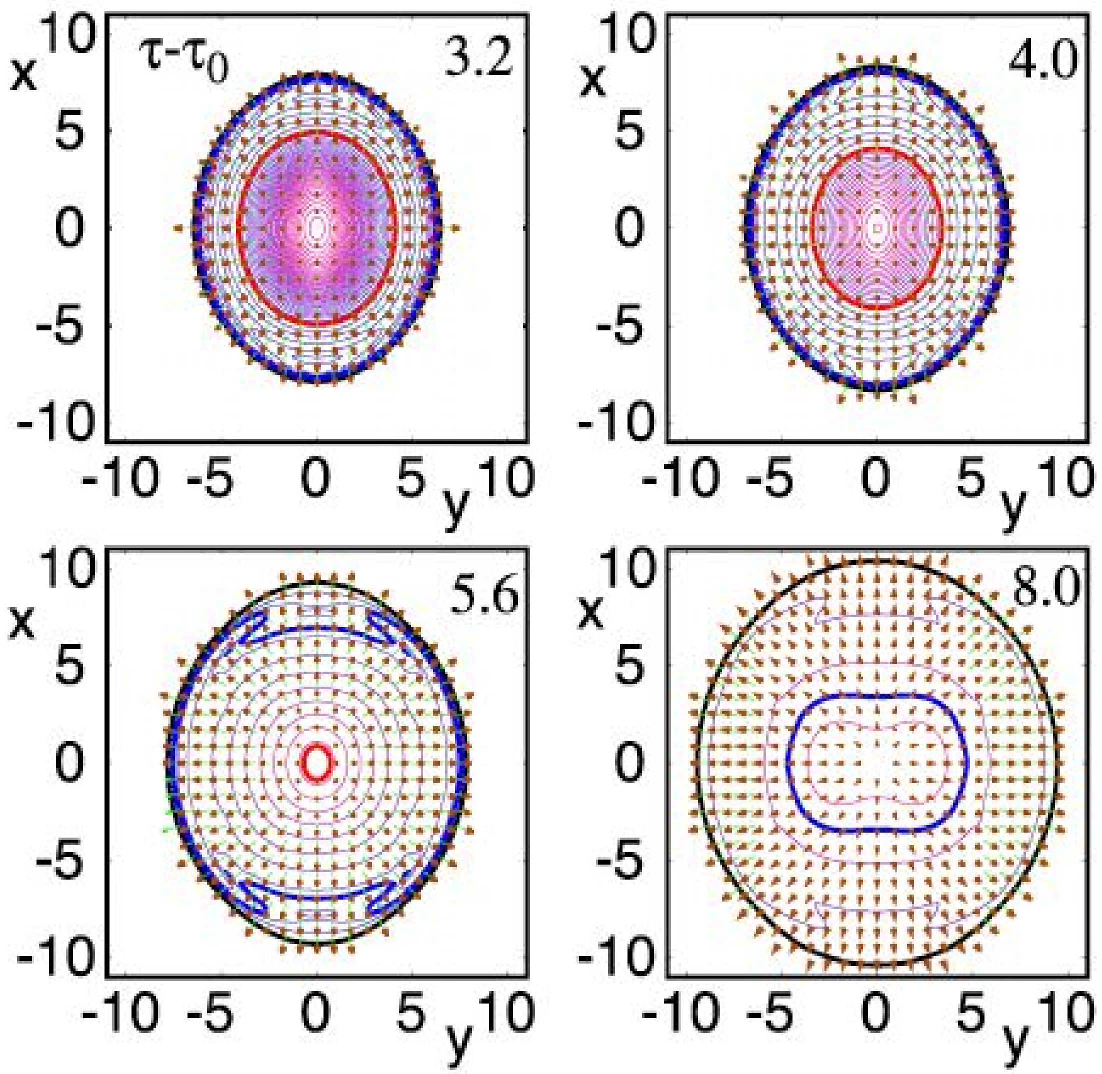
- Gradients of almond-shape surface will lead to preferential expansion in the reaction plane
- Anisotropy of emission is quantified by 2nd Fourier coefficient of angular distribution: v_2

➢ prediction of fluid dynamics



Elliptic flow is created early

time evolution of the energy density:

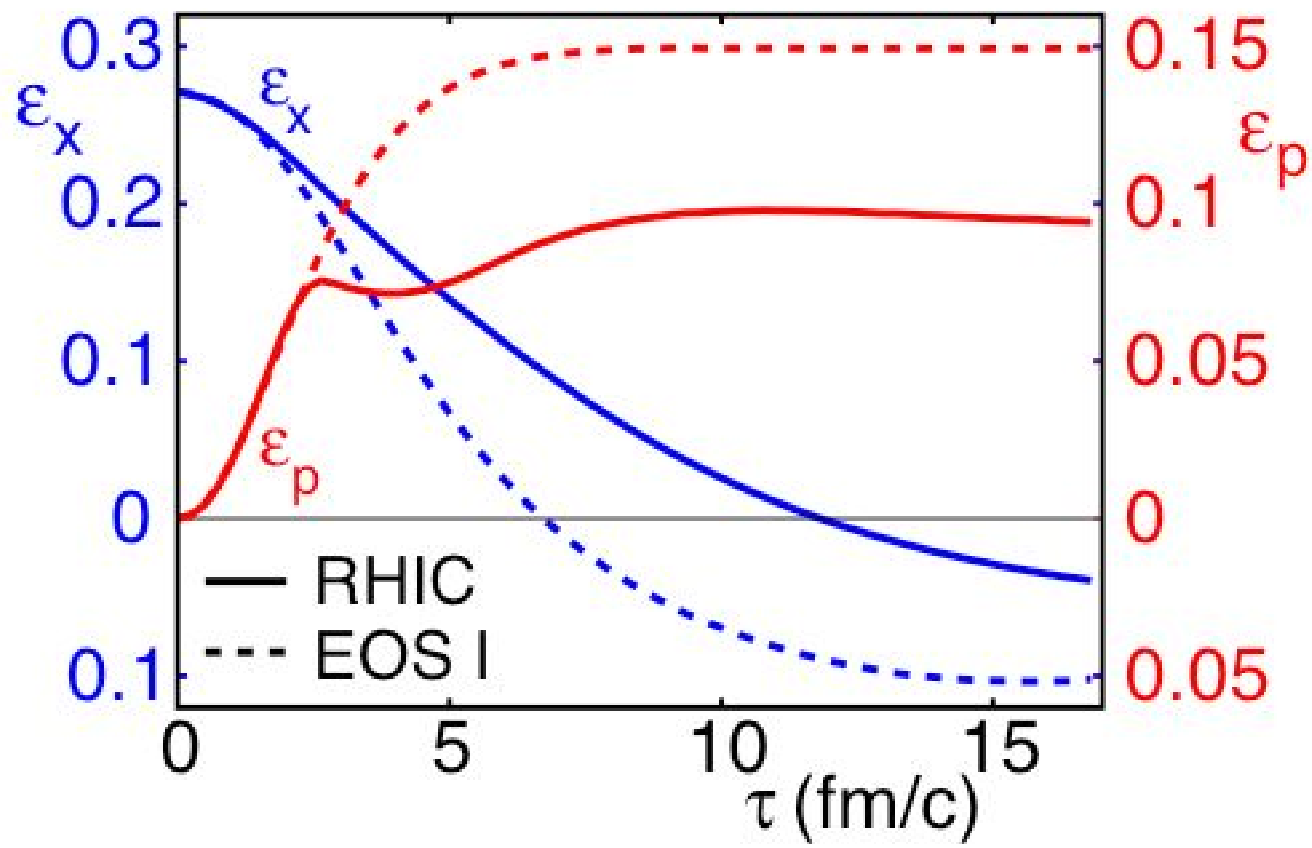


spatial eccentricity

$$\epsilon_x = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

momentum anisotropy

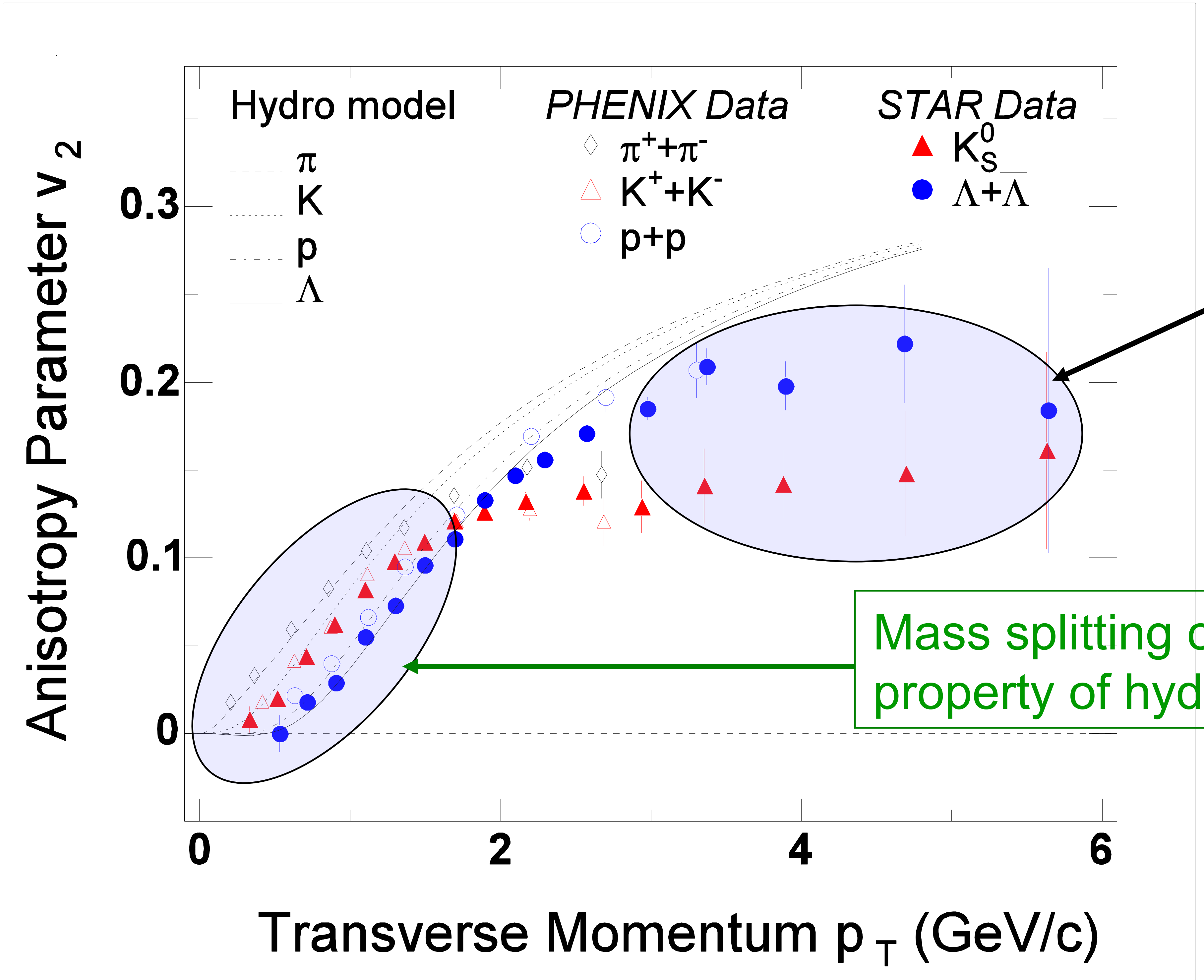
$$\epsilon_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$



P. Kolb, J. Sollfrank and U.Heinz, PRC 62 (2000) 054909

Model calculations suggest that flow anisotropies are generated at the earliest stages of the expansion, on a **time scale of ~ 5 fm/c** if a QGP EoS is assumed.

$v_2(p_T)$ vs. hydrodynamics



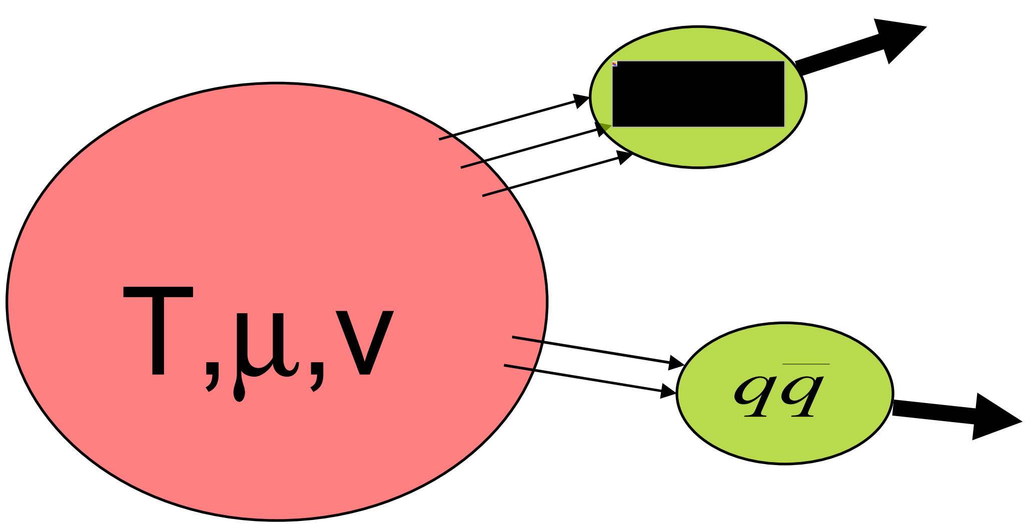
Failure of ideal hydrodynamics - tells us how hadrons form

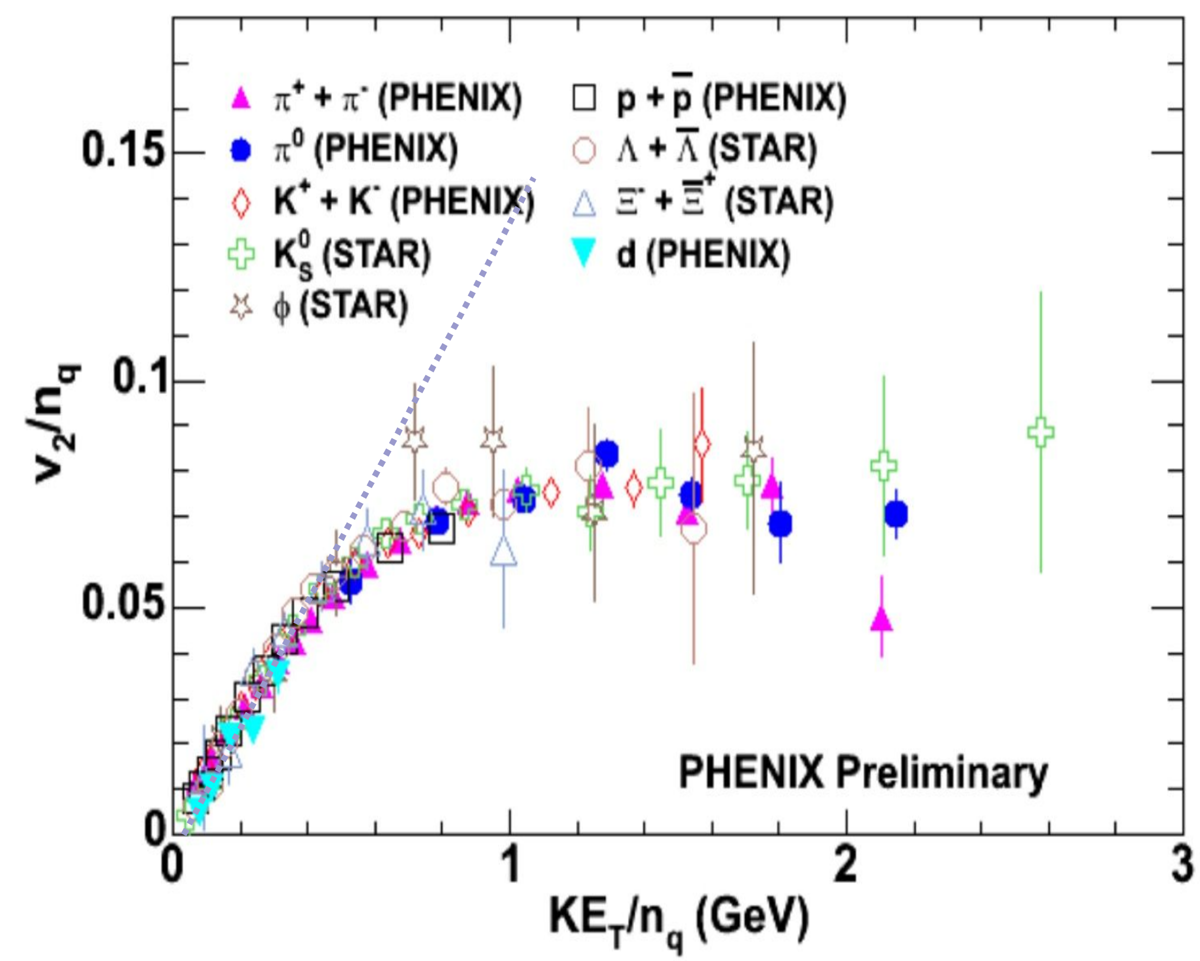
Mass splitting characteristic property of hydrodynamics

Quark number scaling of v_2

In the recombination regime, meson and baryon v_2 can be obtained from the quark v_2 :

$$v_2^M(p_t) = 2v_2^q\left(\frac{p_t}{2}\right) \qquad v_2^B(p_t) = 3v_2^q\left(\frac{p_t}{3}\right)$$



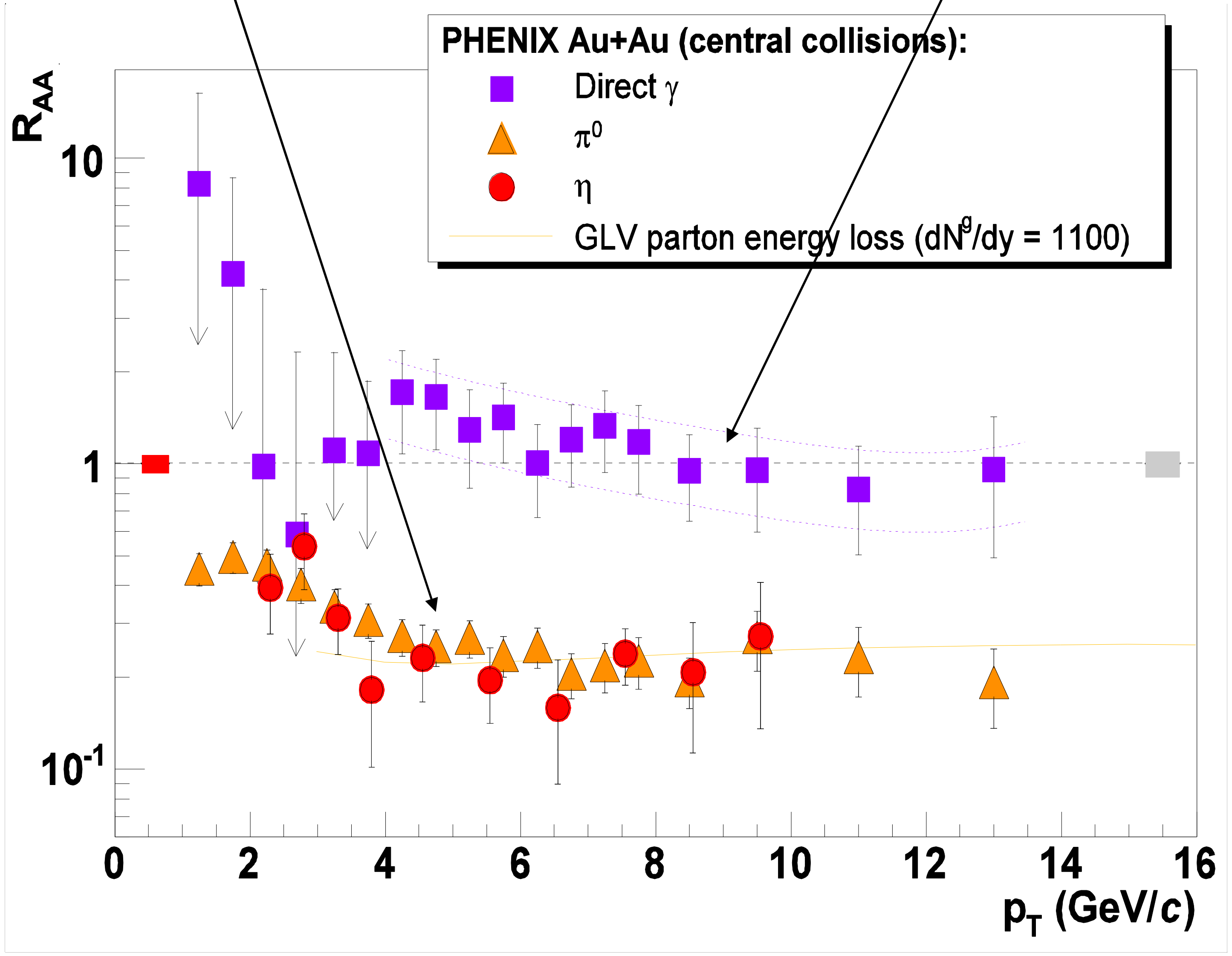


Emitting medium is composed of unconfined, flowing quarks.

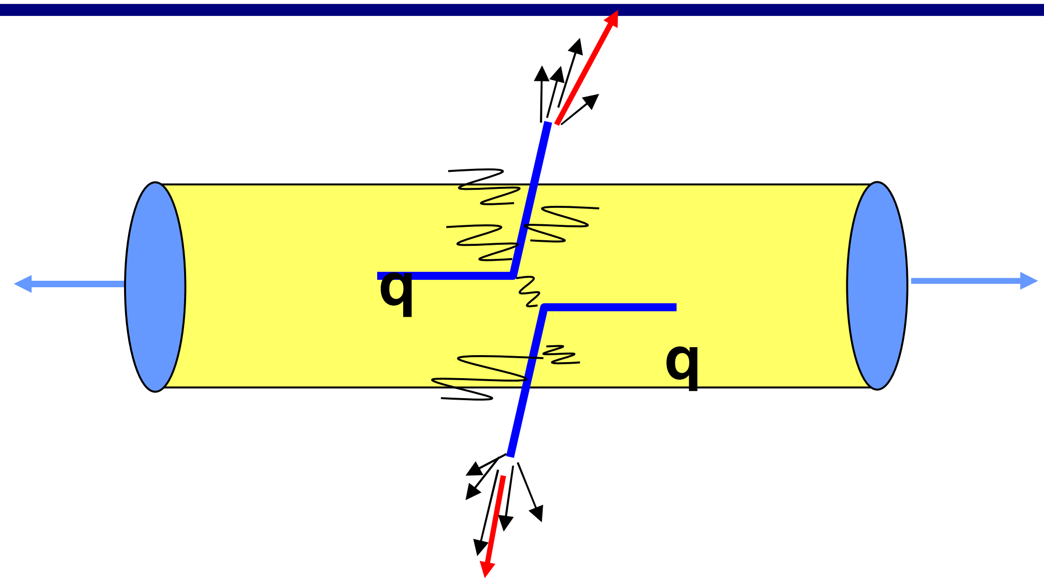
Photons versus hadrons

No suppression for photons

Suppression of hadrons

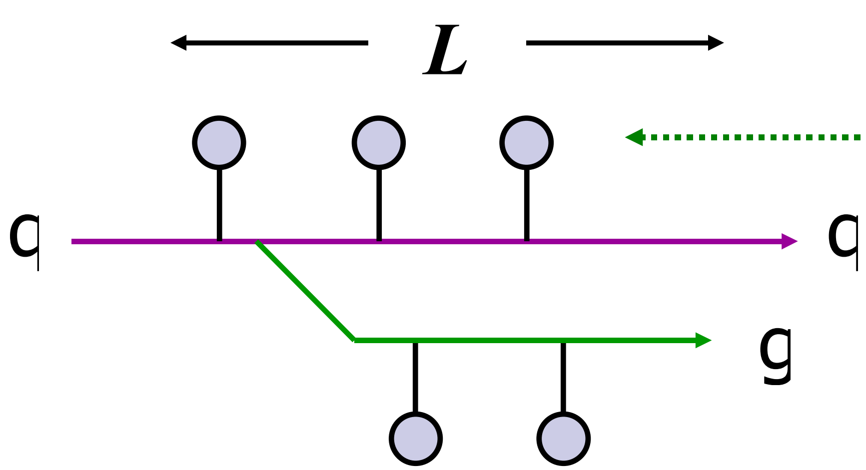


Radiative energy loss



Radiative energy loss:

$$dE / dx : \rho \Lambda \langle \kappa_T^{\ 2} \rangle$$



Scattering centers = color charges

Density of scattering centers

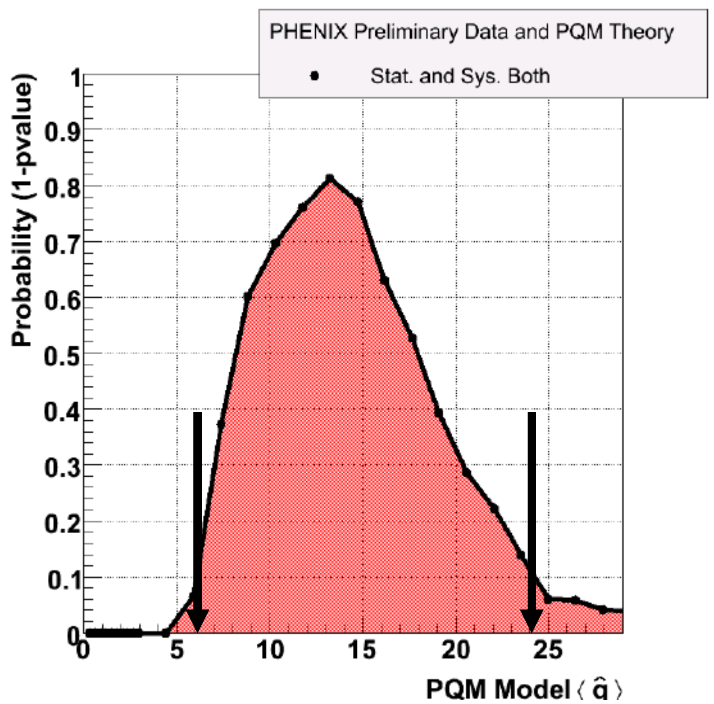
$$\ddot{\varphi} = \rho \check{n} \theta^2 \delta\theta^2 \frac{\delta\sigma}{\delta\theta^2} \text{ ş } \rho\sigma \langle \kappa_T^2 \rangle = \frac{\mu^2}{\lambda_\phi}$$

Scattering power of the QCD medium:

Range of color force

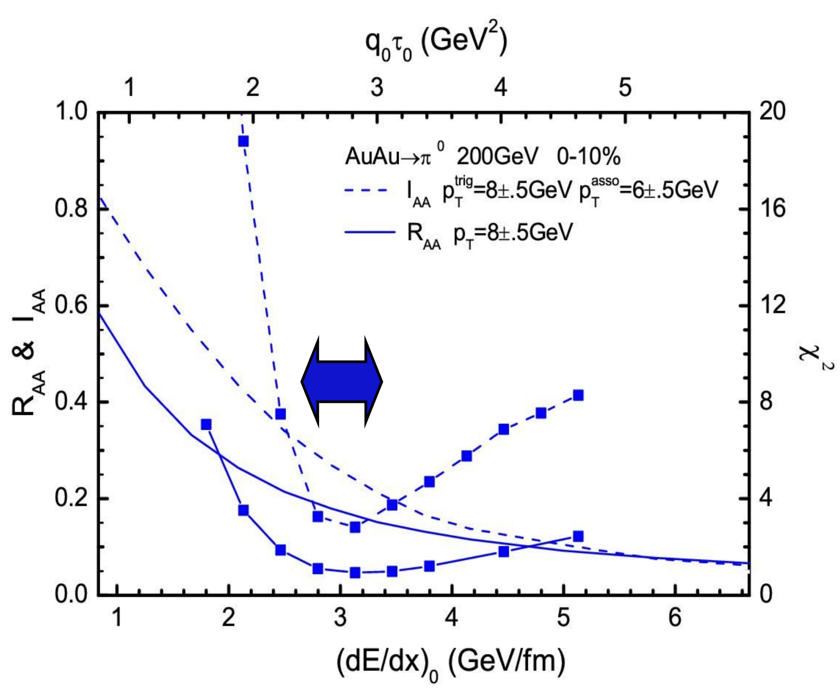
How large is q-hat?

- Data are described by a large loss parameter for central collisions:



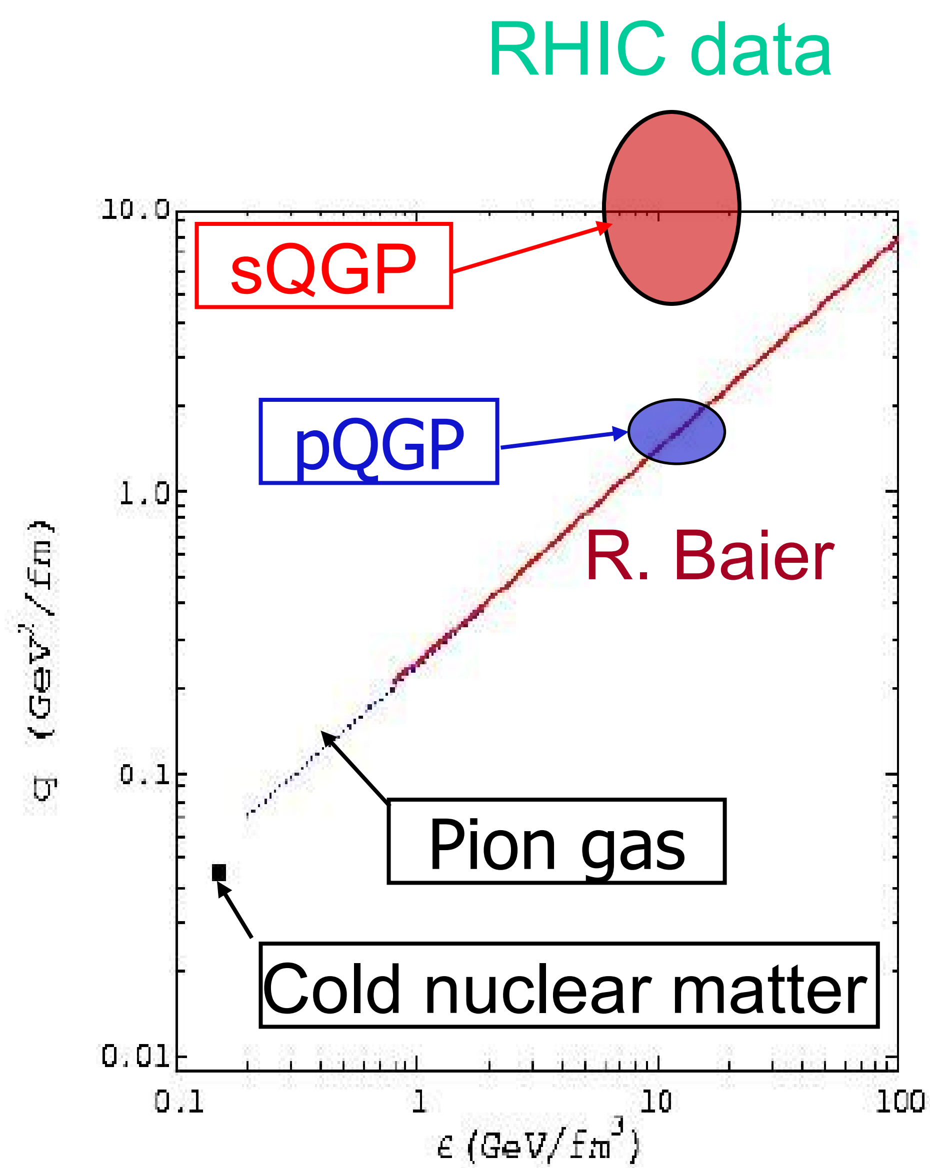
Loizides, hep-ph/0608133

$$\langle \ddot{\varphi} \rangle \approx 5 - 20\ \Gamma\varepsilon\varsigma\ ^{2}/\phi\mu$$



Zhang, Owens, Wang & Wang, nucl-th/0701045

$$\langle \ddot{\varphi} \rangle \approx 1 - 2\ \Gamma\varepsilon\varsigma\ ^{2}/\phi\mu$$



Larger than expected from perturbation theory ?

Part III

Toward the
"Perfect" Liquid

Ideal gas vs. perfect liquid

- An ideal gas is characterized by interactions strong enough to reach thermal equilibrium (on a reasonable time scale), but weak enough to neglect their effect on $P(n,T)$.
  - This ideal can be approached arbitrarily well by diluting the gas and waiting very patiently (limit $t \to \infty$ first, then $n \to 0$).

- A perfect fluid is one that obeys the Euler equations, i.e. a fluid that has negligible viscosity and infinite thermal conductivity (relative to gradients).
  - There is no presumption with regard to the equation of state.

What is viscosity ?

Shear and bulk viscosity are defined as coefficients in the expansion of the stress tensor in gradients of the velocity field:

$$T_{ik} = \varepsilon u_i u_k + P(\delta_{ik} + u_i u_k) - \eta\left(\nabla_i u_k + \nabla_k u_i - \tfrac{2}{3}\delta_{ik}\nabla\cdot u\right) + \zeta\,\delta_{ik}\nabla\cdot u$$

Microscopically, η is given by the rate of momentum transport:

$$\eta \approx \tfrac{1}{3} n\bar{p}\lambda_f = \frac{\bar{p}}{3\sigma_{\rm tr}}$$

Unitarity limit on cross sections suggests that η has a lower bound:

$$\sigma_{\rm tr} \le \frac{4\pi}{\bar{p}^2} \quad \Rightarrow \quad \eta \ge \frac{\bar{p}^3}{12\pi}$$

Viscosity of materials

Temperature dependence of the shear viscosity of typical fluids:

Lower bound on η/s ?

A heuristic argument for $(\eta/s)_{min}$ is obtained using $s \approx 4n$:

$$\eta \approx \frac{1}{3} n \left(\bar{p}\bar{v}\right)\left(\frac{\lambda_f}{\bar{v}}\right) \approx \frac{1}{12} s \left(\frac{\varepsilon}{n}\right)\tau_f$$

The uncertainty relation dictates that $\tau_f(\varepsilon/n) \geq \hbar$, and thus:

$$\eta \geq \frac{\hbar}{12} s \approx \frac{\hbar}{4\pi} s$$

All known materials obey this condition!

For $\mathcal{N}=4$ SU(N_c) SYM theory the bound is saturated at strong coupling:

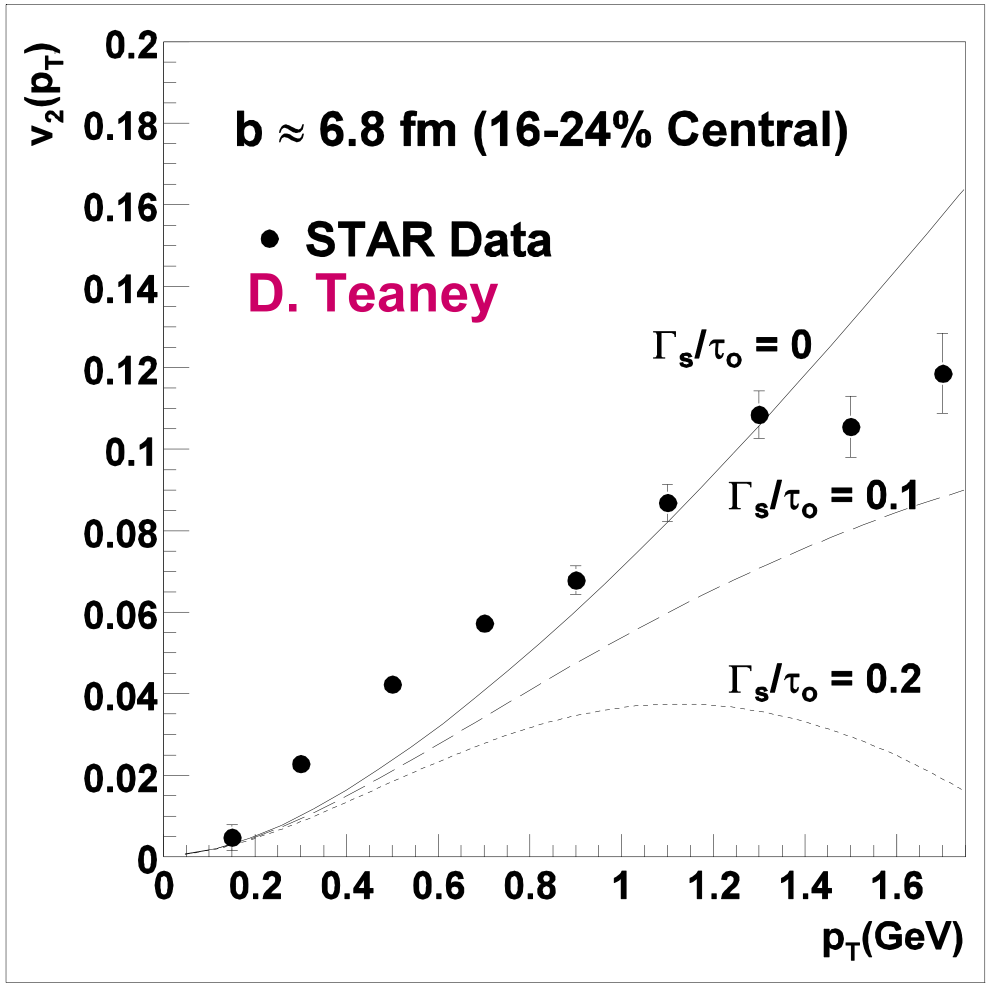
$$\eta = \frac{s}{4\pi}\left[1 + \frac{135\,\zeta(3)}{\left(8 g^2 N_c\right)^{3/2}} + \ldots\right]$$

Bounds on η from v_2

Relativistic viscous hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with}$$

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \eta(\partial^\mu u^\nu + \partial^\nu u^\mu - \text{trace})$$



Boost invar't hydro requires $\eta/s \approx 0.1$.

$$\Gamma_s / \tau_0 \approx \eta / s$$

$\eta/s < 0.3$ confirmed by 2-D viscous hydro calculation (Baier & Romatschke).

N=4 SUSY YM theory ($g^2 N_c$ □ 1):

$\eta/s = 1/4\pi$ (**Policastro, Son, Starinets**).

Absolute lower bound on η/s ?

QGP viscosity – collisions

Baym, Heiselberg, ….

Danielewicz & Gyulassy, Phys.Rev.D31,53 (85)

Arnold, Moore & Yaffe

Classical expression for shear viscosity:

$$\eta \approx \tfrac{1}{3} n \bar{p} \lambda_f$$

Collisional mean free path in medium:

$$\lambda_f^{(C)} = \left(n \sigma_{\mathrm{tr}} \right)^{-1}$$

Transport cross section in QCD medium:

$$\sigma_{\mathrm{tr}} \approx \frac{5\pi}{\bar{p}^2} \alpha_s^2 I(\alpha_s)$$

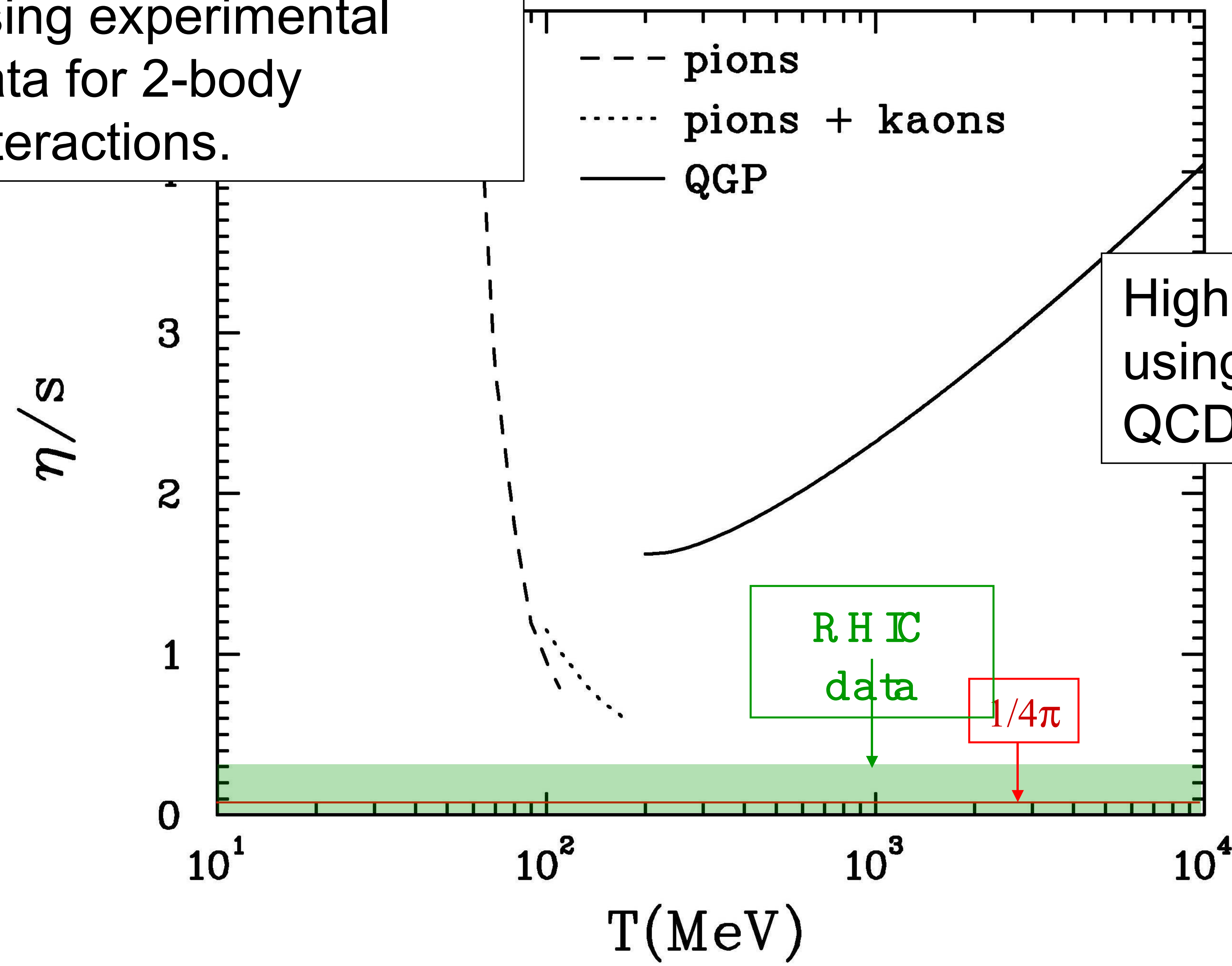
$$I(\alpha_s) = (1 + 2\alpha_s) \ln\left(1 + \frac{1}{\alpha_s} \right) - 2$$

Collisional shear viscosity of QGP:

$$\eta_C \approx \frac{T}{\sigma_{\mathrm{tr}}} \approx \frac{9 s}{100 \pi \alpha_s^2 \ln \alpha_s^{-1}}$$

QCD matter

Low T (*Prakash et al.*) using experimental data for 2-body interactions.



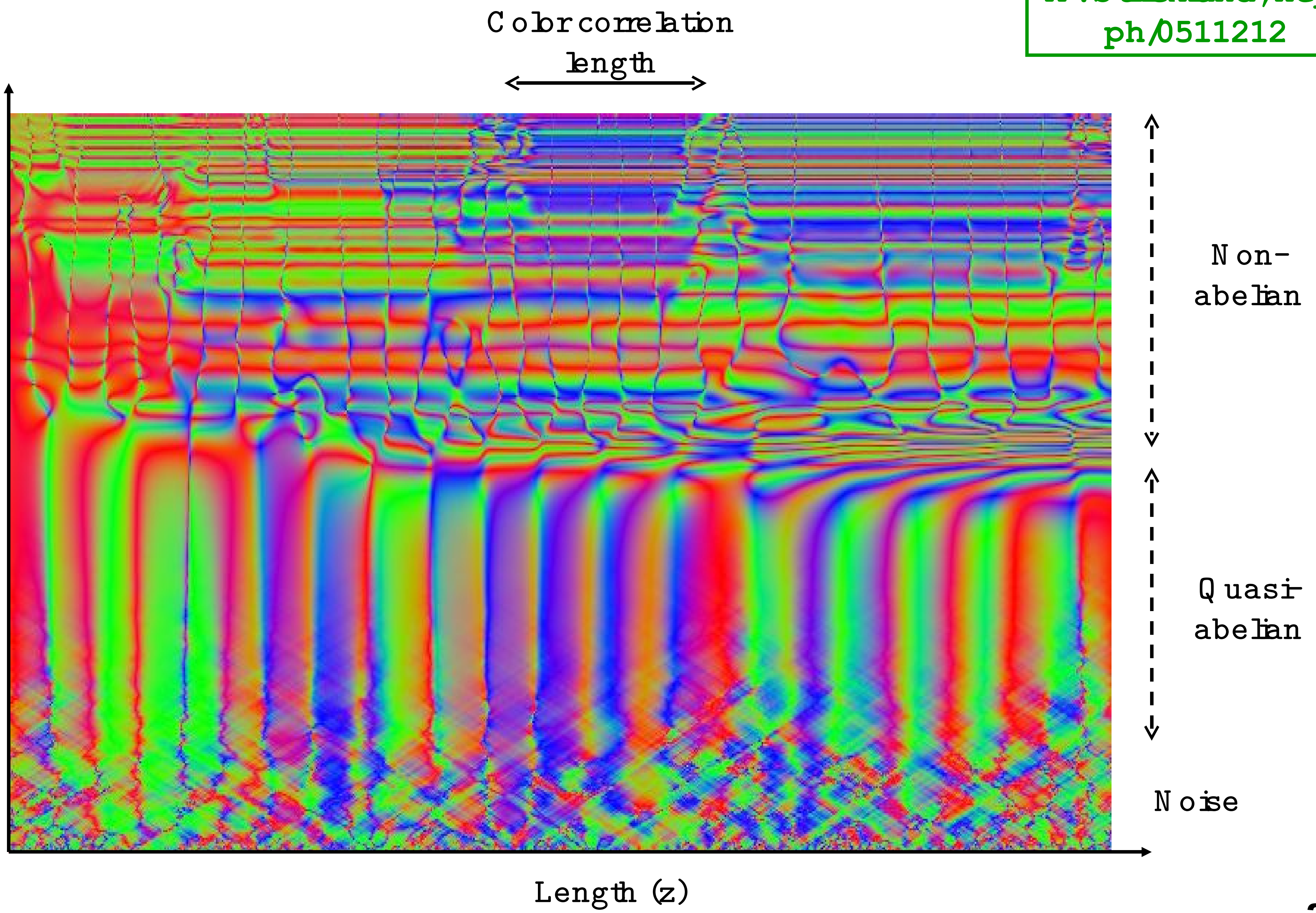
High T (Yaffe *et al.*) using perturbative QCD

Part IV

Anomalous Viscosity

Spontaneous color fields

M. Strickland, hep-ph/0511212



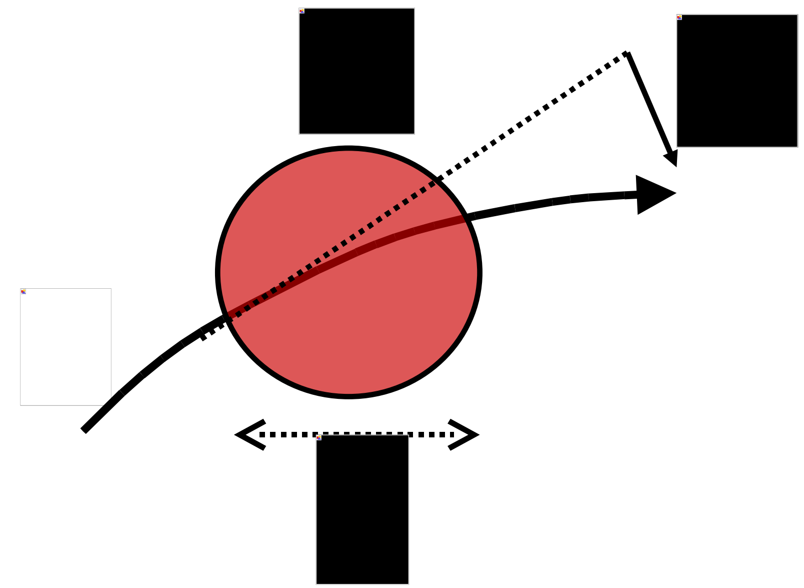
QGP viscosity – anomalous

Classical expression for shear viscosity:

$$h \text{ » } \tfrac{1}{3} n \bar{p} l_f$$

Momentum change in one coherent domain:

$$Dp \text{ » } g Q^a B^a r_m$$



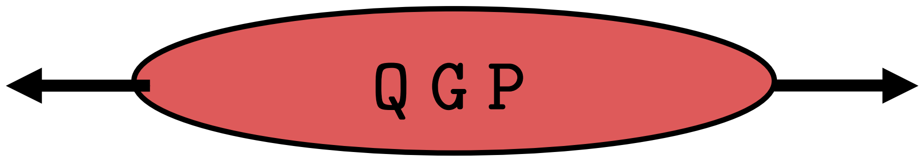
Anomalous mean free path in medium:

$$l_f^{(A)} \text{ » } r_m \left\langle \frac{\bar{p}^2}{(Dp)^2} \right\rangle \text{ » } \frac{\bar{p}^2}{g^2 Q^2 \langle B^2 \rangle r_m}$$

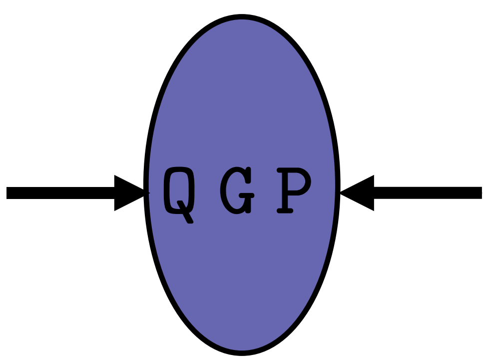
Anomalous viscosity due to random color fields:

$$h_A \text{ » } \frac{n \bar{p}^3}{3 g^2 Q^2 \langle B^2 \rangle r_m} \text{ » } \frac{\frac{9}{4} s T^3}{g^2 Q^2 \langle B^2 \rangle r_m}$$

Expansion ⇔ Anisotropy



X-space



P-space

Perturbed equilibrium distribution:

$$f(p) = f_0(p)\left[1 + f_1(p)\left(1 \pm f_0(p)\right)\right]$$

$$f_0(p) = \exp[-u_\mu p^\mu / T]$$

For shear flow of ultrarelat. fluid:

$$f_1(p) = \frac{5\eta / s}{E_p T^2}\left(p^i p^j - \tfrac{1}{3}\delta_{ij}\right)(\nabla u)_{ij}$$

$$(\nabla u)_{ij} = \tfrac{1}{2}\left(\nabla_i u_j + \nabla_j u_i\right) - \tfrac{1}{3}\delta_{ij}\nabla \cdot u$$

Anisotropic momentum distributions generate instabilities of soft field modes. Growth rate $\Gamma \sim f_1(p)$.

- **Shear flow always results in the formation of soft color fields;**
- **Size controlled by $f_1(p)$, i.e. (∇u) and η/s.**

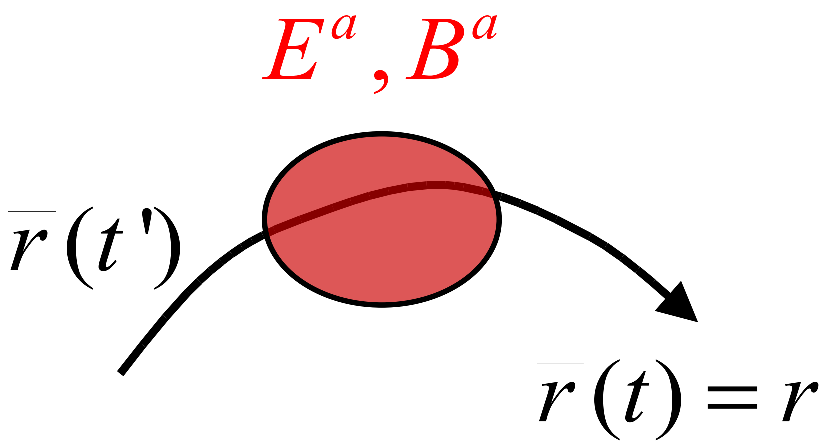
Turbulence ⇒ Diffusion

Vlasov-Boltzmann transport of thermal partons:

$$\left[\frac{\partial}{\partial t} + \frac{p}{E_p}\cdot\nabla_r + F\cdot\nabla_p\right] f(r,p,t) = C[f]$$

with Lorentz force

$$F = gQ^a\left(E^a + v \times B^a\right)$$



Assuming E, B random ⇒ Fokker-Planck eq:

$$\left[\frac{\partial}{\partial t} + \frac{p}{E_p}\cdot\nabla_r - \nabla_p\cdot D(p)\cdot\nabla_p\right] \bar{f}(r,p,t) = C\left[\bar{f}\right]$$

with diffusion coefficient

$$D_{ij}(p) = \int_{-\infty}^{t} dt' \left\langle F_i\left(\bar{r}(t'),t'\right) F_j\left(r,t\right)\right\rangle .$$

Diffusion is dominated by chromo-magnetic fields:

$$\int dt' \left\langle B(t')B(t)\right\rangle \approx \left\langle B^2\right\rangle \tau_m$$

Shear viscosity

Take moments of $\left[\frac{\partial}{\partial t} + \frac{p}{E_p}\cdot\nabla_r - \nabla_p\cdot D(p)\cdot\nabla_p\right] f(r,p,t) = C[f]$ with p_z^2

$$D_{ij}(p) = \int_{-\infty}^{t} d\tau \left\langle F_i^{\alpha}(r(\tau),\tau)\, U_{\alpha\beta}(r,r')\, F_i^{\beta}(r',t)\right\rangle$$

$$\vec{F}^{\alpha} = g\left(\vec{E}^{\alpha} + \vec{v}\times\vec{B}^{\alpha}\right) = \text{color force}$$

$$\frac{1}{\eta} = O(1)\frac{N_c}{N_c^2-1}\frac{\langle F^2\rangle \tau_m}{sT^3} + O\left(10^{-2}\right)\frac{g^4\ln g^{-1}}{T^3} \equiv \frac{1}{\eta_A} + \frac{1}{\eta_C}$$

$$\int dt' \left\langle F_i^{+}(t')F^{+i}(t)\right\rangle = \left\langle F^2\right\rangle \tau_m \equiv \hat{q}$$

= jet quenching parameter !!!

M. Asakawa, S.A. Bass, B.M.,

PRL 96:252301 (2006)

Prog Theo Phys 116:725 (2007)

Who wins?

Smallest viscosity dominates in system with several sources of viscosity

Anomalous viscosity

$$\frac{\eta_A}{\sigma} \sim \left(\frac{1}{\gamma^2 |\nabla u|} \right)^{3/5}$$

Collisional viscosity

$$\frac{\eta_X}{\sigma} \approx \frac{36\pi}{50\,\gamma^4 \ln \gamma^{-1}}$$

$|\nabla u| \sim t^{-1}$ → Anomalous viscosity wins out at small *g* and *t*

Estimate for turbulent color field intensity: $\langle F^2 \rangle \sim t^{-3.1}$

Part IV

Exploring the

"Perfect" Liquid

Connecting jets with the medium

Hard partons probe the medium via the density of colored scattering centers:

$$\ddot{\varphi} = \rho \check{\eta} \theta^2 \delta\theta^2 (\delta\sigma / \delta\theta^2)$$

If kinetic theory applies, thermal gluons are quasi-particles that experience the same medium. Then the shear viscosity is:

$$\eta \approx Cr \langle p\, l_f(p) \rangle = C \left\langle \frac{p}{s_{tr}(p)} \right\rangle$$

In QCD, small angle scattering dominates:

$$\sigma_{tr}(p) » \frac{4\ddot{\varphi}}{\ddot{s}r}$$

With $\langle p^3 \rangle$ ~ T^3 and $s \approx 4\rho$ one finds:

$$\frac{\eta}{s} » \frac{5}{4}\frac{T^3}{\ddot{\varphi}}$$

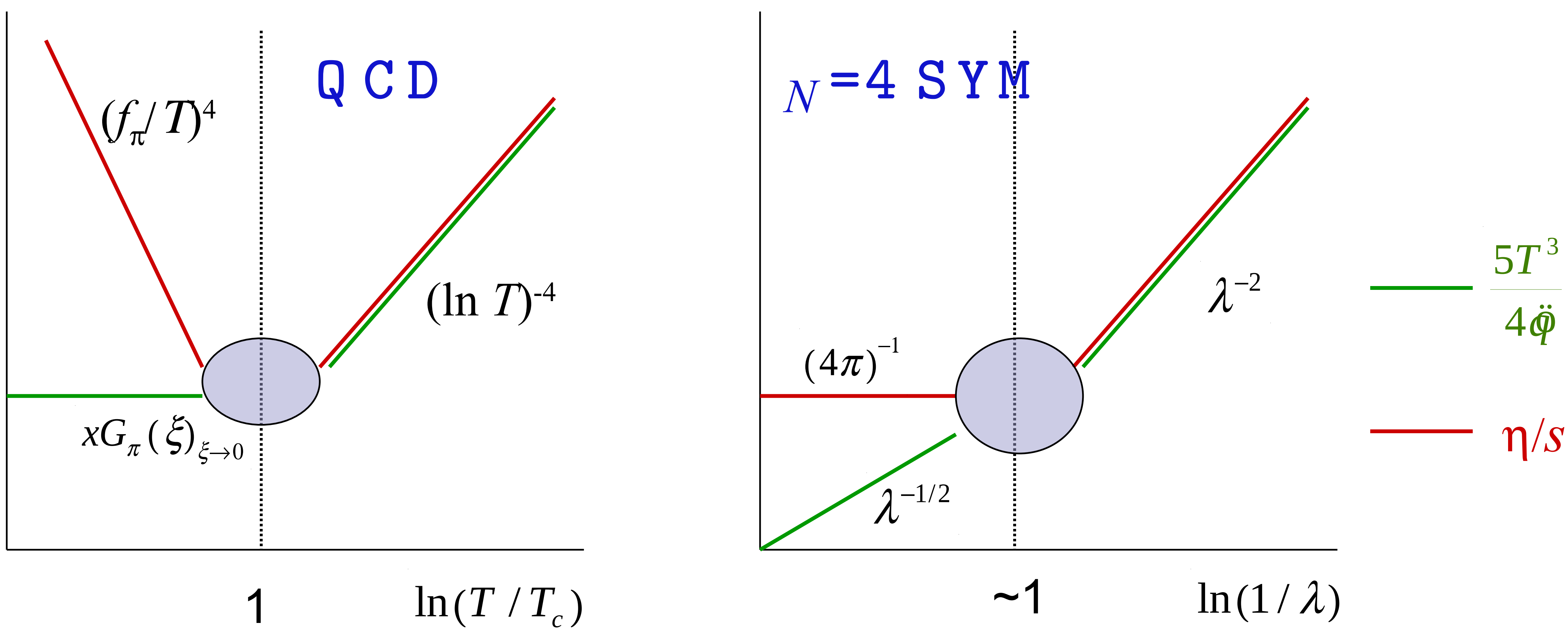
Majumder, BM, Wang, hep-ph/0703085

From RHIC data:

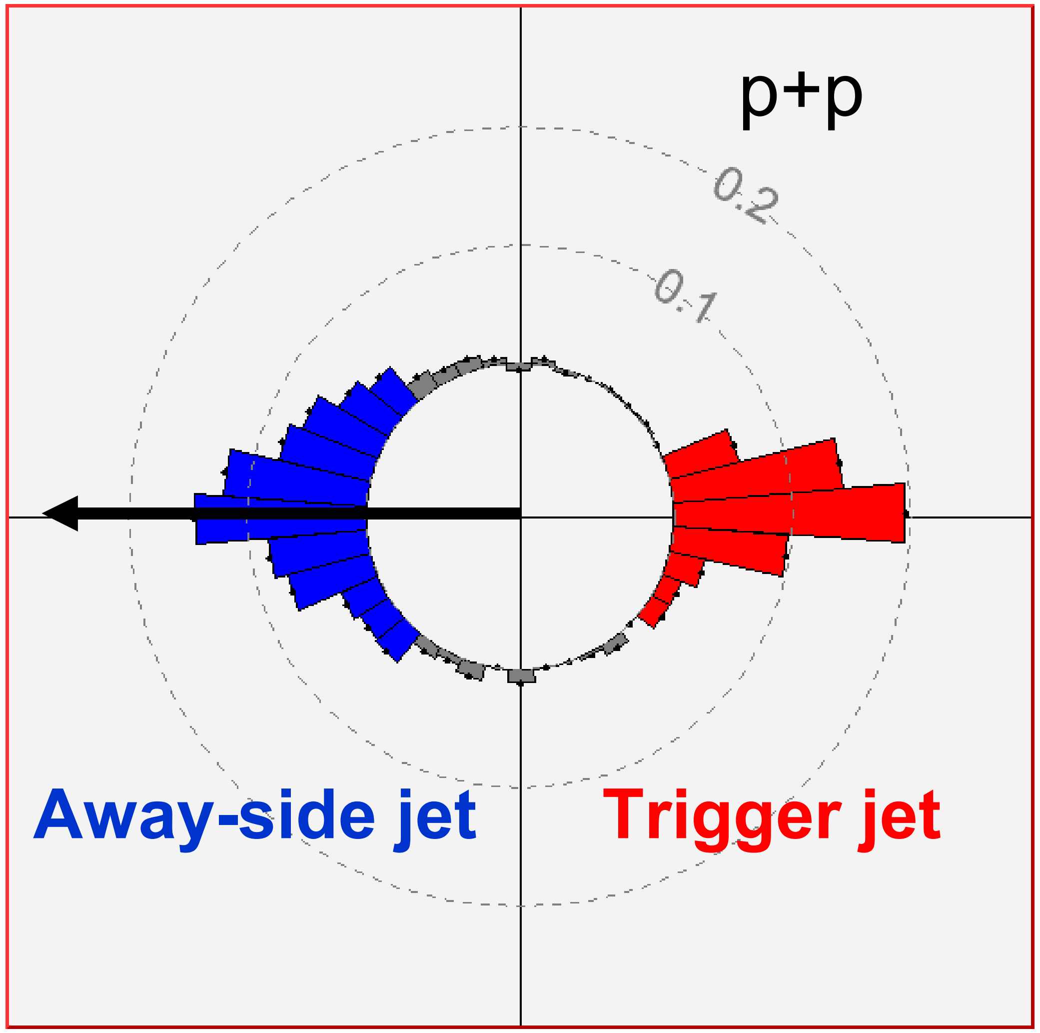
$$T_0 \approx 335 \text{ Me}\varsigma, \; \hat{\theta}_0 \approx 1-2 \text{ }\Gamma\varepsilon\varsigma^2/\text{φμ} \rightarrow \frac{T_0^3}{\hat{\theta}_0} \approx 0.12 - 0.24$$

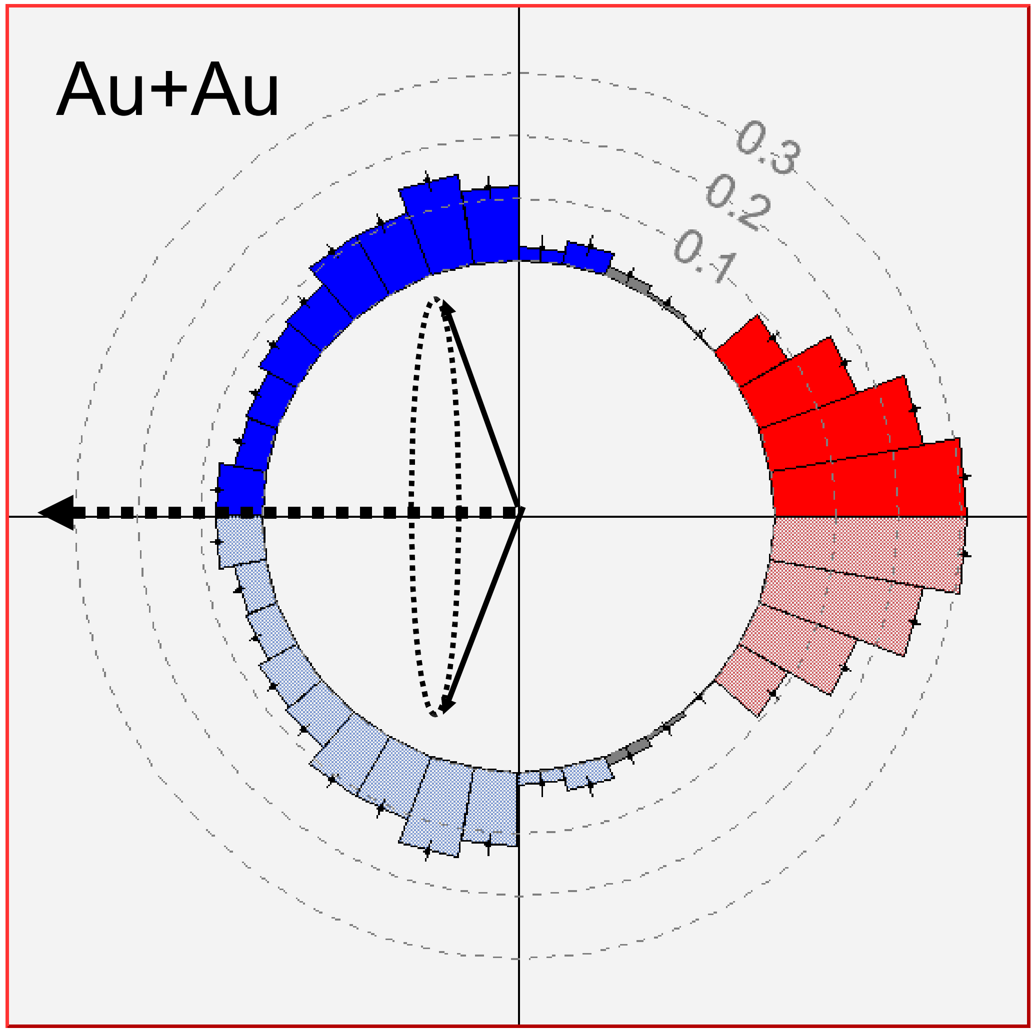
Strong vs. weak coupling

At strong coupling, $\frac{T^3}{\ddot{\varphi}}$ is a more faithful measure of medium "blackness".



Fate of the “lost” energy (I)

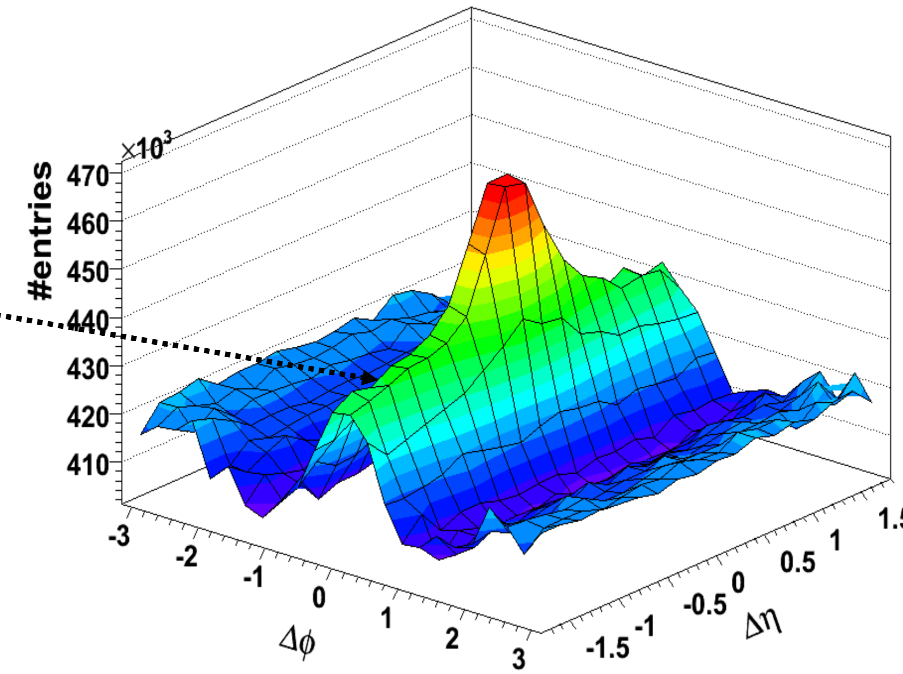




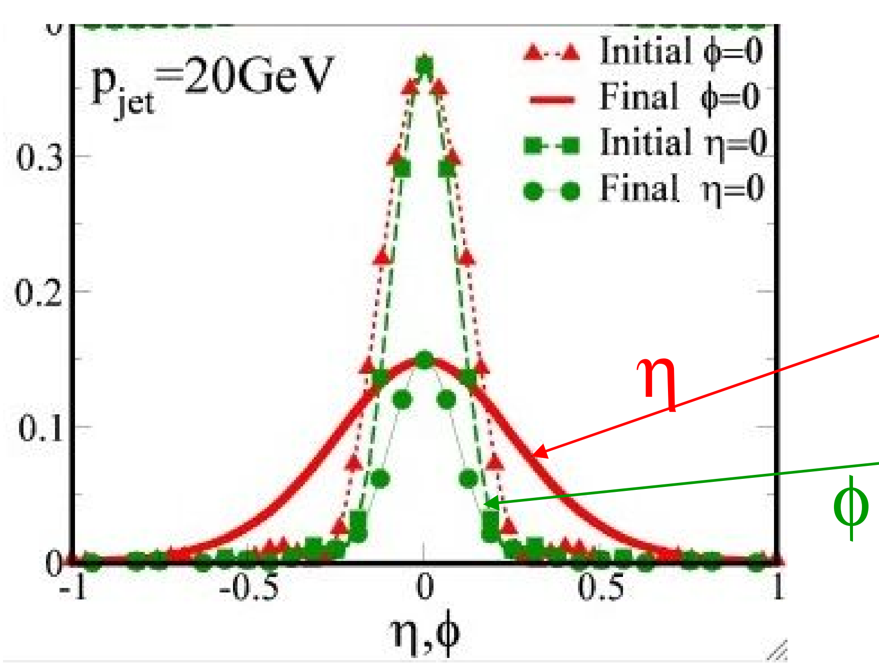
Lost energy of away-side jet is redistributed to angles away from 180° and low transverse momenta p_T < 2 GeV/c (Mach cone?).

Fate of the “lost” energy (II)

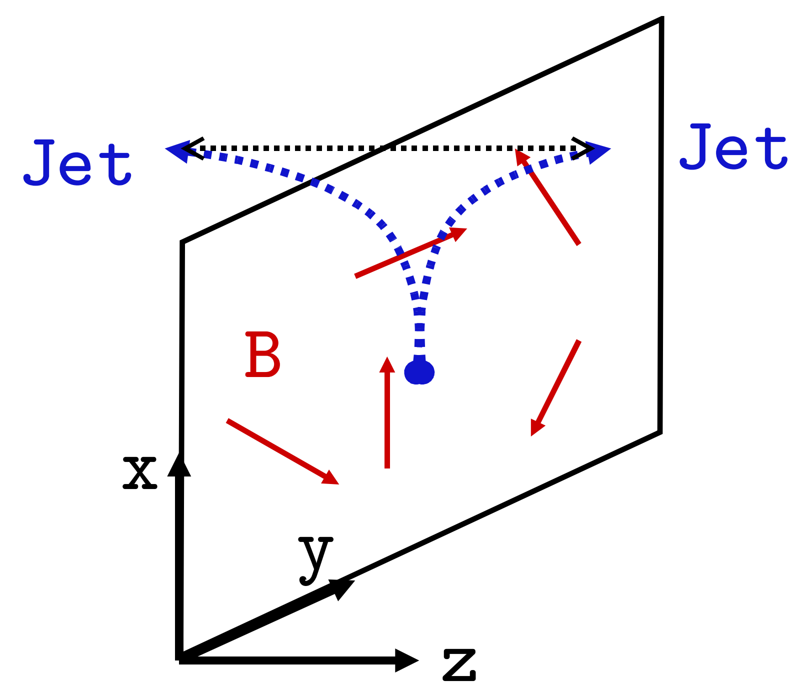
- Near side jet phenomenon:
  - Longitudinal broadening of jet cones.
    “Ridge” contains all additional energy:
    In-medium fragmentation



Possible explanation: Longitudinal diffusion of radiated gluons in random, transverse color-magnetic fields (A. Majumder,...)



Final distribution of in-medium radiated gluons



Summary

The matter created in heavy ion collisions forms a highly color opaque plasma, which has an extremely small shear viscosity. This (nearly) "perfect liquid" behaves like a strongly coupled plasma of quarks and gluons, possibly due to the presence of strong turbulent color fields, especially at early times when the expansion is most rapid.

Penetrating probes, i.e. jets, heavy quarks, photons, are the best way to further probe this medium. The extended dynamic range of RHIC II and LHC, together with detailed theoretical modeling and simulations, will allow us to quantitatively determine its essential transport properties.