AdS/CFT and Second Order Viscous Hydrodynamics

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Roadmap

- Boost-invariant energy-momentum tensors
  - Bjorken hydrodynamics
  - Dissipative hydrodynamics
- Einstein and string frame
- Gravity duals
  - Holography
  - Perfect fluid metric
  - Subasymptotic corrections
- AdS/CFT and Israel-Stewart theory
  - Third order solution
  - Calculation of relaxation time
  - Discussion
- Summary and Outlook

Introduction

- matter created in RHIC is **strongly coupled** and **deconfined**
- studying dynamics of $\mathcal{N} = 4$ SYM may be relevant
- let's use AdS/CFT...
- to study the dynamics on energy-momentum tensor
- ...bearing in mind differences:
  - coupling does not run
  - no hadrons
- simplest dynamics to study

  1D expansion + boost invariance (Bjorken)

From now on we are only within **$\mathcal{N} = 4$ SYM** plasma!

Boost-invariant energy-momentum tensors

- no dependence on transverse coordinates $x^{2,3}$
- 3 nonzero components $T_{\tau\tau},\ T_{yy},\ T_{xx} = T_{x^2x^2} = T_{x^3x^3}$
- boost invariance forces $T_{\mu\nu}(\tau, y) = T_{\mu\nu}(\tau)$
- constraints on energy-momentum $T_{\mu\nu}$ dynamics:
  - conservation $\tau \frac{d}{d\,\tau} T_{\tau\tau} + T_{\tau\tau} + \frac{1}{\tau^2} T_{yy} = 0$
  - tracelessness $T_{\tau\tau} + \frac{1}{\tau^2} T_{yy} + 2\,T_{xx} = 0$

- $T_{\mu\nu}$ can be expressed in terms of a single function

$$\epsilon(\tau) = T_{\tau\tau}$$

- $\epsilon(\tau)$ is plasma's energy density
- perfect fluid case: $\epsilon \sim \frac{1}{\tau^{4/3}}$

Second order viscous hydrodynamics

- Perfect fluid - equation of motion for energy density

$$\partial_\tau \epsilon = -\frac{\epsilon + p}{\tau} = -\frac{4}{3}\frac{\epsilon}{\tau}$$

- we want to include dissipative corrections
- equations of motion in Bjorken regime read

$$\partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{\Phi}{\tau}$$

$$\tau_\pi \partial_\tau \Phi = -\Phi + \frac{4}{3}\frac{\eta}{\tau}$$

- in hydrodynamical simulation formula

$$\tau_\pi^{Boltzmann} = \frac{3}{2}\frac{\eta}{p}$$

  is commonly used
- assumption: $\tau_\pi = r\tau_\pi^{Boltzmann}$ holds for $\mathcal{N} = 4$ SYM plasma
- our goal is to calculate coefficient r

Crucial question

How to determine energy density $\epsilon(\tau)$ and relaxation time τ_π of $\mathcal{N} = 4$ SYM from AdS/CFT correspondence?

String and Einstein frames

Let's consider scalar field (dilaton) coupled to gravity

- equations of motion

$$\begin{aligned} G_{\alpha\beta} = R_{\alpha\beta} + 4g_{\alpha\beta} - \frac{1}{2}\partial_\alpha\phi\,\partial_\beta\phi &= 0 \\ \Box\phi &= 0 \end{aligned}$$

- there are two equivalent frames
  - Einstein frame, with metric g_E entering directly above eqns
  - string frame, with rescaled metric $g_s = e^{\frac{1}{2}\phi} g_E$
- differ only by local rescaling, crucial later on
- string frame - curved geometry probed by string

How to extract $\langle T_{\mu\nu} \rangle$ from 5D geometry?

We need to adopt Fefferman-Graham coordinates

$$ds^2 = \frac{\tilde{g}_{\mu\nu} dx^\mu dx^\nu + dz^2}{z^2}$$

where

$$\tilde{g}_{\mu\nu} dx^\mu dx^\nu = -e^{a(t,z)} d\tau^2 + \tau^2 e^{b(\tau,z)} dy^2 + e^{c(\tau,z)} dx_\perp^2$$

Near-boundary metric expansion takes the form (only even powers of z)

$$\tilde{g}_{\mu\nu} = \tilde{g}^{(0)}_{\mu\nu} + z^2 \tilde{g}^{(2)}_{\mu\nu} + z^4 \tilde{g}^{(4)}_{\mu\nu} + O(z^6)$$

where

- $\tilde{g}^{(0)}_{\mu\nu} = \eta_{\mu\nu}$ (4D Minkowski metric)
- $\tilde{g}^{(2)}_{\mu\nu} = 0$ (consistency condition)
- $\tilde{g}^{(4)}_{\mu\nu} = \frac{N_c^2}{2\pi^2} \langle T_{\mu\nu} \rangle$ (VEV of energy-momentum tensor)

Does the gravity in the bulk specifies uniquely $< T_{\mu\nu} >$?

Reproducing 5D geometry from field theory data

Task:

- find a solution of Einstein eqns

$$R_{\alpha\beta} - \frac{1}{2} R\, g_{\alpha\beta} + \Lambda\, (= -6)\, g_{\alpha\beta} = 0$$

- with boundary conditions

$$\tilde{g}_{\mu\nu} = g^0_{\mu\nu}\, (= \eta_{\mu\nu}) + z^4 g^{(4)}_{\mu\nu}\, (= \frac{N_c^2}{2\pi^2} \langle T_{\mu\nu} \rangle) + O(z^6)$$

Solution:

- one can iteratively find higher order terms $\tilde{g}^{(i)}_{\mu\nu}$ (constraints!)
- asymptotic large proper time formula for energy density $\epsilon \sim \frac{1}{\tau^s}$, where $0 < s < 4$ (energy positivity)
- instead of iterating, let's introduce scaling variable $v = \frac{z}{\tau^{s/4}}$
- keeping v fixed while $\tau \to \infty$ reduces Einstein eqns to ODEs
- these can be solved, but how to determine s?

Perfect fluid metric

Regularity of $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ chooses energy density $\epsilon = \frac{e}{\tau^{4/3}}$

Asymptotic geometry looks like

$$ds^2 = \frac{1}{z^2}\Big(-\frac{(1-\frac{e}{3}\frac{z^4}{\tau^{4/3}})^2}{1+\frac{e}{3}\frac{z^4}{\tau^{4/3}}} d\tau^2 + (1+\frac{e}{3}\frac{z^4}{\tau^{4/3}})(\tau^2 dy^2 + dx_\perp^2) + dz^2 \Big)$$

Similar to standard AdS-Schwarzshild, but with horizon "moving away"

$$z_0 = (\frac{3}{e})^{1/4}\tau^{1/3}$$

Naive extraction of thermodynamical quantities

- $T \sim \frac{1}{z_0} \sim \tau^{-1/3}$ (temperature)
- $S \sim \mathrm{AREA} \sim \frac{\tau}{z_0^3} \sim const$ (entropy per u. rapidity and area$_\perp$)

(Janik, Peschanski [hep-th/0512162]**)**

Subasymptotic solution

- we start with

  $$a(\tau,z) = a_0(v) + \frac{1}{\tau^{2/3}} a_1(v) + \frac{1}{\tau^{4/3}} a_2(v) + \ldots$$

  where $v = \frac{z}{\tau^{1/3}}$. Similar relations for $b(\tau,z)$ and $c(\tau,z)$

- rescaling Einstein tensor $G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} - 6 g_{\alpha\beta}$

  $$\tilde{G} = (\tau^{2/3} G_{\tau\tau}, \tau^{4/3} G_{\tau z}, \tau^{-4/3} G_{yy}, \tau^{2/3} G_{xx}, \tau^{2/3} G_{zz})$$

  leads to systematic expansion in powers of $\frac{1}{\tau^{2/3}}$

  $$\tilde{G} = \tilde{G}_0(v) + \frac{1}{\tau^{2/3}} \tilde{G}_1(v) + \frac{1}{\tau^{4/3}} \tilde{G}_2(v) + \ldots$$

- solving problem in perturbative manner:

  $$\tilde{G}_i = 0$$

- first order ($\frac{1}{\tau^{2/3}}$) solution reads

$$a_1(v) = 2\eta_0 \frac{(9+v^4)v^4}{9-v^8}$$
$$b_1(v) = -2\eta_0 \frac{v^4}{3+v^4} + 2\eta_0 \log \frac{3-v^4}{3+v^4}$$
$$c_1(v) = -2\eta_0 \frac{v^4}{3+v^4} - \eta_0 \log \frac{3-v^4}{3+v^4}$$

- giving regular Riemann squared $\mathfrak{R}^2$
- second order solution = lengthy formulas with two parameters
  - η_0 (viscostity coefficient)
  - C (needed to calculate relaxation time)
- Riemann squared takes the form

$$\mathfrak{R}^2 = \# + \frac{1}{\tau^{4/3}} \frac{\text{polynomial in v}, \eta_0 \text{ and } C}{(3-v^4)^4(3+v^4)^6}$$

- is nonsingular only for

$$\eta_0 = \frac{1}{2^{1/4}\, 3^{3/4}}$$ (**Janik** `[hep-th/0710144]`)

- C cannot be determined in this order

Third order solution

- Riemann squared up to this order gives $\Re^2 =$

  $\# + \frac{1}{\tau^2}\left(\frac{\text{polynomial in } v \text{ and } C}{(v-3^{1/4})^4} + 8\, 2^{1/2}\, 3^{3/4}\, \log\left(3^{1/4} - v\right) + \ldots\right)$

- cancelation of 4th order pole at $v = 3^{1/4}$ fixes

  $C = \frac{-17+6\log 2}{3^{1/2}}$

- logarithmic singularity survives
- idea = in string frame dilaton contribution cancels singularity
- indeed the case for $\phi = \frac{1}{\tau^2}\frac{1}{14}\, 2^{1/2}\, 3^{3/4}\, \log\frac{3-v^4}{3+v^4}$
- regularity restored, but in the string frame

Relaxation time in $\mathcal{N} = 4$ SYM

- we assume

  $$\tau_\pi = r\tau_\pi^{Boltzmann} = \frac{3\,r}{2}\frac{\eta}{p}$$

- in the leading order

  $\epsilon \sim \frac{1}{\tau^{4/3}}$ and $\eta \sim \frac{1}{\tau}$ so let's write

  $$\eta = A\,\epsilon^{3/4}$$

- goal = determine A and r
- second order dissipative hydrodynamics gives

  $$-\frac{4\,A\,\epsilon^{3/4}}{\tau} + \frac{4\,\epsilon}{3} + \tau\epsilon' + \frac{21\,A\,r\,\epsilon'}{2\epsilon^{1/4}} + \frac{9\,A\,r\,t\,\epsilon''}{2\epsilon^{1/4}} = 0$$

- vanishing of this expression for energy density

  $$\epsilon = \frac{1}{\tau^{4/3}} - \frac{\sqrt{2}}{3^{3/4}\tau^2} + \frac{1+2\log 2}{12\sqrt{3}\,\tau^{8/3}}$$

  requires

  $$r = \frac{1-\log 2}{9} \text{ and } A = \frac{1}{\sqrt{2}\,3^{3/4}}$$

- relaxation times than takes the form

  $$\tau_\pi = \frac{1-\log 2}{6\,\pi\,T}$$ (T - temperature)

- nontrivial dilaton profile leads to

$$\mathrm{tr}\, F^2 < 0$$

- this means that

$$\left\langle \vec{E}^2 \right\rangle \neq \left\langle \vec{B}^2 \right\rangle$$

- in fact magnetic modes dominate
- relaxation time is almost 30 x shorter than weak coupling approximation

Summary and outlook

Summary

- studying 1D expansion of strongly coupled plasma using AdS/CFT
- regularity of dual geometry chooses the physical behavior
- results are consistent with second order dissipative hydrodynamics

Perspectives

- applying dynamical horizons framework to calculate the entropy *(work in progress)*
- determining short time behavior from geometry regularity *(work in progress)*
- generalizing dynamics to less symmetric situation