Bifurcation in the Shakura Model

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Motivation

Compact objects

- Are the accretion disk models good enough to distinguish different types of the core objects?
- Do gravastars exist?
- Could we see them?

Inverse problem

What can we say about the compact object knowing the complete set of observational data?

Data collected from observations

A complete set of observational data

- Total luminosity
- Total mass
- Equation of state of the accreting gas
- Asymptotic temperature (speed of sound)
- Gravitational potential at the surface of the compact object

Our model

Assumptions

- Compact object with hard surface surrounded by a cloud of accreting gas
- Spherical symmetry
- Newtonian description
- Stationary accretion
- At the outer boundary of the fluid ball R_∞

$$U_\infty^2 \ll \frac{m(R_\infty)}{R_\infty} \ll a_\infty^2$$

- Sonic point

Definitions

- Mass accretion rate:

$$\dot{M} = -4\pi R^2 \rho U$$

- Gas equation of state (polytropic):

$$p = K\rho^\Gamma$$

$$1 < \Gamma \leq 5/3$$

- Speed of sound:

$$a^2 = \frac{\partial p}{\partial \rho}$$

- The Eddington luminosity:

$$L_E = \frac{GM}{\alpha}$$

$$\alpha = \frac{\sigma_T}{4\pi m_p c}$$

$$L_E = 1.25 \cdot 10^{38} \frac{M}{M_\odot} \frac{\mathrm{erg}}{\mathrm{s}}$$

Definitions

- The total luminosity:

$$L_0 = \dot{M}\phi_0$$

- The gravitational potential:

$$\phi(R) = -\frac{m(R)}{R} - \int_R^{R_\infty} r\rho(r)\mathrm{d}r$$

- Luminosity at a given point $L(R)$:

$$\partial_R \ln L = \frac{\alpha\dot{M}}{R^2}$$

$$L(R) = L_0 \exp\left(\frac{-L_0\tilde{R}_0}{L_E R}\right)$$

Definitions

- Modified size measure of the body:

$$\tilde{R}_0 = \frac{GM}{|\phi(R_0)|}$$

- Quasilocal mass:

$$\partial_R m(R) = 4\pi\rho R^2$$

- Total mass:

$$M = m(R_\infty)$$

Main equations

Modified Shakura model

- Euler equation:

$$U\partial_R U = -\frac{Gm(R)}{R^2} - \frac{1}{\rho}\partial_R p + \alpha\frac{L(R)}{R^2}$$

- Mass conservation:

$$\partial_R \dot{M} = 0$$

- Energy conservation:

$$L_0 - L(R) = \dot{M}\left(\frac{a_\infty^2}{\Gamma - 1} - \frac{a^2}{\Gamma - 1} - \frac{U^2}{2} - \phi(R)\right)$$

Sonic point

Sonic point

At the sonic point the infall velocity of the gas equals the speed of sound. Values measured at that place are denoted with an asterisk: $M_*, a_* = U_*$.

Sonic point variables

- $x = \dfrac{L_0}{L_E}$
- $y = \dfrac{M_*}{M}$
- $\gamma = \dfrac{\tilde{R}_0}{R_*}$

Luminosity as a function of sonic point mass

Total luminosity

$$L_0 = \phi_0 G^2 \pi^2 \chi_\infty \frac{M^3}{a_\infty^3} (1-y)(y - x\exp(-x\gamma))^2 \left(\frac{2}{5-3\Gamma}\right)^{\frac{5-3\Gamma}{2(\Gamma-1)}}$$

Using sonic point variables...

$$x = \beta(1-y)(y - x\exp(-x\gamma))^2.$$

Bifurcation

Theorem

Let us define a function

$$F(x,y) = x - \beta(1-y)(y - x\exp(-x\gamma)).$$

Then, for any β, $0 \leq \gamma < 1$ *and* x *smaller than a certain critical value:*

- *There exists a critical point* $x = a, y = b$ *of* F $(F(a,b) = 0$ *and* $\partial_y F(x,y)|_{(a,b)} = 0)$*. The parameters* a *and* b *satisfy* $0 < a, b < 1$ *and* $3b = 2 + a\exp(-a\gamma)$.
- *For any* $0 < x < a$ *there exist two solutions* $y(x)^{\pm}$*, bifurcating from* (a,b)*. They are locally approximated by:*

$$y^{\pm} = b \pm \frac{\sqrt{(a-x)(b + a\exp(-a\gamma)(1-2a\gamma))}}{\sqrt{\beta(b - a\exp(-a\gamma))(1 - a\exp(-a\gamma))}}.$$

- *The relative luminosity is extremized at the critical point* (a,b).

Bifurcation

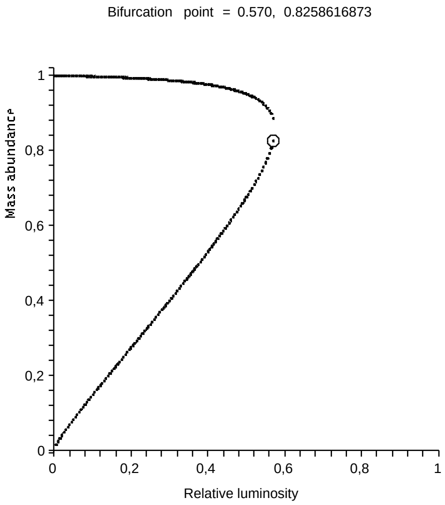

Figure: The plot of the dependence y of x.

Numerical example

Observed values

- Total mass is given in the units of solar mass $M_\odot = 1.989 \times 10^{33}\mathrm{g}$, $M = M_\odot(M/M_\odot)$.
- Eddington luminosity $L_E = 1.3 \times 10^{38}(M/M_\odot)\mathrm{erg/s}$ and for $x = 0.1$ we have $L_0 = 1.3 \times 10^{37}(M/M_\odot)\mathrm{erg/s}$.
- The asymptotic speed of sound $a_\infty = c/50 = 6 \times 10^8\mathrm{cm/s}$, the radius of the sphere enclosing the gas $R_\infty = 1.5 \times 10^{11}(M/M_\odot)\mathrm{cm}$ and the surface potential $\phi(R_0) = -0.25c^2 = -2.25 \times 10^{20}\mathrm{cm}^2/\mathrm{s}^2$.
- The modified size measure $\tilde{R}_0 = 6 \times 10^5(M/M_\odot)\mathrm{cm}$.

Numerical example

Two solutions

- Solution I:
  - (sonic point parameters) $R_* = 8.35 \cdot 10^7 (M/M_\odot)\mathrm{cm}$, $a_* = |U_*| = 8.46 \cdot 10^8 \mathrm{cm/s}$;
  - (size and mass of the hard core) $R_0 = 5.93 \cdot 10^5 (M/M_\odot)\mathrm{cm}$, $M_{\mathrm{core}} = 1.98 \cdot 10^{33} (M/M_\odot)\mathrm{g}$;
  - (asymptotic mass density) $\rho_\infty \approx 6 \cdot 10^{-4} (M_\odot/M)^2 \mathrm{g/cm^3}$.
- Solution II:
  - (sonic point parameters) $R_* = 9.15 \cdot 10^6 (M/M_\odot)\mathrm{cm}$, $a_* = |U_*| = 8.46 \cdot 10^8 \mathrm{cm/s}$;
  - (size and mass of the hard core) $R_0 = 1.18 \cdot 10^5 (M/M_\odot)\mathrm{cm}$, $M_{\mathrm{core}} = 3.92 \cdot 10^{32} (M/M_\odot)\mathrm{g}$;
  - (asymptotic mass density) $\rho_\infty \approx 1.1 \cdot 10^{-1} (M_\odot/M)^2 \mathrm{g/cm^3}$.

Numerical example

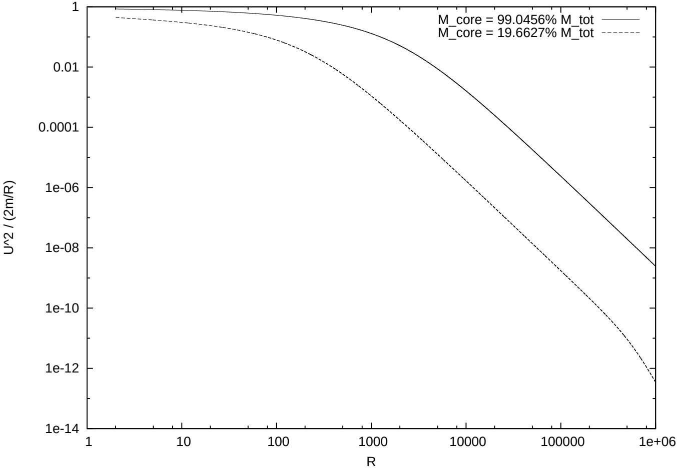

Figure: The speed of the infalling gas in case of the two solutions. $\frac{U^2}{2m(R)/R} = 1$ for the freely falling gas.

Conclusions

Nonuniqueness

- Two different core masses in systems, which have the same total mass and luminosity.
- If the total luminosity is close to the Eddington limit the two core masses are close to each other

Possible extensions

- The stability of the stationary solutions
- Time evolution (stationary initial data)
- Axisymmetry
- Relativistic generalization

Literature

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