

# Sources of Gravitational Waves

2<sup>nd</sup> talk

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# Astrophysical Sources of GWs

- Binary systems (NS/NS, NS/BH, BH/BH)
- Supernova
  - Bounce
  - Fall back
  - Oscillations & Instabilities
- Old and Isolated NS
- Cosmological origin

# GW sources in ground-based detectors



# Sources in LISA



# Binary systems (NS/NS, NS/BH, BH/BH)

# The best candidates and most reliable sources for broad band detectors





### **Coalescence of Compact Binaries**

<ul> <li>During the frequency change</li> </ul>	events/y ear	LIGO-I	LIGO-II
from 100-200Hz GWs carry	NS/NS	~0.05	~60-500
	BH/NS	~0.02	~80
In LIGOs band	BH/BH	~0.8	~2000
<ul> <li>NS/NS (~16000 cycles)</li> </ul>		~0.0	~2000
<ul> <li>NS/BH(~3500 cycles)</li> </ul>	Total	0.8	2000
• BH/BH(~600 cycles) $(2/3)^{2/3}$			
• The GW amplitude is: $h \approx 7.5 \times 10^{-23} \left( \frac{M}{2.8M} \right) \left( \frac{\mu}{0.7M} \right) \left( \frac{f}{100Hz} \right)^{1/2} \left( \frac{100Mpc}{r} \right)$			
<ul> <li>Larger total mass improves detection probability.</li> <li>Phase effects are important, if the signal and the template get out of phase their cross correlation will be reduced.</li> <li>High accuracy templates are needed for accurate detection.</li> </ul>			if the of phase duced. needed

### **Gravitational Waves from Binaries**

### Generically, there are 3 regimes in which black holes radiate:

- Orbital in-spiral: PNapproximations or point-particle orbits.
- Plunge/merger after the last stable orbit: numerical simulations or point-particle orbits.
- Ring-down of the disturbed black hole as it settles down to a Kerr hole: perturbation theory of black holes.





### **BH/BH coalescence**

- The inspiral, merger, and ringdown waves from 50M<sub>o</sub> BH binaries as observed by initial and advanced LIGO.
- The energy spectra are coming from crude estimates (10% of the total mass energy is radiated in merger waves and 3% in ringdown waves).
- We observe that the inspiral phase is not visible with initial LIGO, for this case Numerical Relativity is important.



### Possible First Source: Binary Black Hole Coalescence

- 10M<sub>☉</sub> + 10 M<sub>☉</sub>
   BH/BH binary
- Event rates based on population synthesis,
- mostly globular cluster binaries.
- Totally quiet!!



### **NS-BH** inspiral and NS Tidal Disruption



Based on Population Synthesis

- Initial interferometers
  - Range: 43 Mpc
  - 1/1000 yrs to 1per yr
- Advanced interferometers
  - Range: 650 Mpc
  - 2 per yr to several per day



### **Neutron Star Binary Inspiral**

#### NS-NS coalescence event rates

- Initial interferometers
  - Range: 20 Mpc
  - 1 per 40 yrs to 1 per 2 yrs
- Advanced interferometers
  - Range: 300Mpc
  - few per yr to several per day
- The discovery of a new binary pulsar have increased the rate upwards by an order of magnitude





# Core-collapse Supernova

The most spectacular astronomical event with exciting physics



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### Supernovae/gravitational collapse

Supernova core collapse was the primary source of GW detectors. GW amplitude uncertain by factors of 1,000's?

#### Rate 1/30yr in typical galaxy

Detection would provide unique insight into SN physics:

– optical signal hours after collapse

- neutrinos after several seconds
- GWs emitted <u>during</u> collapse

Simulations suggest low level of radiation (<10<sup>-6</sup> M<sub>o</sub>c<sup>2</sup>?), but

- rotational instabilities possible
- observational evidence for asymmetry from speeding final neutron stars (release of 10<sup>-6</sup> M<sub>☉</sub> c<sup>2</sup> could explain 1000 km/s?)
- convective "boiling" observable to LMC



# Core-Collapse Supernovae I

- Stars more massive than ~8M<sub>☉</sub> end in core collapse (~90% are stars with masses ~8-20M<sub>☉</sub>).
- Most of the material is ejected
- If M>20M<sub>o</sub> more than 10% falls back and pushes the PNS above the maximum NS mass leading to the formation of BHs (type II collapsars).
- If M>40M<sub>o</sub> no supernova is launched and the star collapses to form a BH (type I collapsars)
- Formation rate:
  - 1-2 per century in the Galaxy
  - 5-40% of them produce BHs through the fall back material
  - Limited knowledge of the rotation rate! Initial periods probably <20ms.</li>
  - **Chernoff & Cordes** fit the initial spin with a Gaussian distribution peaked at 7ms. This means that 10% of pulsars are born spinning with millisecond periods.

# Core-Collapse Supernovae II



# Core-Collapse Supernovae III

• GW amplitude

$$h \approx 10^{-23} \frac{10 Mpc}{r}$$

- Signals from Galactic supernova detectable.
- Frequencies ~1 kHz
- The numerical estimates are not conclusive.
- Kicks suggest that a fraction of newly born NSs (and BHs) may be strongly asymmetric.
- Polarization of the light spectra in SN indication of asymmetry.



### Fragmentation and Fallback

- A significant amount of remnants can fallback, subsequently spinning up and reheating the nascent NS.
- Instabilities can be excited again during such a process.
- BH-QNMs can be excited for as long as the process lasts.
- "Collapsars" accrete initially (for about ~2-3s) at rates ~1-10M\_ $_{\odot}$  /sec ! Later at a rate ~0.1M\_ $_{\odot}$  /sec for a few tenths of secs.
- Typical frequencies: ~2kHz.
- Oscillation of matter surrounding the black hole (Zanetti et al 2002)
- If disk mass is :~1M<sub>☉</sub> self-gravity becomes important and gravitaional instabilities (spiral arms, bars) might develop and radiate GWs (Davies et al 2002, Fryer et al 2002)
- The collapse material might fragments into clumps, which orbit for some circles like a binary system (Fragmentation Instability). Needs density distribution to peaks off the center (maybe in Population III stars).

# **Black-Hole Ringing I**

- The newly formed BH is ringing till settles down to the stationary Kerr state (QNMs).
- The ringing due to the excitation by the fallback material might last for secs
- Typical frequencies: ~1-3kHz



$$f_{m=2} \approx 3.2 \text{kHz} \ M_{10}^{-1} \Big[ 1 - 0.63 (1 - a/M)^{3/10} \Big]$$
  
 $Q = \pi f \tau \approx 2(1 - a)^{-9/20}$ 

- The amplitude of the ringdown waves and their energy depends on the distortion of the BH.
- Energy emitted in GWs by the falling material: ΔE>0.01µc<sup>2</sup>(µ/M)

$$h_c \approx 2 \times 10^{-21} \left(\frac{\varepsilon}{0.01}\right) \left(\frac{d}{10Mpc}\right)^{-1} \left(\frac{\mu}{M}\right)$$



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# **Oscillations & Instabilities**

The end product of gravitational collapse

### **Neutron Stars**

- Suggested:
- Discovered: 1967

1932

- Known: 1070+
- Mass: ~ 1.3-1.8 M<sub>☉</sub>
- Radius: ~ 8-14 Km
- Density: ~10<sup>15</sup>gr/cm<sup>3</sup>





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### An extreme challenge

#### Neutron star modelling involves the very extremes of physics:



Can GW, x-ray, γ-ray observations constrain the theoretical models?

### **Neutron Star "ringing**

- p-modes main restoring force is the pressure (f-mode) (can become unstable)
   f<sup>2</sup> ~ M/R<sup>3</sup> (>1.5 kHz)
- Inertial modes (r-modes) main restoring force is the Coriolis force (can become unstable) f~Ω
- **Torsional modes** (t-modes)  $f^2 \sim u_s/R$  (>20 Hz) shear deformations, divergence-free, with no-radial components. Restoring force, the weak Coulomb force of the crystal ions.
- w-modes: pure space-time modes (only in GR) (can become unstable) f ~ 1/R (>5kHz)





# F-mode-(I)

- F-mode is the fundamental pressure mode of the star
- It corresponds to polar perturbations
- Frequency for uniform density stars

$$\omega^{2} = \frac{2l(l-1)}{2l+1} \frac{GM}{R^{3}}$$
growth time(if unstable)  

$$t_{GW} \approx f(l)R \left(\frac{R}{M}\right)^{l+1} \sim 0.07 \left(\frac{1.4M_{\odot}}{M}\right)^{3} \left(\frac{R}{10km}\right)^{4} \sec l$$
For  $\ell = 2$  is  $\sim 2-4$ kHz

- Rotation breaks the symmetry: the various - ℓ ≤m≤ℓ decouple
- There is coupling between the polar and axial modes
- The frequency shifts:

$$\omega_{\rm inert}(\Omega) = \omega(\Omega = 0) + \kappa m \Omega$$



### The r-mode-(I)

- A non-rotating star has only trivial axial • modes
- Rotation provides a restoring force (Coriolis) and leads in the appearance of the inertial modes.
- The *I=m=2* inertial mode is called r-mode
- In a frame rotating with the star, the rmodes have frequency

$$\omega_{\rm rot} = \frac{2m}{l(l+1)}\Omega$$

 Meanwhile in the inertial frame  $\frac{\omega_{\text{inertial}}}{\omega_{\text{rot}}} = -\frac{\omega_{\text{rot}}}{\omega_{\text{rot}}} + \Omega = \Omega \left(1 - \frac{2}{\omega_{\text{rot}}}\right)$ 

m

l(l+1)ograde in the r-modes are ap rotating system while in the inertial frame the prograde at all rotation rates!



#### **R-modes have:**

$$\delta u^{\varphi} \sim \Omega$$

$$\delta u^r, \delta u^{\theta}, \delta \rho \sim \Omega^2$$

m

# Torsional (t-) modes



Giant flares in SGRs

- Up to now, three giant flares have been detected.
  - SGR 0526-66 in 1979, SGR 1900+14 in 1998, SGR 1806-20 in 2004
- Peak luminosities : 10<sup>44</sup>–10<sup>46</sup> erg/s
- A decaying tail for several hundred seconds follows the flare.
- QPOs in decaying tail (Israel *et al.* 2005; Watts & Strohmayer 2005, 2006)

SGR 1900+14 : 28, 54, 84 & 155 Hz

SGR 1806-20 : 18, 26, 29, 92.5, 150, 626.5, and 1837 Hz

$$h \approx 10^{-25} - 10^{-28} \left( \frac{10 \, kpc}{r} \right)$$

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### GW and SGRs

- The energy relesed during the 2004 hyperflare is of the order of
  - 8x10<sup>46</sup> 3x10<sup>47</sup> erg or
  - 4x10<sup>-8</sup> 2x10<sup>-7</sup> M<sub>☉</sub>c<sup>2</sup>
  - If the same amount of energy was released in GWs then the signal would have been marginally detectable by LIGO
  - Sensitivity in 2004 about 8 times smaller than H1



$$E_{GW}^{iso,90\%} = 4.3 \times 10^{-8} M_{sun} c^2 \times \left(\frac{r}{10 \text{kpc}}\right)^2 \left(\frac{f_{QPO}}{92.5 \text{Hz}}\right)^2 \left(\frac{h_{rss-\text{det}}^{90\%}}{4.5 \times 10^{-22} \text{ strain Hz}^{-1/2}}\right)$$

# Grav. Wave Asteroseismology



### Grav. Wave Asteroseismology



### **Stability of Rotating Stars**

Non-Axisymmetric Perturbations

A general criterion is:

$$\beta = \frac{T}{W}$$

T : rot. kinetic energyW : grav. binding energy

#### Dynamical Instabilities

- Driven by hydrodynamical forces (bar-mode instability)
- Develop at a time scale of about one rotation period

$$\beta \ge 0.27$$

#### **Secular Instabilities**

- Driven by dissipative forces (viscosity, gravitational radiation)
- Develop at a time scale of several rotation periods.
- Viscosity driven instability causes a Maclaurin spheroid to evolve into a non-axisymmetric Jacobi ellipsoid.
- Gravitational radiation driven instability causes a Maclaurin spheroid to evolve into a stationary but nonaxisymmetric Dedekind ellipsoid.



### The bar-mode instability I

#### For rapidly (differentially!) rotating stars with:

 $\beta \ge 0.27$ 

the "<u>bar-mode</u>" grows on a dynamical timescale.

$$h \approx 9 \times 10^{-23} \left(\frac{\varepsilon}{0.2}\right) \left(\frac{f}{3 \text{ kHz}}\right)^2 \left(\frac{15 \text{ Mpc}}{\text{d}}\right) M_{1.4} R_{10}^2$$

If the bar persists for many (~10-100) rotation periods, the signal will be easily detectable from at least Virgo cluster.

-A considerable number of events per year in Virgo:  $\leq 10^{-2}$  /yr/Galaxy

#### -Frequencies ~1.5-3.5kHz



Remember mini–Grail: f<sub>0</sub>~3.2kHz

# Bar Modes IV

- Bars can be also created during the merging of NS-NS, BH-NS, BH-WD and Collapsars (type II).
- GR enhances the onset of the instability ( $\beta_{dyn} \ge 0.24$ ) and  $\beta$  decreases with increasing *M*/*R*.
- Bar-mode instability might happen for much smaller  $\beta$  if centrifugal forces produce a peak in the density off the source's rotational center.
- Highly differentially rotating stars are shown to be dynamically unstable for significantly lower  $\beta$  (even when  $\beta \sim 0.01$ ).

$$h_{eff} = 3 \times 10^{-22} \left(\frac{f}{800 Hz}\right)^{1/2} \left(\frac{R_{eq}}{30 km}\right) \left(\frac{M}{1.4 M_{\odot}}\right)^{1/2} \left(\frac{100 Mpc}{d}\right)$$



### The pattern speed

The pattern speed σ of a mode is:

$$\frac{d\varphi}{dt} = -\frac{\omega}{m} = \sigma$$

$$\begin{split} &\omega_{\rm inert} = \omega_{\rm rot} + m\Omega \\ &\sigma_{\rm inert} = \sigma_{\rm rot} + \Omega \end{split}$$

 If a star rotates very fast, a backward moving mode, might change to move forward, according to an inertial observer.



### **Growth vs Damping**

- Viscosity tends to suppress a GW instability.
- An instability is only relevant if it grows sufficiently fast that is not completely damped by viscosity
- **Bulk viscosity**: arises because the pressure and density variations associated with the mode oscillation drive the fluid away from beta equilibrium. It corresponds to an estimate of the extent to which energy is dissipated (via neutrino emission) from the fluid motion as the weak interaction tries to re-establish equilibrium.
- Shear viscosity: in matter hotter than superfluid transition temperature T~10<sup>9</sup> K, due to neutron-neutron scattering, and for superfluids, due to <u>electron-electron scattering</u>

$$\frac{1}{2E}\frac{dE}{dt} = \frac{1}{\tau_{GW}} + \frac{1}{\tau_{BV}} + \frac{1}{\tau_{SV}}$$
$$E = \frac{1}{2}\int \rho \left|\dot{\xi}\right|^2 dV$$

### Timescales

Dissipation due to bulk viscosity

$$\frac{dE}{dt}\Big|_{_{\rm BV}} = \int \zeta \left|\delta\sigma\right|^2, \ \delta\sigma = -i(\omega + m\Omega)\frac{\Delta p}{\Gamma p}, \ \zeta \sim \left(\frac{T}{10^9 K}\right)^6$$

• Dissipation due to shear viscosity

$$\left(\frac{dE}{dt}\right)_{sv} = -2\int \eta \delta \sigma^{ab} \delta \sigma^*_{ab} dV$$
  
$$\delta \sigma = -i \frac{(\omega + m\Omega)}{2} \left(\nabla_a \xi_b + \nabla_b \xi_a - 2g_{ab} \nabla_c \xi^c\right), \quad \eta \sim \left(\frac{T}{10^9 K}\right)^{-2}$$

Dissipation/growth due to gravitational radiation

$$\left(\frac{dE}{dt}\right)_{GW} = -(\omega + m\Omega)\sum_{l=2}^{\infty} N_l \omega^{2l+1} \left(\left|\delta D_{lm}\right|^2 + \left|\delta J_{lm}\right|^2\right)$$
$$\delta D_{lm} = \int \delta \rho r^l Y_{lm}^* dV, \quad \delta J_{lm} = 2\left(\frac{l}{l+1}\right)^{1/2} \int r^l \left(\rho \delta \upsilon + \upsilon \delta \rho\right) \overline{Y}_{lm}^{*B} dV$$

### R-mode: Instability window





### R-modes (astrophysics)

- GW amplitude depends on  $\alpha$  (the saturation amplitude).
- Mode coupling might not allow the growth of instability to high amplitudes
- The existense of crust, hyperons in the core, magnetic fields, affect the efficiency of the instability.
- For newly born neutron stars might be quite weak ; unless we have the creation of a strange star
- Old accreting neutron (or strange) stars, probably the best source! (400-600Hz)



#### Lindblom-Vallisneri-Tohline

$$h(t) \approx 10^{-22} \alpha \left(\frac{\Omega}{1 \text{ kHz}}\right) \left(\frac{1 \text{Mpc}}{d}\right)$$
$$\alpha \approx 10^{-2} - 10^{-3}$$

### F-mode (astrophysics)

- F-mode is naturally excited in any process.
- In GR the *m*=2 mode becomes unstable for  $\Omega > 0.85\Omega_{Kepler}$  or  $\beta > 0.06-0.08$
- The instability window significantly smaller than the r-mode
- Detectable from as far as 15Mpc (LIGO-I), 100Mpc (LIGO-II) (*depending on the saturation amplitude*).
- Differential rotation affects the onset of the instability
- Recent non-linear calculations by Shibata & Karino (2004) suggest that:
  - Up to 10% of energy and angular momentum will be dissipated by GWs.
  - Amplitude (ar ~500Hz):



# Isolated & Old NS





### Slowdown from pulsar



•Upper limits on amplitudes from known pulsars, set by assuming spindown due to the emission of gw energy. The points represent all pulsars with gravitational wave frequencies above 7 Hz and amplitudes above 10<sup>-27</sup>.

• Expected sensitivities of three first-generation interferometers in a one-year observation, and the thermal noise limits on narrow-banding (dotted lines).

### The Wagoner mechanism (1984) Papaloizou & Pringle (1978)

Key idea: Emission of GW balances accretion torque. Strength of waves can be inferred from Xray flux. Requires deformation:

$$\varepsilon = 4.5 \times 10^{-8} \left( \frac{\dot{M}}{10^{-9} M / yr} \right)^{1/2} \left( \frac{300 \text{ Hz}}{v_s} \right)^{5/2}$$

Observational evidence (?):

clustering of spin-frequencies in LMXB (250–590 Hz)

Possible GW mechanisms:

- accretion induced asymmetry

 unstable r-modes: strong bulk viscosity may shift instability window to lower temperatures; accreting stars can reach quasi-equilibrium state

Variable accretion rate: coherent integration of signal only meaningful for 20 days or so.



# LIGO narrow banding



# Stochastic Background



### Stochastic Background of Grav. Waves

• Energy density:

$$\rho_{GW} = \frac{c^2}{32\pi G} < \dot{h}_{ab} \dot{h}^{ab} >$$

• Characterized by logfrequency spectrum:

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}(f)}{d\ln f}$$

$$S(f) = \frac{3H_0^2}{10\pi^2} \frac{\Omega_{GW}(f)}{f^3}$$

• Strain scale:  $h(f) = 6.3 \times 10^{-22} \sqrt{\Omega_{GW}(f)} \left(\frac{100 \text{ Hz}}{f}\right)^{3/2} \text{ Hz}^{-1/2}$ 

### GWs from the Big Bang

Stochastic background reflecting fundamental physics in the early universe;

- Phase transitions
- Inflation
- Topological defects
- String-inspired cosmology
- Higher dimensions

After the Big Bang, photons decoupled after 10<sup>5</sup> years, neutrinc after 1s, **GWs before10**<sup>-24</sup> s!

Strength expressed as fraction of closure energy density;

 $10^{-14} \approx \Omega_{gw} < 10^{-5}$ 

simple inflation nucleosynthesis



**Detection:** Requires cross-correlation of detectors. Best window, free of "local" GW sources, is around 0.1-1 Hz. Need LISA follow-on mission?

$$\Omega_{gw} = \frac{f}{\rho_c} \frac{d\rho_{gw}}{df} \qquad \rho_c = \frac{3H_0^2}{8\pi G} \qquad h_c \approx 10^{-18} \left(\frac{1Hz}{f}\right) \sqrt{h_0^2 \Omega_{gw}(f)}$$

One of the most fundamental observations possible!

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