Gravitational Waves

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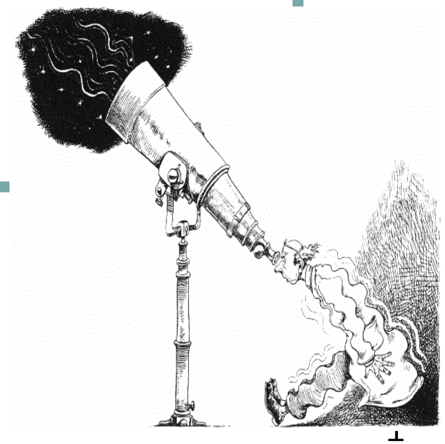


About the lectures…

- Theory of Gravitational Waves
- Gravitational Wave Detectors
  - Signal Analysis
- Sources of Gravitational Waves

Gravitational Waves

- **Why Gravitational Waves?**
  - Fundamental aspect of General Relativity
  - Originate in the **most violent events** in the Universe
  - Major **challenge to present technology**
- **Why we have not seen them yet?**
  - They **carry enormous amount of energy but**
  - They **couple very weakly** to detectors.
- **How we will detect them?**
  - **Resonant Detectors** (Bars & Spheres)
  - **Interferometric Detectors** on Earth
  - **Interferometers** in Space

Gravitational vs E-M Waves

- **EM waves are radiated by individual particles**, **GWs are due to non-spherical bulk motion of matter**. I.e. the information carried by EM waves is stochastic in nature, while the GWs provide insights into coherent mass currents.
- **The EM will have been scattered many times**. **In contrast, GWs interact weakly with matter and arrive at the Earth in pristine condition**. Therefore, GWs can be used to probe regions of space that are opaque to EM waves. Still, the weak interaction with matter also makes the GWs fiendishly hard to detect.
- Standard astronomy is based on **deep imaging of small fields of view**, while **gravitational-wave detectors cover virtually the entire sky**.
- **EM radiation has a wavelength smaller than the size of the emitter**, **while the wavelength of a GW is usually larger than the size of the source**. Therefore, we cannot use GW data to create an image of the source. GW observations are more like audio than visual.

Neutrinos: are more like EM waves than GW in most respects, except…
Propagate through most things like GW, so you can see dense centers
But neutron stars don’t generate so many ν after first few minutes

Uncertainties and Benefits

- **Uncertainties**
  - The strength of the sources (may be orders of magnitude)
  - The rate of occurrence of the various events
  - The existence of the sources

- **Benefits**
  - **Information about the Universe that we are unlikely ever to obtain in any other way**
  - Experimental tests of fundamental laws of physics which cannot be tested in any other way
  - **The first detection of GWs will directly verify their existence**
  - By comparing the arrival times of EM and GW bursts we can measure their speed with a fractional accuracy $\sim 10^{-11}$
  - **From their polarization properties of the GWs we can verify GR prediction that the waves are transverse and traceless**
  - **From the waveforms we can directly identify the existence of black-holes.**

Information carried by GWs

- **Frequency**

$$f \sim 10^4 Hz \Rightarrow \rho \sim 10^{16} gr/cm^3$$

$$f \sim 10^{-4} Hz \Rightarrow \rho \sim 1 gr/cm^3$$

$$f_{dyn} \sim \left(\frac{GM}{R^3}\right)^{1/2} \sim (G\rho)^{1/2}$$

- **Rate of frequency change**

$$\dot{f}/f \sim (M_1, M_2)$$

- **Damping**

$$\tau \sim M^3/R^4$$

- **Polarization**
  - Inclination of the symmetry plane of the source
  - Test of general relativity
- **Amplitude**
  - Information about the strength and the distance of the source (h~1/r).
- **Phase**
  - Especially useful for detection of binary systems.

GW Frequency Bands

- High-Frequency: 1 Hz – 10 kHz
  - (Earth Detectors)
- Low-Frequency: 10^{-4} – 1 Hz
  - (Space Detectors)
- Very-Low-Frequency: 10^{-7} – 10^{-9} Hz
  - (Pulsar Timing)
- Extremely-Low-Frequency: 10^{-15}–10^{-18} Hz
  - (COBE, WMAP, Planck)

Gravitation & Spacetime Curvature

Newton

$$\nabla^2 U = 4\pi G\rho$$

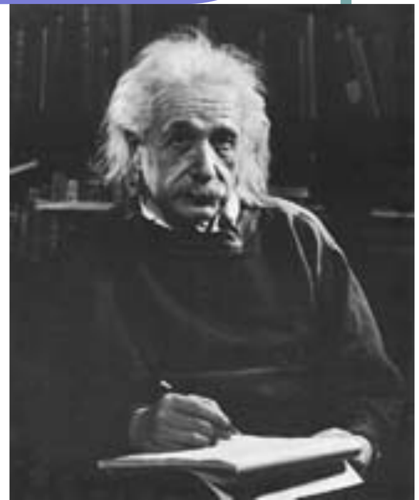
$$\frac{d^2\vec{x}}{dt^2} = \nabla U$$

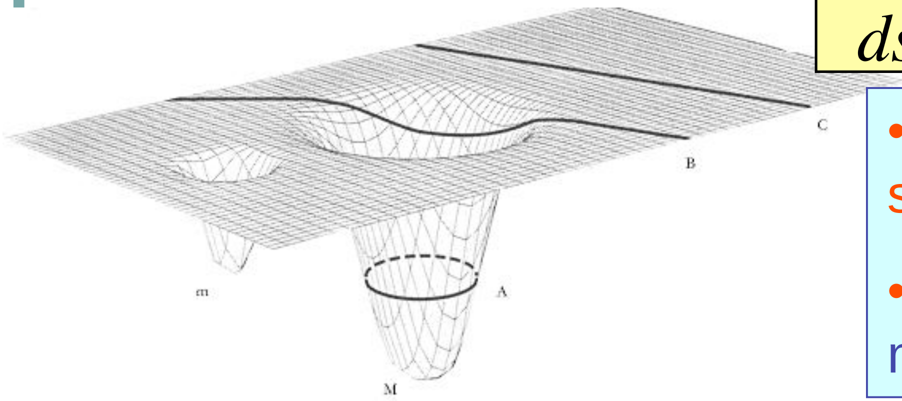
Einstein

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

$$\frac{d^2x^\mu}{ds^2} \sim f(g^{\mu\nu})$$





- Matter dictates the degree of spacetime deformation.
- Spacetime curvature dictates the motion of matter.

GWs fundamental part of Einstein's theory

Linearized Gravity

- Assume a **small perturbation** on the background metric:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| << 1$$

- The **perturbed Einstein's equations** are:

$$h_{\alpha\beta;\mu}{}^{;\mu} + g_{\alpha\beta} h^{\mu\nu}{}_{;\nu\mu} - 2h_{\mu(\alpha}{}^{;\mu}{}_{;\beta)} + 2R_{\mu\alpha\nu\beta} h^{\mu\nu} - 2R_{\mu(\alpha} h_{\beta)}{}^{\mu} = kT_{\alpha\beta}$$

- Far from the source (**weak field limit**)…
- And by **choosing a gauge**:

$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} n_{\mu\nu} h_{\alpha}{}^{\alpha}$$

$$h^{\mu\nu}{}_{;\nu} = 0$$

- **Simple wave equation**:

$$(-\frac{\partial^2}{\partial^2 t} + \nabla^2) h^{\mu\nu} = \partial_\lambda \partial^\lambda h^{\mu\nu} = kT^{\mu\nu}$$

Lorentz or De Donder gauge

Transverse-Traceless (TT)-gauge

- **Plane wave solution**

$$\tilde{h}^{\mu\nu} = A^{\mu\nu} e^{ik_a x^a}$$

$$A^{\mu\nu} k_\mu = 0$$

$$k^\mu k_\mu = 0$$

- TT-gauge (wave propagating in the z-direction)

$$A^{\mu\nu} = h_+ \varepsilon_+^{\mu\nu} + h_\times \varepsilon_\times^{\mu\nu}$$

$$\varepsilon_+^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \varepsilon_\times^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- **Riemann tensor**
- **Geodesic deviation**

$$R_{j0k0}{}^{TT} = -\frac{1}{2}\frac{\partial^2}{\partial t^2} h_{jk}{}^{TT}$$

$$\frac{d^2\xi_k}{dt^2} \approx -R_{k0j0}{}^{TT}\xi^j = \frac{1}{2}\frac{\partial^2 h_{jk}{}^{TT}}{\partial t^2}\xi^j$$

- **…and the tidal force**

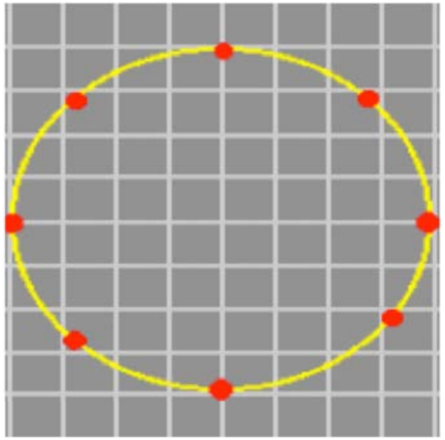
$$f^k \sim m \cdot R^k_{0j0} \cdot \xi^j$$

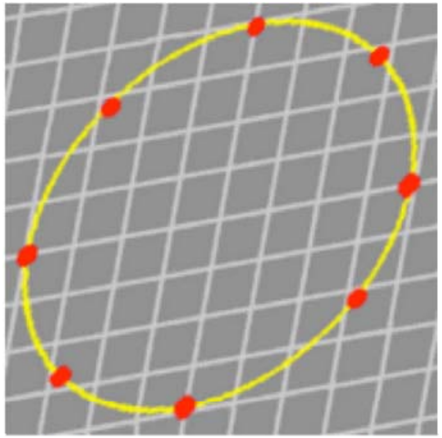
Gravitational Waves II

$$h^{\mu\nu} = h_{+}\varepsilon_{+}^{\mu\nu}\cos[\omega(t-z)]$$

$$\Delta x = -\frac{1}{2}h_{+}\cos\left[\omega(t-z)\right]x$$

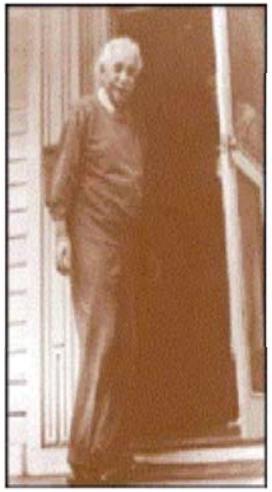
$$\Delta y = \frac{1}{2}h_{+}\cos\left[\omega(t-z)\right]y$$

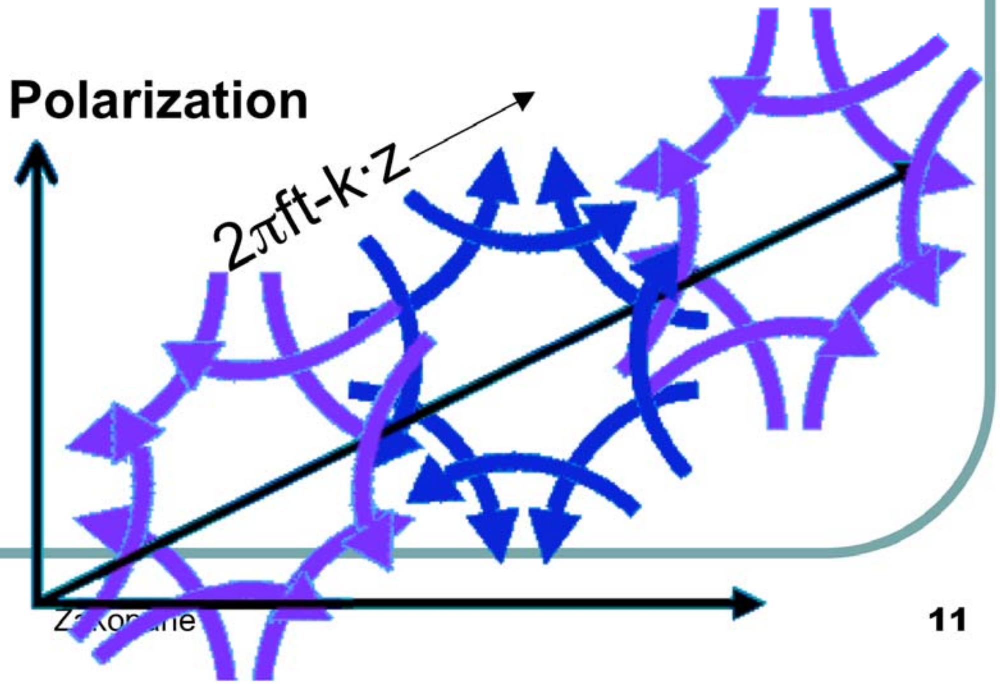




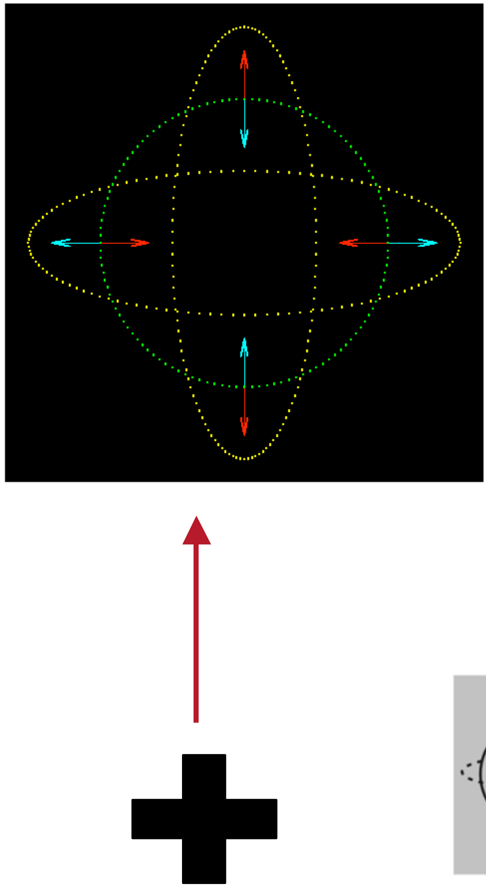
…in other words

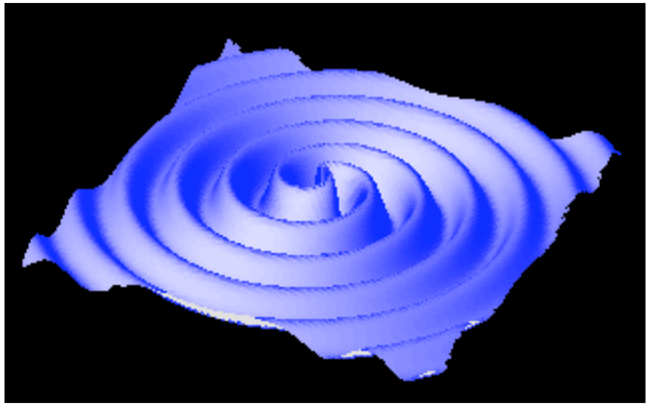
$$\frac{\Delta\ell}{\ell} = h$$

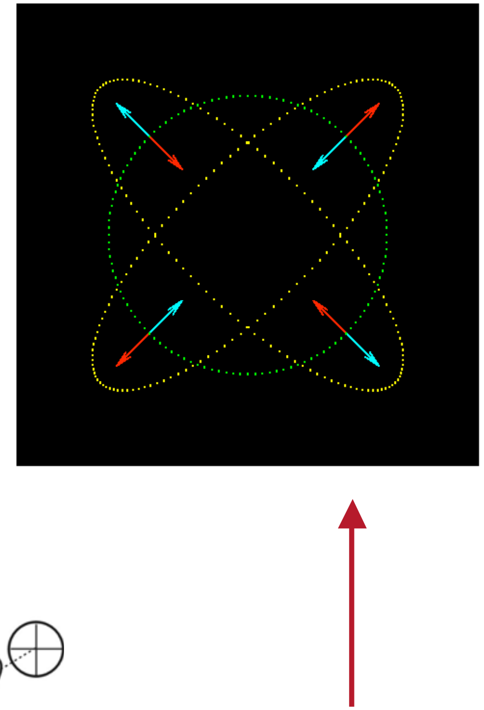




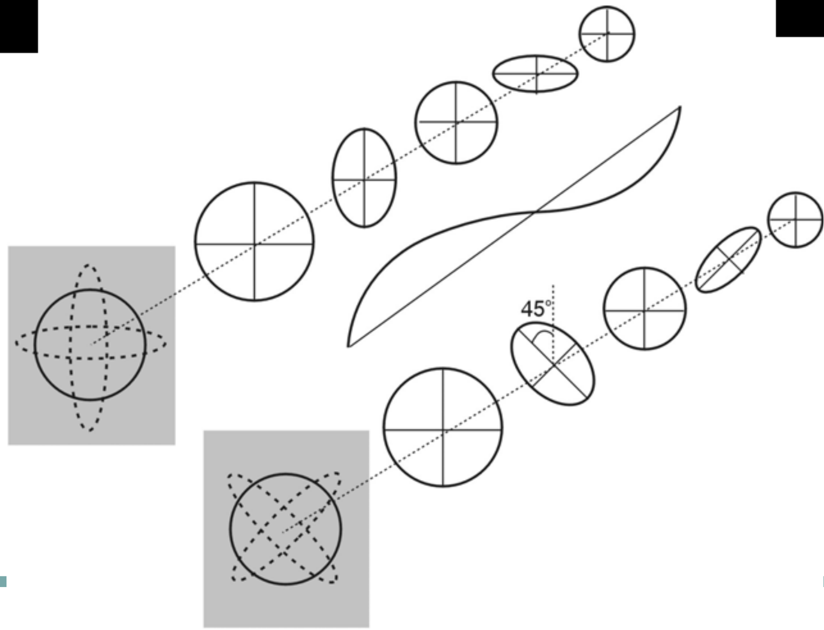
GW Polarizations







X



Stress-Energy carried by GWs

GWs exert forces and do work, they must carry energy and momentum

- The energy-momentum tensor in an arbitrary gauge

$$t_{\mu\nu}{}^{(GW)} = \frac{1}{32\pi}\left\langle \tilde{h}_{\alpha\beta;\mu}\tilde{h}^{\alpha\beta}{}_{;\nu} - \frac{1}{2}\tilde{h}_{;\mu}\tilde{h}_{;\nu} - \tilde{h}^{\alpha\beta}{}_{;\beta}\tilde{h}_{\alpha\mu;\nu} - \tilde{h}^{\alpha\beta}{}_{;\beta}\tilde{h}_{\alpha\nu;\mu}\right\rangle$$

- …in the TT-gauge:

$$t_{\mu\nu}{}^{(GW)} = \frac{1}{32\pi}\left\langle \tilde{h}^{jk}{}_{;\mu}{}^{TT} \cdot \tilde{h}_{jk;\nu}{}^{TT}\right\rangle$$

- …it is divergence free

$$t^{\nu}_{\mu;\nu}{}^{(GW)} = 0$$

- For waves propagating in the z-direction

$$t_{00}{}^{(GW)} = -\frac{1}{c}t_{0z}{}^{(GW)} = \frac{1}{c^2}t_{zz}{}^{(GW)} = \frac{1}{16\pi}\frac{c^2}{G}\left\langle \dot{h}_+{}^2 + \dot{h}_\times{}^2\right\rangle$$

- for a SN exploding in Virgo cluster the energy flux on Earth

$$t_{0z}{}^{(GW)} \approx \frac{\pi}{4}\frac{c^3}{G}f^2\left\langle h_+^2 + h_\times^2\right\rangle = 320\times\left(\frac{f}{1kHz}\right)^2\left(\frac{h}{10^{-21}}\right)^2\frac{\text{ergs}}{\text{cm}^2\text{sec}}$$

- The corresponding EM energy flux is:

$$\sim 10^{-9}\,\text{erg}\cdot\text{cm}^{-2}\cdot\text{sec}^{-1}$$

Wave-Propagation Effects

GWs affected by the large scale structure of the spacetime exactly as the EM waves

- The magnitude of h_{jk}^{TT} falls of as $1/r$
- The polarization, like that of light in vacuum, is parallel transported radially from source to earth
- The time dependence of the waveform is unchanged by propagation except for a frequency-independent **redshift**

$$\frac{f_{\text{received}}}{f_{\text{emitted}}} = \frac{1}{1+z}$$

We expect

- Absorption, scattering and dispersion
- Scattering by the background curvature and tails
- Gravitational focusing
- Diffraction
- Parametric amplification
- Non-linear coupling of the GWs (frequency doubling)
- Generation of background curvature by the waves

The emission of grav. radiation

If the energy-momentum tensor is varying with time, GWs will be emitted

- The retarded solution for the linear field equation

$$\left(-\frac{\partial^2}{\partial^2 t}+\nabla^2\right)h^{\mu\nu} = kT^{\mu\nu}$$

$$h^{\mu\nu} = 2\int \frac{T^{\mu\nu}(t-|\vec{x}-\vec{x}'|,\vec{x}')}{|\vec{x}-\vec{x}'|}d^3x'$$

- For a point in the radiation zone in the slow–motion approximation

$$h^{\mu\nu} \approx \frac{2}{r}\int T^{\mu\nu}(t-r,\vec{x}')d^3x' \sim \frac{2}{r}\frac{\partial^2}{\partial t^2}\left[Q^{jk}(t-r)\right]^{TT}$$

- Where Q_{kl} is the quardupole moment tensor

$$Q^{kl} \equiv \int \rho(t,x^k)\left(x^k x^l - \frac{1}{3}r^2\delta^{kl}\right)d^3x$$

- **Power emitted in GWs**

$$L_{GW} = -\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\sum_{ij}\left\langle \dddot{Q}_{ij}\cdot\dddot{Q}_{ij}\right\rangle$$

Angular and Linear momentum emission

- **Angular momentum** emission

$$\frac{dJ_i^{GW}}{dt} = \frac{2}{5}\sum_{jkl} \varepsilon_{ijk} \left\langle \ddot{Q}_{jl} \cdot \dddot{Q}_{lk} \right\rangle$$

- **Linear momentum** emission

$$\frac{dP_i^{GW}}{dt} = \frac{2}{63}\sum_{jk} \left\langle \dddot{Q}_{jk} \cdot \ddddot{Q}_{jki} \right\rangle + \frac{16}{45}\sum_{jkl} \varepsilon_{ijk} \left\langle \dddot{Q}_{jl} \cdot \dddot{P}_{lk} \right\rangle$$

Q_{ijk} : mass octupole moment

P_{ij} : current quadrupole moment

Back of the envelope calculations!

- **Characteristic time-scale** for a mass element to move from one side of the system to another is:

$$T \sim \frac{R}{\upsilon} \sim \frac{R}{(M/R)^{1/2}} = \left(\frac{R^3}{M}\right)^{1/2}$$

- The **quadrupole moment** is approximately:

$$\dddot{Q}_{ij} \sim \frac{MR^2}{T^3} \sim \frac{M\upsilon^2}{T} \sim \frac{E_{ns}}{T} \sim \left(\frac{M}{R}\right)^{5/2}$$

- **Luminosity**

$$L_{GW} \sim \frac{G}{c^5}\left(\frac{M}{R}\right)^5 \sim \frac{G}{c^5}\left(\frac{M}{R}\right)^2 \upsilon^6 \sim \frac{c^5}{G}\left(\frac{R_{Sch}}{R}\right)^2\left(\frac{\upsilon}{c}\right)^6$$

$$\frac{c^5}{G} = 3.63\times 10^{59}\, erg/s = 2.03\times 10^5 M_{\odot} c^2/s$$

- The **amplitude** of GWs at a distance r ($R \sim R_{Schw} \sim 10$Km and $r \sim 10$Mpc$\sim 3\times 10^{19}$km):

$$h \sim \frac{\ddot{Q}}{r} \sim \frac{1}{r}\left(\frac{MR^2}{T^2}\right) \sim \frac{1}{r}\frac{M^2}{R} \sim \ldots \sim 10^{-19}$$

- **Radiation damping**

$$\tau_{react} = \frac{E_{kin}}{L_{GW}} \sim \left(\frac{R}{M}\right)^{5/2} T \sim \left(\frac{\upsilon}{c}\right)\left(\frac{R}{R_{Schw}}\right)^3 T$$

What we should remember…

- **Length variation**

$$\frac{\Delta\ell}{\ell} = h$$

- **Amplitude**

$$h^{jk} \approx \frac{2}{r}\ddot{Q}^{jk}$$

- **Power emitted**

$$L_{GW} = -\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\sum_{ij}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle$$

Vibrating Quadrupole

- The position of the two masses

$$x = \pm[x_0 + \xi \sin(\omega t)] \quad , \quad x_0 \gg \xi$$

- The quadrupole moment of the system is:

$$Q^{kl}(t-r) \approx \left[1 + \frac{2\xi}{x_0}\sin\omega(t-r)\right] Q_0{}^{kl}$$

$$Q_0{}^{kl} = \begin{pmatrix} -2mx_0{}^2 & 0 & 0 \\ 0 & -2mx_0{}^2 & 0 \\ 0 & 0 & 4mx_0{}^2 \end{pmatrix}$$

- The radiated gravitational field is:

$$h^{kl} = \frac{2}{3}\left(\frac{\xi}{x_0}\right)\frac{\omega^2}{r}\sin[\omega(t-r)]Q_0{}^{kl}$$

- The emitted power

$$-\frac{dE}{dt} = \frac{G}{45c^5}\left\langle \dddot{Q}_{kl}\dddot{Q}_{kl} \right\rangle = \frac{16}{15}\frac{G}{c^5}(mx_0\xi)^2\omega^6$$

- And the damping rate of the oscillator is:

$$\gamma_{rad} = -\frac{1}{E}\left\langle \frac{dE}{dt} \right\rangle = \frac{16}{15}\frac{G}{c^5}mx_0{}^2\omega^4$$

Two-body collision

- The radiated power

$$-\frac{dE}{dt}=\frac{8}{15}\frac{G}{c^5}m^2(3\dot{z}\,\ddot{z}+z\,\dddot{z})^2$$

- The energy radiated during the plunge from $z=\infty$ to $z=-R$

$$-\Delta E=\frac{4}{105}\frac{1}{R^{7/2}}\frac{G}{c^5}m^2(2GM)^{5/2}$$

- If $R=R_{Schw}$ ($M=10M_o$ & $m=1M_o$)

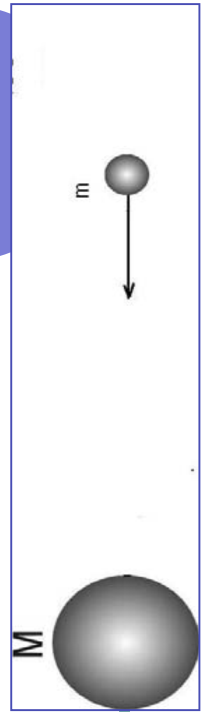
$$-\Delta E=0.019mc^2\frac{m}{M}$$

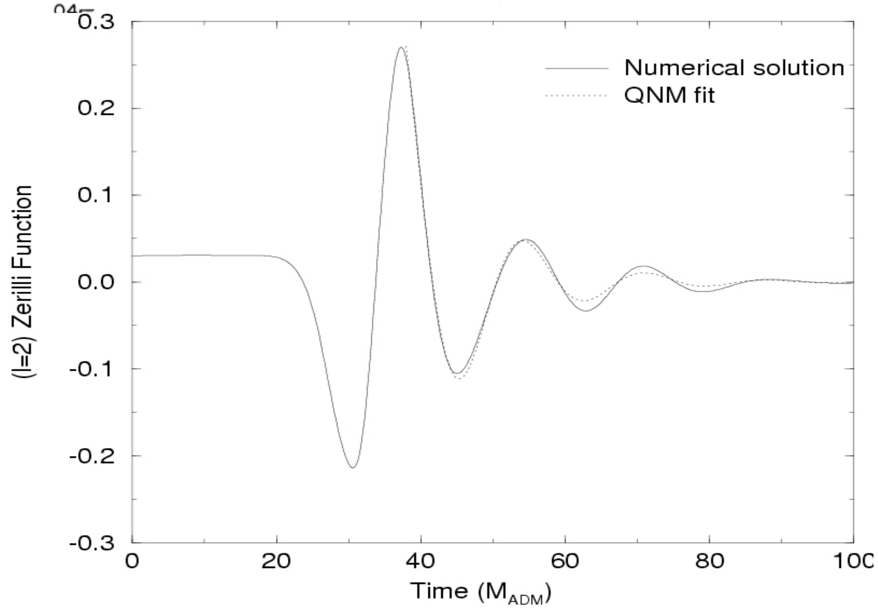
$$-\Delta E_{true}=0.0104mc^2\frac{m}{M}\rightarrow 2\times10^{51}erg$$

- Most radiation during **2R->R** phase

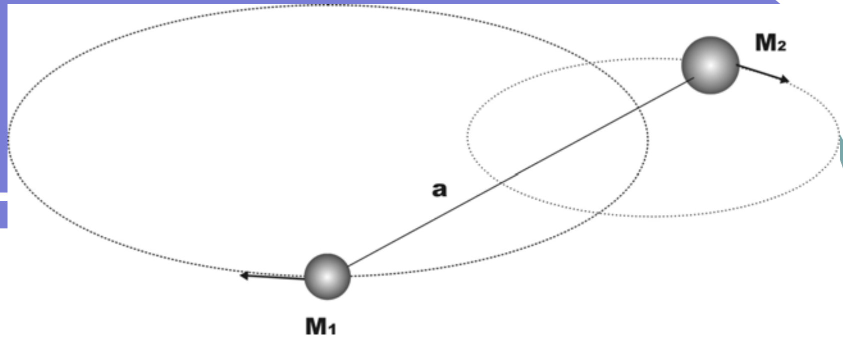
$$\Delta t\sim R/v\sim R/c\sim 30km/c\sim 10^{-4}s$$

$$f\sim 10^4\,\mathrm{Hz}$$





Rotating Quadrupole (a binary system)



THE BEST SOURCE FOR GWs

- **Radiated power**

$$-\frac{dE}{dt} = \frac{32}{5}\frac{G}{c^5}\mu^2 a^4 \omega^6 = \frac{32}{5}\frac{G}{c^5}\frac{M^3\mu^2}{a^5}$$

- Energy loss leads to **shrinking of their orbital separation**

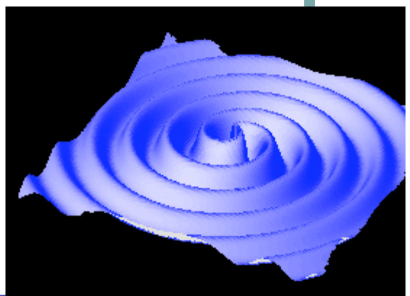
$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^3}$$

- **Period changes** with rate

$$\frac{\dot{P}}{P} = -\frac{96}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^4}$$

- ...and the system **will coalesce** after

$$T_{inspiral} = \frac{5}{256}\frac{c^5}{G^3}\frac{a_0^4}{\mu M^2}$$



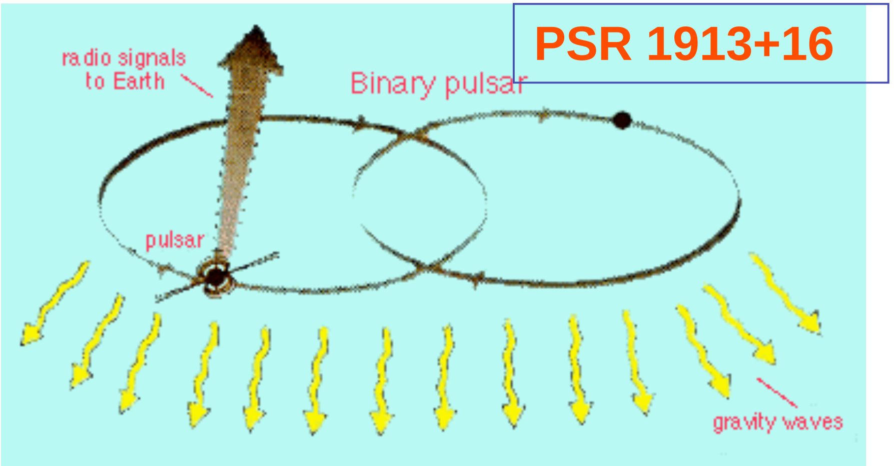
- The **total energy loss** is

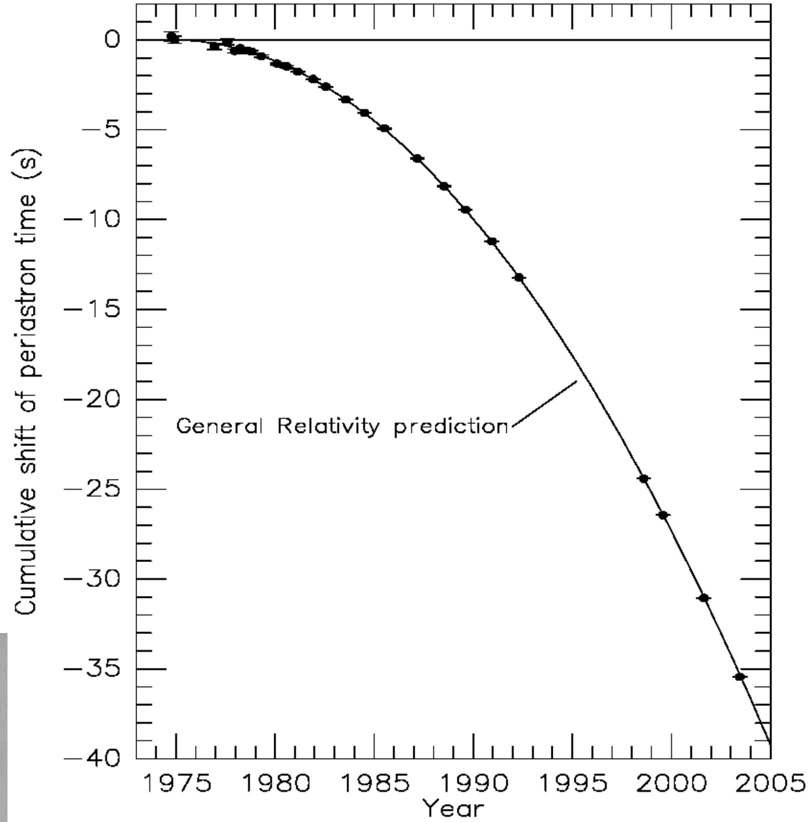
$$\Delta E_{rad} = \frac{G}{2}\mu M\left(\frac{1}{a_0} - \frac{1}{a}\right)$$

- Typical **amplitude** of GWs

$$h \approx 5\times10^{-22}\left(\frac{M}{2.8M_\odot}\right)^{2/3}\left(\frac{\mu}{0.7M_\odot}\right)\left(\frac{f}{100Hz}\right)^{2/3}\left(\frac{15Mpc}{r}\right)$$

First verification of GWs





Nobel 1993

Hulse & Taylor

$$\frac{\dot{P}_{b,corrected}}{\dot{P}_{b,GR}} = 1.0013 \pm 0.0021$$

Binary systems (examples)

PSR 1913+16

$M_1 = M_2 \sim 1.4 M_e$, P=7h 45m 7s, r=5kpc,

$h_{earth} \sim 10^{-20}$, $f \sim 10^{-4}$Hz, $T_{insp} \sim 3 \times 10^8$yr

$dP_{theo}/dt = -7.2' \; 10^{-12}$s/yr $dP_{obs}/dt = -(6.9 \pm 06) \times 10^{-12}$s/yr

The LIGO/VIRGO binary (10-1000Hz)

$M_1 = M_2 \sim 1.4 M_e$, $f_0 = 10$Hz, $f_{final} = 1000$Hz,

$T_{insp} \sim 15$min, after ~15000 cycles (inspiral/merging 300Mpc)

$M_1 = 50 M_e$, $M_2 \sim 50 M_e$, $f_0 = 10$Hz,

$f_{final} = 100$Hz, (inspiral/merging 400Mpc)

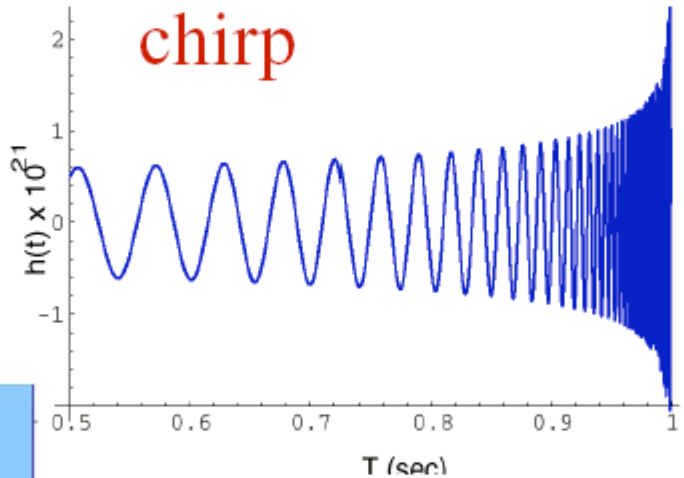
The LISA binary (10^{-5}-10^{-2}Hz)

$M_1 = M_2 \sim 10^6 M_e$, $f_0 = 10^{-4}$Hz, $f_{final} = 0.01$Hz, (inspiral/merging at r~3Gpc)

$M_1 = M_2 \sim 10^5 M_e$, $f_0 = 10^{-4}$Hz, $f_{final} = 0.1$Hz, (inspiral/merging at r~3Gpc)

$M_1 = M_2 \sim 10^4 M_e$, $f_0 = 10^{-3}$Hz, $f_{final} = 1$Hz, (inspiral at r~3Gpc)

Smaller Stars/BHs plunging into super-massive ones



$$f \sim 1kHz \left(\frac{10 M_o}{M} \right)$$

An interesting observation

- The observed frequency change will be:

$$\dot{f} \sim f^{11/3} \mathcal{M}_{chirp}^{5/3}$$

$$\mathcal{M}_{chirp}^{5/3} = \mu M^{2/3}$$

-

- The corresponding amplitude will be :

$$h \sim \frac{\mathcal{M}_{chirp}^{5/3} f^{2/3}}{r} = \frac{\dot{f}}{f^3 r}$$

•Since both frequency and its rate of change are measurable quantities, we can immediately compute the chirp mass.

•The third relation provides us with a direct estimate of the distance of the source

•Post-Newtonian relations can provide the individual masses

2nd Part

DETECTION of GWs

A Quadrupole Detector

Tidal force is the driving force of the oscillator

- Plane wave

$$h^{\mu\nu} = h_+ \varepsilon^{\mu\nu} e^{i(\omega t - kz)}$$

- Displacement & Tidal force

$$\xi_x \approx L h_+ e^{i\omega t} \qquad f_x \approx m L h_+ \omega^2 e^{i\omega t}$$

- Equation of motion

$$\ddot{\xi} + \dot{\xi}/\tau + \omega_0{}^2 \xi = -\frac{1}{2}\omega^2 L h_+ e^{i\omega t}$$

- Solution

$$\xi = \frac{\omega^2 L h_+ e^{i\omega t}/2}{\omega_0^2 - \omega^2 + i\omega/\tau}$$

$$\xi_{\max} = \omega_0 \tau \cdot L \cdot h_+ / 2 = Q \cdot L \cdot h / 2$$

- **Cross section**

$$\sigma = \frac{32\pi}{15} \frac{G}{c^3} \omega_0 \cdot Q \cdot M \cdot L^2$$

- Weber's detector: ***M=1410kg, L=1.5m, d=66cm,*** ***ω_0=1660Hz, Q=2x10^5.***

$$\sigma_{\mathrm{Weber}} \sim 3 \times 10^{-19}\,\mathrm{cm}^2$$

Quadrupole Detector Limitations

Problems

- Very small cross section ***~3x10^{-19}cm^2.***
- Sensitive to periodic GWs **tuned** in the right frequency of the detector
- Sensitive to bursts **only** if the pulse has a substantial component at the resonant frequency
- The **width of the resonance** is:

$$\Delta\nu \sim \gamma / 2\pi \sim 10^{-2}\,Hz$$

- Thermal noise limits our ability to detect the energy of GWs.
- The excitation energy has to be greater than the thermal fluctuations $E \geq kT$

$$h_{\min} \geq \frac{1}{\omega_0 LQ}\sqrt{\frac{15kT}{M}} \sim 10^{-20}$$

BURSTS

- Periodic signals which match the resonant frequency of the detector are extremely rare.
- A great number of events produces short pulses which spread radiation over a wide range of frequencies.
- The minimum detectable amplitude is

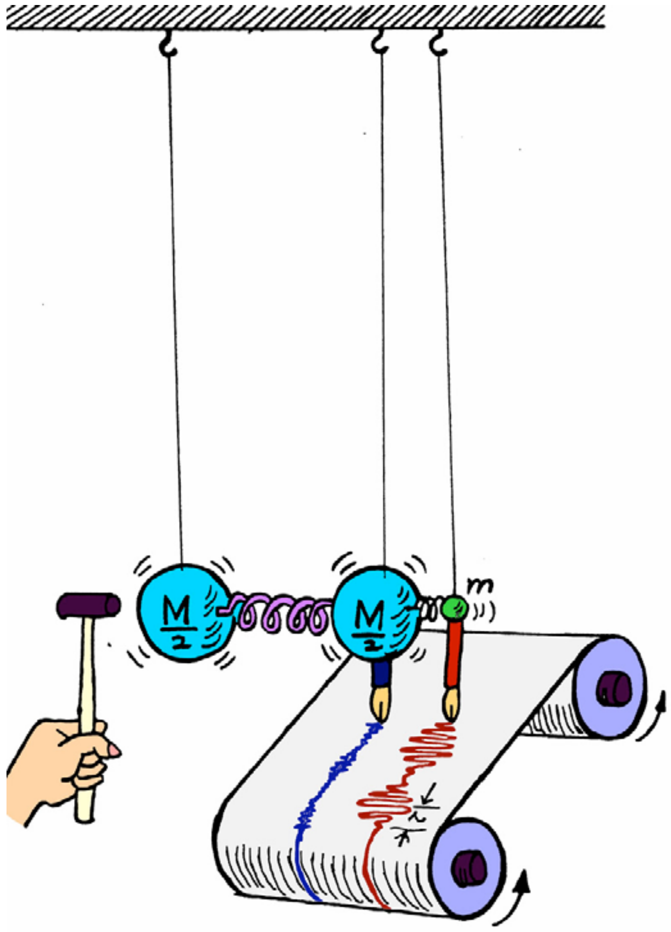
$$h_{\min} \geq \frac{1}{\omega_0 L}\sqrt{\frac{30kT_{eff}}{\pi M}} \sim 10^{-16}$$

The total energy of a pulse from the Galactic center (r=10kpc) which will provide an amplitude of $h \sim 10^{-16}$ or energy flux $\sim 10^9$ erg/cm^2.

$$4\pi r^2 \times 10^9\, erg / cm^2 = 10^{55}\, erg$$
$$\approx 10 M_{\odot} c^2 \,!!!$$

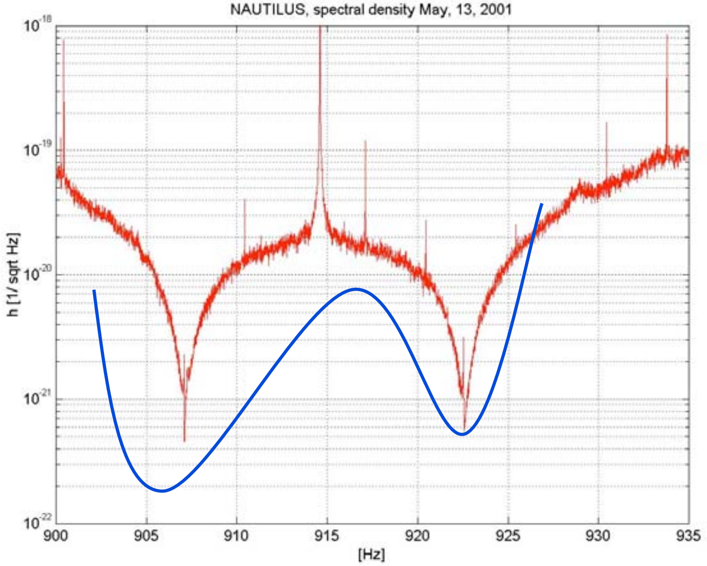
Improvements to resonant detectors

- Have higher Q
- Operate in extremely low temperatures (mK)
- Larger masses
- Different geometry
- Better electronic sensors



Modern Bar Detectors

	WEBER	NAUTILUS
mass(kg)	1410	2270
Length(m)	1.53	2.97
ω_0(Hz)	1660	910
$Q = \omega/\gamma$	2×10^5	2.3×10^6
$\sigma\ (\omega_0)_{\mathrm{abs}}\ (\mathrm{cm}^2)$	2×10^{-19}	70×10^{-19}
Typical pulse sensitivity h	10^{-16}	9×10^{-19}



International Gravitational Events Collaboration

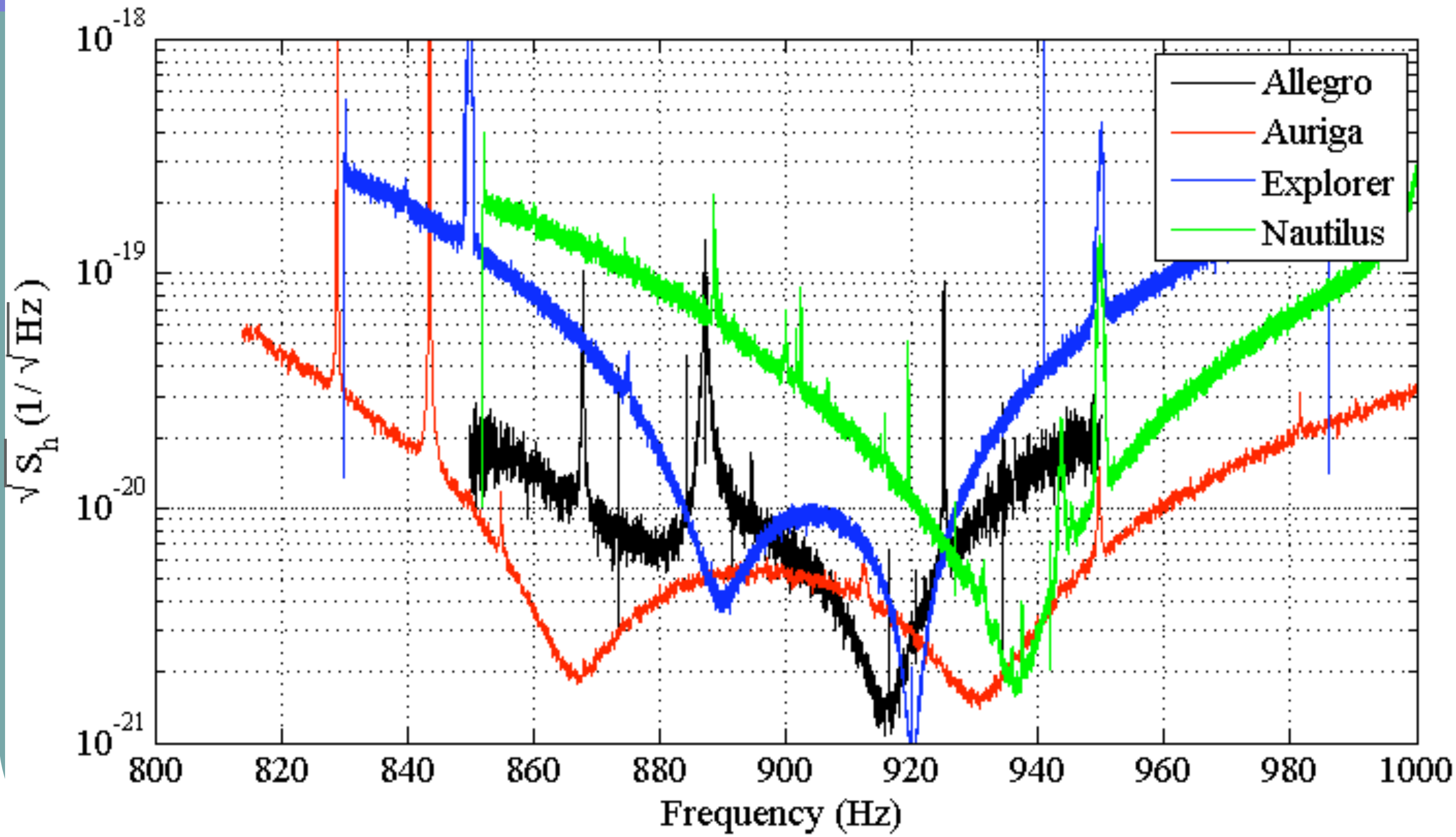
ALLEGRO- AURIGA - ROG (EXPLORER-NAUTILUS)

- The "oldest" resonant detector EXPLORER started operations about 16 years ago.
- This kind of detector has reached a high level of reliability.
- The duty factor is greater than 90% .



New Advanced Acoustic Detectors are in RD phase

Sensitivity of Resonant Detectors

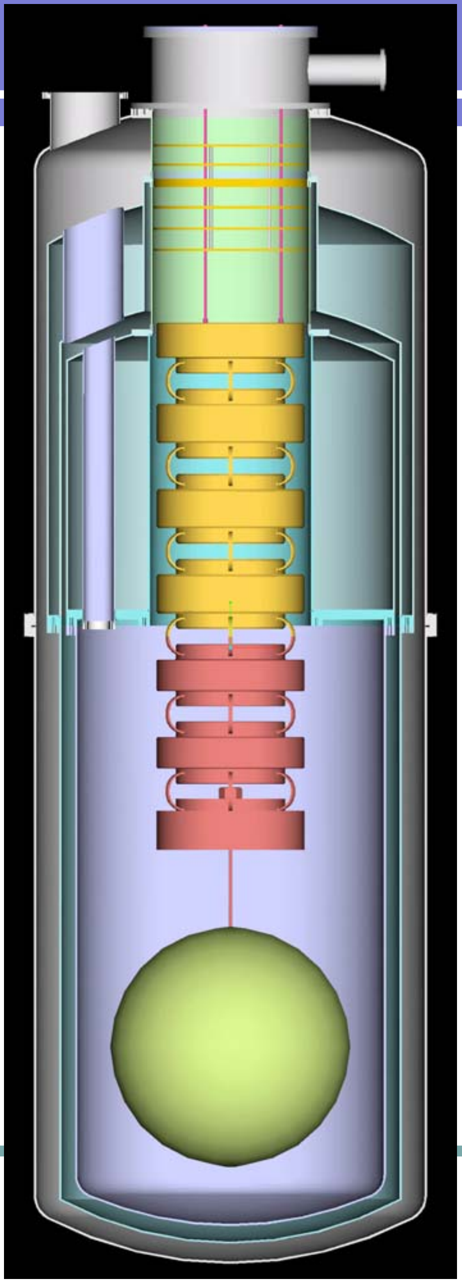


Exploiting the resonant-mass detector technique: the spherical detector

M = 1-100 tons

Sensitivity:
10^{-23} – 10^{-24} $Hz^{-1/2}$;
h ~ 10^{-21} – 10^{-22}

- **Omnidirectional**
- Capable of **detecting source position**
- Capable of **measuring polarization**



MINIGRAIL
Leiden (Netherlands)

MARIO SHENBERG
Sao Paulo (Brasil)

SFERA
Frascati (Italy)

CuAl(6%) sphere
Diameter= 65 cm
Frequency = 3 kHz
Mass = 1 ton

Laser Interferometers

- The output of the detector is

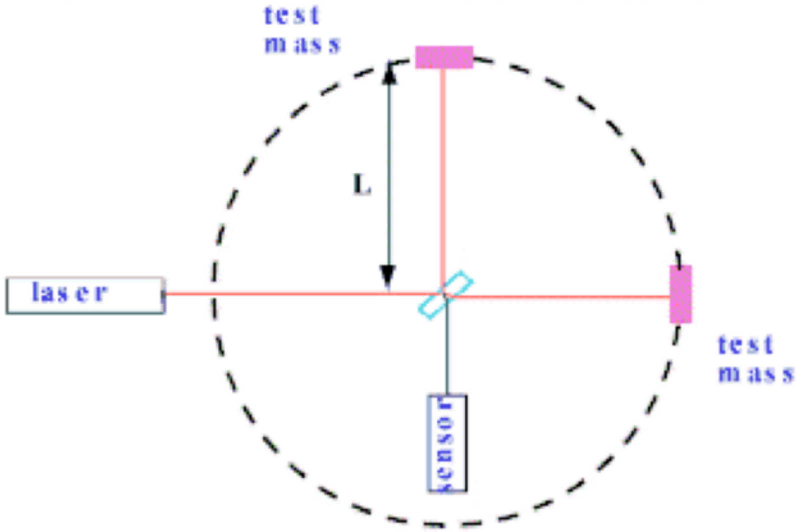
$$\frac{\Delta L}{L} = F_+ h_+(t) + F_\times h_\times(t) = h(t)$$

- Technology allows measurements ***$\Delta L \sim 10^{-16}$cm.***
- For signals with ***$h \sim 10^{-21}$-10^{-22}*** we need arm lengths ***L~1-10km***.
- Change in the arm length by ***ΔL*** corresponds to a phase change

$$\Delta\varphi = \frac{4\pi b \Delta L}{\lambda} \sim 10^{-9}\,\mathrm{rad}$$

- The number of photons reaching the photo-detector is proportional to laser-beam's intensity *[~$\sin^2(\Delta\varphi/2)$]*

$$N_{\mathrm{out}} = N_{\mathrm{input}} \sin^2(\Delta\varphi / 2)$$



OPTIMAL CONFIGURATION

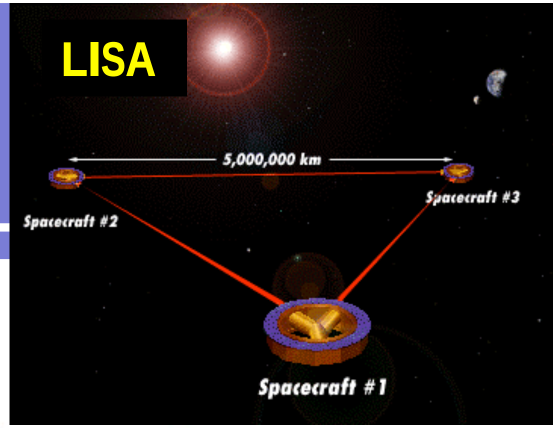
•Long arm length L

•Large number of reflections b

•Large number of photons (but be aware of radiation pressure)

•Operate at interface minimum *$\cos(2\pi b \Delta L/\lambda)=1$*.

Interferometric Projects



Noise Sources I

- ## Photon Shot Noise
  - The number of emerging photons is subject to statistical fluctuations
  - Implies an uncertainty in the measurement of *ΔL*.

$$\delta N_{out} \sim \sqrt{N_{out}}$$

$$h_{\min} = \frac{\delta(\Delta L)}{L} = \frac{\Delta L}{L} \sim \frac{1}{bL}\sqrt{\frac{\hbar c \lambda}{2\pi\tau I_0}}$$

- ## Radiation Pressure Noise
  - Lasers produce radiation pressure on the mirrors
  - Uncertainty in the measurement of the deposited momentum leads to an uncertainty in the position of the mirrors

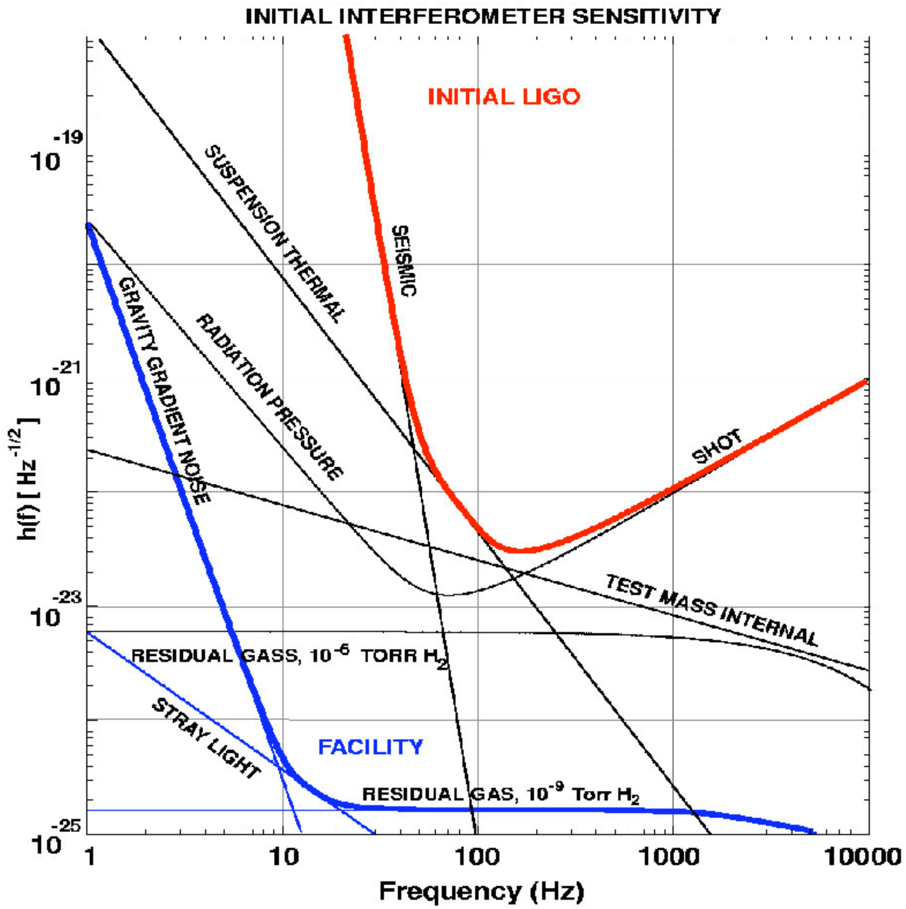
$$h_{\min} = \frac{\tau}{m}\frac{b}{L}\sqrt{\frac{\tau\hbar I_0}{c\lambda}}$$

- ## Quantum Limit
  - If we try to minimize PSN and RPN with respect to laser power we get a minimum detectable stain
  - Heisenberg's principle sets an additional uncertainty in the measurement of ΔL
  - ***(ΔL Δp*** $\geq \hbar$, if ***Δp~m ΔL/τ*** …) and the minimum detectable strain is

$$h_{\min} = \frac{1}{L}\sqrt{\frac{\tau\hbar}{m}}$$

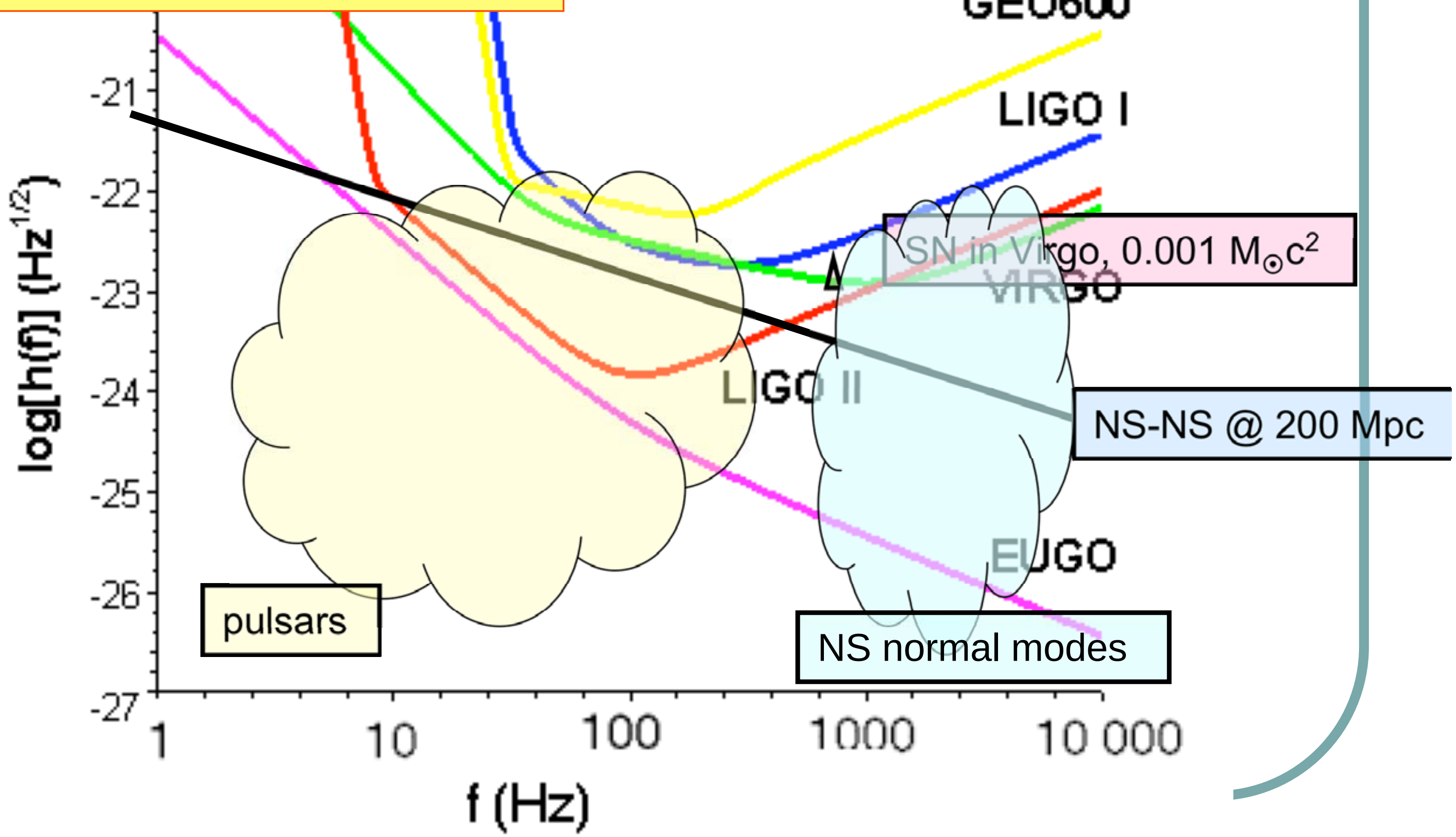
LIGO/Virgo Sensitivity

- Seismic Noise
  - Important below 60Hz. Dominates over all other types of noise.
- Residual gas-phase noise
  - Statistical fluctuations in the residual gas density induce fluctuations of the refraction index and as a result on the monitored phase shift.
  - Vacuum pipes **(~10^{-9} torr)**

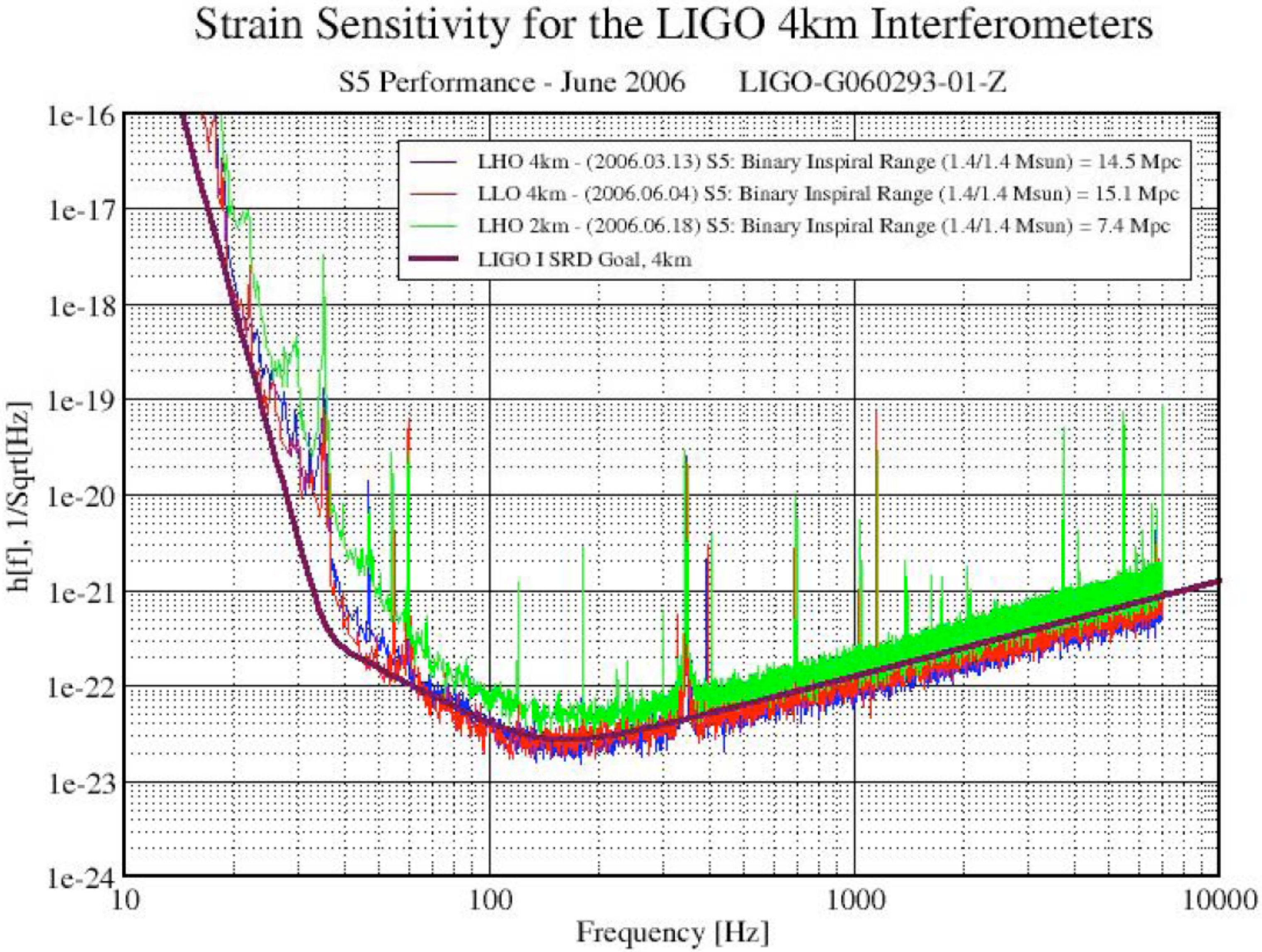


Interferometers and Sources

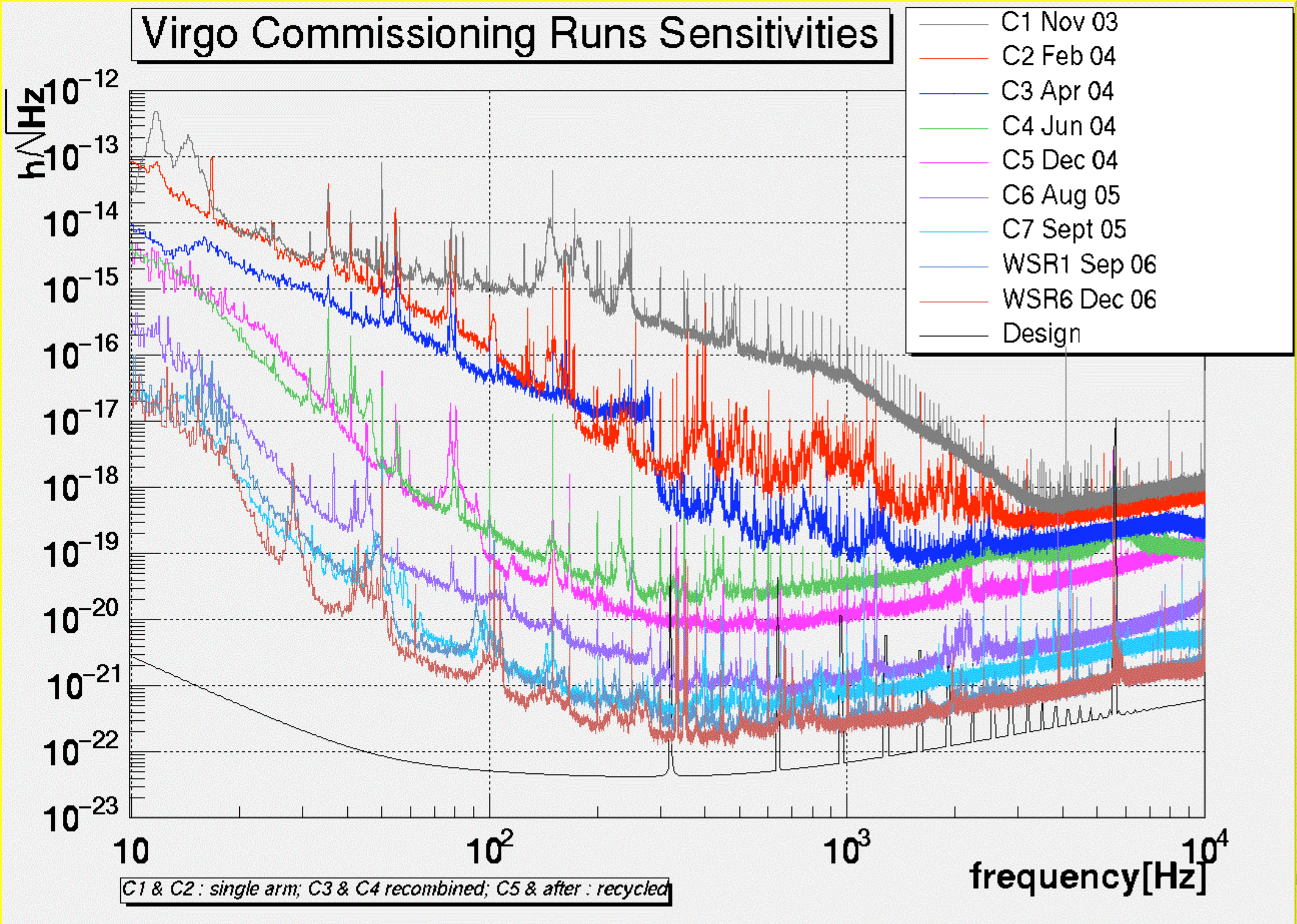
×100 in 10-20 years is possible



LIGO now at design sensitivity

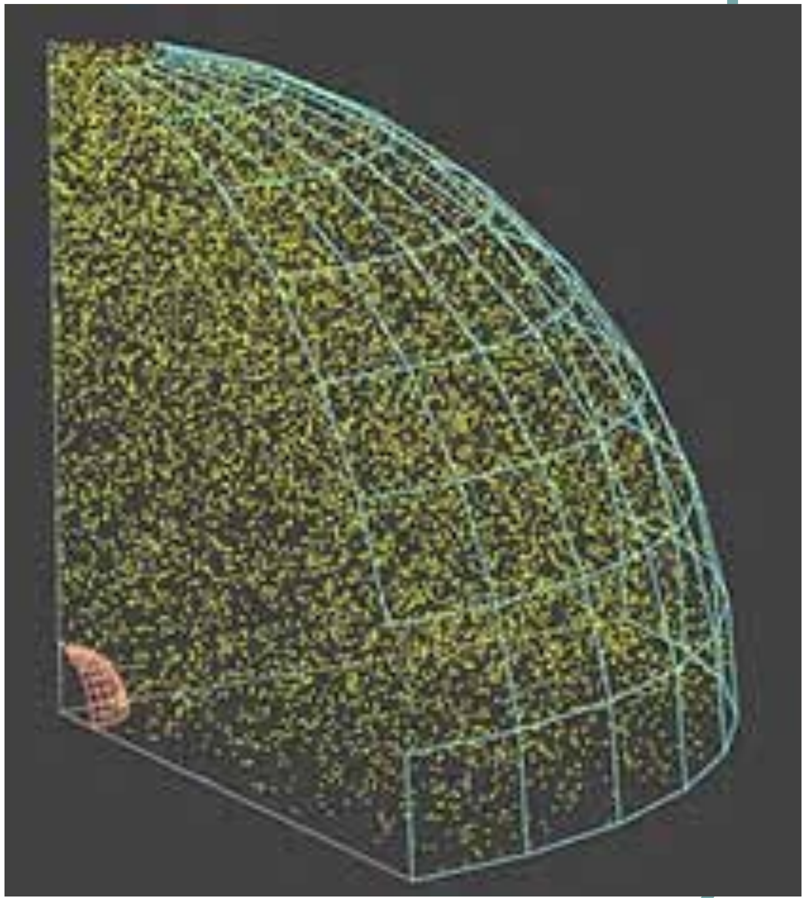


VIRGO is reaching its designed sensitivity



Future Gravitational Wave Antennas

- **Advanced LIGO**
  - 2008-2010; planning under way
  - 10-15 times more sensitive than initial LIGO
- **High Frequency GEO**
  - 2008? Neutron star physics, BH quasi-normal modes
- **EGO: European Gravitational Wave Observatory**
  - 2012? Cosmology



Towards GW astronomy

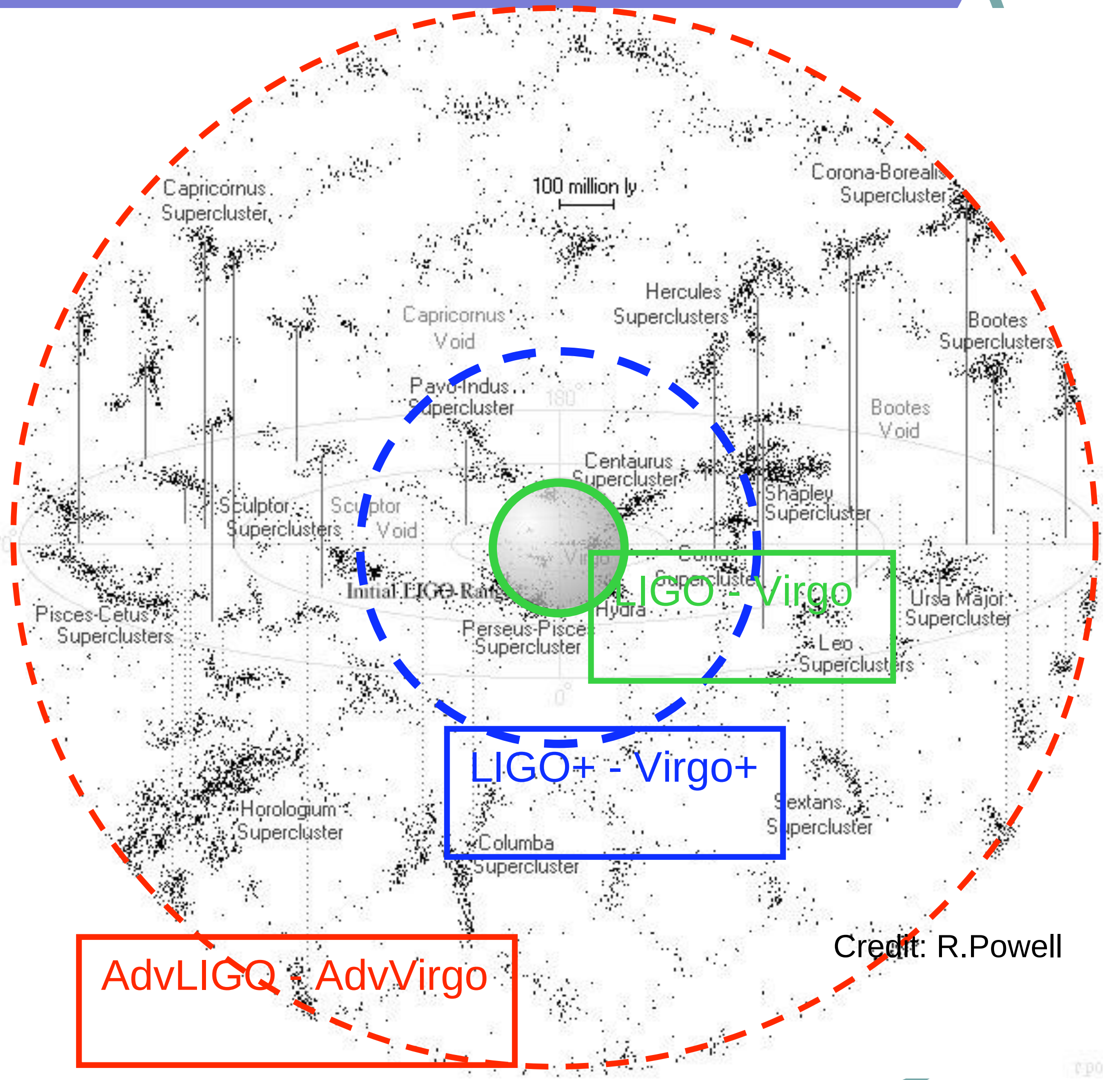
- Present detectors will test
- upper limits

- Even in the optimistic case rate too low to start GW astronomy

- Need to improve the sensitivity

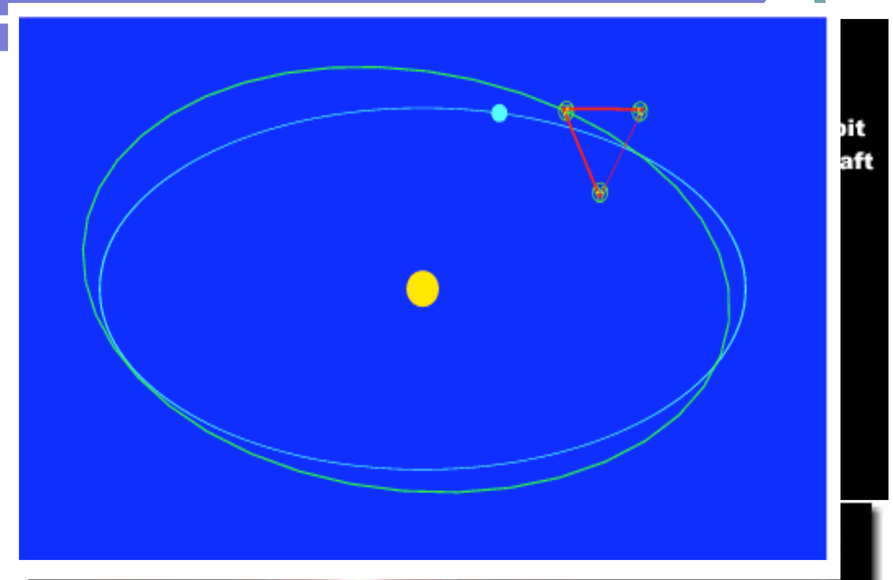
- Increase the sensitivity by 10 ⇒ increase the probed volume by 1000

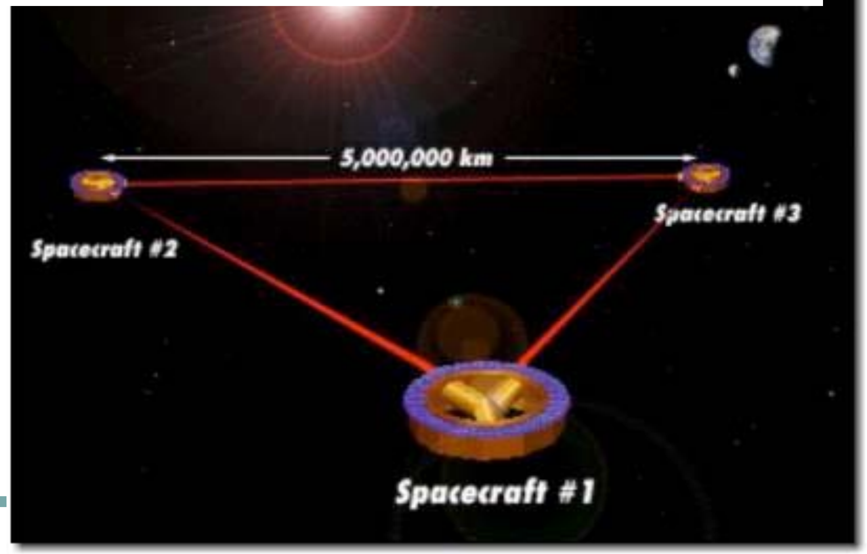
- Plans to improve the present detectors



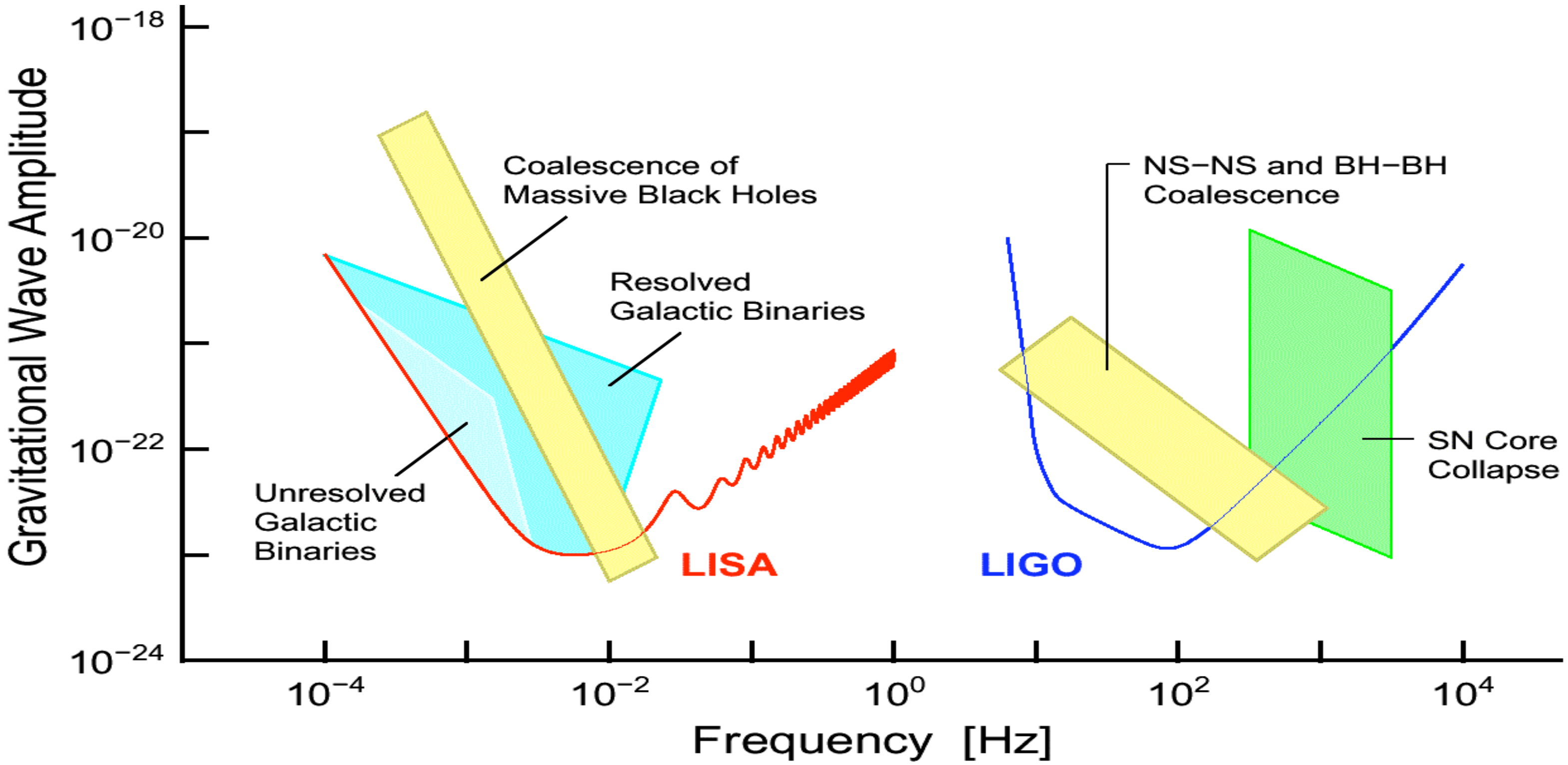
LISA the space interferometer

- LISA is low frequency detector.
- With arm lengths 5,000,000 km targets at 0.1mHz – 0.1Hz.
- Some sources are very well known (close binary systems in our galaxy).
- Some other sources are extremely strong (SM–BH binaries)
- LISA's sensitivity is roughly the same as that of LIGO, but at 10^5 times lower frequency.
- Since the gravitational waves energy flux scales as $F \sim f^2 \cdot h^2$, it has **10 orders** better energy sensitivity than LIGO.





Gravitational Wave Spectrum…



Complementary observations, different frequency bands, and different astrophysical sources…

The END of the 1st day