Diagrammatic calculation of thermodynamical quantities in nuclear matter

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Zakopane, 27 May - 5 June 2006

Introduction

- (infinite and symmetric) many body system of strongly interacting nucleons
- its thermodynamic properties can be investigated starting from the free N-N potential
- self-consistent finite-temperature Green's function approach

$$G^{<}(\mathbf{p},\omega) = A(\mathbf{p},\omega) f(\omega)$$

- thermodynamical consistency

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Energy Summation of diagrams (in the T-matrix approximation) vs. Galitskii-Koltun's sum rule.

Pressure Including the contribution of the generating functional.

Entropy Full calculation vs. quasiparticle approximation.

Energy of the interacting system

The (total) internal energy is the expectation value of the Hamiltonian

$$\frac{E}{N} = \frac{1}{\rho} \left[\frac{\langle H_{kin} \rangle}{V} + \frac{\langle H_{pot} \rangle}{V} \right] .$$

- kinetic term

$$\langle H_{kin} \rangle = V \int \frac{d^3p}{(2\pi)^3} \frac{d\omega}{2\pi} \frac{\mathbf{p}^2}{2m} A(\mathbf{p}, \omega) f(\omega) ,$$

- potential term

$$\langle H_{pot} \rangle = \frac{V}{2} \int \frac{d^3P}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \frac{d\Omega}{2\pi} V(\mathbf{k}, \mathbf{k}') \langle \mathbf{k}' | G_2^<(\mathbf{P}, \Omega) | \mathbf{k} \rangle .$$

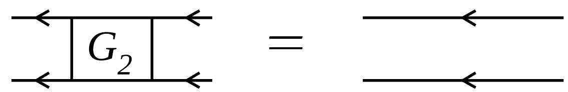
Alternatively, it is possible to estimate the internal energy through the Galitskii-Koltun's sum rule

$$\frac{E}{N} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \frac{d\omega}{2\pi} \left[\frac{\mathbf{p}^2}{2m} + \omega \right] A(\mathbf{p}, \omega) f(\omega) .$$

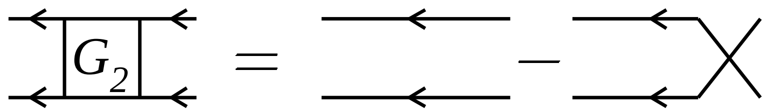
T-matrix or ladder approximation

Whenever there is a two body interaction the equations of motion for the Green's functions couple N-particle and N+1-particle propagators.

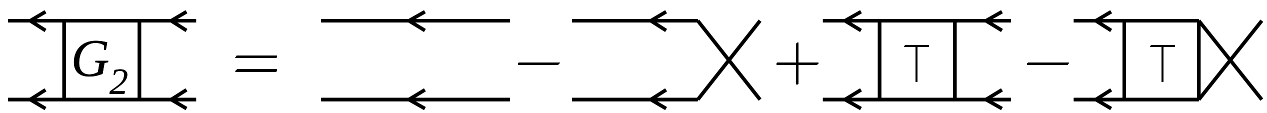
Hartree $\mathcal{G}_2 = G_2^{nc}$



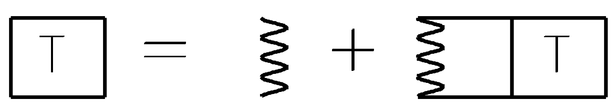
Hartree-Fock $\mathcal{G}_2 = G_2^{nc} + ex$



T-matrix $\mathcal{G}_2 = G_2^{nc} + G_2^{nc}\,T\,G_2^{nc} + ex$



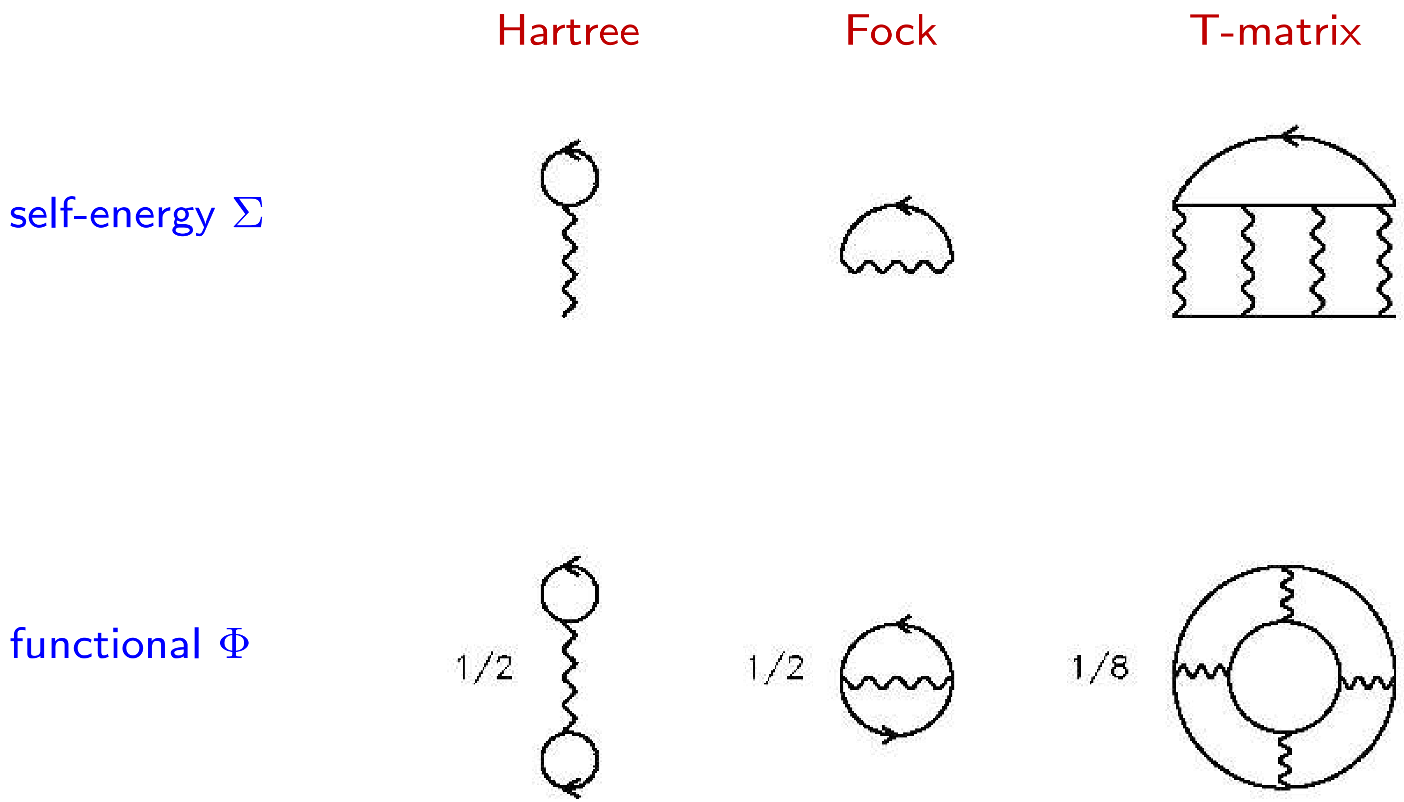
where the in-medium two-particle scattering matrix T is introduced:



The functional Φ

All the previous approximations are Φ-derivable. It means they can be constructed by introducing a generating functional $\Phi[G, V]$ defined as a set of two-particle irreducible diagrams:

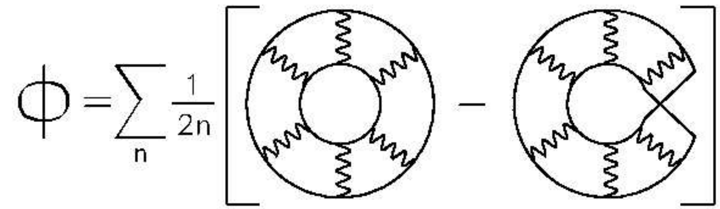
$$\Sigma(x, y) = \frac{\delta \Phi}{\delta G(x, y)} \qquad \text{(Baym, Kadanoff 1961)}$$



Self-consistent T-matrix approach

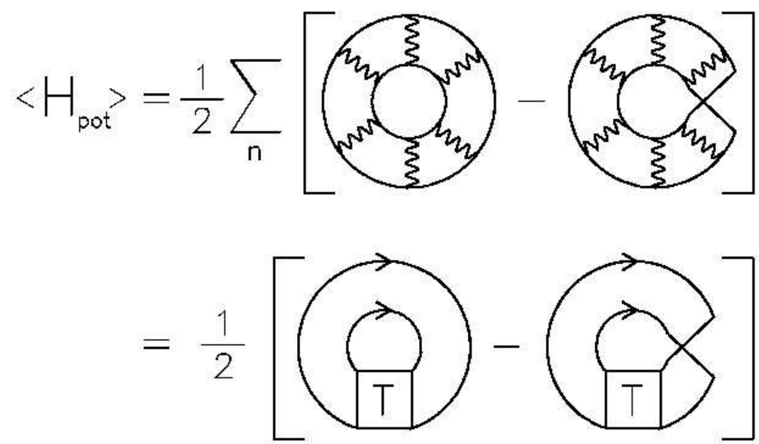
The following iterative scheme si employed:

- calculation of the T-matrix
- calculation of the self-energy
- use of the Dyson equation

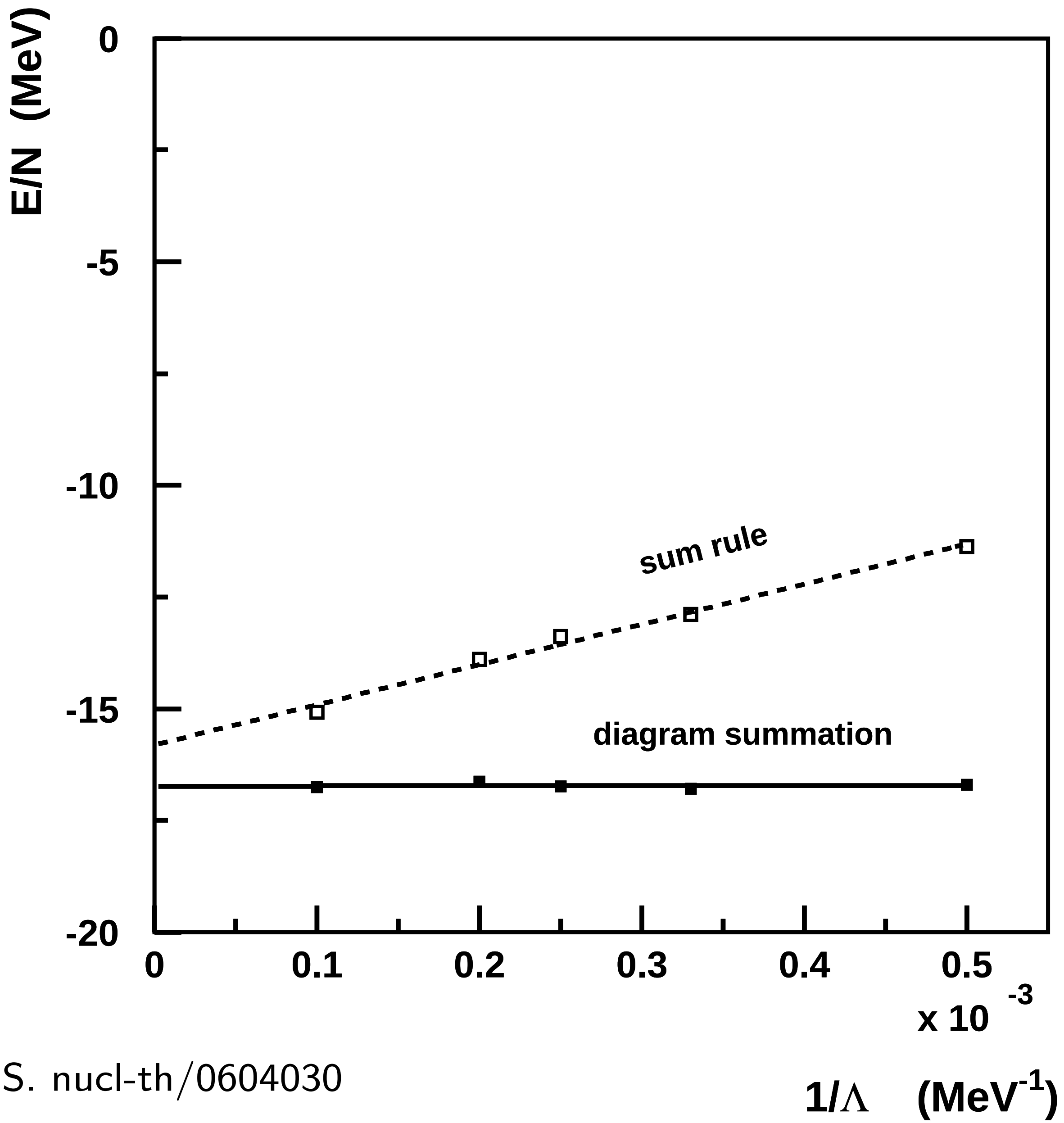


In order to calculate the potential term, one needs VG_2:

$$
\begin{aligned}
V\mathcal{G}_2 &= V\,G_2^{nc} + V\,G_2^{nc}\,T\,G_2^{nc} \\
&= [V + V\,G_2^{nc}\,T]\,G_2^{nc} \\
&= TG_2^{nc}\,.
\end{aligned}
$$



Results: energy



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Pressure

The pressure is related to the thermodynamical potential

$$\Omega(T,\mu,V) = -PV \ .$$

In the scheme of a self-consistent approximation

$$\Omega = -\mathrm{Tr}\{\ln[G^{-1}]\} - \mathrm{Tr}\{\Sigma G\} + \Phi \ .$$

The first contribution to the pressure is

$$P_I = \frac{1}{V}\left[\mathrm{Tr}\{\ln[G^{-1}]\} + \mathrm{Tr}\{\Sigma G\}\right]$$

$$= T\int \frac{d^3p}{(2\pi)^3}\frac{d\omega}{2\pi}\ln(1+e^{-\beta\omega})\,[\ A(\mathbf{p},\omega)$$

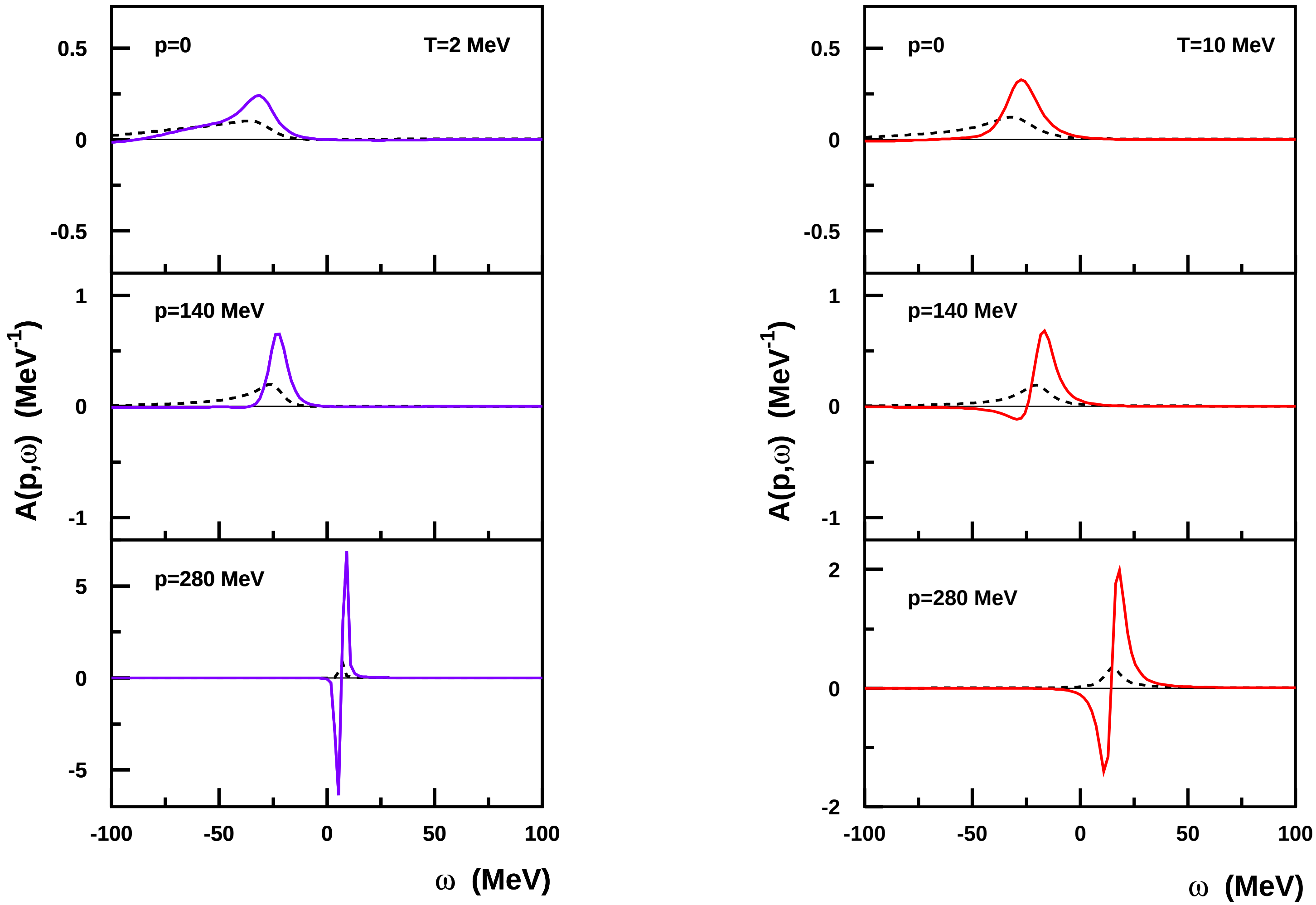
$$+\frac{\partial A(\mathbf{p},\omega)}{\partial\omega}\mathrm{Re}\Sigma^R(\mathbf{p},\omega) - 2\mathrm{Im}\Sigma^R(\mathbf{p},\omega)\frac{\partial \mathrm{Re}G^R(\mathbf{p},\omega)}{\partial\omega}\ ]$$

$$= T\int \frac{d^3p}{(2\pi)^3}\frac{d\omega}{2\pi}\ln(1+e^{-\beta\omega})B(\mathbf{p},\omega) \ .$$

The spectral function $B(\mathbf{p}, \omega)$

A new spectral function, $B(\mathbf{p}, \omega)$, is introduced; it is normalized exactly as the *ordinary* spectral function $A(\mathbf{p}, \omega)$:

$$\int \frac{d\omega}{2\pi} B(\mathbf{p}, \omega) = 1 \, .$$



The contribution from Φ

The second contribution to the pressure comes from the functional Φ. It is calculated by introducing an integration over the (artificial) parameter λ:

$$\begin{aligned}
\Phi &= \sum_n \frac{1}{2n} \mathrm{Tr}\{(V\,G_2)^n\} \\
&= \int_0^1 \frac{d\lambda}{\lambda} \sum_n \frac{1}{2} \mathrm{Tr}\{(\lambda V\,G_2)^n\} \\
&= \int_0^1 \frac{d\lambda}{\lambda} < H_{pot}(\lambda V, G_{\lambda=1}) > \quad .
\end{aligned}$$

Since $< H_{pot}(V,G) > \sim \frac{1}{2} \mathrm{Tr}\{T\,G_2^{nc}\}$ and $T = \frac{V}{1 - V\,G_2^{nc}}$, finally

$$\int_0^1 \frac{d\lambda}{\lambda} < H_{pot}(\lambda V, G_{\lambda=1}) > \sim \frac{1}{2} \int_0^1 d\lambda \frac{\mathrm{Tr}\{V\,G_2^{nc}\}}{1 - \mathrm{Tr}\{\lambda V\,G_2^{nc}\}} \, .$$

Results: pressure

The results are compared with the pressure of a gas of quasiparticles in a mean-field potential

$$P_{qp} = \int \frac{d^3p}{(2\pi)^3} f(\omega_p - \mu) \left[\frac{p}{3} \frac{d\omega_p}{dp} + \frac{1}{2} \Sigma_p \right] ,$$

with $\Sigma_p = \Sigma(p, \omega_p) = \omega_p - p^2/2m + \mu$.

Results are displayed in the table below (in MeV):

T	E_{GK}/N	E_{diag}/N	P_I/ρ	P_{II}/ρ	P_{tot}/ρ	P_{qp}/ρ
0	-15.80	-16.63	-40.19	32.50	-7.69	-0.85
2	-15.15	-16.29	-38.40	32.54	-5.86	-0.78
5	-14.40	-15.24	-37.83	32.35	-5.48	-0.74
10	-11.15	-11.72	-34.02	31.36	-2.66	-0.49
20	-1.29	-1.21	-24.43	30.92	6.49	0.17

Entropy

The entropy is estimated through the thermodynamic relation

$$\frac{S}{N} = \frac{1}{T}\left[\frac{E}{N} + \frac{P_{tot}}{\rho} - \mu\right] .$$

It is then compared with analytical expressions:

1. the reduced formula

$$\frac{S_{red}}{N} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3}\frac{d\omega}{2\pi}\sigma(\omega)B(\mathbf{p},\omega),$$

   where

$$\sigma(\omega) = -f(\omega)\ln[f(\omega)] - [1-f(\omega)]\ln[1-f(\omega)] \; ;$$

2. the entropy for a free Fermi gas

$$\frac{S_{free}}{N} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3}\sigma(\omega_p),$$

   in the low T limit $\quad \dfrac{S_{free}}{N} = \dfrac{\pi^2 mT}{{p_F}^2} \; ;$

3. the entropy calculated as for a free Fermi gas but using the effective mass m^* instead of the rest mass m

$$\text{(in the low T limit)} \qquad \frac{S_{free^\star}}{N} = \frac{\pi^2 m^* T}{{p_F}^2}.$$

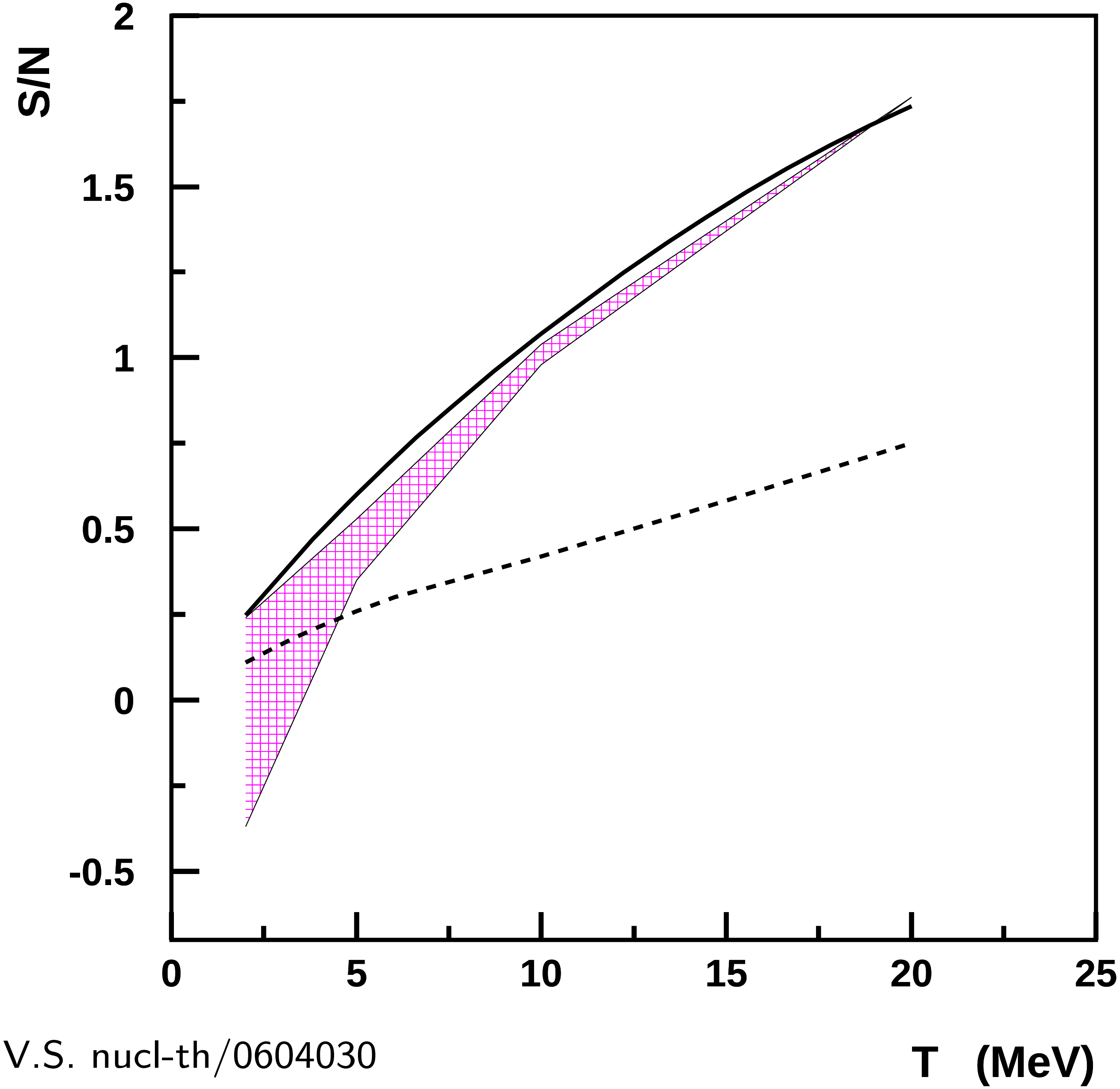
The effective mass m^* is determined at each temperature by

$$\left(\frac{\partial \omega_p}{\partial p^2}\right)_{p=p_F} = \frac{1}{2m^\star} \, .$$

Results are displayed in the table below:

T (MeV)	S_{GK}/N	S_{diag}/N	S_{free}/N	$S_{free^\star}/N$	S_{red}/N
2	0.24	−0.37	0.27	0.24	0.11
5	0.53	0.35	0.66	0.60	0.26
10	1.04	0.98	1.22	1.07	0.42
20	1.76	1.76	2.02	1.74	0.75

Results: entropy



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Summary

- *Internal energy, pressure* and *entropy* calculated in the thermodynamically consistent T-matrix approximation in nuclear matter.

- For what concerns the internal energy, the Galitskii-Koltun's sum rule and the summation of diagrams yield similar results (up to 1 MeV difference).

- The pressure was estimated from the summation of diagrams contributing to the functional Φ; three-body forces needed.

- Accordingly the entropy was calculated; the entropy of a free Fermi gas turns out to be close to the result of the full calculation (if $m \to m^*$).

- Possible applications are the modeling of neutron stars and intermediate-energy heavy ion collisions.

References

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