Cosmological inflation

David Langlois
(APC, Paris)

Astroparticules
et Cosmologie

Friedmann equations

- Homogeneous and isotropic Universe

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$$

$$\gamma_{ij}dx^i dx^j = \frac{dr^2}{1-\kappa r^2} + r^2\left(d\theta^2 + \sin^2\theta\ d\phi^2\right)$$

- Einstein's equations $G_{\mu\nu} = 8\pi G\ T_{\mu\nu}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)\,.$$

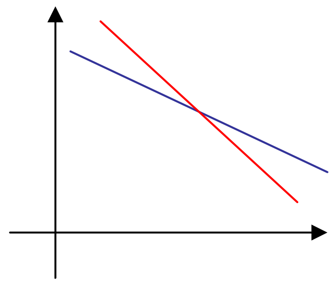
$$H \equiv \frac{\dot{a}}{a} \qquad T^{\mu}{}_{\nu} = \mathrm{Diag}\,(-\rho, P, P, P)$$

The Universe in the Past

The energy densities dilute at various rates:

- pressureless matter

$$\rho_m \propto \frac{1}{a^3} \qquad \Rightarrow a(t) \propto t^{2/3}$$



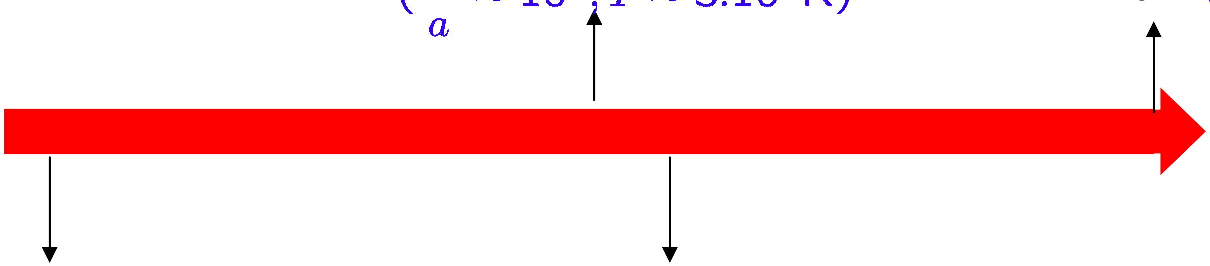
- radiation

$$\rho_r \propto \frac{1}{a^4} \qquad \Rightarrow a(t) \propto t^{1/2}$$

$$T \propto 1/a$$

matter-radiation equality
$(\frac{a_0}{a} \sim 10^4, T \sim 3.10^4\text{K})$

now $(T \sim 3\text{K})$



Time

nucleosynthesis
$(T \sim 0.1\text{MeV})$

last scattering $(T \sim 3.10^4\text{K})$

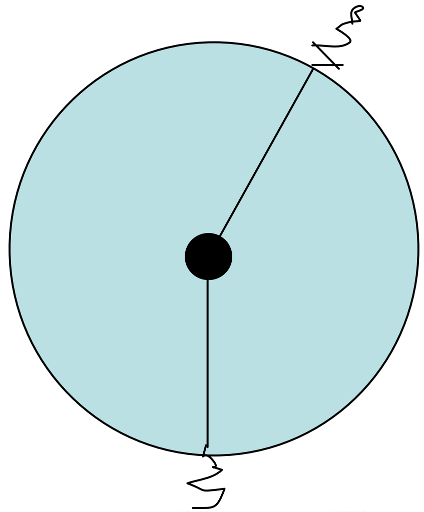
Cosmic Microwave Background (CMB)

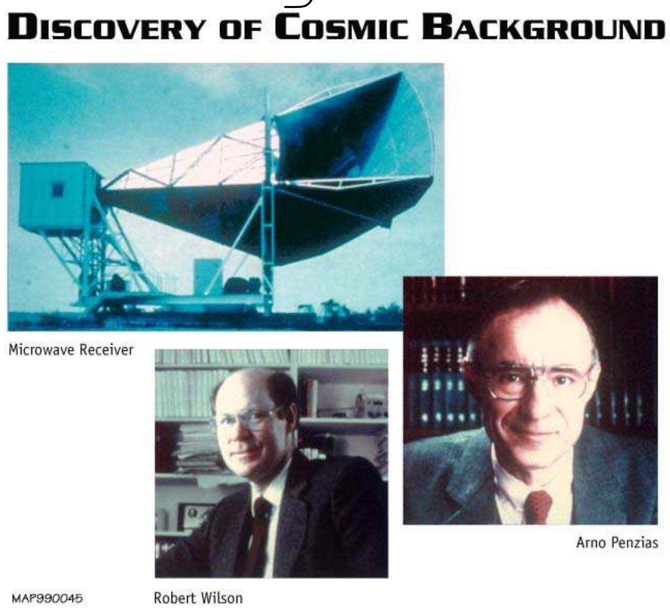
- T> 3.10^3 K: atoms are ionised (H$\rightarrow$ p+e$^-$)
  $\Rightarrow$ **opaque Universe**

- T< 3.10^3 K: “recombination” (p+e$^-$$\rightarrow$ H)
  $\Rightarrow$ **transparent Universe**

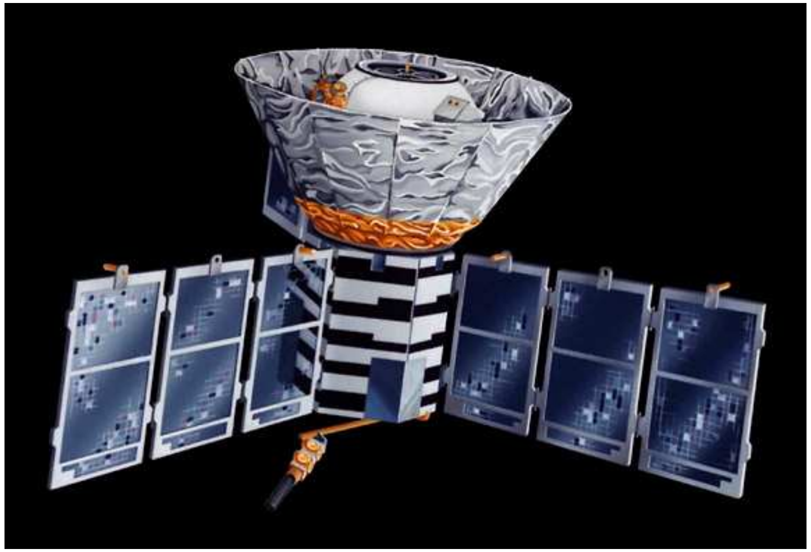
- “Fossil” background radiation
  - predicted in 1948,
  - discovered in 1965 by Penzias and Wilson.

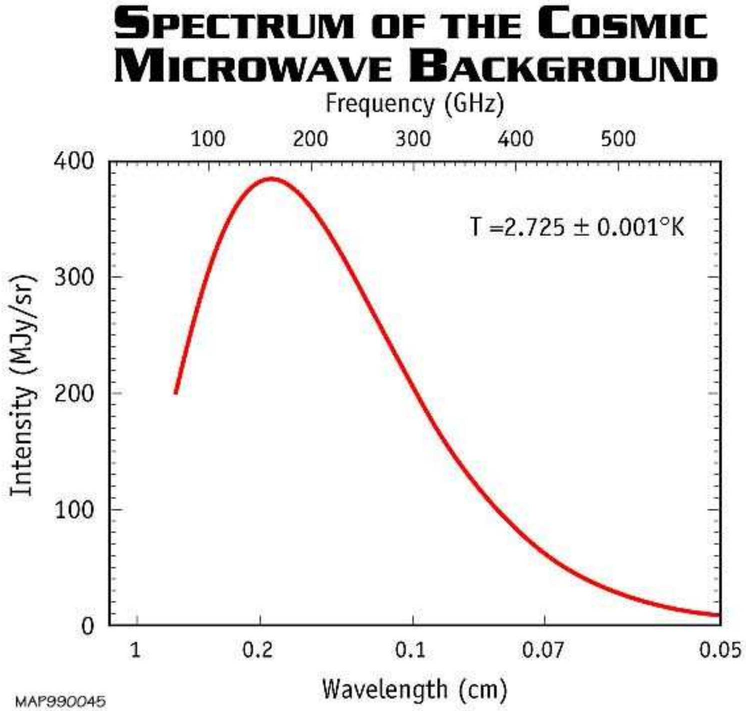
T$\simeq$ 2.7 K





COBE satellite





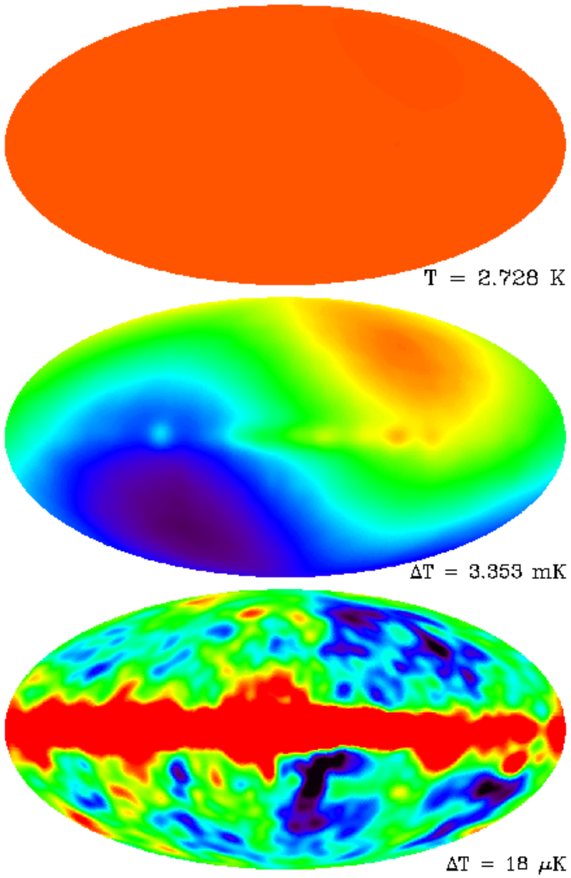
John C. Mather

George F. Smoot

"for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation"

Nobel Prize in Physics 2006

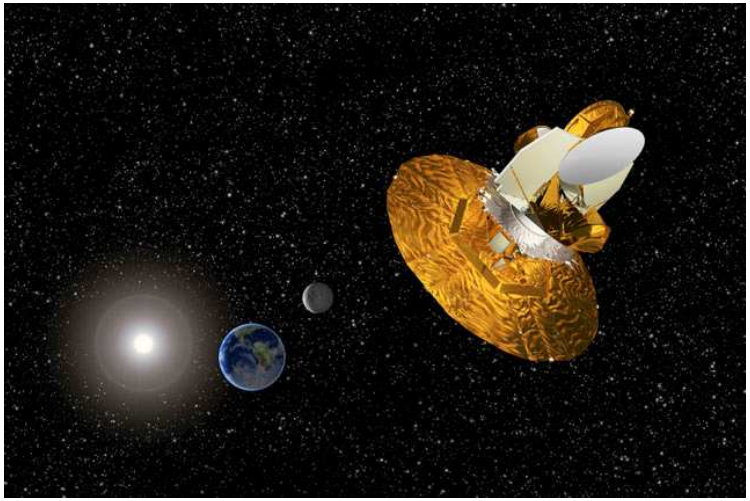
CMB seen by COBE



$$v_{\text{Earth/CMB}} = 371 \pm 1 \text{ km/s}$$

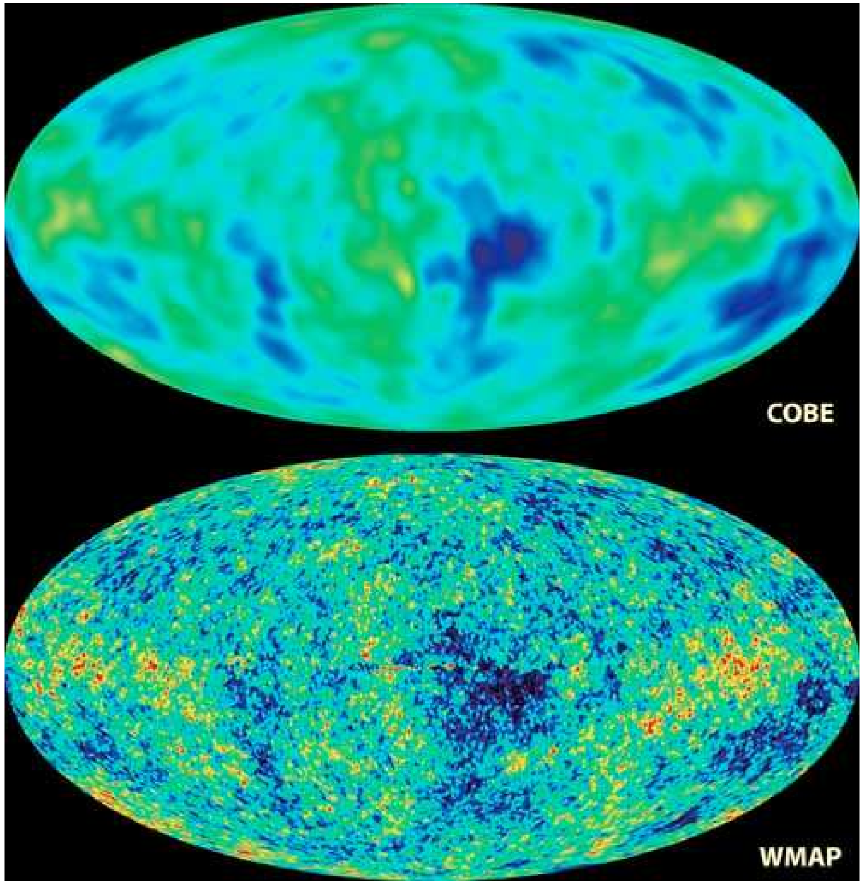
$$\frac{\delta T}{T} \sim 10^{-5}$$

CMB seen by WMAP

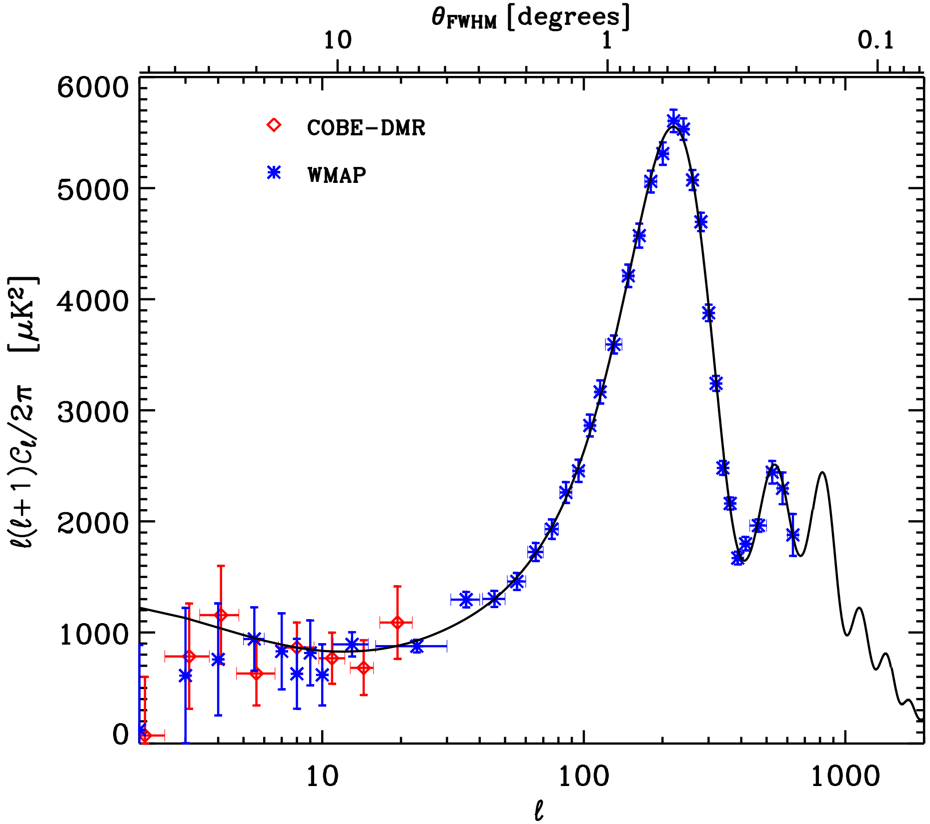


WMAP

Planck (2008 ?)



CMB seen by … theorists



$$\frac{\delta T}{T}(\theta,\phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi)$$

$$C_l = \langle |a_{lm}|^2 \rangle$$

Oscillations = sound waves

Horizon problem

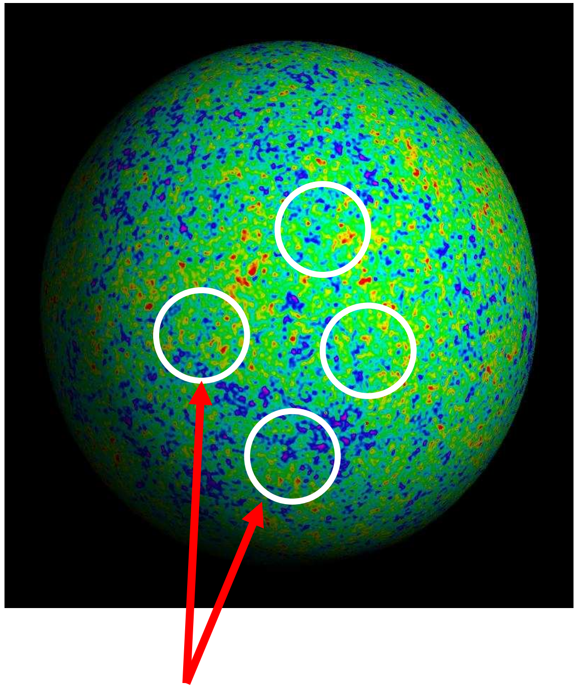
- Horizon = maximal distance covered by a particle
  - matter domination: $d_H = H^{-1}$ (Hubble radius)
  - radiation domination: $d_H = 2H^{-1}$
- Causal size in comoving space

$$\lambda_{\text{phys}} = a(t)\tilde{\lambda}_{\text{com}}$$

$$\tilde{\lambda}_H = (aH)^{-1} = 1/\dot{a}$$

- The comoving Hubble radius increases in standard cosmology

  (when the scale factor decelerates)

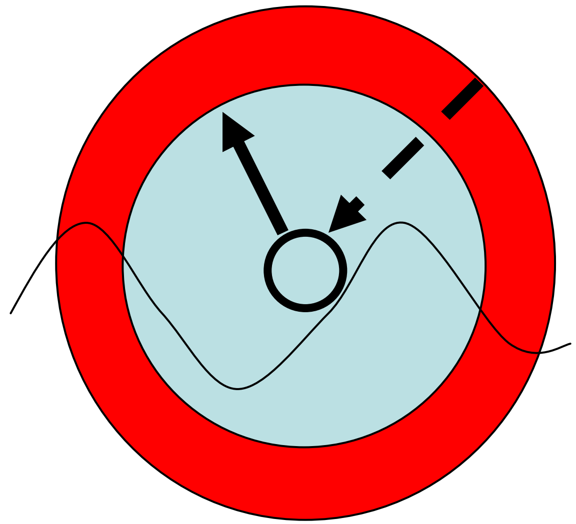


Causally disconnected regions ?

Inflation

- A period of acceleration in the early Universe

$$\ddot{a} > 0 \Leftrightarrow (aH)^{-1} \text{decreases}$$



- Inflation also solves the flatness problem .

- Inflation also provides an explanation for the origin of the primordial perturbations, which will give birth to structures in the Universe.

Scalar field inflation

- How to get inflation ? $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)\,.$
- Scalar field $S_\phi = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right)$
- Homogeneous equations

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

- Slow-roll motion $\frac{1}{2}\dot{\phi}^2 \ll V(\phi), \quad \ddot{\phi} \ll 3H\dot{\phi}$

$$P \simeq -\rho$$

Slow-roll regime

- Slow-roll equations

$$H^2 \simeq \frac{8\pi G}{3} V \qquad\qquad 3H\dot{\phi} + V' \simeq 0$$

- Slow-roll parameters

$$\epsilon_V \equiv \frac{m_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \qquad\qquad \eta_V \equiv m_P^2 \frac{V''}{V} \ll 1$$

- Number of e-folds

$$N = \ln \frac{a_{end}}{a} \qquad\qquad N(\phi) \simeq \int_{\phi}^{\phi_{end}} \frac{V}{m_P^2 V'} d\phi$$

Cosmological perturbations

- Matter: perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$

- Perturbed metric (with only scalar perturbations)

$$\begin{aligned} ds^2 &= -(1+2A)dt^2 + 2a(t)\nabla_i B\, dx^i dt \\ &+ a^2(t)\left[(1-2\psi)\gamma_{ij} + 2\nabla_i\nabla_j E\right] dx^i dx^j \end{aligned}$$

- ψ related to the intrinsic curvature of constant time spatial hypersurfaces ${}^{(3)}R = \frac{4}{a^2}\nabla^2\psi \quad [\kappa = 0]$

- Change of coordinates, e.g. $t \rightarrow t + \delta t$

$$\delta\rho \rightarrow \delta\rho - \dot{\rho}\delta t \qquad \psi \rightarrow \psi + H\delta t, \qquad H \equiv \frac{\dot{a}}{a}$$

Gauge-invariant quantities

- Curvature perturbation on uniform energy density hypersurfaces [Bardeen et al (1983)]

$$-\zeta \equiv \psi + \frac{H}{\dot{\rho}}\delta\rho = \psi - \frac{\delta\rho}{3(\rho + p)}$$

gauge-invariant

- Comoving curvature perturbation $\mathcal{R} = \psi - \frac{H}{\rho + p}\delta q$

- Using linearized Einstein's equations

$$\zeta = -\mathcal{R} - \frac{2\rho}{3(\rho + P)}\left(\frac{k}{aH}\right)^2 \Psi \qquad \left[A(t,\vec{x}) = \int \frac{d^3x}{(2\pi)^{3/2}} e^{i\vec{k}.\vec{x}} A_{\vec{k}}\right]$$

⇒ $\zeta \simeq -\mathcal{R}$ on large scales

Linear conserved quantity

- The time component of $\nabla_\mu T^{\mu\nu} = 0$ yields

$$\dot{\zeta} = -\frac{H}{\rho + P}\delta P_{\rm nad} - \frac{1}{3}\nabla^2\left(\dot{E} + v\right)$$

[Wands et al ‘00]

$$\delta P_{\rm nad} \equiv \delta p - \frac{\dot{p}}{\dot{\rho}}\delta\rho = \delta p - c_s^2\delta\rho$$

- For adiabatic perturbations, ζ and $\mathcal{R}$ are conserved on large scales

Perturbations during inflation

- Scalar field fluctuations: $\phi(t,\vec{x}) = \bar{\phi}(t) + \delta\phi(t,\vec{x})$
- The quantum fluctuations of the scalar field are amplified at **Hubble crossing** $(k = aH)$ $\delta\phi \simeq \frac{H}{2\pi}$
- This generates geometrical perturbations
$$\Phi \sim \frac{\delta a}{a} \sim \frac{\dot{a}}{a}\delta t \sim H\delta t \sim H\frac{\delta\phi}{\dot{\phi}} \sim \frac{H^2}{\dot{\phi}} \sim \frac{V^{3/2}}{m_P^3 V'}$$
- CMB: $\frac{\delta T}{T} \sim \Phi$

Example: $V = \frac{1}{2}m^2\phi^2 \quad \Rightarrow m \sim 10^{-5} m_P,\ V^{1/4} \sim 10^{16}\text{GeV}$

Some details…

- Massless scalar field

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) = \int d\tau\ d^3x\ a^4\left[\frac{1}{2a^2}\phi'^2 - \frac{1}{2a^2}\vec{\nabla}\phi^2\right]$$

using the conformal time $d\tau = dt/a$

- De Sitter spacetime $a(\tau) = -\frac{1}{H\tau}$

- New function $u = a\phi$

$$S = \frac{1}{2}\int d\tau\ d^3x\left[u'^2 - \vec{\nabla}u^2 + \frac{a''}{a}u^2\right]$$

$$m_{eff}^2 = -\frac{a''}{a} = -\frac{2}{\tau^2}$$

Quantum fluctuations

- Quantum field

$$\hat{u}(\tau,\vec{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k \left\{\hat{a}_{\vec{k}} u_k(\tau) e^{i\vec{k}.\vec{x}} + \hat{a}^{\dagger}_{\vec{k}} u^*_k(\tau) e^{-i\vec{k}.\vec{x}}\right\}$$

with $$\left[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}'}\right] = \left[\hat{a}^{\dagger}_{\vec{k}}, \hat{a}^{\dagger}_{\vec{k}'}\right] = 0, \quad \left[\hat{a}_{\vec{k}}, \hat{a}^{\dagger}_{\vec{k}'}\right] = \delta(\vec{k} - \vec{k}')$$

- The function u_k satisfies

$$u''_k + \left(k^2 - \frac{a''}{a}\right) u_k = 0$$

- Minkowski-like vacuum on small scales $k\tau \gg 1$

$$u_k = \sqrt{\frac{\hbar}{2k}} e^{-ik\tau}\left(1 - \frac{i}{k\tau}\right)$$

Quantum fluctuations

- Correlation function

$$\langle 0|\hat{\phi}(\vec{x}_1)\hat{\phi}(\vec{x}_2)|0\rangle = \int d^3k \; e^{i\vec{k}.(\vec{x}_1-\vec{x}_2)}\frac{\mathcal{P}_\phi(k)}{4\pi k^3}$$

- Power spectrum

$$\mathcal{P}_\phi(k) = \frac{k^3}{2\pi^2}\frac{|u_k|^2}{a^2} \simeq \hbar\left(\frac{H}{2\pi}\right)^2 \qquad (k \ll aH)$$

- With metric perturbations

$$\begin{aligned} S[\bar{\phi}+\delta\phi, g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}] \;&=\; S^{(0)}[\bar{\phi},\bar{g}_{\mu\nu}] + S^{(1)}[\delta\phi, h_{\mu\nu};\bar{\phi},\bar{g}_{\mu\nu}] \\ &\quad + S^{(2)}[\delta\phi, h_{\mu\nu};\bar{\phi},\bar{g}_{\mu\nu}], \end{aligned}$$

$$v = a\left(\delta\phi + \frac{\dot{\phi}}{H}\psi\right) = a\frac{\dot{\phi}}{H}\mathcal{R} \qquad\qquad \mathcal{P}_\mathcal{R} \simeq \frac{\hbar}{4\pi^2}\left(\frac{H^4}{\dot{\phi}^2}\right)_{k=aH}$$

Scalar perturbations from inflation

- The spectrum is quasi-scale invariant …

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 \simeq \frac{1}{24\pi^2}\left(\frac{V}{m_{\mathsf{P}}^4\epsilon}\right)_{k=aH}$$

$$\epsilon = \frac{m_{\mathsf{P}}^2}{2}\left(\frac{V'}{V}\right)^2$$

- but not quite …

$$n_s = \frac{d\,\mathsf{ln}\mathcal{P}_{\mathcal{R}}}{d\,\mathsf{ln}k} = -6\epsilon + 2\eta$$

$$\eta = m_{\mathsf{P}}^2\frac{V''}{V}$$

- Hint of a deviation
  from flat spectrum

WMAP: $n_s = 0.95 \pm 0.02$

Gravitational waves from inflation

- Metric fluctuations: gravitational waves

$$ds^2 = -dt^2 + a(t)^2 \left[\delta_{ij} + E^{TT}_{ij}\right] dx^i dx^j$$

2 polarisations

$$S_g = \int d^4x \sqrt{-g} \frac{R}{16\pi G}$$

- Spectrum: $\mathcal{P}_T = 2 \times \frac{4}{m_P^2} \left(\frac{H}{2\pi}\right)^2 = \frac{2H^2}{\pi^2 m_{\mathrm{P}}^2} \simeq \frac{2}{3\pi^2} \left(\frac{V}{m_{\mathrm{P}}^4}\right)_{k=aH}$

- Scale dependence: $n_T = \frac{d\ln\mathcal{P}_T}{d\ln k} = -2\epsilon$

- Consistency relation: $r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = -8n_T$

Observations: $r < 0.3$

Multi-field inflation

- So far, the data are compatible with the simplest models of inflation. Will it remain so in the future data ?

- Many high energy physics models involve several scalar fields.

- If several scalar fields are light enough during inflation
  ⇨ **multi-field inflation** !

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} \gamma_{IJ}(\phi)\, \partial^\mu \phi^I \partial_\mu \phi^J - V(\phi) \right)$$

Two-field inflation: double inflation

[Polarski & Starobinsky '92; D.L. '99]

- Two massive scalar fields minimally coupled to gravity

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{1}{2}\partial_\mu\phi_l\partial^\mu\phi_l - \frac{1}{2}\partial_\mu\phi_h\partial^\mu\phi_h - \frac{1}{2}m_l^2\phi_l^2 - \frac{1}{2}m_h^2\phi_h^2$$

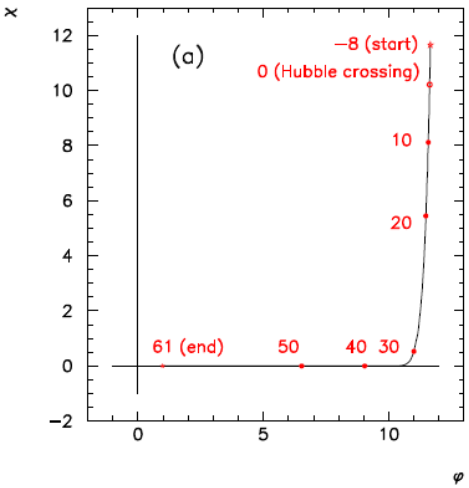
- Background equations of motion

$$3H^2 = 4\pi G(\dot\phi_l^2 + \dot\phi_m^2 + m_l^2\phi_l^2 + m_h^2\phi_h^2)$$
$$\ddot\phi_l + 3H\dot\phi_l + m_l^2\phi_l = 0, \qquad \ddot\phi_h + 3H\dot\phi_h + m_h^2\phi_h = 0.$$

- Slow-roll approximation

$$\phi_h = \sqrt{\frac{N}{2\pi G}}\sin\alpha, \qquad \phi_l = \sqrt{\frac{N}{2\pi G}}\cos\alpha$$

$$N = -\ln(a/a_e) = N_0\frac{(\sin\alpha)^{\frac{2}{R^2-1}}}{(\cos\alpha)^{\frac{2R^2}{R^2-1}}}, \qquad R \equiv \frac{m_h}{m_l}$$



Double inflation: perturbation equations

- Metric: $ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Phi)\delta_{ij}dx^i dx^j$

- Linearized Einstein and Klein-Gordon equations yield

$$
\begin{aligned}
\dot{\Phi} + H\Phi &= 4\pi G\left(\dot{\phi}_h \delta\phi_h + \dot{\phi}_l \delta\phi_l\right), \\
\delta\ddot{\phi}_h + 3H\delta\dot{\phi}_h + \left(\frac{k^2}{a^2} + m_h^2\right)\delta\phi_h &= 4\dot{\phi}_h\dot{\Phi} - 2m_h^2\phi_h\Phi, \\
\delta\ddot{\phi}_l + 3H\delta\dot{\phi}_l + \left(\frac{k^2}{a^2} + m_l^2\right)\delta\phi_l &= 4\dot{\phi}_l\dot{\Phi} - 2m_l^2\phi_l\Phi,
\end{aligned}
$$

- In the slow-roll approximation and for large scales,

$$
\begin{aligned}
\Phi &\simeq -\frac{C_1\dot{H}}{H^2} + 2C_3\frac{(m_h^2 - m_l^2)m_h^2\phi_h^2 m_l^2\phi_l^2}{3(m_h^2\phi_h^2 + m_l^2\phi_l^2)^2}, \\
\frac{\delta\phi_l}{\dot{\phi}_l} &\simeq \frac{C_1}{H} - 2C_3\frac{Hm_h^2\phi_h^2}{m_h^2\phi_h^2 + m_l^2\phi_l^2}, \qquad \frac{\delta\phi_h}{\dot{\phi}_h} \simeq \frac{C_1}{H} + 2C_3\frac{Hm_l^2\phi_l^2}{m_h^2\phi_h^2 + m_l^2\phi_l^2},
\end{aligned}
$$

From quantum fluctuations to "primordial" perturbations

- At Hubble radius crossing,

$$\mathcal{P}_{\delta\phi_h}(k) = \left(\frac{H}{2\pi}\right)^2_{k=aH}, \qquad \mathcal{P}_{\delta\phi_l}(k) = \left(\frac{H}{2\pi}\right)^2_{k=aH}$$

$$\left[\langle \delta\phi_{\vec{k}} \delta\phi^*_{\vec{k}'} \rangle = 2\pi^2 k^{-3} \mathcal{P}_{\delta\phi}(k) \delta(\vec{k} - \vec{k}')\right]$$

- Assume:
  - the light scalar field decays into ordinary matter
  - the heavy scalar field decays into dark matter

This will produce **adiabatic** and **isocurvature** perturbations in the post-inflation radiation era.

Adiabatic and isocurvature perturbations

- In the radiation era:
  - Adiabatic / curvature perturbations

$$\mathcal{R} \simeq -\zeta$$

  - Entropy / isocurvature perturbations

$$S \equiv \frac{\delta n_c}{n_c} - \frac{\delta n_\gamma}{n_\gamma} = \frac{\delta \rho_c}{\rho_c} - \frac{3}{4}\frac{\delta \rho_\gamma}{\rho_\gamma}$$

- They can be related to the perturbations during inflation:

$$\mathcal{R}_{\mathrm{rad}} \simeq -\frac{40\pi G}{9}\left(\phi_h \delta\phi_h + \phi_l \delta\phi_l\right)$$

$$S_{\mathrm{rad}} \simeq -\frac{2}{3H} m_h^2 \left(\frac{\delta\phi_h}{\dot{\phi}_h} - \frac{\delta\phi_l}{\dot{\phi}_l}\right)$$

⇨ **Correlation between adiabatic and isocurvature perts !**

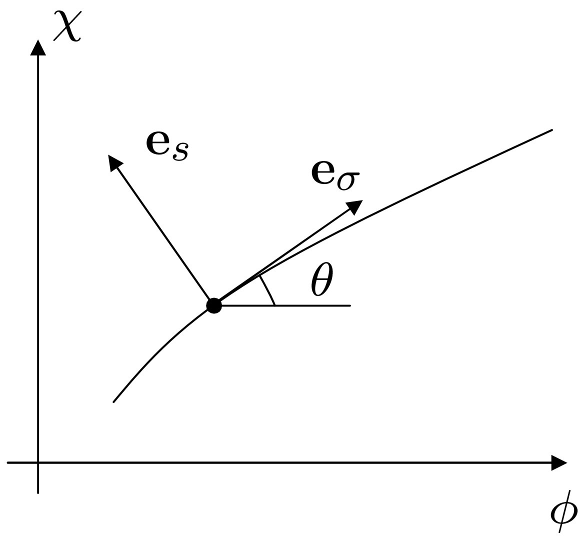
Generic two-field inflation

- Two scalar fields ϕ and χ with generic potential $V(\phi, \chi)$
- Introduce θ defined by [Gordon et al. PRD ’00]

$$(\dot\phi, \dot\chi) = \dot\sigma\,(\cos\theta, \sin\theta)\,, \qquad \dot\sigma \equiv \sqrt{\dot\phi^2 + \dot\chi^2}$$

- Decomposition on parallel and orthogonal directions with respect to the trajectory

$$\mathbf{e}_\sigma = (\cos\theta, \sin\theta)\,, \qquad \mathbf{e}_s = (-\sin\theta, \cos\theta)$$



- Background equations of motion

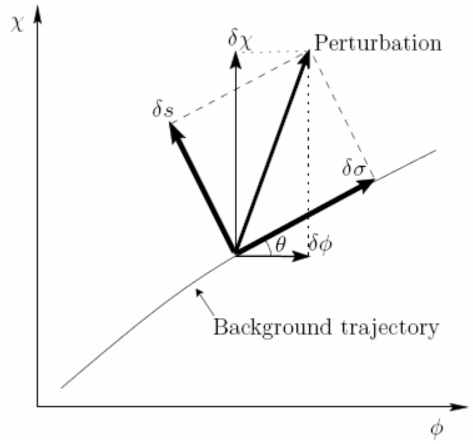
$$\ddot\sigma + 3H\dot\sigma + V_{,\sigma} = 0$$

$$\dot\sigma\dot\theta + V_{,s} = 0$$

Linear perturbations

- Similarly, the perturbations can be decomposed as

$$\delta\sigma = \cos\theta\ \delta\phi + \sin\theta\ \delta\chi,$$
$$\delta s = -\sin\theta\ \delta\phi + \cos\theta\ \delta\chi$$



[From Gordon et al. '00]

- Equations of motion for $Q \equiv \delta\sigma + \frac{\dot\sigma}{H}\psi$ and δs

$$\ddot{Q} + 3H\dot{Q} + \left[\frac{k^2}{a^2} + V_{\sigma\sigma} - \dot\theta^2 - \frac{8\pi G}{a^3}\left(\frac{a^3\dot\sigma^2}{H}\right)^{\cdot}\right] Q = 2(\dot\theta\delta s)^{\cdot} - 2\left(\frac{V_\sigma}{\dot\sigma} + \frac{\dot H}{H}\right)\dot\theta\delta s\,.$$

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{ss} + 3\dot\theta^2\right)\delta s = \frac{\dot\theta}{\dot\sigma}\frac{k^2}{2\pi G a^2}\Phi$$

Evolution on large scales

- Isocurvature perturbations decoupled from curvature perturbations

- Curvature perturbation is sourced by the isocurvature perturbation

$$\mathcal{R} = \frac{H}{\dot{\sigma}} Q = \psi + \frac{H}{\dot{\sigma}} \delta\sigma \qquad \dot{\mathcal{R}} = \frac{2H}{\dot{\sigma}} \dot{\theta} \delta s + \mathcal{O}\left(\frac{k^2}{a^2 H^2}\right)$$

  - Single field inflation: $\mathcal{R}$ conserved on large scales

$$\mathcal{P}_{\mathcal{R}}^{\mathrm{sf}}(k) \simeq \left(\frac{H^4}{4\pi^2 \dot{\sigma}^2}\right)_{k=aH}$$ evaluated at Hubble crossing

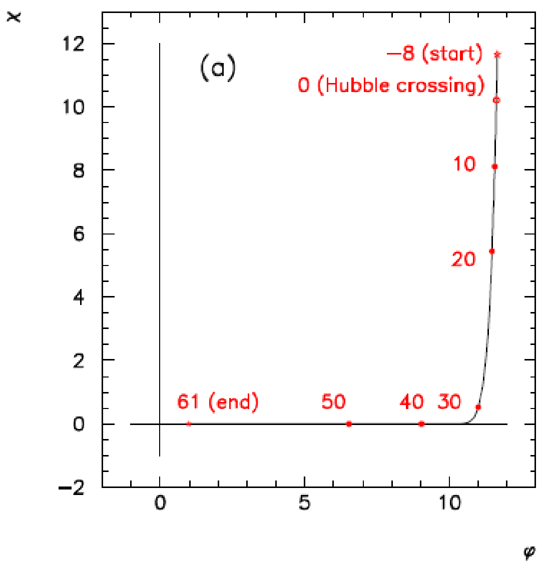
  - Multi-field inflation: $\mathcal{R}$ not conserved on large scales

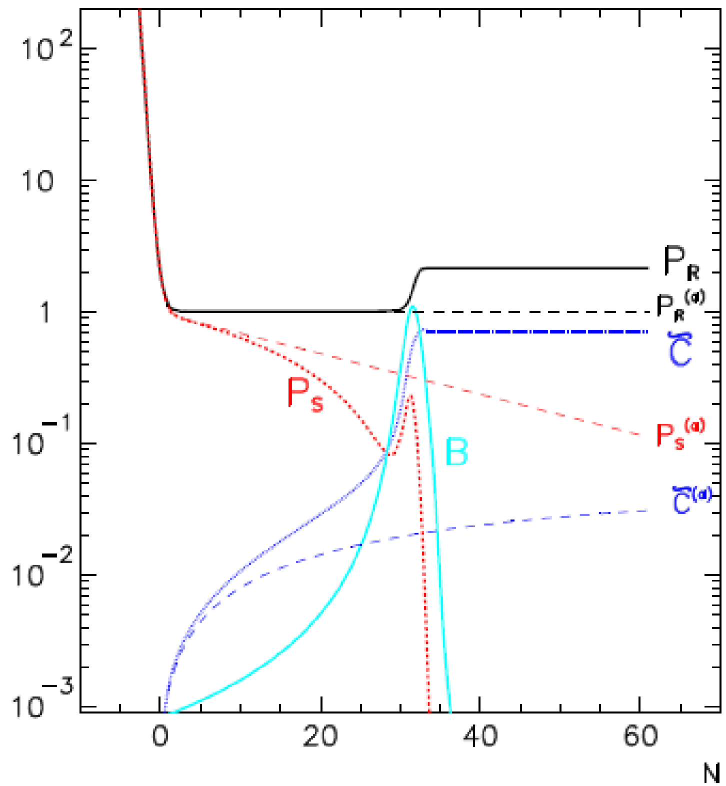
[Starobinsky, Yokayama ’95]

Numerical analysis

Double inflation model

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$



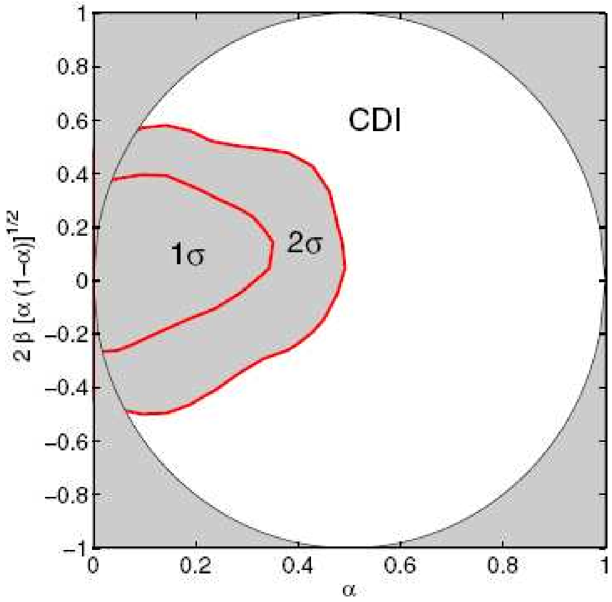


Lalak, DL, Pokorski, Turzynski ‘07

$$\mathcal{S} = \frac{H}{\dot\sigma}\delta s$$

Observational constraints

- Adiabatic and isocurvature produce different peak structures in the CMB
- Sachs-Wolfe effect $\frac{\delta T}{T} \simeq \frac{1}{5}(\mathcal{R}_{\rm rad} - 2\mathcal{S}_{\rm rad})$



$$\Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2}\langle \mathcal{R}^2_{\rm rad}\rangle = \frac{k_0^3}{2\pi^2}A^2\left(\frac{k}{k_0}\right)^{n_{\rm ad}-1},$$

$$\Delta^2_{S}(k) \equiv \frac{k^3}{2\pi^2}\langle S^2_{\rm rad}\rangle = \frac{k_0^3}{2\pi^2}B^2\left(\frac{k}{k_0}\right)^{n_{\rm iso}-1},$$

$$\Delta^2_{\mathcal{R}S}(k) \equiv \frac{k^3}{2\pi^2}\langle \mathcal{R}_{\rm rad} S_{\rm rad}\rangle = \frac{k_0^3}{2\pi^2}AB\cos\Delta_{k_0}\left(\frac{k}{k_0}\right)^{n_{\rm cor}+(1/2)(n_{\rm ad}+n_{\rm iso})-1}.$$

$$f_{\rm iso} = B/A \qquad \alpha = f^2_{\rm iso}/(1+f^2_{\rm iso}), \qquad \beta = \cos\Delta_{k_0}$$

$\alpha < 0.4$

[Beltran et al. PRD '05]

- **Hint of isocurvature modes ?**

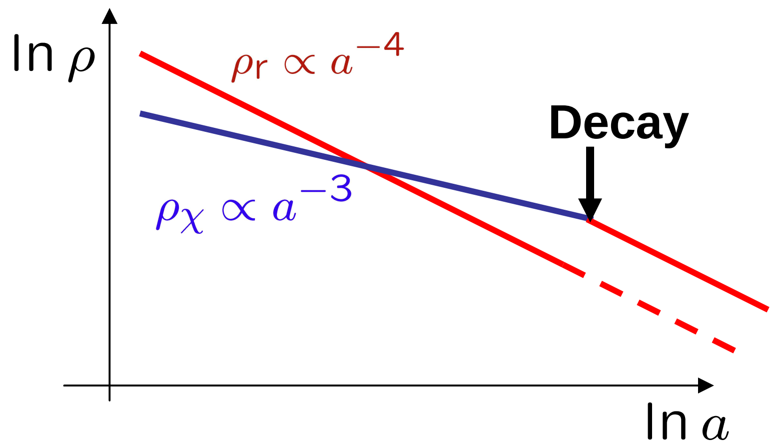
Keskitalo et al. astro-ph/0611917

The curvaton scenario

Mollerach (1990); Linde & Mukhanov (1997) ;
Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (2001)

Light scalar field during inflation (when H > m)
which later oscillates (when H < m),
and finally decays.

$$\rho_\chi = \frac{1}{2} m^2 \chi^2$$



The curvaton

- During inflation: fluctuations $\delta\chi$ with $\mathcal{P}_{\delta\chi} = \left(\frac{H}{2\pi}\right)^2$
- Oscillating phase: $\rho_\chi = \frac{1}{2}m^2\chi^2$

$$\zeta = \frac{\delta\rho}{3(\rho+p)} - \psi \qquad \zeta_\chi = \left(\frac{\delta\rho_\chi}{3\rho_\chi}\right)_{\text{flat}} \qquad \Rightarrow \mathcal{P}_{\zeta_\chi} \simeq \left(\frac{H}{3\pi\chi}\right)^2$$

- Decay: $\zeta = r_\chi \zeta_\chi$

$r_\chi \simeq 1$ if the curvaton dominates when it decays.

Mixed inflaton and curvaton perturbations

[DL, Vernizzi, PRD'04]

$$\zeta = \frac{H}{2\sqrt{2}\pi\, m_P}\left[\frac{1}{\sqrt{\epsilon}}\;\hat{e}_\phi + \frac{3}{\sqrt{2}} f(\chi)\hat{e}_\chi\;\right]$$

$$\langle \hat{e}_\phi\;\hat{e}_\chi \rangle = 0$$

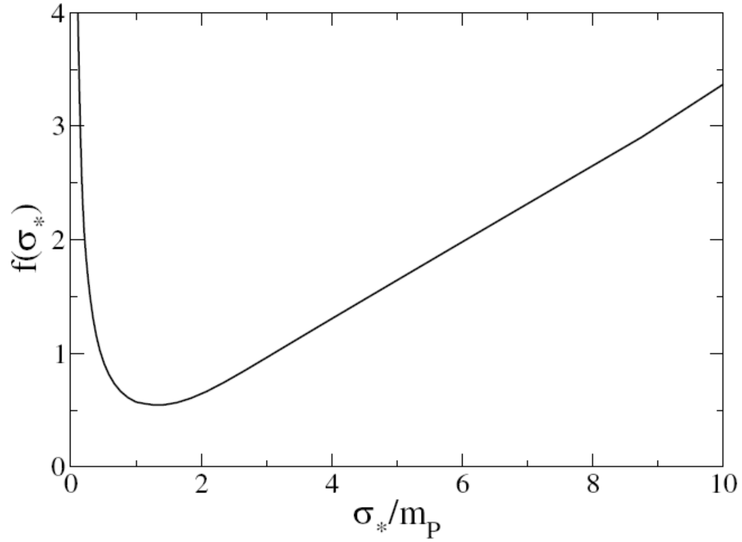
Interpolates between

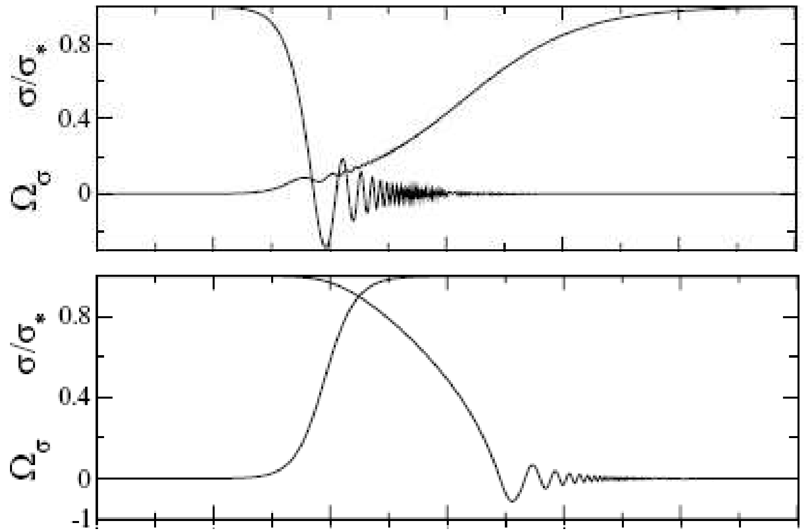
- pure curvaton case

$$f(\chi) \simeq \frac{4\, m_P}{9\,\chi}$$

- secondary inflaton case

$$f(\chi) \simeq \frac{\chi}{3\, m_P}$$





Conclusions

Multi-field inflation generates isocurvature perturbations in addition to adiabatic perturbations

- Isocurvature perturbations affect the evolution of the curvature perturbation (if the trajectory is bent).
- Depending on the models (reheating), the isocurvature perturbations can survive after inflation.
- In this case, the primordial adiabatic and isocurvature are in general correlated.
- An isocurvature contribution in the primordial perturbations can in principle be detected in cosmological observations.

⇨ additional window on the early universe physics