# Matter stability in the neutron star interior

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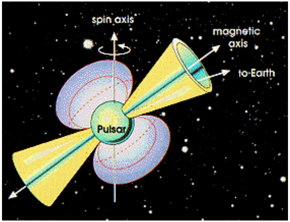
Cracow School of Theoretical Physics, 2007

- NS properties and internal structure
- parameters of nuclear matter, symmetry energy
- $E_s$ and the crust-core transition
- $E_s$ and the phase separation in the inner core



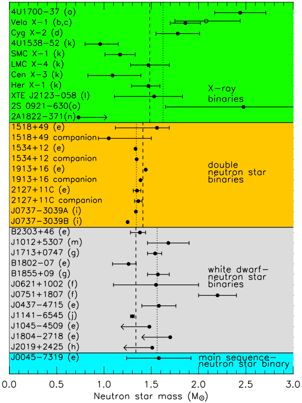

radio pulsars: period $P = 1$ ms - 5 s

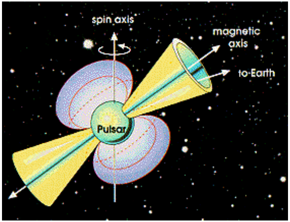
magn. dipole $B = 10^{9-12}$ Gs

mass $M = 1 - 2\ M_{\odot}$

radius $R = 10 - 20$ km

$\Rightarrow \overline{\rho} > \rho_{nuc}$



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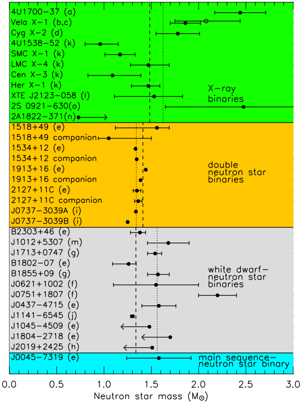
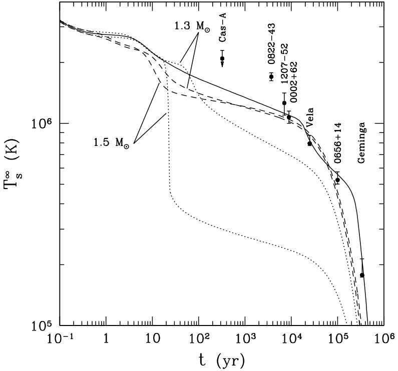



thermal X-ray emission

surface $T = 10^{5-6}$ K



| $g/cm^3$ | |
| --- | --- |
| $10^1$ | the edge of NS, *bcc* lattice of $^{56}Fe$ |
| $10^7$ | $e$ - relativistic, $\mu_e > m_n - m_p$ beta equilibrium starts (in nuclei)<br>neutron-rich nuclei, proton fraction $x \searrow$<br>$x(^{56}_{26}Fe) = 0.46$ $Ni$, $Se$, $Ge$, $\cdots$ $x(^{118}_{36}Kr) = 0.30$ |

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| $10^{11}$ | neutrons drip out of nuclei<br>two-phase system:<br>I: e + $^A_Z\mathcal{X}$, $(x > 0)$ II: e + neutrons (x=0)<br><br>partial Gibbs conditions for two-phase system:<br>$\mu^I_e = \mu^{II}_e$ , $\mu^I_n = \mu^{II}_n$ , $\mu^I_p < \mu^{II}_p$<br>protons confined in nuclei |
| $10^{13}$ | $\mu^I_p = \mu^{II}_p$ - proton drip, two phases with : $x^I \neq x^{II}$<br><br>nuclei dissolve *or* non-spherical structure: rods, plates ...<br>the lattice disappear - the inner edge of the crust |

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| $> 10^{14}$ | liquid *npl* matter, one-phase system<br>95% of NS mass |

## Basics on nuclear matter

binding energy of $(A, Z)$ nucleus

$$\frac{E_b}{A} = -w_0 + a_s(1 - 2\frac{Z}{A})^2 + a_s A^{-1/3} + a_C \frac{Z^2}{A^{4/3}} + \cdots$$

infinite nuclear matter with density $n_0$

$$\left.\frac{E}{A}\right|_{A\to\infty} = m_N - w_0 + a_s(1 - 2x)^2$$

for any barion density n

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NS matter - degenerated $n, p, e$ under beta equilibrium

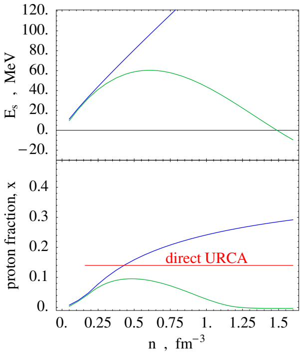
$$n \leftrightarrow e + p + \bar{\nu}_e \nearrow \quad \Rightarrow \quad \mu_n - \mu_p = \mu$$

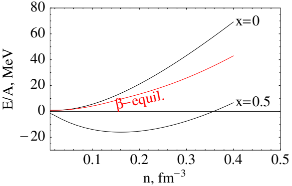
proton fraction at $n_0$ in $npe$ matter (NS liquid core)

$$4(1 - 2x)a_s = \mu \quad , \quad a_s = 30 \text{ MeV} \quad \Rightarrow \quad x(n_0) \approx 4\%$$

$E_s(n) \longrightarrow$ the chemical composition of matter
$V(n) \longrightarrow$ the stiffness of EoS

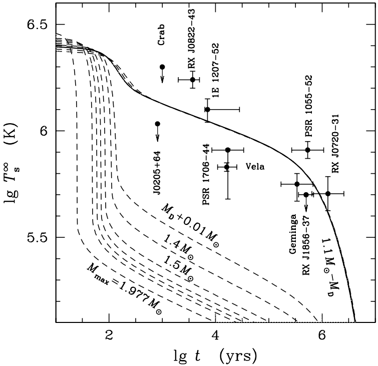
# The importance of the symmetry energy




slow or fast NS cooling

*Yakovlev'01*




saturation: $P = 0 \Rightarrow$ clusterization of symmetric matter (nuclei)

NS matter unstable under $P > 0$
need for proper stability conditions

## Stability conditions

the core $\longrightarrow$ crust transition - the instability point for homogeneous phase

$$du = -P\,dv \quad \Rightarrow \quad -\frac{\partial P}{\partial v} > 0 \qquad (T = 0)$$

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NS matter two component system: $B$ and $Q$

$$\longrightarrow du = -P\,dv - \mu\,dq \quad , \quad u = U/B \ , \ v = V/B \ , \ q = Q/B$$

stability of a single phase against n and q fluctuations
$\longrightarrow$ convexity of $u(v, q)$

$$-\left(\frac{\partial P}{\partial v}\right)_q > 0 \ , \ -\left(\frac{\partial \mu}{\partial q}\right)_P > 0 \qquad \text{or} \qquad -\left(\frac{\partial P}{\partial v}\right)_\mu > 0 \ , \ -\left(\frac{\partial \mu}{\partial q}\right)_v > 0$$

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the compressibility

$$K_i = \left(\frac{\partial P}{\partial n}\right)_i$$

the electrical capacitance

$$\chi_j = -\left(\frac{\partial q}{\partial \mu}\right)_j$$

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$$K_\mu < K_q \quad , \quad \chi_v < \chi_P$$

the symmetry energy $E_s \longrightarrow K_\mu$ , $\chi_v$

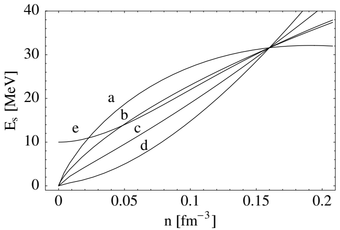
$$K_\mu = n^2\left(E_s''(1-2x)^2+V''\right)+2\,n\left(E_s'(1-2x)^2+V'\right) - \frac{2(1-2x)^2E_s'^2n^2}{E_s}$$

$$\chi_v = \frac{1}{8E_s(n)}+\frac{\mu(k_e+k_\mu)}{n\pi^2} \quad , \quad E_s(n)\searrow 0 - \text{specially interesting}$$

unknown behavior of $E_s$ / $E_s(n_0) = a_s \approx 30$ MeV /

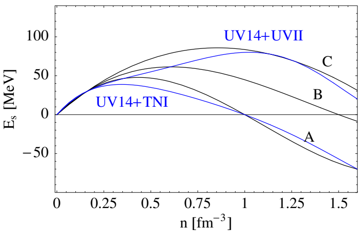
at low densities:

a-d - Chen,Ko,Li '05
e - Kowalski *et.al* '06

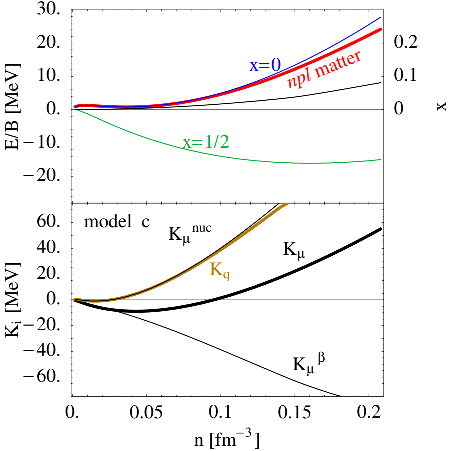


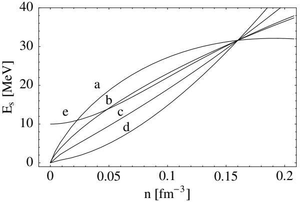

at high densities

realistic potentials
(Argonne, Urbana '88)



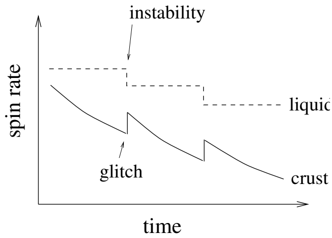
## the instability point at low density



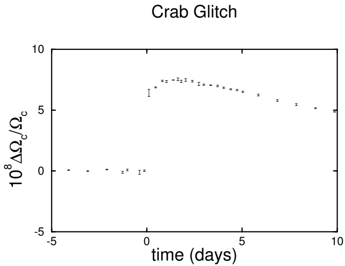




| model | $n_c$ fm$^{-3}$ | $n_c/n_0$ |
|---|---|---|
| a | 0.119 | 0.7 |
| b | 0.092 | 0.57 |
| c | 0.095 | 0.6 |
| d | 0.160 | 1. |
| e | 0.053 | 0.33 |

## Pulsar glitching and the crust size




*Lyne'92*




superfluid vortex unpinning
$\longrightarrow$ sudden increase in $\Omega$ - *glitch*

crustal moment of inertia
for Vela pulsar *Link'99*

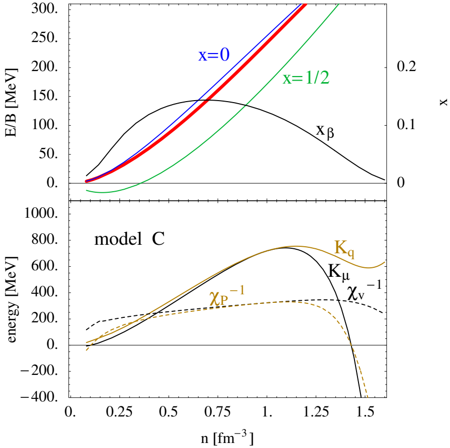
$$\frac{\Delta I_c}{I} > 1.4\%$$

different nuclear models /for $1.44 M_\odot$/

$$\frac{\Delta I_c}{I} = 1\% - 5\%$$

- critical density sensitive for $E_s$
- the NS crust size may constrain $E_s$ for $n < n_0$

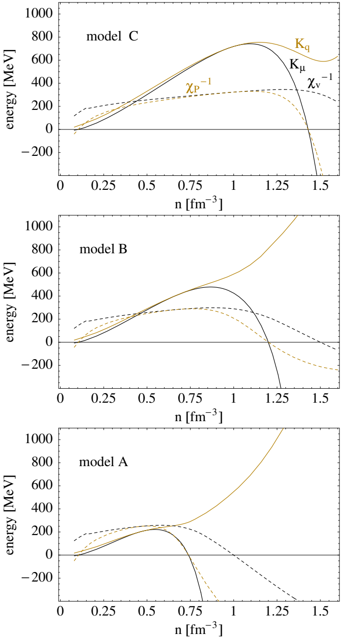
# Instability at high densities

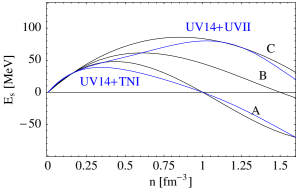



$K_q > 0$ - always
instability overlooked !

$K_\mu < 0$ above $n_c$, two phases
with different charge

## high densities *(contd)*




| | A | B | C |
|---|---|---|---|
| soft | | | |
| $n_c$ | 0.74 | 1.20 | 1.43 |
| $n_{cen}$ | 1.92 | 1.32 | 1.21 |
| $M_{max}/M_\odot$ | 1.64 | 1.73 | 1.84 |
| stiff | | | |
| $n_c$ | 0.85 | 1.40 | $> 1.6$ |
| $n_{cen}$ | 1.35 | 1.22 | 1.17 |
| $M_{max}/M_\odot$ | 2.02 | 2.08 | 2.13 |

- phase separation available in a sufficiently massive star

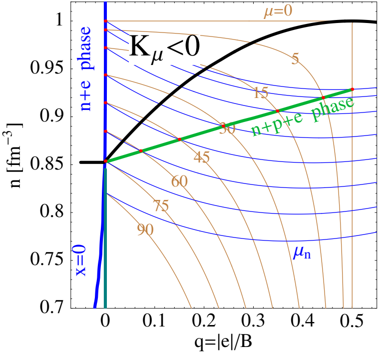
# Two-phase system

/nuclear model: stiff EoS + symmetry energy: A/

Construction of phase equilibrium

Gibbs conditions:

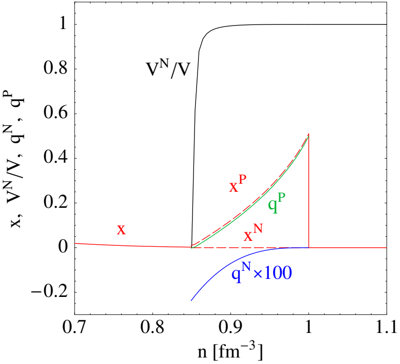
$\mu_e^N = \mu_e^P$ , $\mu_n^N = \mu_n^P$ , $\mu_p^N > \mu_p^P$




Mixed phase region:

$V^N/V$ - volume fraction of pure n phase

$q$ - charge per barion, $x$ - proton fraction



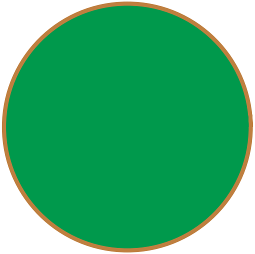

lattice formation (solid core ?)
or funny phases

## Massive neutron stars

multi-layer structure of NS core (model: stiff + A)

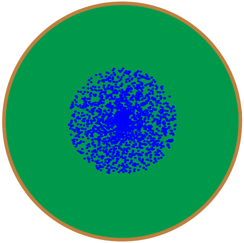
$R_{NS} = 10.5 - 9.2$ km $\quad \Delta R_{crust} \approx 0.3$ km

$M/M_{\odot} < 1.74$



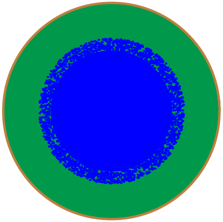
homogeneous, liquid core

$1.74 < M/M_{\odot} < 1.90$



mixed phase:
$<$ 20% of total mass

$1.90 < M/M_{\odot} < 2.02$



mixed phase $<$ 20%
pure n matter $<$ 35%

## SUMMARY

- simple, analytical relation between the $E_s$ and the critical density for phase separation
- glitching observations (crust thickness) may constraint the behavior of $E_s$ below $n_0$
- at high densities low $E_s$ leads to the phase separation, more rich structure of NS core, possible observational consequences:
  - new effects in NS rotation (second component in the NS core) relevant for pulsar precession and glitching
  - another type of cooling (*lattice URCA)*