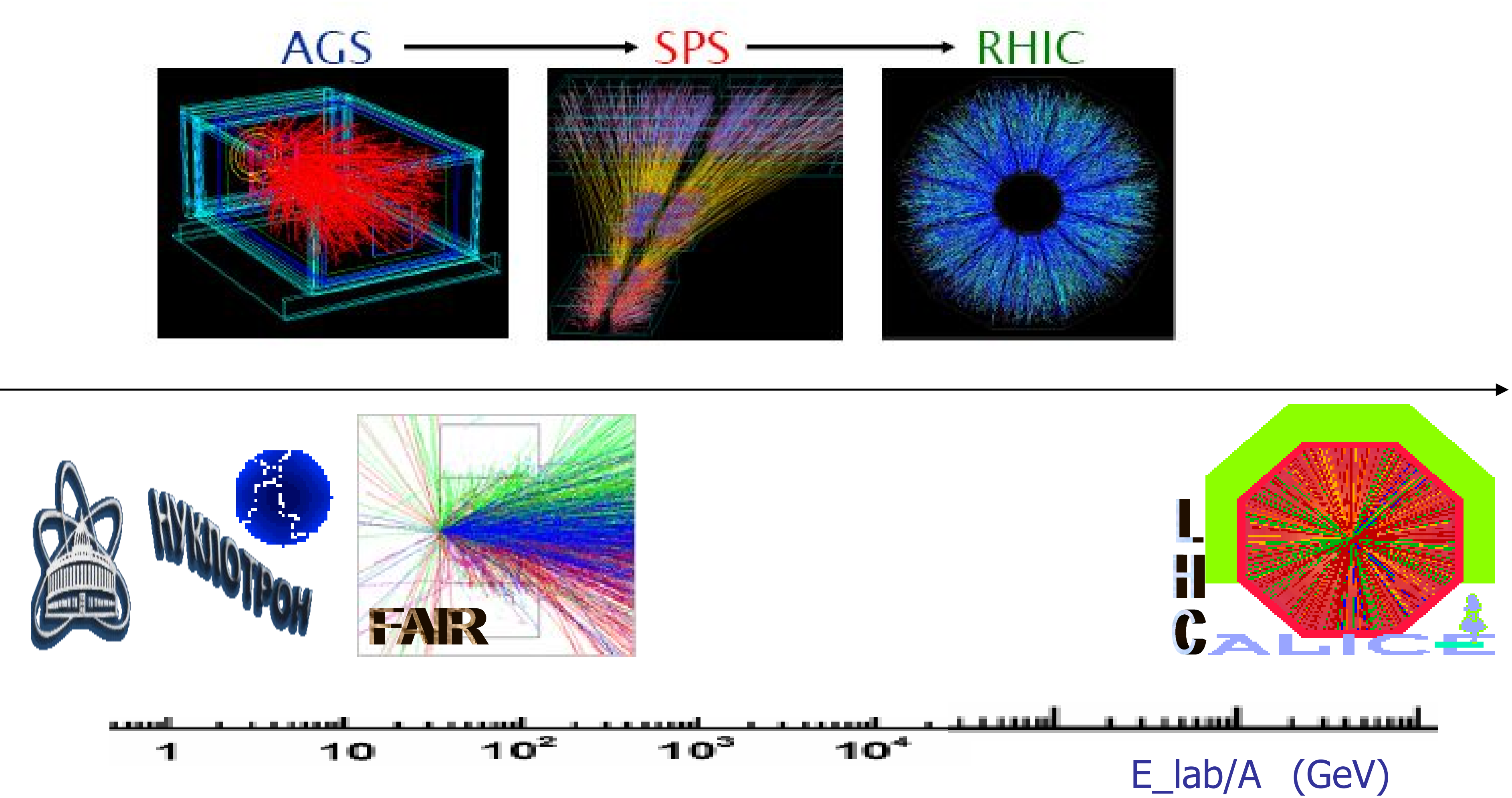
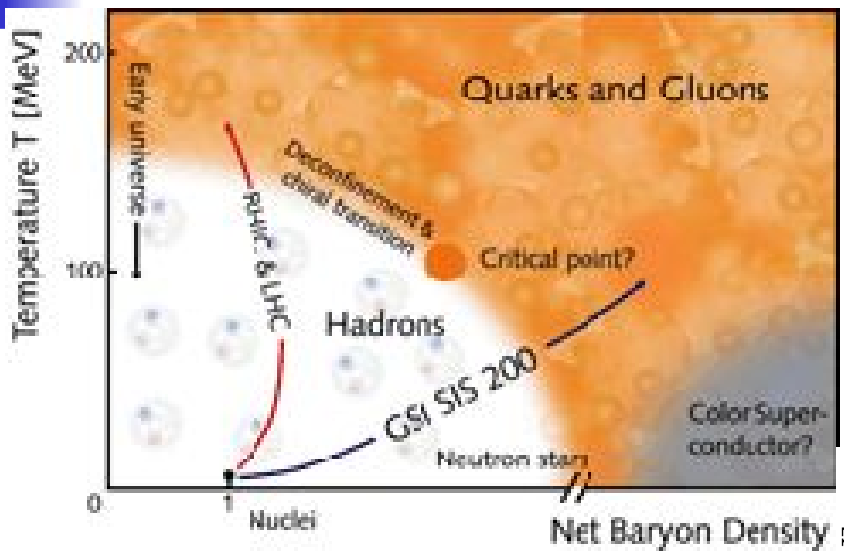
Matter evolution and soft physics in A+A collisions

Yu. Sinyukov, BITP, Kiev

Heavy Ion Experiments



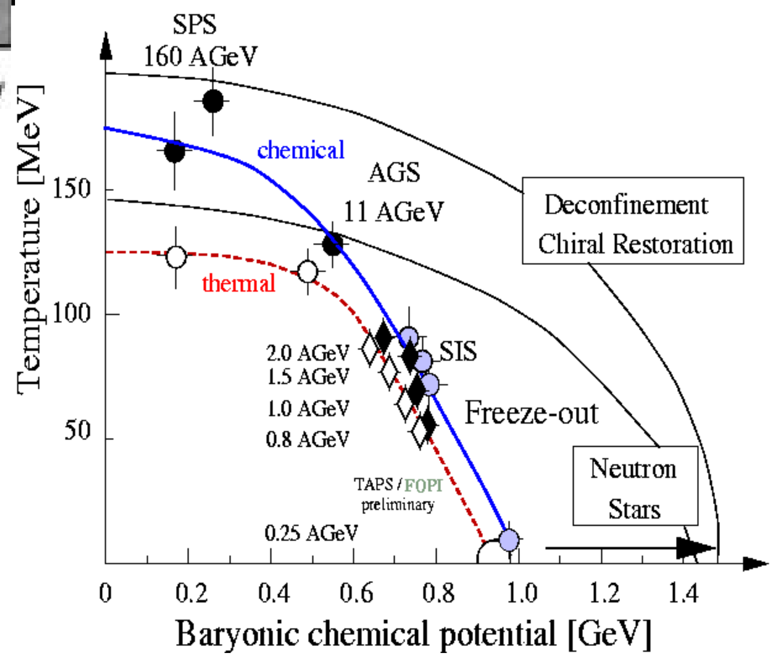
Thermodynamic QCD diagram of the matter states



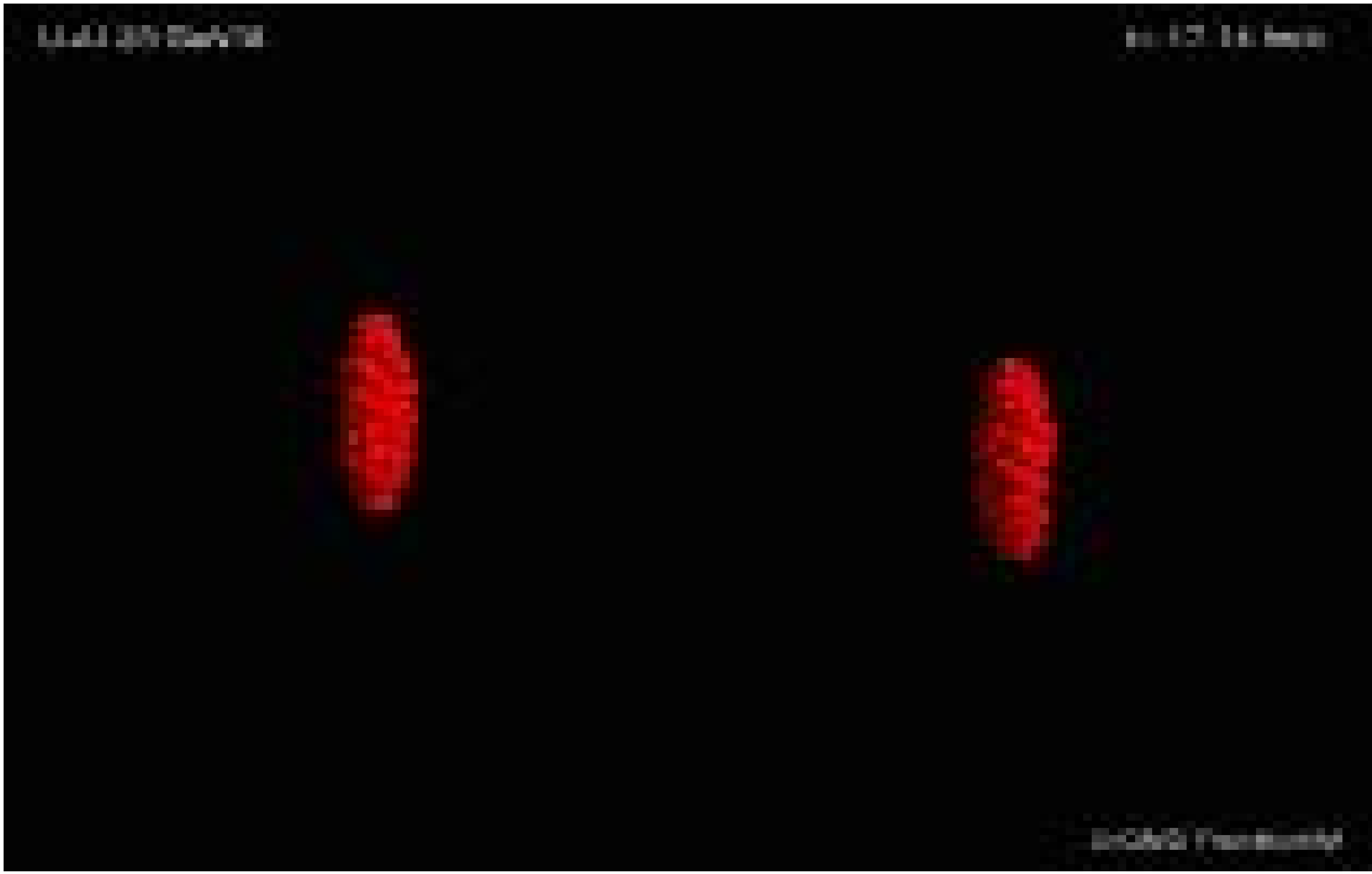
The thermodynamic arias

occupied by different forms of matter

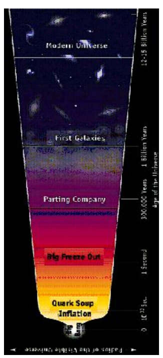
Theoretical expectations vs the experimental estimates

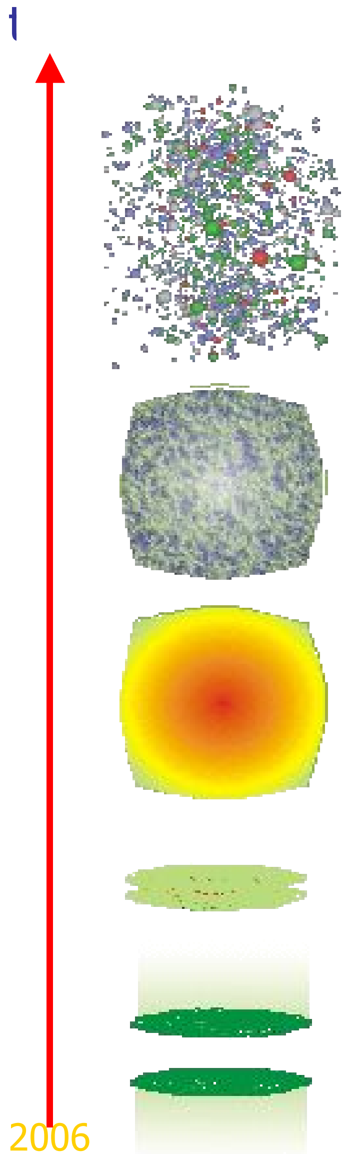


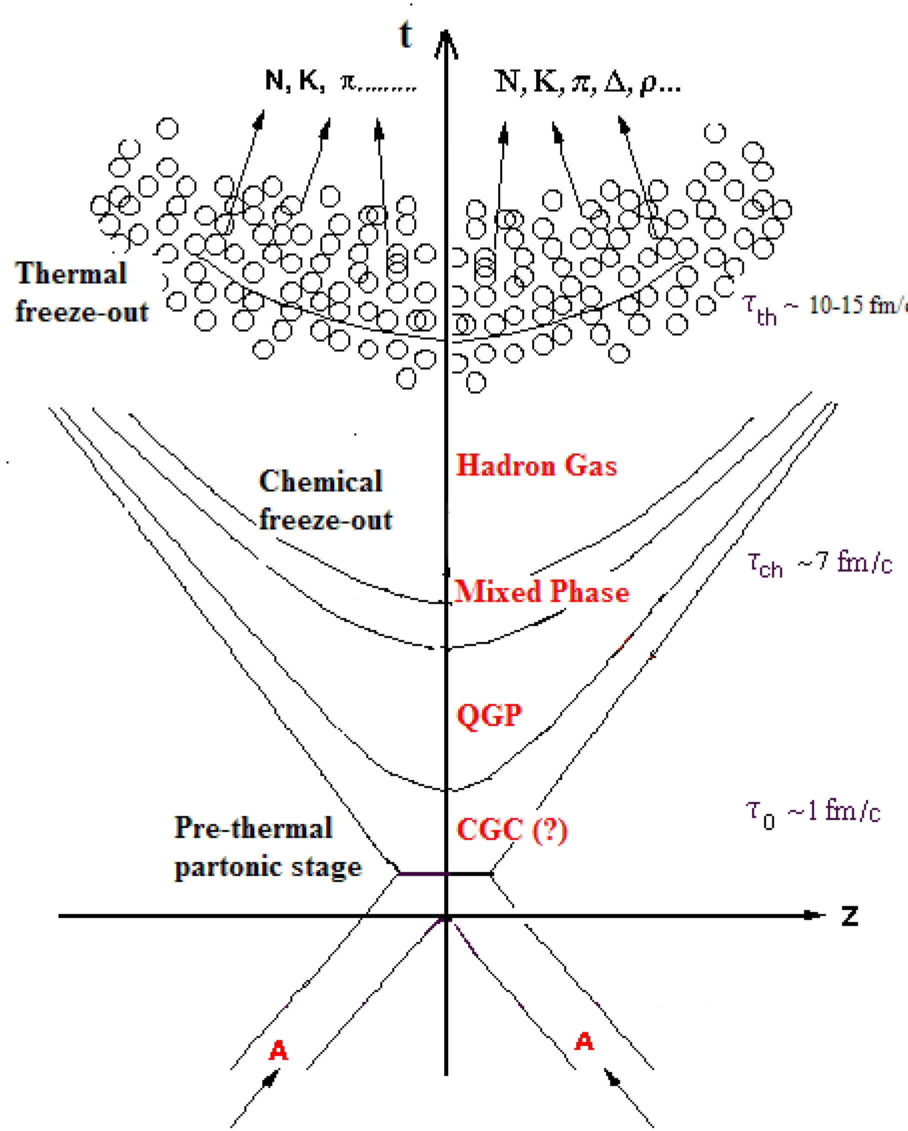
UrQMD Simulation of a U+U collision at 23 AGeV



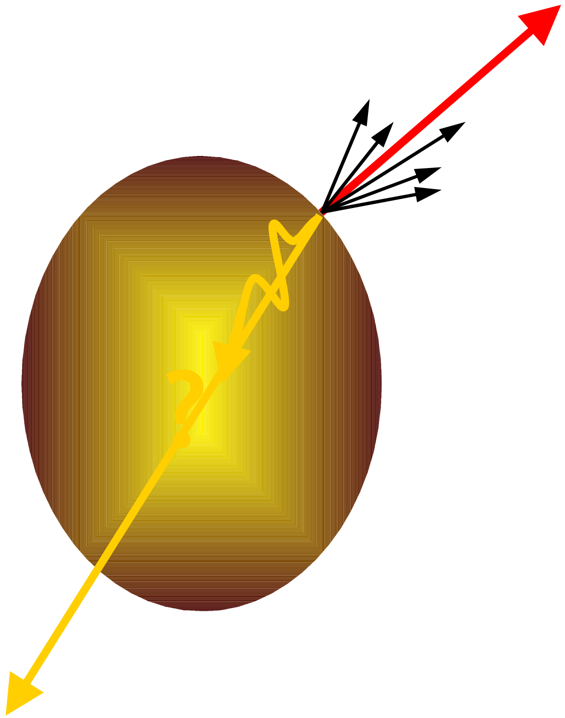
Expecting Stages of Evolution in Ultrarelativistic A+A collisions



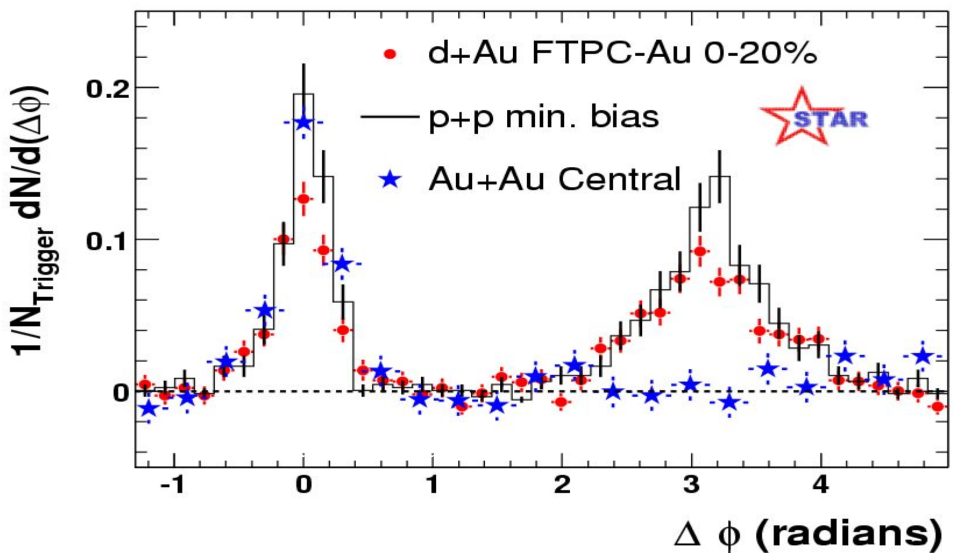




Jet quenching as a signature of very dense matter

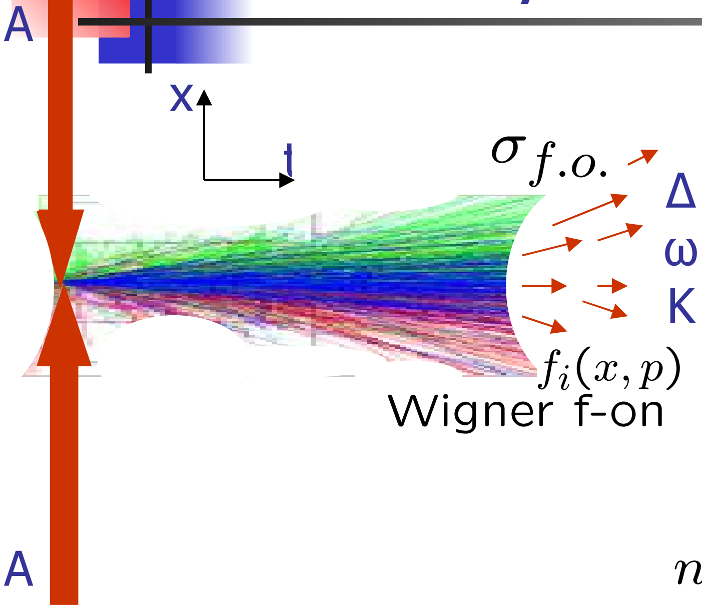


Phys. Rev. Lett. 91, 072304 (2003).



"… was observed *jet quenching* predicted to occur in a hot deconfined envi-ronment 100 times dense than ordinary nuclear matter" (BNL RHIC, June 2003).

"Soft Physics" measurements



$$N_i = \int \frac{d^3p}{p^0} d\sigma_\mu p^\mu f_i(x,p)$$

$$n_i(p) \equiv p^0 \frac{d^3 N_i}{d^3 p} = \int d\sigma_\mu p^\mu f_i(x,p) \sim e^{-m_{i,T}/T_{eff,i}}$$

$$n_i(p_1,p_2) \equiv p_1^0 p_2^0 \frac{d^6 N_i}{d^3p_1 d^3p_2} = C(p,q)\, n(p_1) n(p_2)$$

$p=(p_1+p_2)/2$

$q=p_1-p_2$

$\{\frac{N_i}{N_j}\}$ ⇒ T_{ch} and μ_{ch} soon after hadronization (chemical f.o.)

$$\frac{d^3N}{dp_t\, dy\, d\varphi} = \frac{d^2N}{dp_t\, dy} \frac{1}{2\pi}(1+2v_1\cos(\varphi)+2v_2\cos(2\varphi)+\ldots)$$

Directed flow

Elliptic flow

Radial flow ⇒ $T_{eff,i} \approx T_{f.o.} + m_i \frac{\langle v^2 \rangle}{2}$

(QS) Correlation function

$1+\exp(R_L^2 q_L^2 + R_T^2 q_T^2)$

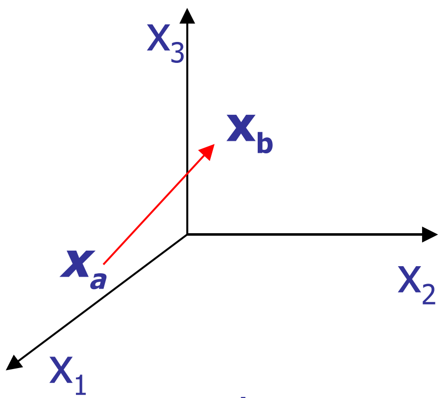
⇓

Space-time structure of the matte

$$\tau \approx R_L \sqrt{\frac{m_T}{T_{f.o.}}}$$

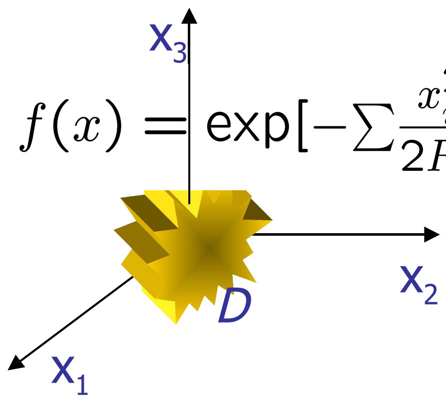
Interferometry microscope: GGLP -1960, *Kopylov/Podgoretcky* -1971

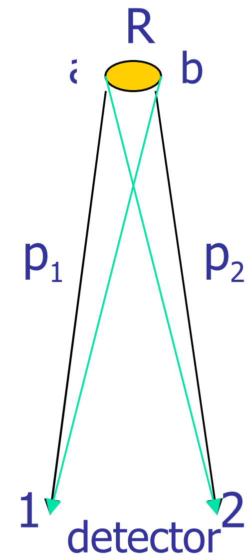
The idea of the correlation femtoscopy is based on an impossibility to distinguish between registered particles emitted from different points.



t=0

$$f(x) = \exp[-\sum \frac{x_i^2}{2R_i^2}]$$





Momentum representation

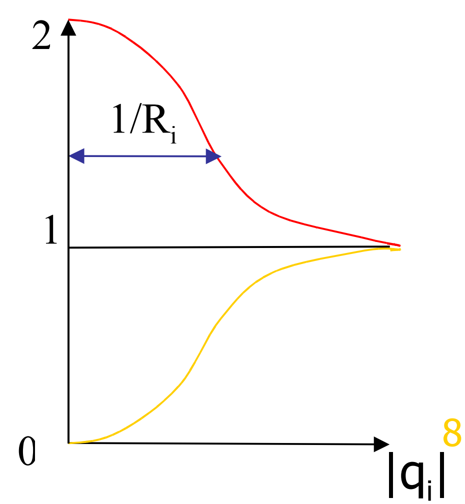
$$\Psi_{x_a,x_b}(p_1,p_2) = \frac{1}{\sqrt{2}}\left[e^{-i\mathbf{p}_1\cdot\mathbf{x}_a}e^{-i\mathbf{p}_2\cdot\mathbf{x}_b} \pm e^{-i\mathbf{p}_2\cdot\mathbf{x}_a}e^{-i\mathbf{p}_1\cdot\mathbf{x}_b}\right]$$

Probabilities:

$$W_{x_a,x_b}(p_1,p_2) = \left|\Psi_{x_a,x_b}(p_1,p_2)\right|^2 = 1 \pm \cos\left[\underbrace{(\mathbf{p}_1 - \mathbf{p}_2)}_{\mathbf{q}} \cdot (\mathbf{x}_a - \mathbf{x}_b)\right]$$

$$W_D(p_1,p_2) = \int d^3x_a d^3x_b f(x_a) f(x_b) W_{x_a,x_b}(p_1,p_2) =$$

$$1 \pm \left|\int d^3x f(x) e^{i\mathbf{q}\mathbf{x}}\right|^2 = 1 \pm \exp\left[-\sum q_i^2 R_i^2\right]$$



THE DEVELOPMENT OF THE FEMTOSCOPY

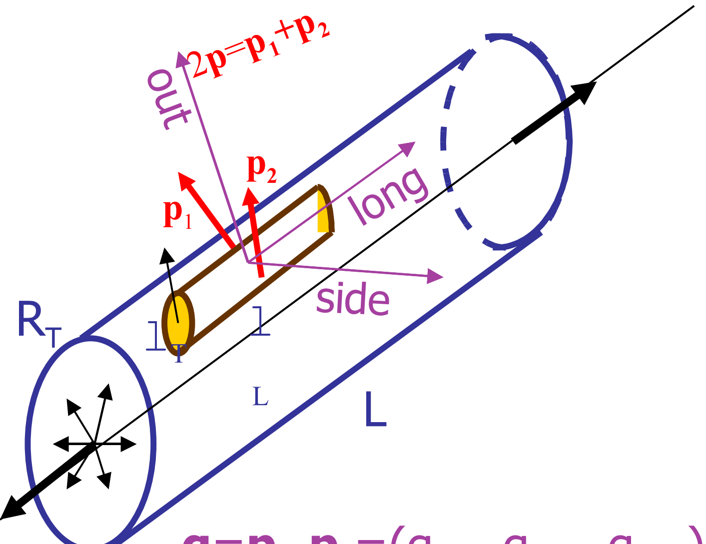
- **Even ultra small systems can have an internal structure.**

  Then the distribution function *f(x,p)* and emission function of such an objects are inhomogeneous and, typically, correlations between the momentum *p* of emitted particle and its position x appear.

- **In this case and in general the interferometry microscope measure the homogeneity lengths in the systems** [Yu. Sinyukov , 1986, 1993-1995].

$$\frac{\left|f(p,x_0+\lambda)-f(p,x_0)\right|}{f(p,x_0)}=\frac{1}{2} \quad \text{at} \quad \left.\frac{\partial f(p,x)}{\partial x_i}\right|_{x_0(p)}=0 \quad \Longrightarrow \quad \lambda_i^2=\frac{f(x_0,p)}{\left|f''_{x_i}(x_0,p)\right|}$$

- **Idea of femtoscopy scanning of a source over momentum:** Averchenkov/Makhlin/Yu.S.



Interferomety radii:

$$R_L(p_T)\approx\lambda_L=\tau\sqrt{\frac{T_{f.o.}}{m_T}},\, m_T=\sqrt{m^2+p_T^2}$$

$$R_S\approx\lambda_T=R_T/\sqrt{1+Im_T/T_{f.o.}},\; I\propto grad(v_T)$$

$$R_o^2\approx\lambda_T^2+v^2\langle\triangle t^2\rangle_p-2v\langle\triangle x_o\triangle t\rangle_p,\, v=\frac{p_{out}}{p_0}$$

$q=p_1-p_2=(q_{out}, q_{side}, q_{long})$ in $C(p,q)=\dfrac{d^6N/d^3p_1d^3p_2}{d^3N/d^3p_1d^3N/d^3p_2}\approx 1+e^{R_L^2(p)q_L^2+R_s^2(p)q_s^2+R_O^2(p)q_O^2}$

Resonance and Coulomb effects: "Bowler-Sinyukov treatment".

- Bose-Einstein correlations are seriously distorted by two factors:

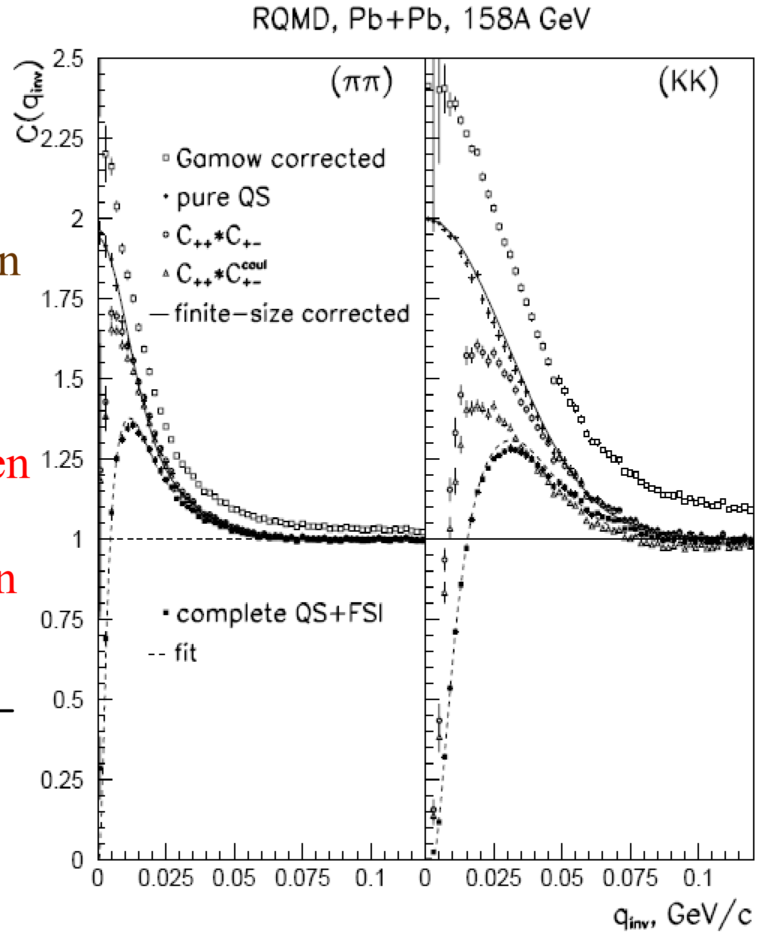
L decays of long-lived resonances: width of the CF then much less then detector resolution. It leads to suppression of the correlations.

K Long-scale Coulomb forces between charged identical particles which also depend on an extension of pion source

$$C(q,p) = (1-\Lambda) + \Lambda K_{coul}[1+\exp(R_o^2 q_o^2 + R_s^2 q_s^2 + R_l^2 q_l^2)]$$

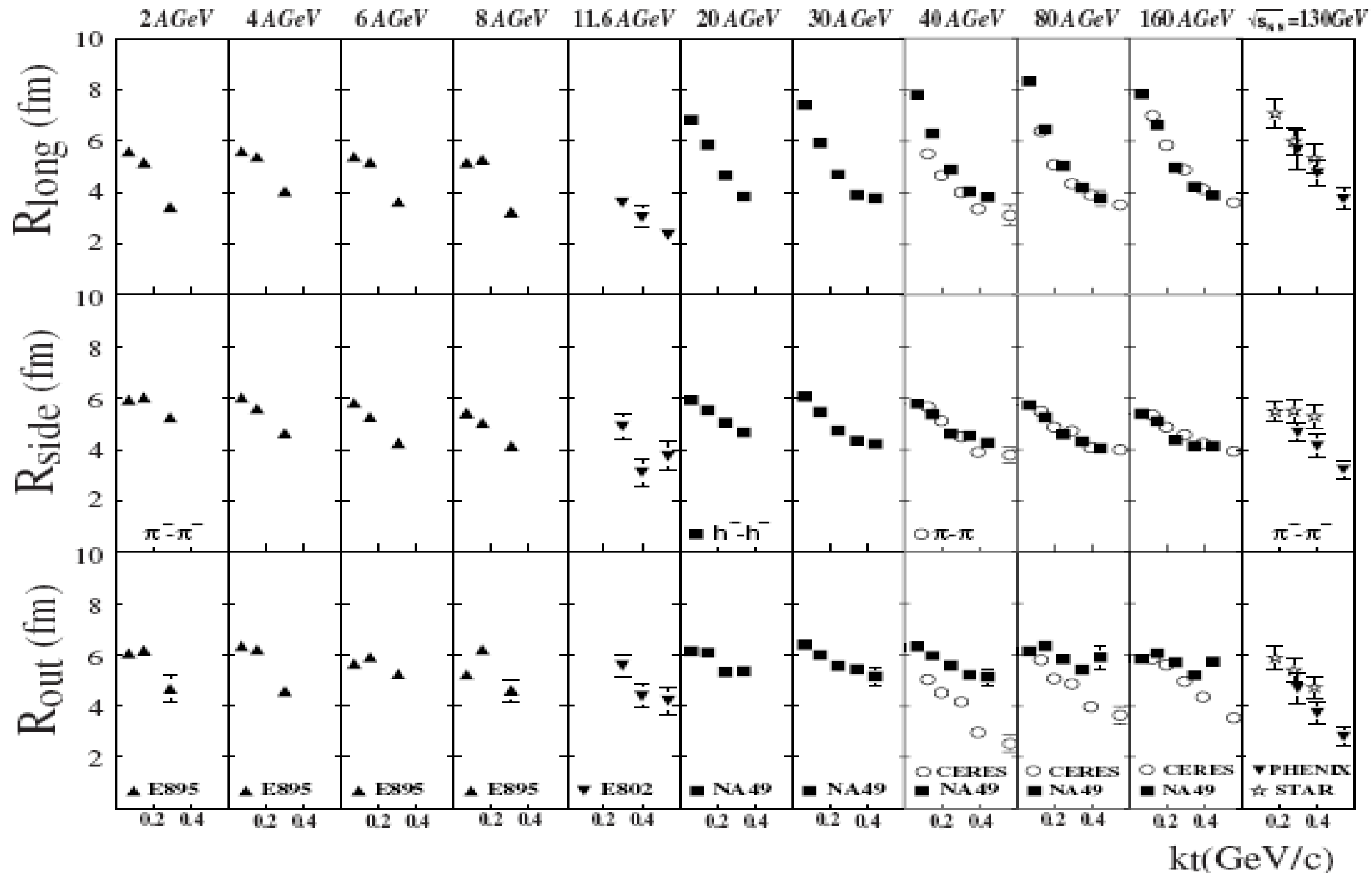
$$K_{coul} = K_{coul}(q_{inv}a, \frac{<r*>}{a}) = <|\psi^c_{-\mathbf{q}*}(\mathbf{r}*)|^2>$$

a is Bohr radius, $<r*> = <r*>_{[R_i,\Lambda]}$



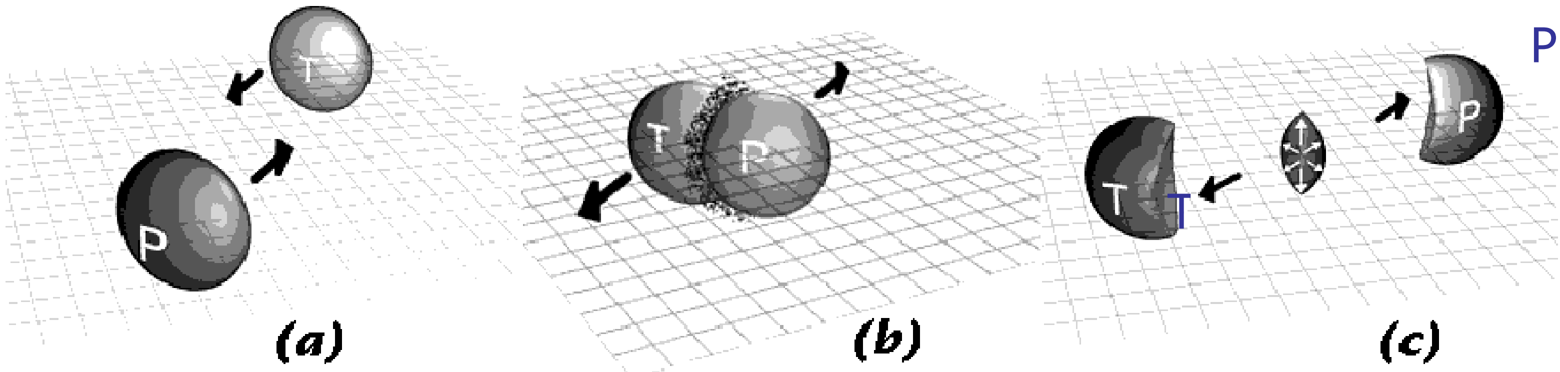
Y. Sinyukov, R. Lednicky, S. V. Akkelin, J. Pluta, B. Erazmus, Phys. Lett., **B432** (1998) 248–257.

Energy dependence of the interferometry radii



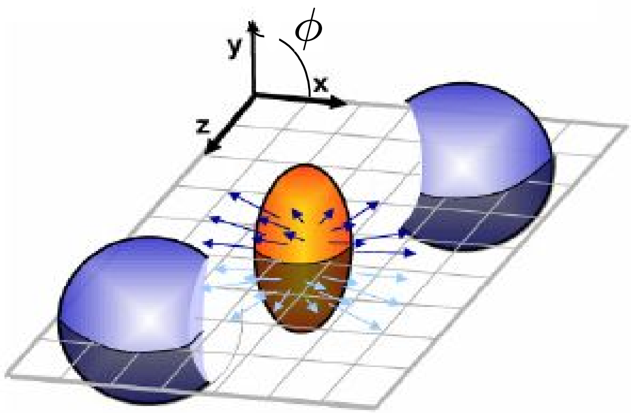
Energy- and Pt-dependence of the radii Rlong, Rside, and Rout for central Pb+Pb (Au+Au) collisions from AGS to RHIC experiments measured near midrapidity.
S. Kniege et al. (The NA49 Collaboration), J. Phys. G30, S1073 (2004).

Collective flows

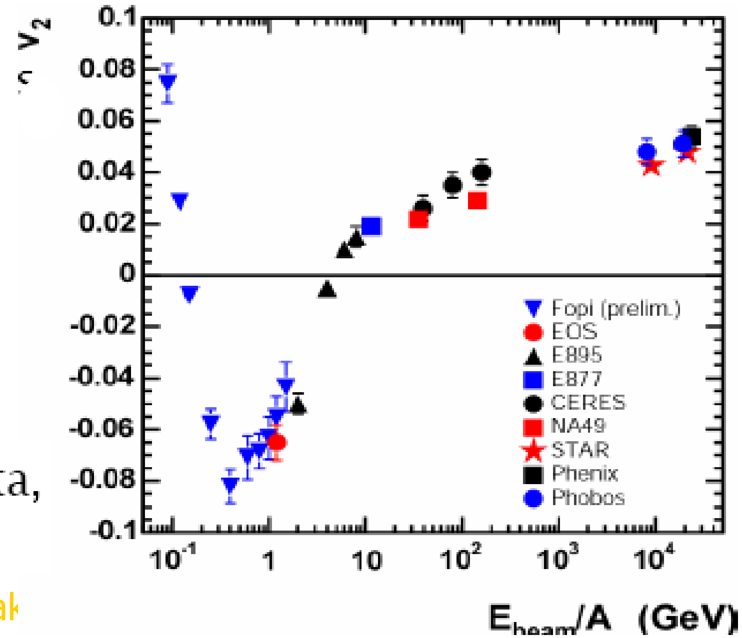


Initial spatial anisotropy → different pressure gradients → momentum anisotropy v2

$$\frac{dN}{d\phi} \sim [1 + 2v_1\cos(\phi) + 2v_2\cos(2\phi)]$$



Mid-rapidity data,
p_t integrated



Empirical observations and theoretical problems (1)

EARLY STAGES OF THE EVOLUTION

- An satisfying description of elliptic flows at RHIC requires the earlier thermalization, $\tau_{th} \simeq 0.6 fm/c$, and perfect fluidity.

- The letter means an existence of a new form of thermal matter: asymptotically free QGP $\rightarrow$ strongly coupled sQGP.

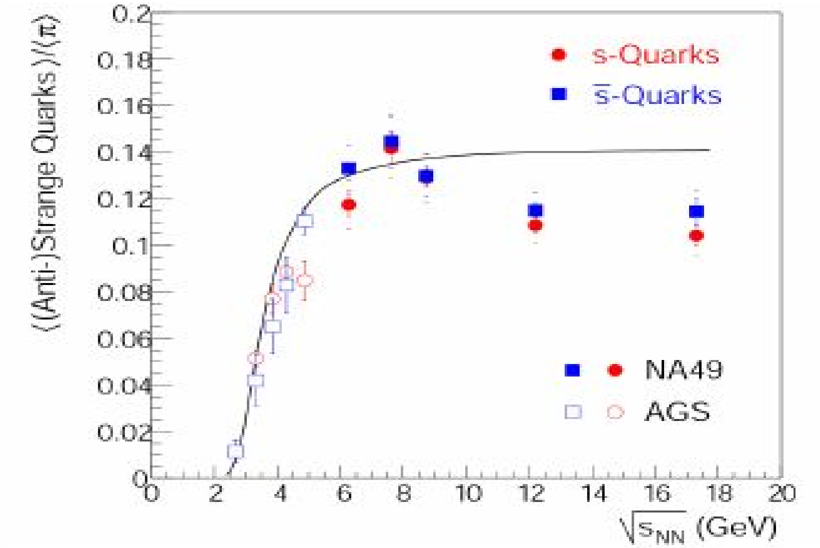
? PROBLEM:

How does the initially coherent state of partonic matter – CGC transform into the thermal sQGP during extremely short time ~ ½ fm/c (problem of thermalization).

Empirical observations and theoretical problems (2)

LATE STAGES OF THE EVOLUTION:

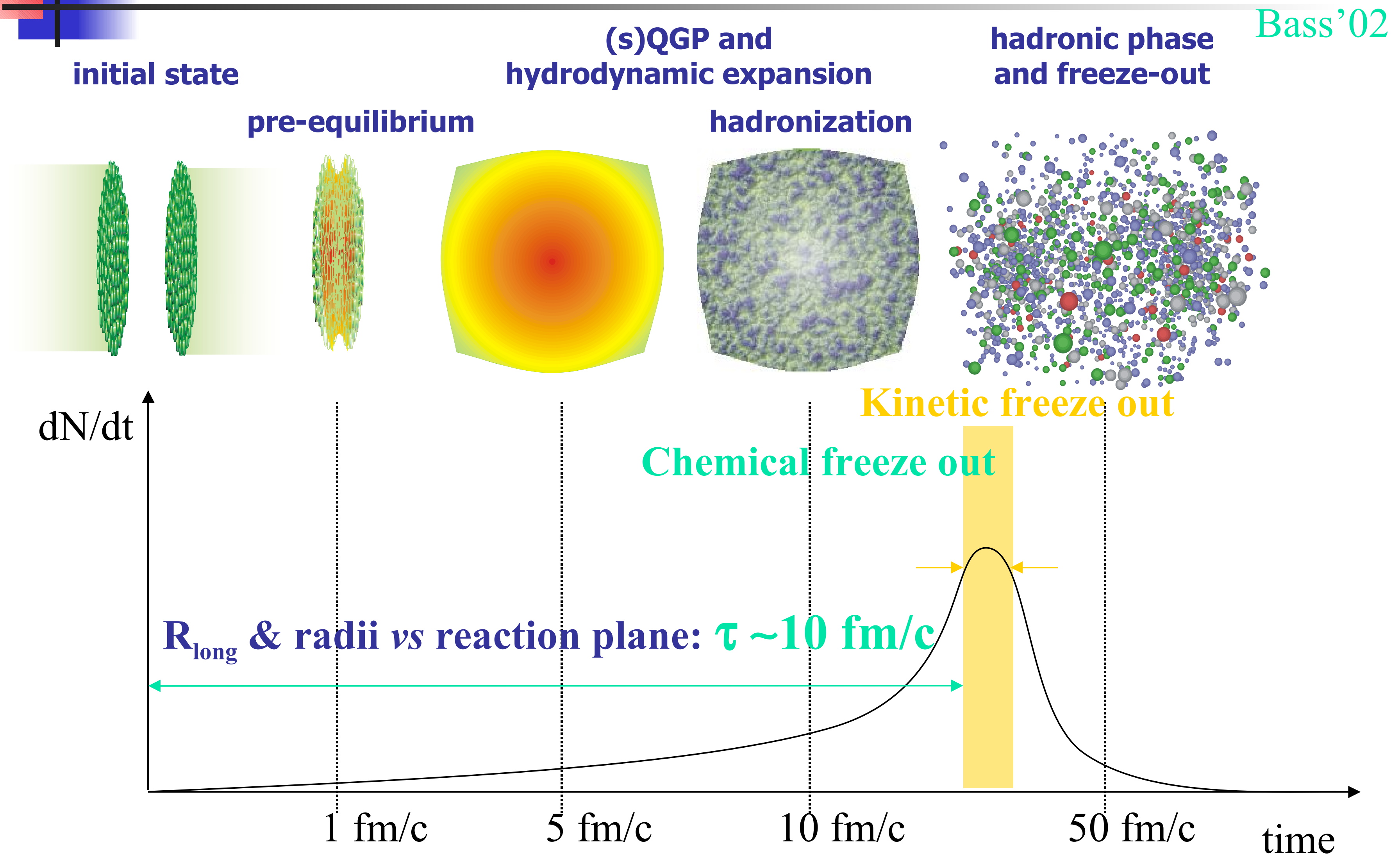
- No direct evidence of (de)confinement phase transition in “soft physics” except (**?**) for strange particles: NA49
  + Gadzidzki/Gorenstein
  However: it needs asymp. free QGP (+ light quarks)



- HBT PUZZLE. The behavior of the interferometry volume are only slightly depends on the collision energy: R_L slightly grows with $\sqrt{s}$ and
  $R_{out} \approx R_{side} \cong R_T \approx const$

- Realistic hydro (or hydro + cascade) models does not describe the interferometry radii – space-time structure of the collisions.

Evolution in hadronic cascade models (UrQMD) vs Hydro



Problems of Evolution

- Is Landau's idea of multiparticle production through hydro (with universal freeze-out at $T \approx m_\pi$) good?

- Or, under which condition is it good?

- What can we learn from a general analysis of Boltzmann equations?

Way to clarify the problems

Analysis
of evolution of observables in hydrodynamic and kinetic models of A+A collisions

Yu.M. Sinyukov, S.V.Akkelin, Y. Hama: Phys. Rev. Lett. 89, 052301 (2002);

S.V.Akkelin. Yu.M. Sinyukov: Phys. Rev. C **70** , 064901 (2004);
Phys.Rev. C **73**, 034908 (2006);
Nucl. Phys. A (2006) in press

N.S. Amelin, R. Lednicky, L. V. Malinina, T. A. Pocheptsov and Yu.M. Sinyukov:
Phys.Rev. C **73**, 044909 (2006)

Particle spectra and correlations

- Inclusive spectra $$p^0 \frac{dN}{d\mathbf{p}} \equiv n(p) = \langle a_p^+ a_p \rangle, \quad p_1^0 p_2^0 \frac{dN}{d\mathbf{p}_1 d\mathbf{p}_2} \equiv n(p_1, p_2) = \langle a_{p_1}^+ a_{p_2}^+ a_{p_1} a_{p_2} \rangle$$
- Chaotic source $$n(p_1, p_2) = \langle a_{p_1}^+ a_{p_1} \rangle \langle a_{p_2}^+ a_{p_2} \rangle + \langle a_{p_1}^+ a_{p_2} \rangle \langle a_{p_2}^+ a_{p_1} \rangle$$
- Correlation function $$C(p_1, p_2) = n(p_1, p_2) / n(p_1) n(p_2)$$
- Irreducible operator averages:

$$\langle a_{p_1}^+ a_{p_2} \rangle = \int_{\sigma_{out}} d\sigma_\mu p^\mu \exp(iqx) f(x, p); \quad p = (p_1 + p_2)/2, \ q = p_1 - p_2$$

Escape probability

- **Boltzmann Equation:**

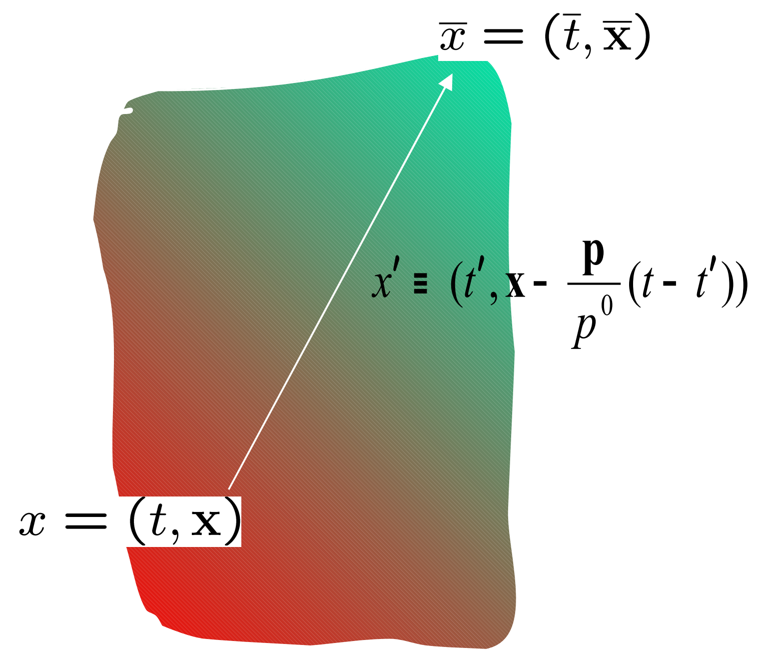
$$\frac{p^\mu}{p^0}\frac{\partial f(x,p)}{\partial x^\mu} = C^{gain}(x,p) - C^{loss}(x,p)$$

rate of collisions

$$F^{loss}(x,p) = R(x,p)f(x,p) \quad \text{where} \quad R(x,p) = \langle \sigma v_{rel} \rangle n(x)$$

- **Escape probability (at $\bar{t} \to \infty$):**

$$\mathcal{P}_{\bar{t}}(x,p) = \exp\left(- \int_t^{\bar{t}} dt' R(x',p) \right)$$



Distribution and emission functions

- Integral form of Boltzmann equation

$$f(t,\mathbf{r},\mathbf{p}) = \mathbf{f}(\mathbf{t_0},\mathbf{x}-\mathbf{v}(\mathbf{t}-\mathbf{t_0}),\mathbf{p})\mathcal{P}_{\mathbf{t}}(\mathbf{x}-\mathbf{v}(\mathbf{t}-\mathbf{t_0}),\mathbf{p})+$$

$$\int_{t_0}^{t} C_{gain}(\tau,\mathbf{x}-\mathbf{v}(t-\tau),p)\mathcal{P}_t(\mathbf{x}-\mathbf{v}(t-\tau),p)d\tau$$

- Operator averages

Distribution function

$$\langle a^{+}_{p_1} a_{p_2}\rangle_{|\sigma} = \int_\sigma d^3\sigma_\mu(x) p^\mu e^{iqx} f(x,p) =$$

$$\int_{\sigma_0} d^3\sigma_\mu(x_0) p^\mu f(x_0,p)\mathcal{P}_\sigma(x,p) e^{iqx_0} + \int_{\sigma_0}^{\sigma} d^4x e^{iqx} p^0 C_{gain}(x,p)\mathcal{P}_\sigma(x,p)$$

Emission function

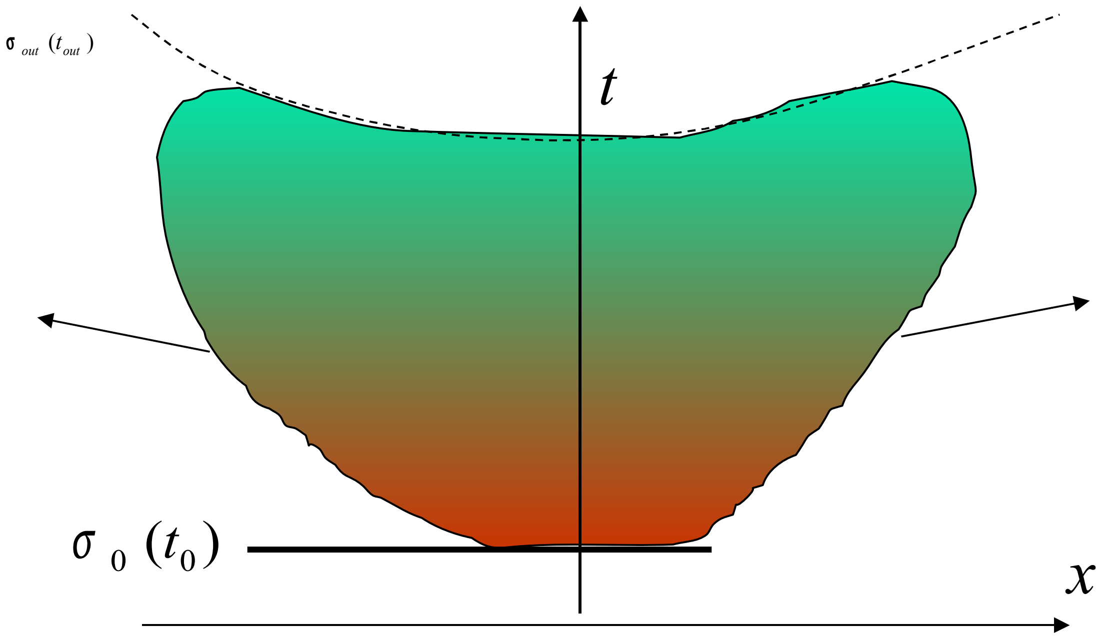
$S_0^\mu(x_0,p)$ Initial emission

$S(x,p)$ Emission density

Dissipative effects & Spectra formation

$$\frac{p^\mu}{p^0}\frac{\partial f(x,p)}{\partial x^\mu} = C^{gain}(x,p) - C^{loss}(x,p)$$

$$\partial_\mu \left[p^\mu \exp(iqx)\right] = 0$$

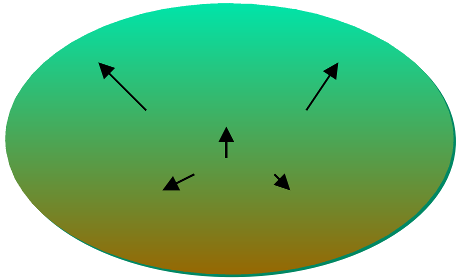


$$\langle a^+_{p_1} a_{p_2}\rangle = p^\mu \int_{\sigma_0} d\sigma_\mu f(x,p)\, e^{iqx} + p^0 \int_{\sigma_0}^{\sigma_{out}} d^4x \left(C^{gain}(x,p) - C^{loss}(x,p)\right) e^{iqx}$$

Simple analytical models

Akkelin, Csorgo, Lukacs, Sinyukov (2001)

Ideal HYDRO solutions with initial conditions at $t = t_0 = 0$.



The n.-r. ideal gas has ellipsoidal symmetry, Gaussian density and a self-similar velocity profile u(x).

$$f(t,\mathbf{x},\mathbf{v}) = \frac{N}{V}\left(\frac{m}{(2\pi)^2 T}\right)^{\frac{3}{2}} \exp\left(-\frac{m(\mathbf{v}-\mathbf{u}(x))^2}{2T} - \sum_{1}^{3}\frac{x_i^2}{2X_i^2}\right)$$

where

$$\mathbf{v} = \mathbf{p}/m,\ V = X_1 X_2 X_3,\ X_i X_i = \frac{T}{m},\ T = T_0\left(\frac{V_0}{V}\right)^{\frac{2}{3}},\ u_i = \frac{\dot{X}_i}{X_i}x_i$$

Spherically symmetric solution:

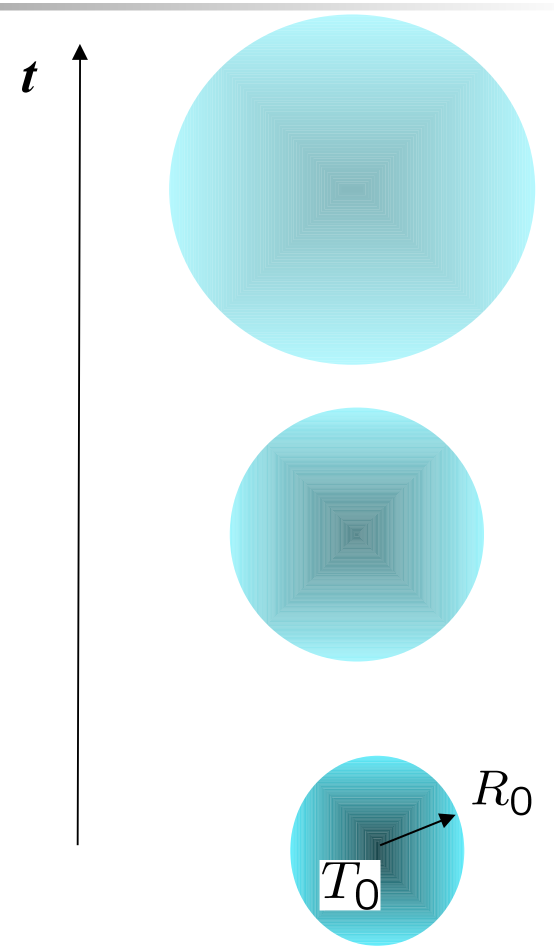
Csizmadia, Csorgo, Lukacs (1998)

$$X_1 = X_2 = X_3 = R$$

Solution of Boltzmann equation for locally equilibrium expanding fireball

$$f(t,\mathbf{x},\mathbf{v}) = \frac{N}{(2\pi R_0)^3}\left(\frac{m}{T_0}\right)^{\frac{3}{2}} \exp\left(-\frac{m\mathbf{v}^2}{2T_0} - \frac{(\mathbf{x}-\mathbf{v}t)^2}{2R_0{}^2}\right)$$

G. E. Uhlenbeck and G. W. Ford, Lectures in Statistical Mechanics (1963)



The spectra and interferometry radii do not change:

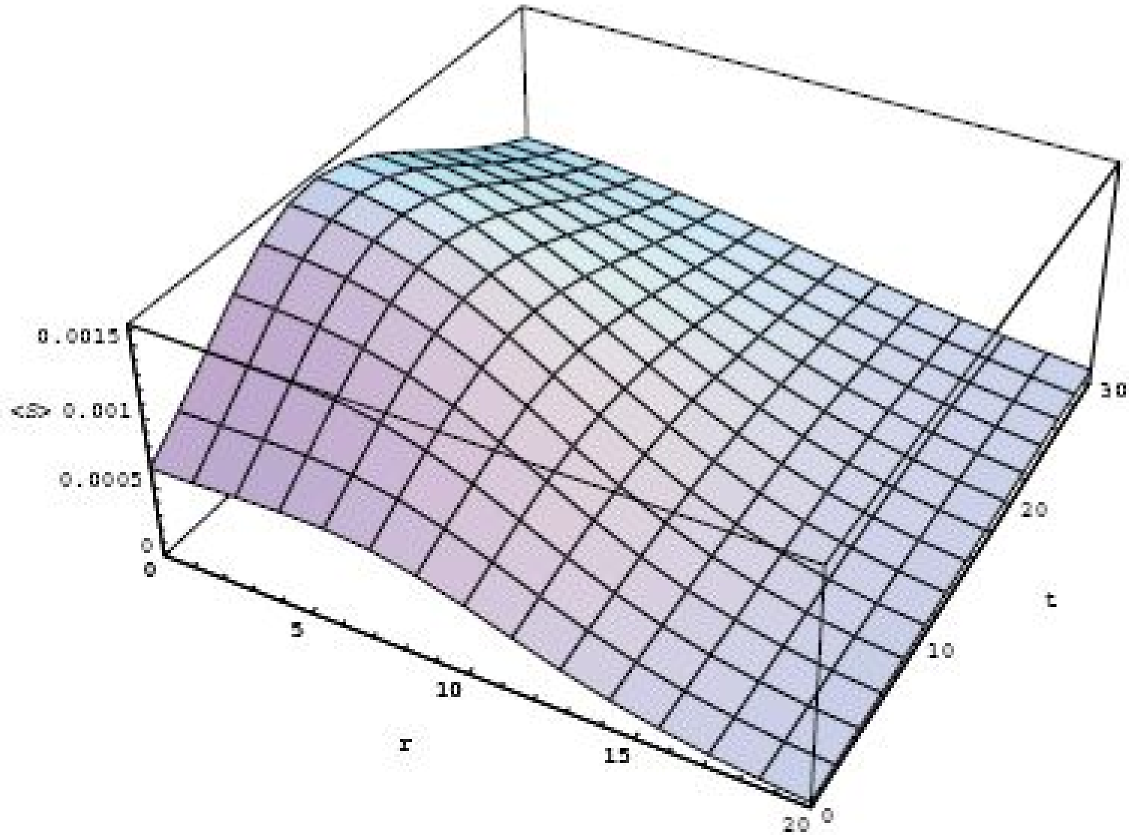
- One particle velocity (momentum) spectrum

$$f(t,\mathbf{v}) = N\left(\frac{m}{2\pi T_0}\right)^{\frac{3}{2}} \exp\left(-\frac{m\mathbf{v}^2}{2T_0}\right) = \underline{f(t=0,\mathbf{v})}$$

- Two particle correlation function

$$C(t,q) = 1 + \frac{\left|\left\langle a^+_{p_1} a_{p_2}\right\rangle\right|^2}{\left\langle a^+_{p_1} a_{p_1}\right\rangle\left\langle a^+_{p_2} a_{p_2}\right\rangle} = 1 + \exp(-q^2 R_0^2) = \underline{C(t=0,q)}$$

Emission density for expanding fireball



The space-time *(t,r)* dependence of the emission function <S(x,p)>, averaged over momenta, for an expanding spherically symmetric fireball containing 400 particles with mass m=1 GeV and with cross section σ = 40 mb, initially at rest and localized with Gaussian radius parameter R =7 fm and temperature T = 0.130 GeV.

Duality in hydrokinetic approach to A+A collisions

Sudden freeze-out, based on Wigner function $f(x,p)$,
vs continuous emission, based on emission function $S(x,p)$:

- Though the process of particle liberation, described by the emission function, is, usually, continuous in time, the observable spectra can be also expressed by means of the Landau/Cooper-Frye prescription. It does not mean that the hadrons stop to interact then at post hydrodynamic stage but momentum spectra do not change significantly, especially if the central part of the system reaches the spherical symmetry to the end of hydrodynamic expansion, so the integral of $(C^{gain} - C^{loss})$ is small at that stage.

- The Landau prescription is associated then with lower boundary of a region of applicability of hydrodynamics and should be apply at the end of (perfect) hydrodynamic evolution, before the bulk of the system starts to decay.

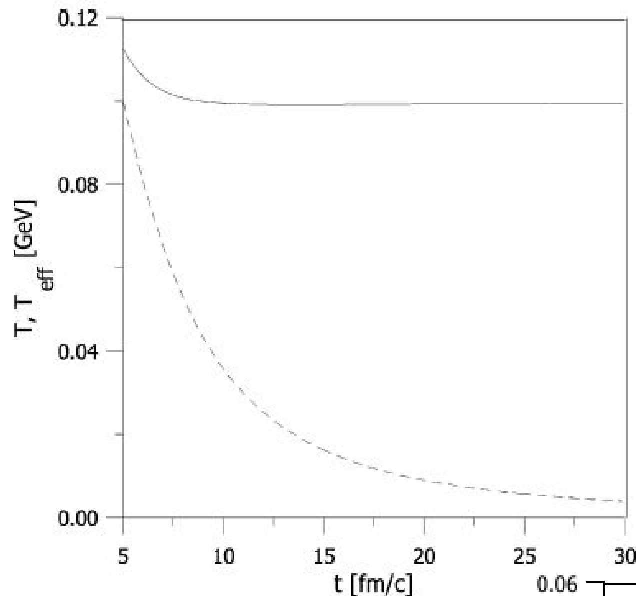
- Such an approximate duality results from the momentum-energy conservation laws and spherically symmetric properties of velocity distributions that systems in A+A collisions reach to the end of chemically frozen hydrodynamic evolution

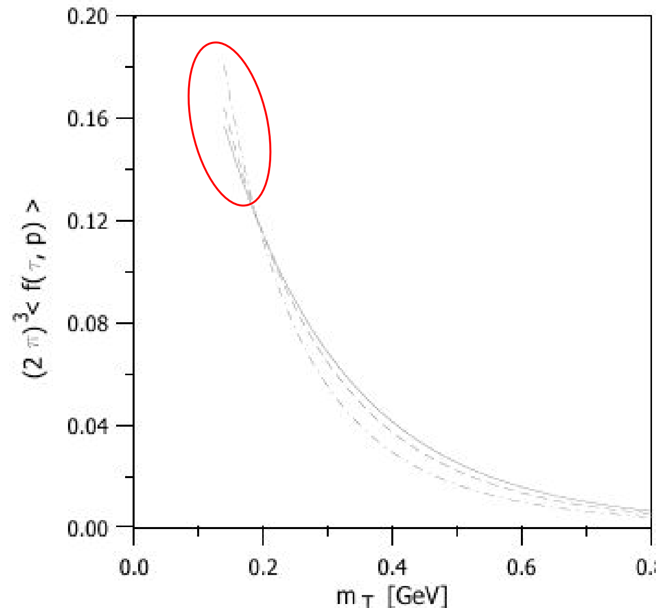
(2+1) n.-r. model with longitudinal boost-invariance

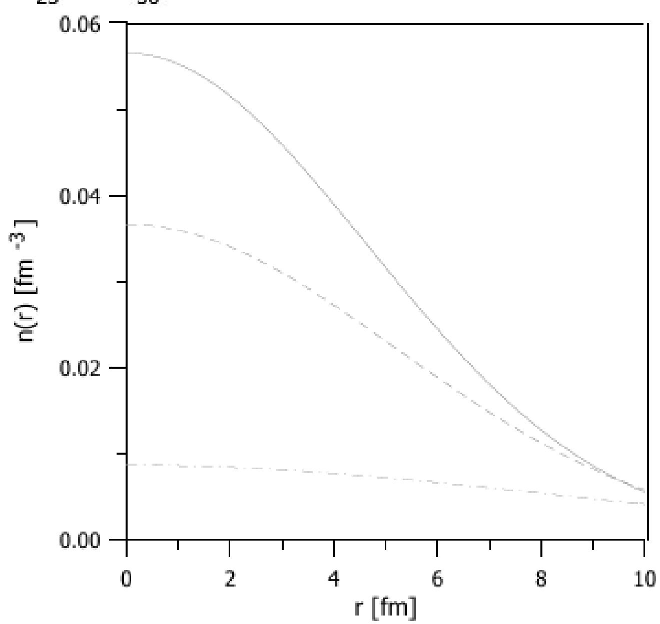
[Akkelin, Braun-Munzinger, Yu.S. Nucl.Phys. A (2002)]

- Momentum spectrum $$\frac{d^3N}{d^3p} = \frac{n_0 R_0^2 t_0}{m^2 T_{eff}(t)} \exp(-\frac{p_T^2}{2mT_{eff}(t)})$$
- Effective temperature $$T_{eff} = mv^2(t) + T(t)$$
- Interferometry volume $$V_{int} \equiv R_O R_S R_L = \frac{R_0^2 t_0 (T_0)^{3/2}}{T_{eff}\sqrt{m}}$$
- Spatially averaged PSD $$\langle f(t,\mathbf{p})\rangle = n_0 \left(\frac{1}{4\pi m T_0}\right)^{3/2} \exp(-\frac{p_T^2}{2m}\frac{1}{T_{eff}(t)})$$
- Averaged PSD (APSD) $$\langle f(t,y)\rangle = const$$

Evolution of T_{eff}, APSD and particle density





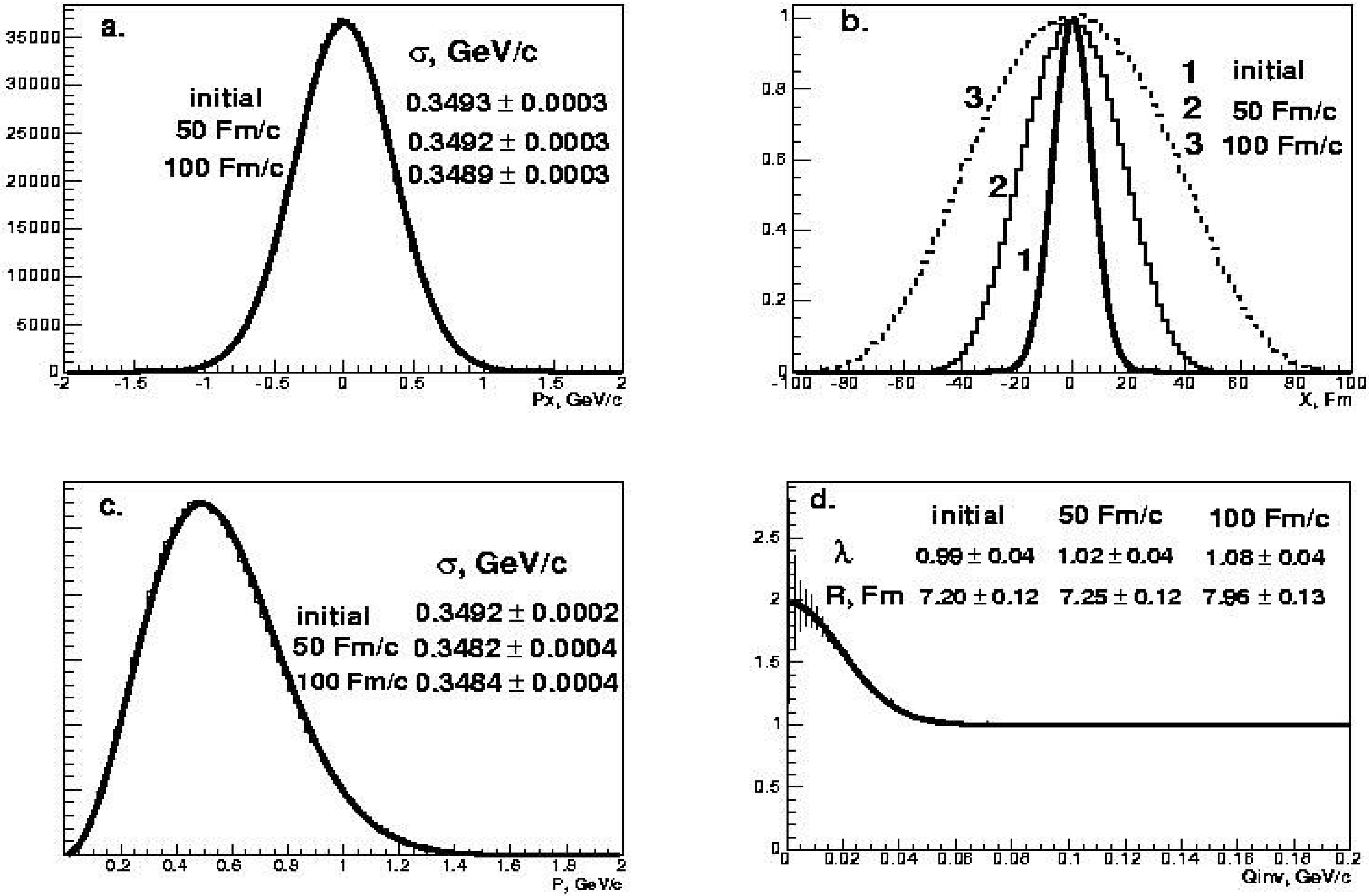


$$n(t,0) \stackrel{t\to\infty}{\longrightarrow} \frac{1}{t^3}$$

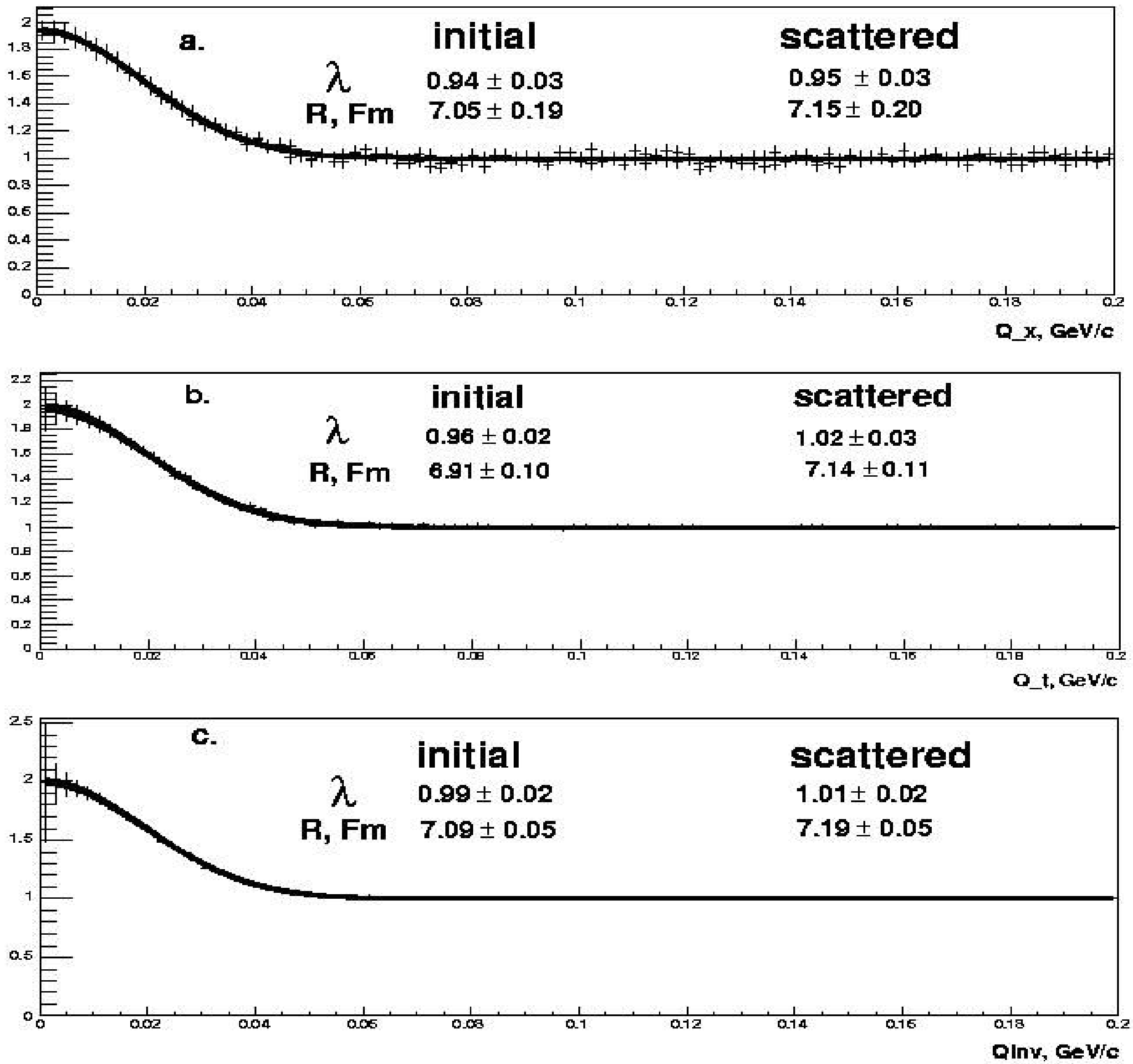
APSD and part. densities at hadronization time =7.24 fm/*c* (solid line) and at kinetic freeze -out =8.9 fm/*c* (dashed line). The dot-dashed line corresponds to the "asymptotic" time =15 fm/*c* of hydrodynamic expansion of hadron-resonance gas

[Akkelin, Braun-Munzinger, Yu.S. Nucl.Phys. A2002]

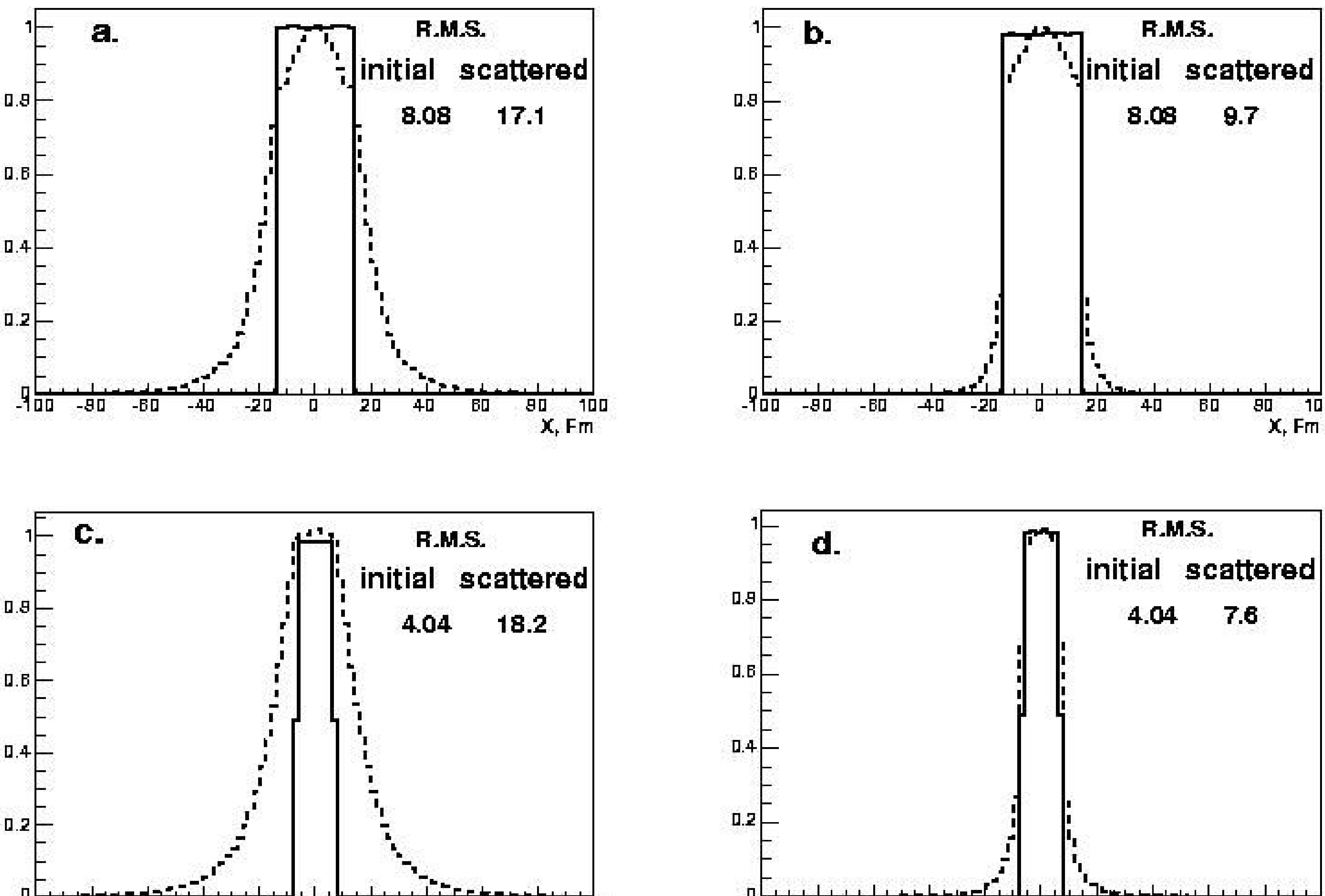
Numerical UKM-R solution of B.Eq. with symmetric IC for the gas of massive (1 GeV) particles [Amelin,Lednicky,Malinina, Yu.S. (2005)]



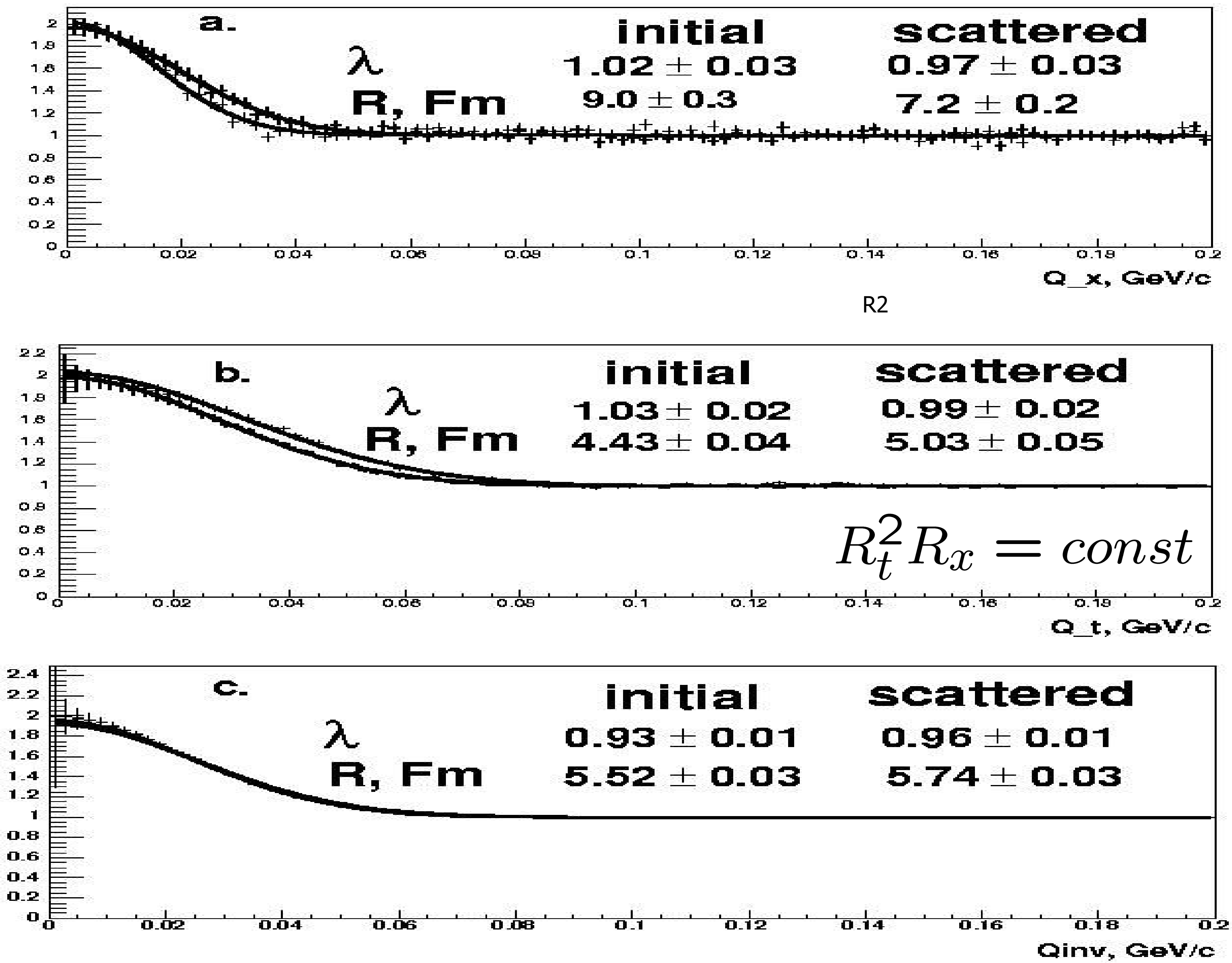
A numerical solution of the Boltzmann equation with the asymmetric initial momentum distribution.



Asymmetric initial coordinate distribution and scattered R.M.S.



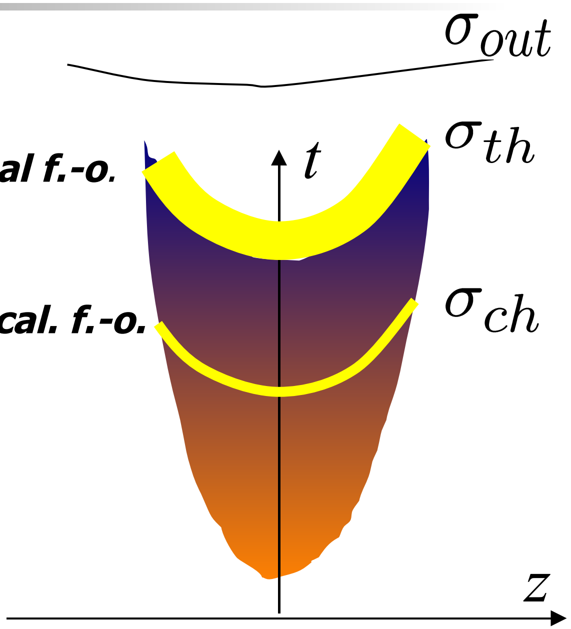
Longitudinal (x) and transverse (t) CF and correspondent radii for asymmetric initial coordinate distribution.



Results and ideas

- The approximate hydro-kinetic duality can be utilized in A+A collisions.
- Interferometry volumes does not grow much *even* if ICs are quite asymmetric: less then 10 percent increase during the evolution of fairly massive gas.
- Effective temperature of transverse spectra also does not change significantly since heat energy transforms into collective flows.
- The APSD do not change at all during non-relativistic hydro- evolution, also in relativistic case with non-relativistic and ultra-relativistic equation of states and for free streaming.
- **The main idea to study early stages of evolution is to use integrals of motion - the "conserved observables" which are specific functionals of spectra and correlations functions.**

Approximately conserved observables



- **APSD - Phase-space density averaged over some hypersurface $\sigma = \sigma_{out}$, where all particles are already free *and* over momentum at fixed particle rapidity,** y=0. (Bertsch)

$n(p)$ is single- , $n(p_1, p_2)$ is double (identical) particle spectra,

correlation function is $C = n(p_1, p_2)/n(p_1)n(p_2)$

$$\langle f(\sigma, y)\rangle_{y\simeq 0} = \frac{\int (f(x,p))^2 p^\mu d\sigma_\mu d^2 p_T}{\int f(x,p) p^\mu d\sigma_\mu d^2 p_T} = \frac{(2\pi)^{-3}\int p_0^{-1} n^2(p)(C(p,q)-1) d^3 q d^2 p_T}{dN/dy}$$

$p=(p_1+p_2)/2$

$q=p_1-p_2$

- **APSD is conserved during isentropic and chemically frozen evolution:**

$$\langle f(\sigma_{ch}, y)\rangle \simeq \langle f(\sigma_{th}, y)\rangle = \langle f(\sigma_{out}, y)\rangle$$

S. Akkelin, Yu.S. Phys.Rev. C 70 064901 (2004):

Approximately conserved observables

- **(1) ENTROPY and (2) SPECIFIC ENTROPY**

$$S = (2J+1)\int \frac{d\sigma^{\mu}p_{\mu}d^{3}p}{(2\pi)^{3}p^{0}}[-(2\pi)^{3}f\ln((2\pi)^{3}f)\pm(1\pm(2\pi)^{3}f)\ln(1\pm(2\pi)^{3}f)] \quad (1)$$

$$\frac{S_i}{N_i} \quad (i = \text{pion}) \quad (2)$$

For spin-zero (*J*=0) bosons in locally equilibrated state:

$$f = f_{l.eq.}(x,p) = (2\pi)^{-3}(\exp(\frac{u_{\nu}(x)p^{\nu}-\mu(x)}{T(x)})-1)^{-1}$$

On the face of it the APSD and (specific) entropy depend on the freeze-out hypersurface and velocity field on it, and so it seems that these values cannot be extracted in a reasonably model independent way.

"Model independent" analysis of pion APSD and specific entropy

- The thermal freeze-out happens at some space-time hypersurface with *T=const* and μ=*const*.

- Then, the integrals in APSD and Specific Entropy contain the common factor, "effective volume " $V_{eff} = \int \frac{d\sigma_\mu}{d\eta} u^\mu$ (η is rapidity of fluid), that completely absorbs the flow $u^\mu(x)$ and form of the hypersurface $\sigma(x)$ in mid-rapidity.

$$\int p^\mu d\sigma_\mu d^2 p_T F_i(f_{l.eq.}) = V_{eff} \int d^3 p \frac{F_i(\overline{f}_{eq})}{(2\pi)^3} = G_i(T,\mu) V_{eff}$$

If $F_{(j)}(f_{l.eq.}) = f_{l.eq.}$ then $G_i(T,\mu) = n_{th}$ is thermal density of equilibrium B-E gas. $F_{(j)}(f_{l.eq.}) = f^2_{l.eq.}$ (APSD-numerator) and $F_{(j)}(f_{l.eq.}) = \ln(1 + f_{l.eq})$ (entropy).

- Thus, the effective volume is cancelled in the corresponding ratios: APSD and specific entropy.

Pion APSD and specific entropy as observables

- **The APSD will be the same as the totally averaged phase-space density in the static homogeneous Bose gas:**

$$(2\pi)^3 \langle f(\sigma,y)\rangle_{y=0} = \frac{\int d^3p \overline{f}^2_{eq}}{\int d^3p \overline{f}_{eq}} = \kappa \frac{2\pi^{5/2} \int \left(\frac{1}{R_O R_S R_L} \left(\frac{d^2N}{2\pi m_T dm_T dy} \right)^2 \right) dm_T}{dN/dy}$$

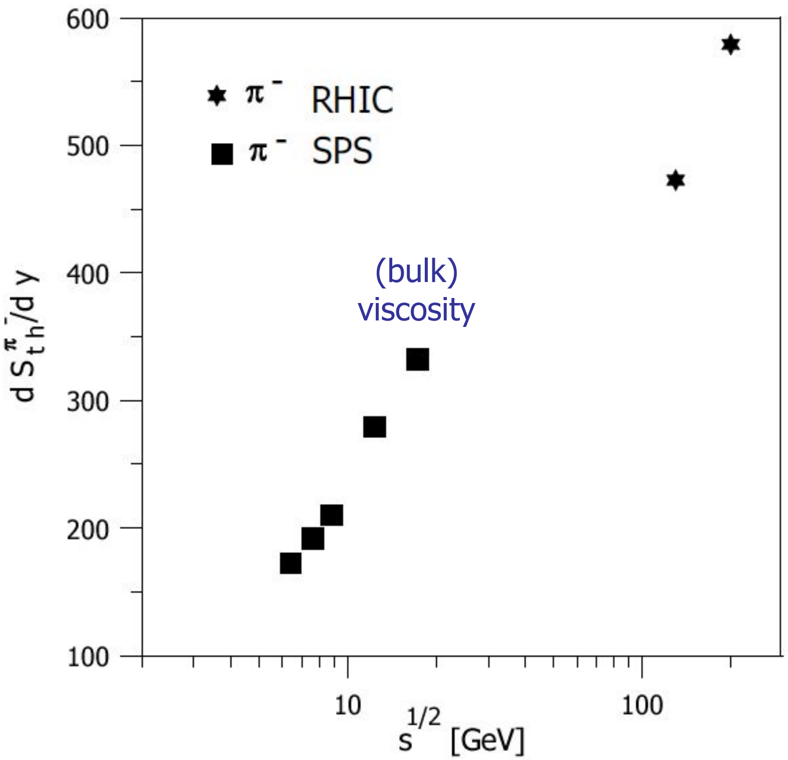
where $\overline{f}_{eq} \equiv (\exp(\beta(p_0 - \mu) - 1)^{-1}$, κ =0.6-0.7 accounts for resonances

- **Spectra + BE correlations + $T_{f.o.}$** → $\langle f(\sigma, y)\rangle$ → **Chemical potential** μ

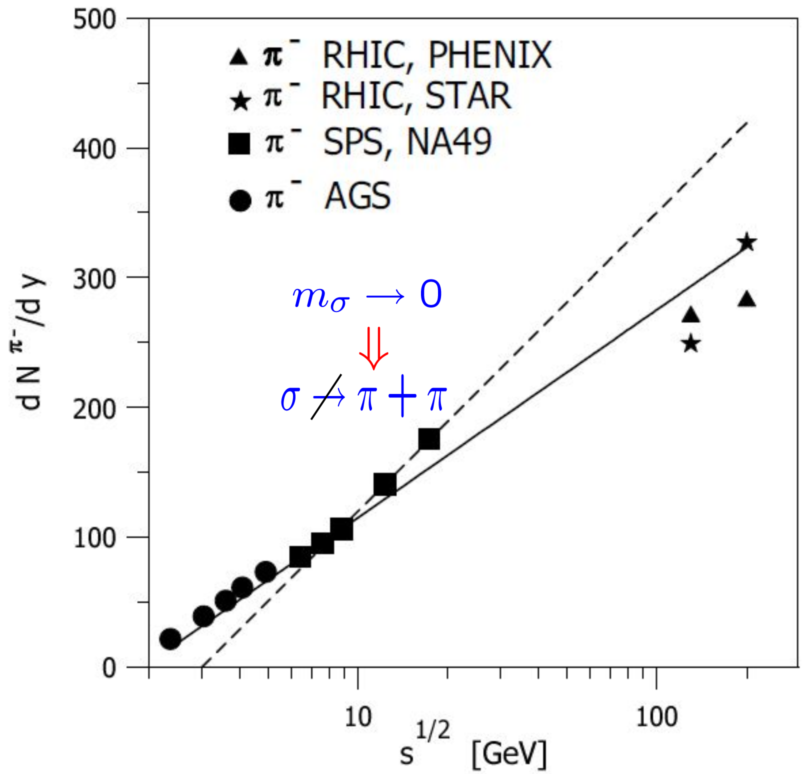
- **Pion specific entropy:**

$$\frac{dS/dy}{dN/dy} = \frac{\int d^3p \, [-\overline{f}_{eq} \ln \overline{f}_{eq} + (1+\overline{f}_{eq}) \ln(1+\overline{f}_{eq})]}{\int d^3p \overline{f}_{eq}}.$$

Rapidity densities of entropy and number of thermal pions vs collision energy

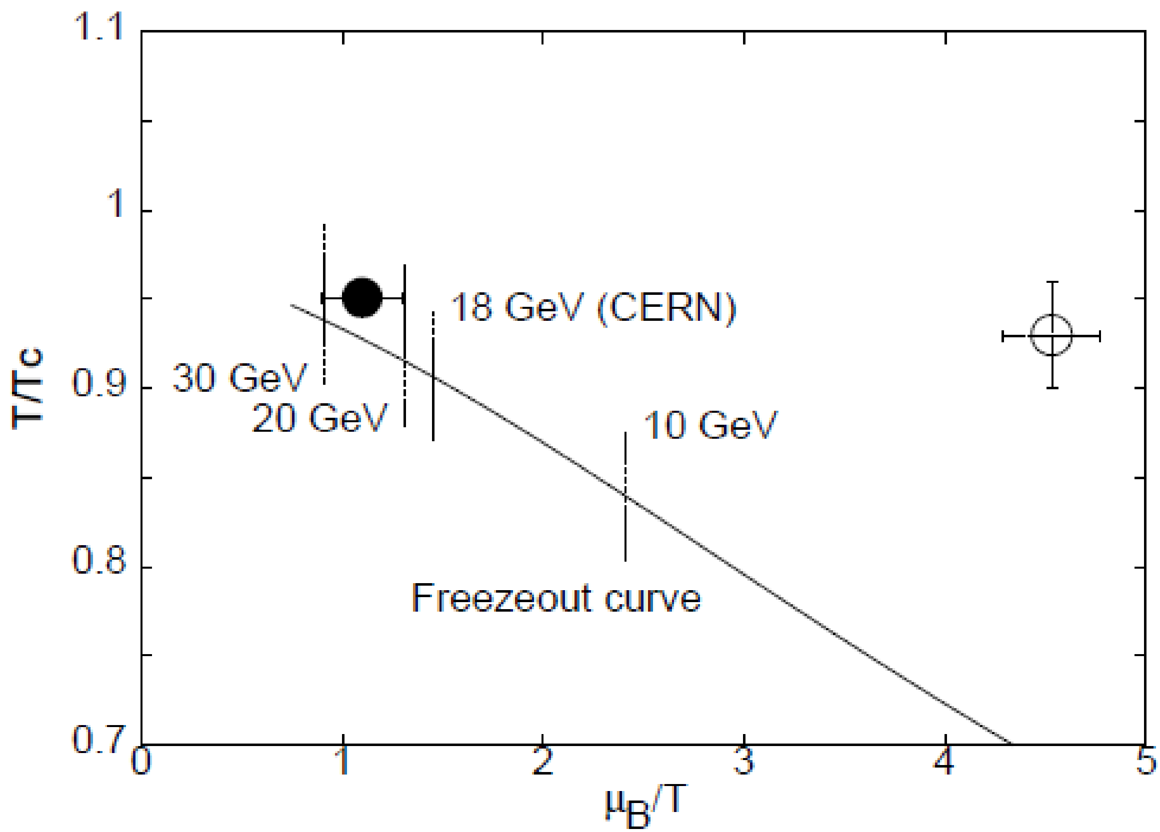


The rapidity density of entropy for negative thermal pions, $dS^{\pi^-}_{th}/dy$, (squares and stars) as function of c.m. energies per nucleon in heavy ion collision.



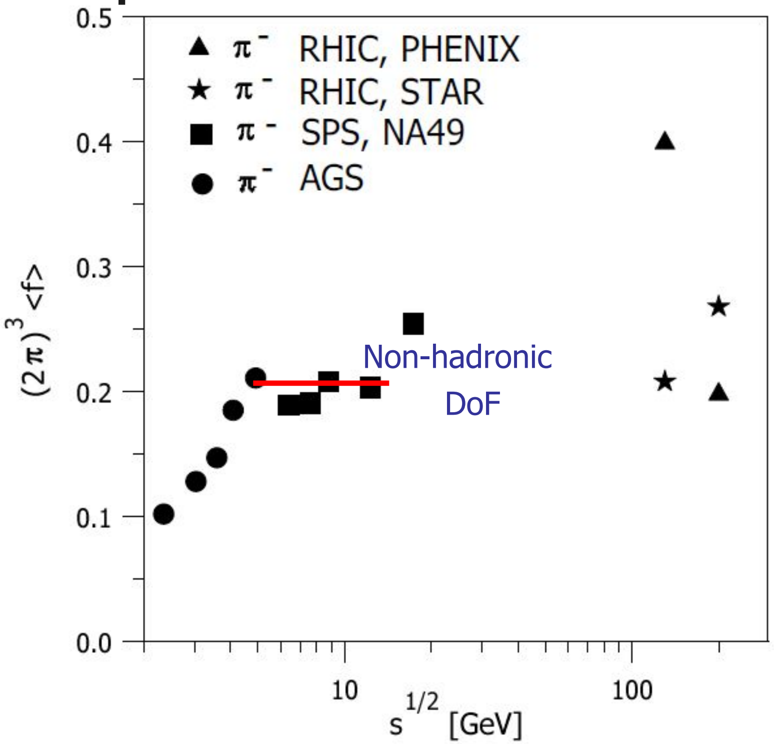
The rapidity density of negative pions, dN^{π^-}/dy, as function of c.m. energy per nucleon in heavy ion collisions.

Anomalous rise of pion entropy/multiplicities and critical temperature

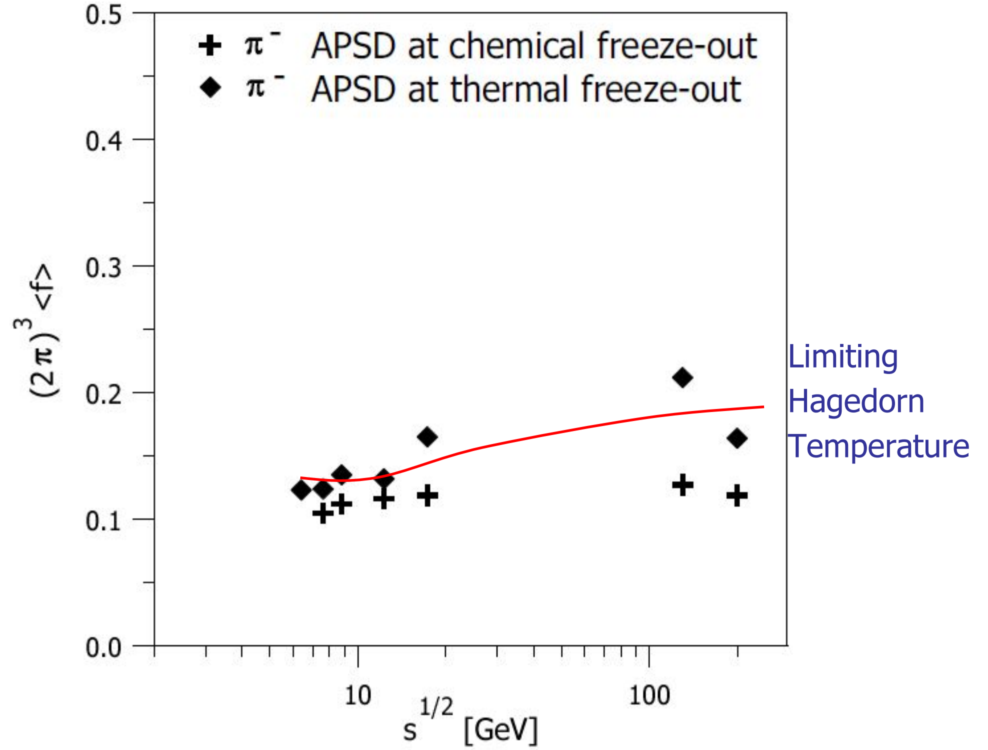


R.V. Gavai: hep-ph/0302130. The QCD phase diagram with a chemical freeze-out curve superimposed. The filled circle denotes the estimate of the critical point which has been obtained in Ref. R.V.Gavai,S.Gupta hep-lat 0412035. The open circle is an earlier estimate from Ref. Z.Fodor, S. Katz, JHEP 0203(2002)014 using smaller lattices and nearly the same quark mass.

The averaged phase-space density

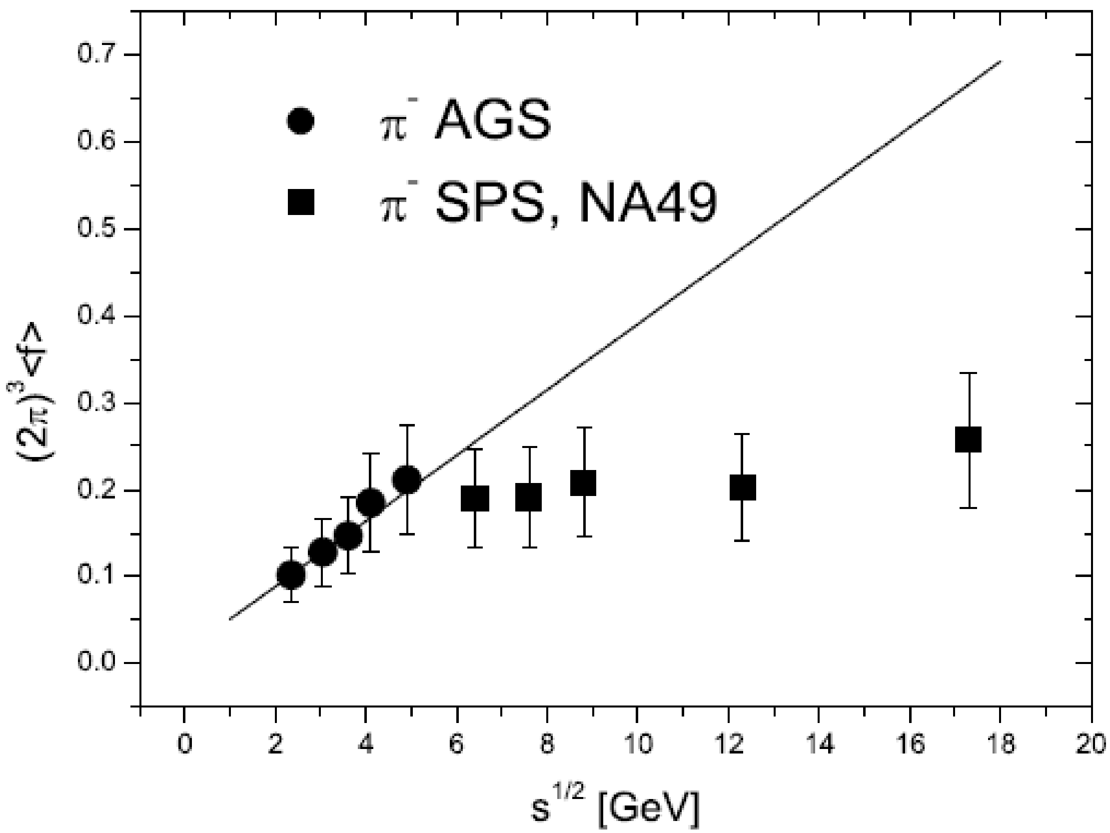


The average phase-space densities of all negative pions at midrapidity, $(2\pi)^3 \langle f(y) \rangle$, (circles, squares, stars and triangles) as functions of c.m. energies per nucleon in heavy ion central collisions.



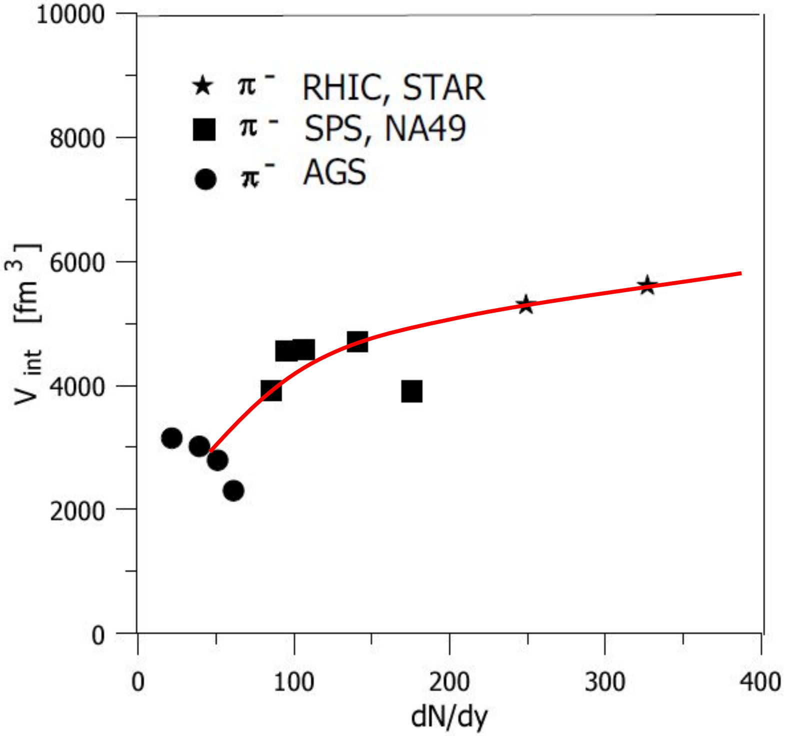
The average phase-space densities of thermal ("direct") negative pions, $(2\pi)^3 \langle f(y) \rangle^{th}$ (rhombus), and the average phase-space densities of negative pions at the stage of chemical freeze-out, $(2\pi)^3 \langle f(y) \rangle^{ch}$ (crosses), as functions of c.m. energies per nucleon in heavy ion central collisions.

The statistical errors

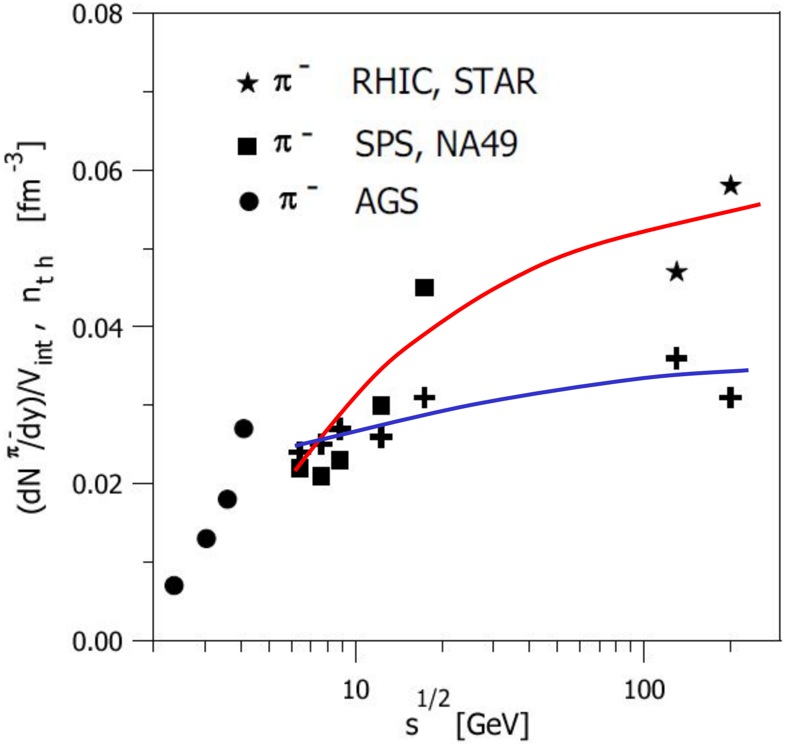


The statistical uncertainties caused by the experimental errors in the interferometry radii in the AGS-SPS energy domain. The results demonstrate the range of statistical signicance of nonmonotonic structures found for a behavior of pion averaged phase-space densities as function of c.m. energy per nucleon in heavy ion collisions.

Interferometry volumes and pion densities at different (central) collision energies

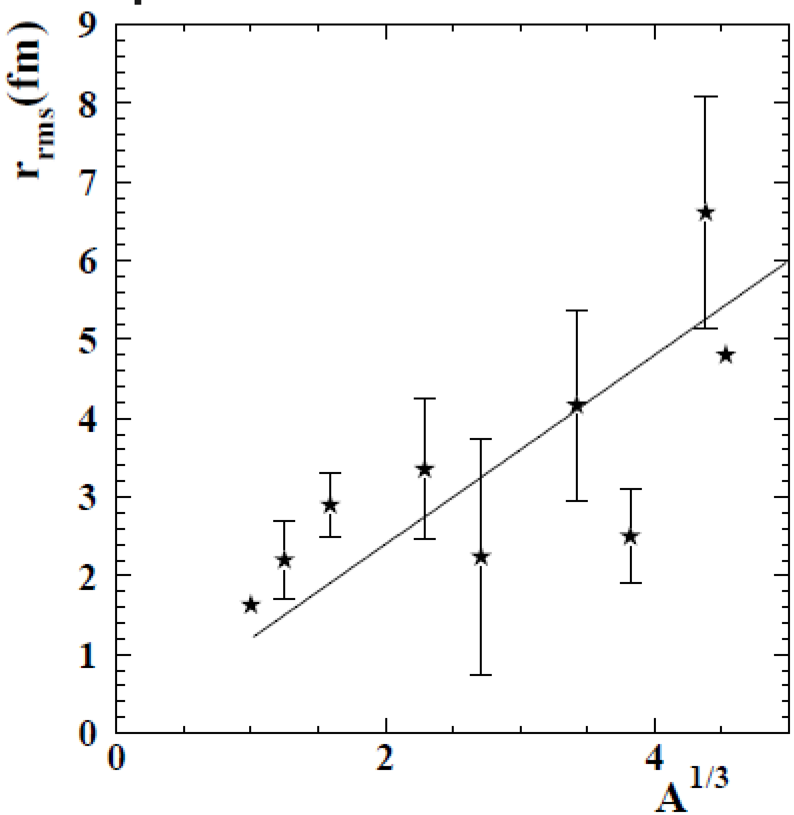


The interferometry volumes
$V_{int} = (2\pi)^{3/2} R_O R_S R_L$
(circles, squares, and stars) of negative pions at $p_T \simeq 0.06 \div 0.07$ GeV vs rapidity densities of the negative pions, dN^{π^-}/dy, at mid-rapidity in central heavy ion collision at different energies.

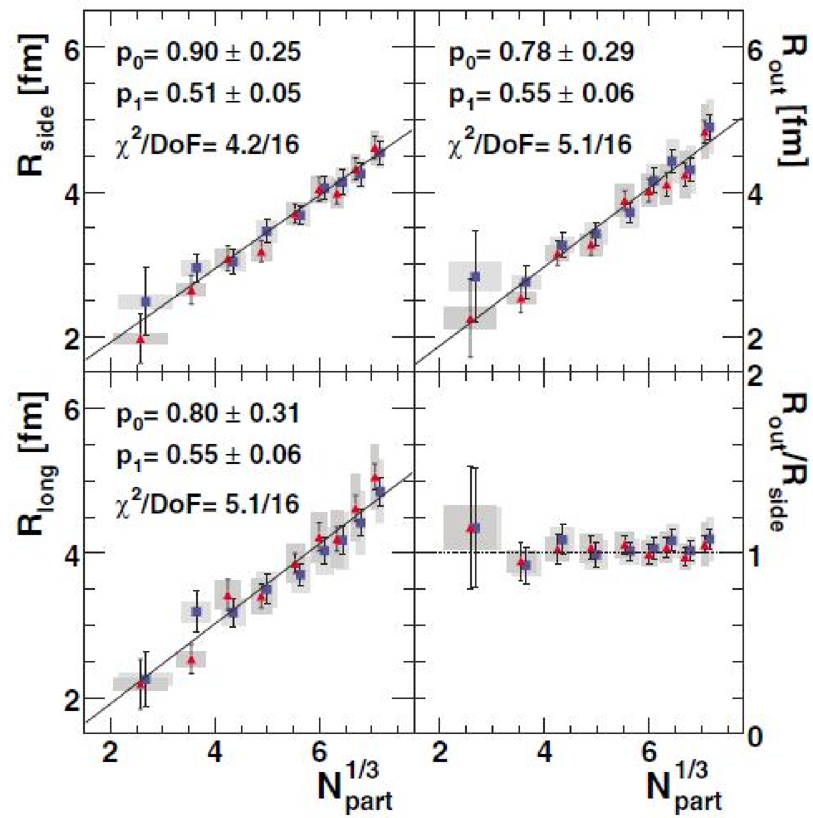


The ratio of rapidity densities of all negative pions to the corresponding interferometry volumes, $(dN^{\pi^-}/dy)/V_{int}$, (circles, squares and stars) and the ratio of rapidity densities of negative thermal pions to their effective volumes, that is thermal densities n_{th}, (crosses) vs c.m. energies per nucleon in heavy ion collisions.

The interferometry radii vs initial system sizes



An early compilation of values obtained for the emitter dimension r_{rms}, as a function of $A^{1/3}$ where A is the atomic number of the projectile: G.Alexander: hep-ph 0302130 and references there. The solid line: $r_{rms} = 1.2 \times A^{1/3}$ fm.



RHIC, Au+Au $\sqrt{s}$=200 GeV, PHENIX: Centrality dependance of interferometry radii is well defined by a linear function: $p_0 + p_1 N_{part}^{1/3}$.

The interferometry radii vs initial system sizes

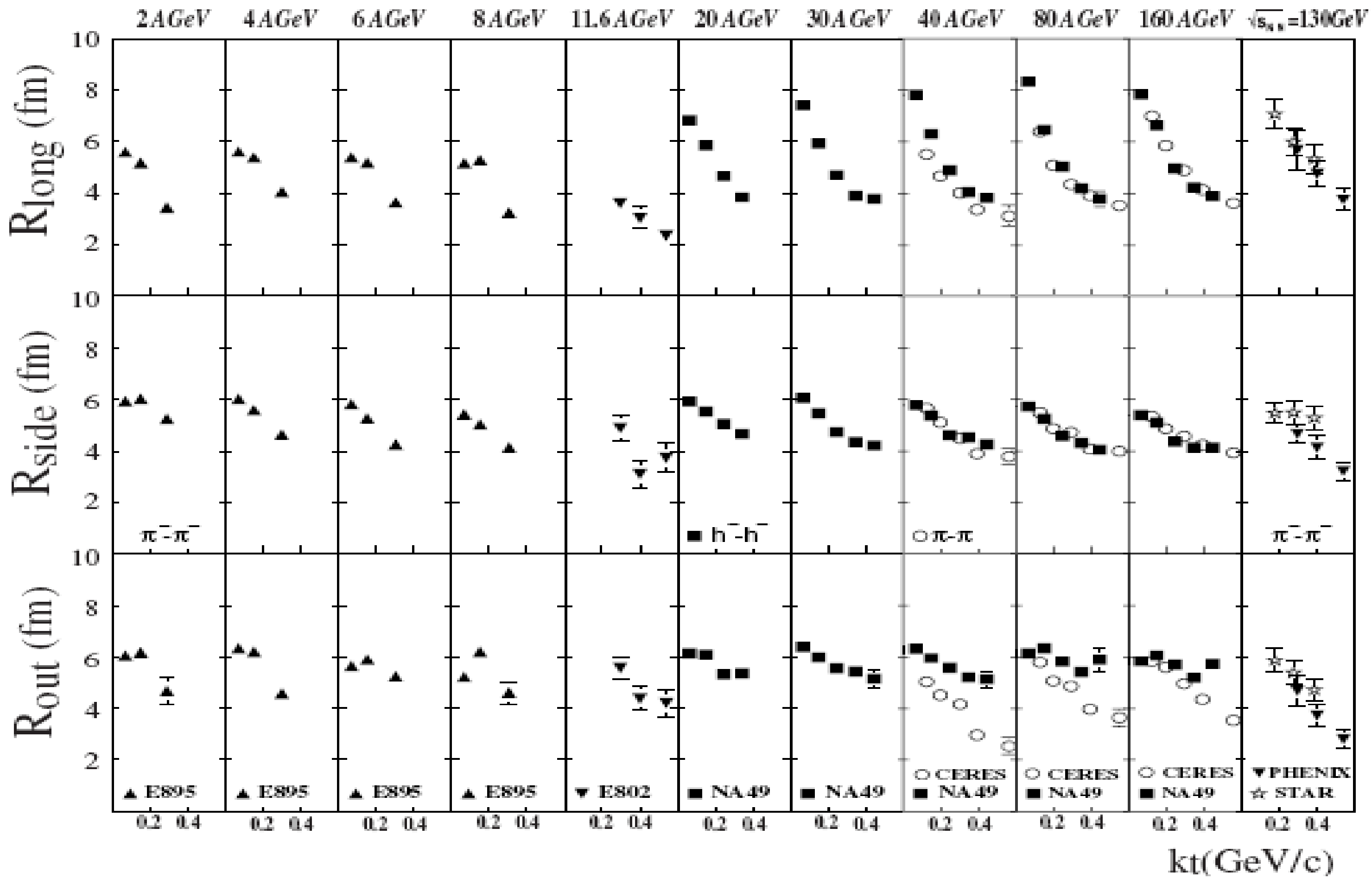
- Let us consider time evolution (in $\boldsymbol{\tau}$) of the interferometry volume if it were measured at corresponding time:

$$V_{int}(\tau) \simeq C \frac{dN/dy(\tau)}{\langle f \rangle_\tau T_{eff}^3(\tau)} \qquad \sqrt{s} \text{ is fixed}$$

- T_{eff} for pions does not change much since the heat energy transforms into kinetic energy of transverse flows (S. Akkelin, Yu.S. Phys.Rev. C 70 064901 (2004));
- The *<f>* is integral of motion;
- dN/dy is conserved because of chemical freeze-out.

Thus the pion interferometry volume will approximately coincide with what could be found at initial time of hadronic matter formation and is associated with initial volume

Energy dependence of the interferometry radii



Energy- and kt-dependence of the radii Rlong, Rside, and Rout for central Pb+Pb (Au+Au) collisions from AGS to RHIC experiments measured near midrapidity.
S. Kniege et al. (The NA49 Collaboration), J. Phys. G30, S1073 (2004).

HBT PUZZLE

- The interferometry volume only slightly increases with collision energy (due to the long-radius growth) for the central collisions of the same nuclei.

Explanation: Roughly, $\frac{dN}{m_T dm_T dy} \propto \exp\left(-m_T/T_{eff}\right)$. Then, estimating APSD and assuming that integral C over dimensionless variable m_T/T_{eff} depends on energies of collisions fairly smoothly, one can write

$$V_{int}(\sqrt{s}) \simeq C \frac{dN/dy}{\langle f \rangle T_{eff}^3}$$

A is fixed

- $\langle f \rangle$ only slightly increases and is saturated due to limiting Hagedorn temperature $T_H = T_c$ ($\mu_B = 0$).
- dN/dy grows with $\sqrt{s}: \frac{dN}{dy}(\sqrt{s} = 200\ \text{GeV})/\frac{dN}{dy}(\sqrt{s} \simeq 10\ \text{GeV}) \simeq 3$
- $T_{eff}^3(\sqrt{s} = 200\ \text{GeV})/T_{eff}^3(\sqrt{s} \simeq 10\ \text{GeV}) \simeq 2$

HBT PUZZLE & FLOWS

- Possible increase of the interferometry volume with $\sqrt{s}$ due to geometrical volume grows is mitigated by more intensive transverse flows at higher energies:

$$R_S = R_T/\sqrt{1 + I\beta m_T},\ I \propto grad(v_T),$$ β is inverse of temperature

- Why does the intensity of flow grow?

More $\sqrt{s}$ ⇨ more initial energy density ε ⇨ more (max) pressure p_{max}

BUT **the initial acceleration** $a = grad(p)/\epsilon \propto p_{max}/\epsilon$ **is ≈ the same !**

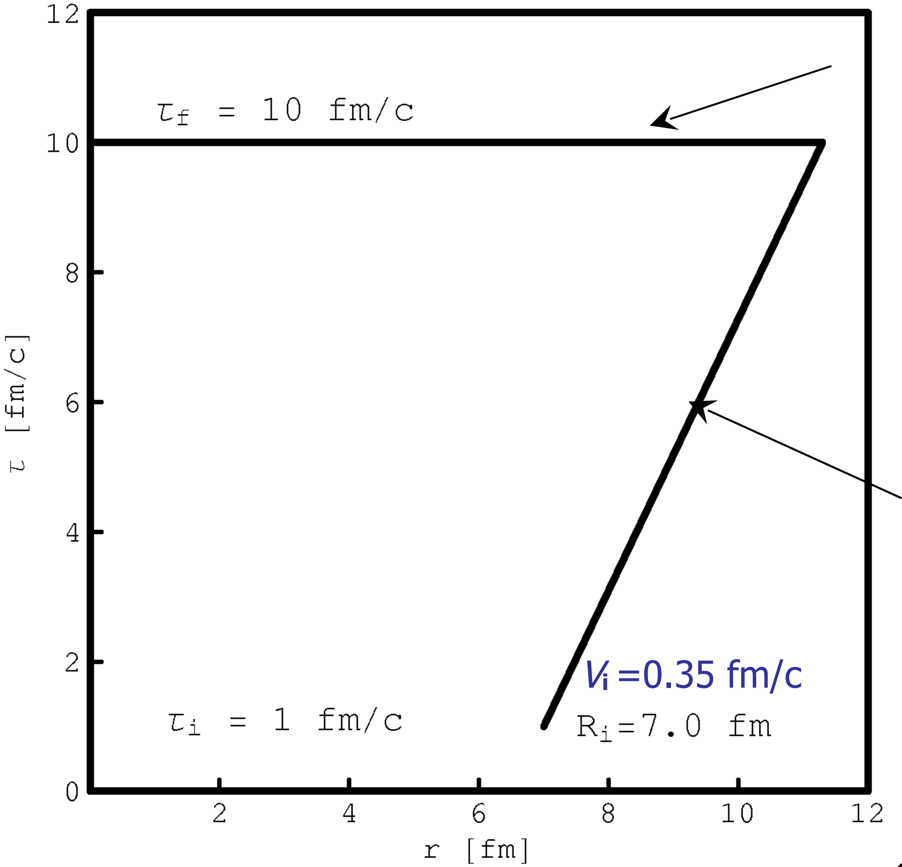
HBT puzzle ➡ **puzzling developing of initial flows at τ< 1 fm/c.**

Dynamical realization of general results

- Description of the hadronic observables within hydrodynamically motivated parametrizations of freeze-out.
  (M.S.Borysova, Yu.M. Sinyukov, S.V.Akkelin, B.Erazmus, Iu.A.Karpenko, Phys.Rev. C 73, 024903 (2006))
- Peculiarities of the final stage of the matter evolution.
- Hydrodynamic realizations of the final stages.
  (Yu.M. Sinyukov, Iu.A. Karpenko. Heavy Ion Phys. 25/1 (2006) 141–147).
- Peculiarities of initial thermodynamic conditions for corresponding dynamic models.
- How to reach these initial conditions at pre-thermal (partonic) stage of ultra-relativistic heavy ion collisions
  (Akkelin, Gyulassy, Nazarenko, Yu.S.

The model of continuous emission

(M.S.Borysova, Yu.S., S.V.Akkelin, B.Erazmus, Iu.A.Karpenko,
Phys.Rev. C 73, 024903 (2006))



$$\tau = \sqrt{t^2 - z^2}$$
$$\tau_v = const$$
$$\tau_s(r) = a \cdot r + b$$

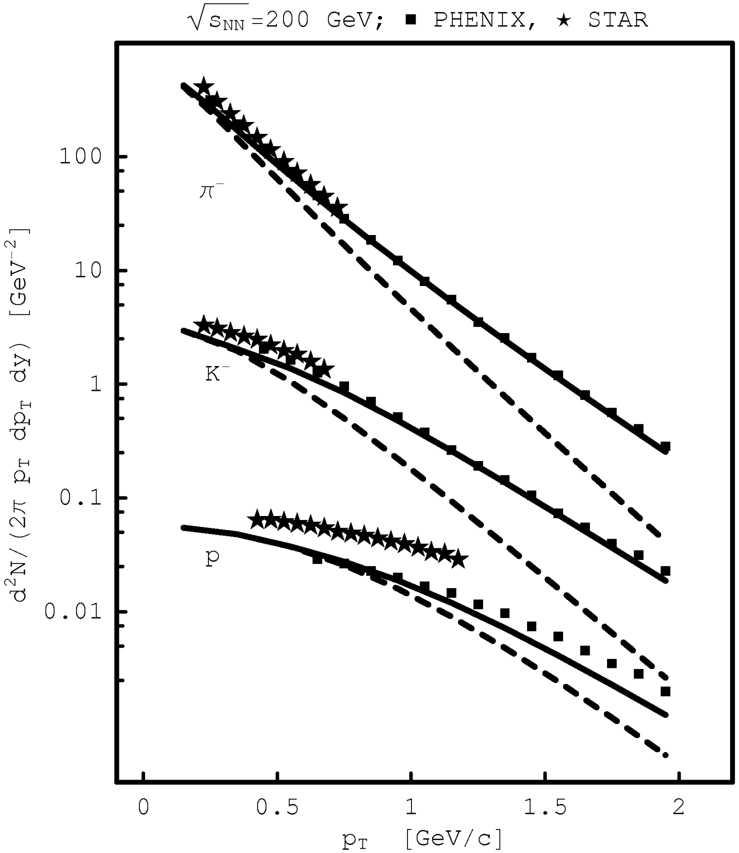
Induces space-time correlations for emission points

$$R_o^2 \approx \lambda_T^2 + v^2 \langle \triangle t^2 \rangle_p - 2v \langle \triangle x_o \triangle t \rangle_p, v = \frac{p_{out}}{p_0}$$

$$f_i = f_{leq,i}(x,p) = (2\pi)^{-3} \left(\exp\left(\frac{u_\nu(x) p^\nu - \mu_i(x)}{T(x)} \right) \pm 1 \right)^{-1}$$

Results : spectra

$$u^{\mu}_{v,s}(r,\eta) = (\cosh\eta \cosh\eta_T^{v,s}, \sinh\eta_T^{v,s}\cos\phi, \sinh\eta_T^{v,s}\sin\phi, \sinh\eta\cosh\eta_T^{v,s})$$



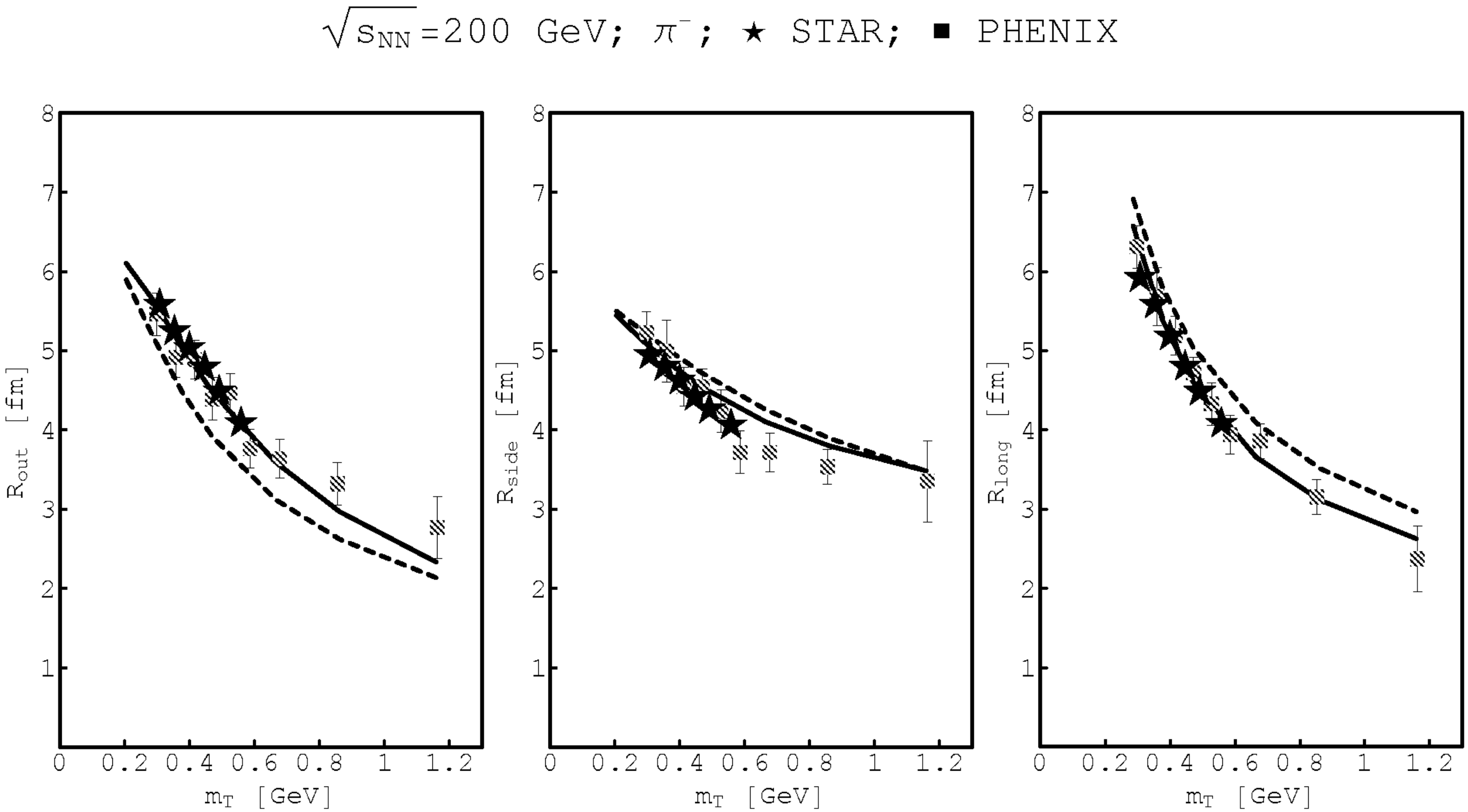
$$\eta_T^s(r,\tau_s(r)) = \eta_T^{max}\frac{\sqrt{(\tau_s(r)-\tau_i)^2+r^2}}{\sqrt{(\tau_f-\tau_i)^2+R_f^2}}$$

$$\eta_T^v(r,\tau_f) = \eta_T^{max}\frac{r}{R_f}$$

	μ_v MeV	μ_s MeV	$\frac{dN^{reg}/dy}{dN^{th}/dy}$
π^-	53	0	2.2
K^-	45	0	2.2
p	280	40	3.5

τ_i fm/c	τ_f fm/c	R_i fm	R_f fm	T_v MeV	T_s MeV	η_T^{max}
1	10	7	11.3	110	150	0.73

Results : interferometry radii



R_o/R_s

Using gaussian approximation of CFs,

$$R_i^2(p) = \langle \Delta r_i^2 \rangle_p + v_i^2 \langle \Delta t^2 \rangle_p - 2 v_i \langle \Delta r_i \Delta t \rangle_p,$$

where $v_i = \frac{p^i}{p^0}$

In the Bertsch-Pratt frame $v_{side} = 0, v_{out} = v_t,$

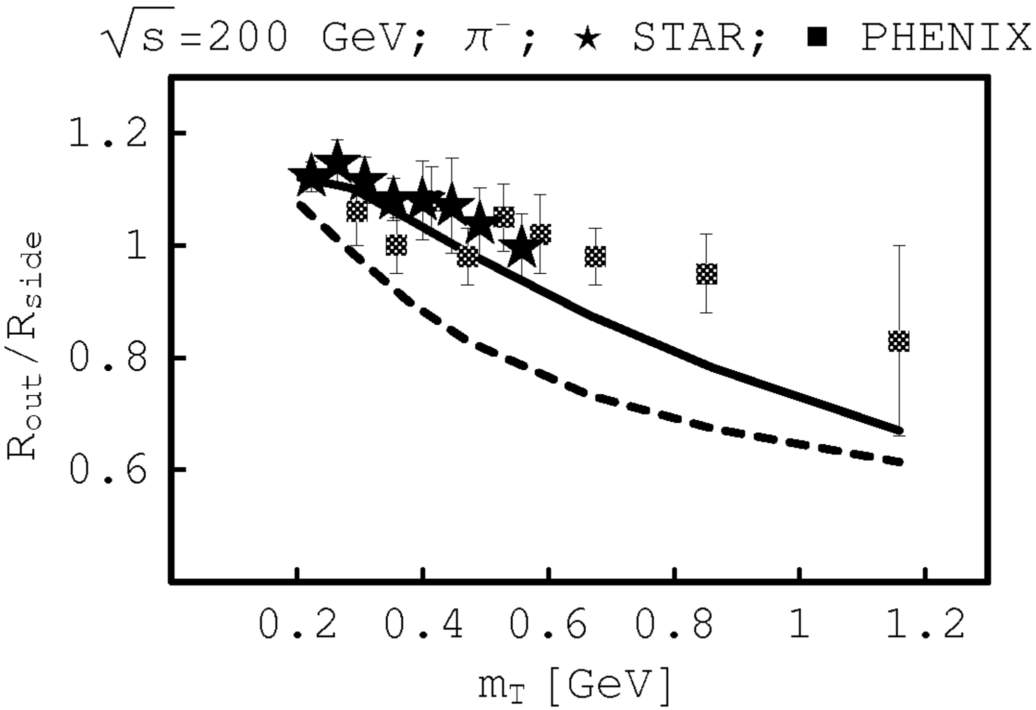
↓

✓Long emission time results in positive contribution to ***Ro/Rs*** ratio

✓Positive r_{out}-t correlations give negative contribution to R_o/R_s ratio

Experimental data : ***Ro/Rs~1***

Results : R_o/R_s



New hydro solutions: Yu.S., Karpenko: Heavy Ion Phys. 25/1 (2006) 141–147.

The new class of analytic (3+1) hydro solutions

$$u^\mu = \{\frac{t}{\sqrt{t^2 - \sum a_i^2(t)x_i^2}}, \frac{a_k(t)x_k}{\sqrt{t^2 - \sum a_i^2(t)x_i^2}}\}, a_i(t) = \frac{t}{t+T_i}$$

$$v_i = \frac{x_i}{t+T_i}$$

For "soft" EoS, ***p=const***

Is a generalization of known Hubble flow and Hwa/Bjorken solution with ***c_s=0*** :

$$T_i = 0 \Rightarrow v_i = \frac{x_i}{t} \text{ (Hubble)}$$

$$T_{1,2} \to \infty, T_3 = 0 \Rightarrow v_t = 0, v_z = z/t \text{ (Bjorken)}$$

Thermodynamical quantities

Density profile for energy and quantum number (particle number, if it conserves):

$$\varepsilon + p_0 = \frac{F_\varepsilon(\frac{x_1}{t+T_1}, \frac{x_2}{t+T_2}, \frac{x_3}{t+T_3})}{(t+T_1)(t+T_2)(t+T_3)}$$

$$n = \frac{F_n(\frac{x_1}{t+T_1}, \frac{x_2}{t+T_2}, \frac{x_3}{t+T_3})}{(t+T_1)(t+T_2)(t+T_3)}$$

with corresponding initial conditions.

Dynamical realization of freeze-out parameterization.

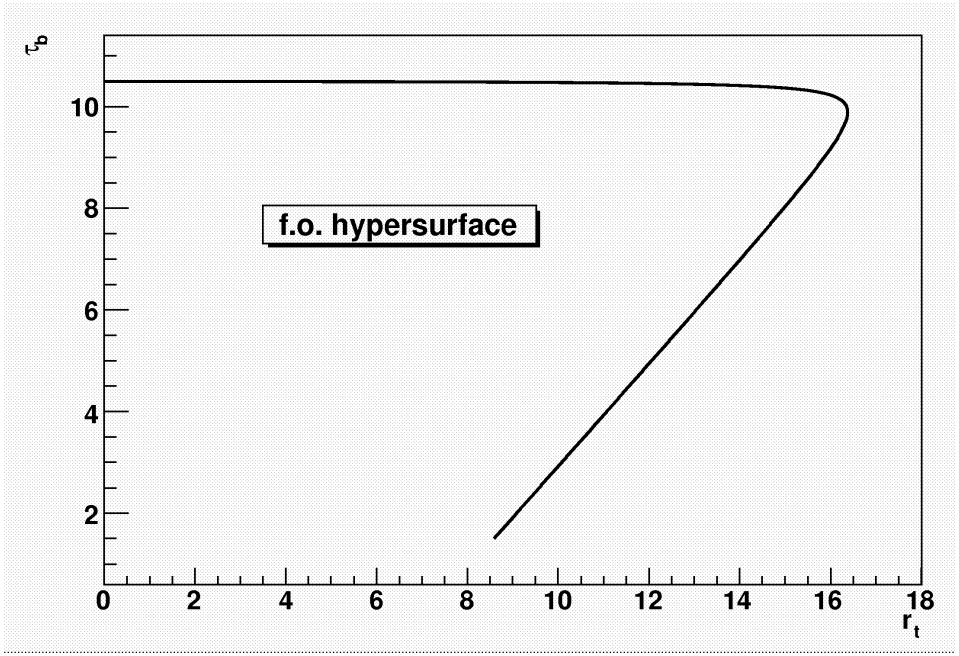
(Yu.S., Iu.A. Karpenko. Heavy Ion Phys. 25/1 (2006) 141–147)

✓Particular solution for energy density:

System is a finite in the transverse direction and is an approximately boost-invariant in the long-direction at freeze-out.

$$\epsilon = \frac{C_\epsilon (1+d(1-z^2/t^2))^2}{t(t+T)^2\sqrt{1-z^2/t^2}} \exp\left(-\frac{b_\epsilon^2}{1-\lambda^2\left(\frac{x_1^2+x_2^2}{(t+T)^2}\right)\frac{1}{1-z^2/t^2}}\right)$$

Dynamical realization of enclosed f.o. hypersurface



Geometry :

$\tau_f = 10.5 fm/c$

$\tau_i = 1.5 fm/c$

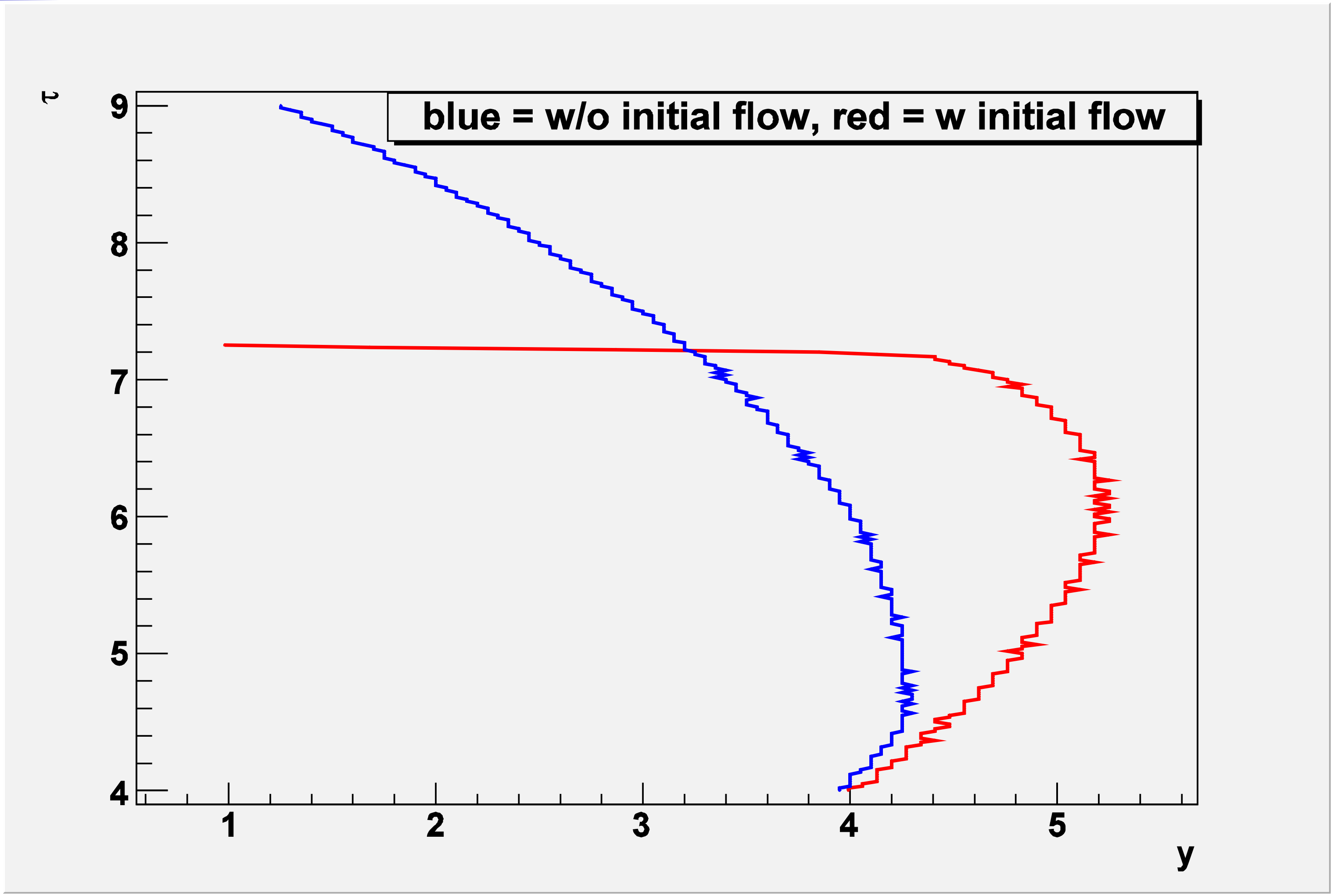
$R_{t,max} = 13.5 fm$

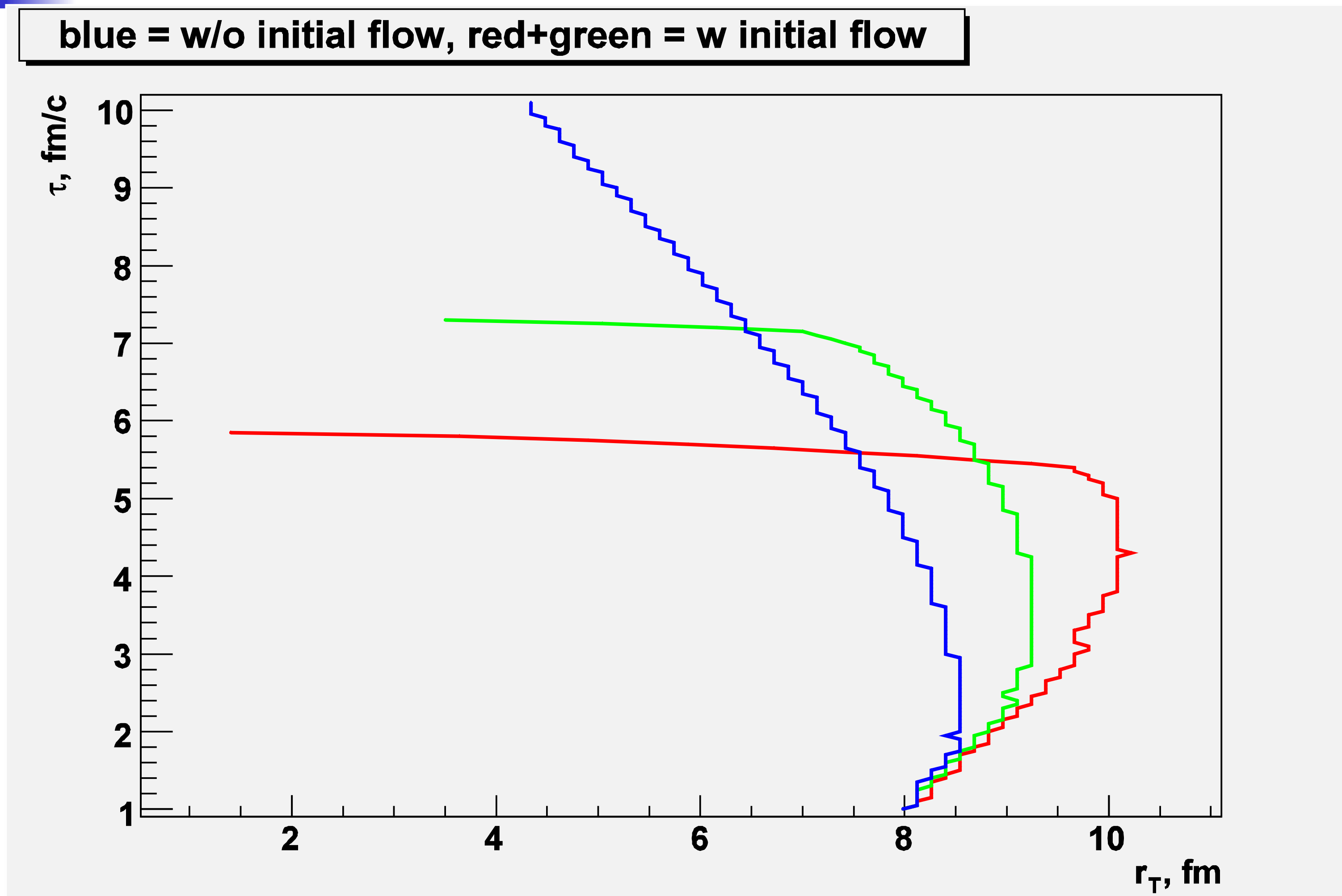
$R_{t,0} = 8 fm$

$\mathrm{R_{t,max}}$ $\mathrm{R_{t,0}}$ decreases with rapidity increase.

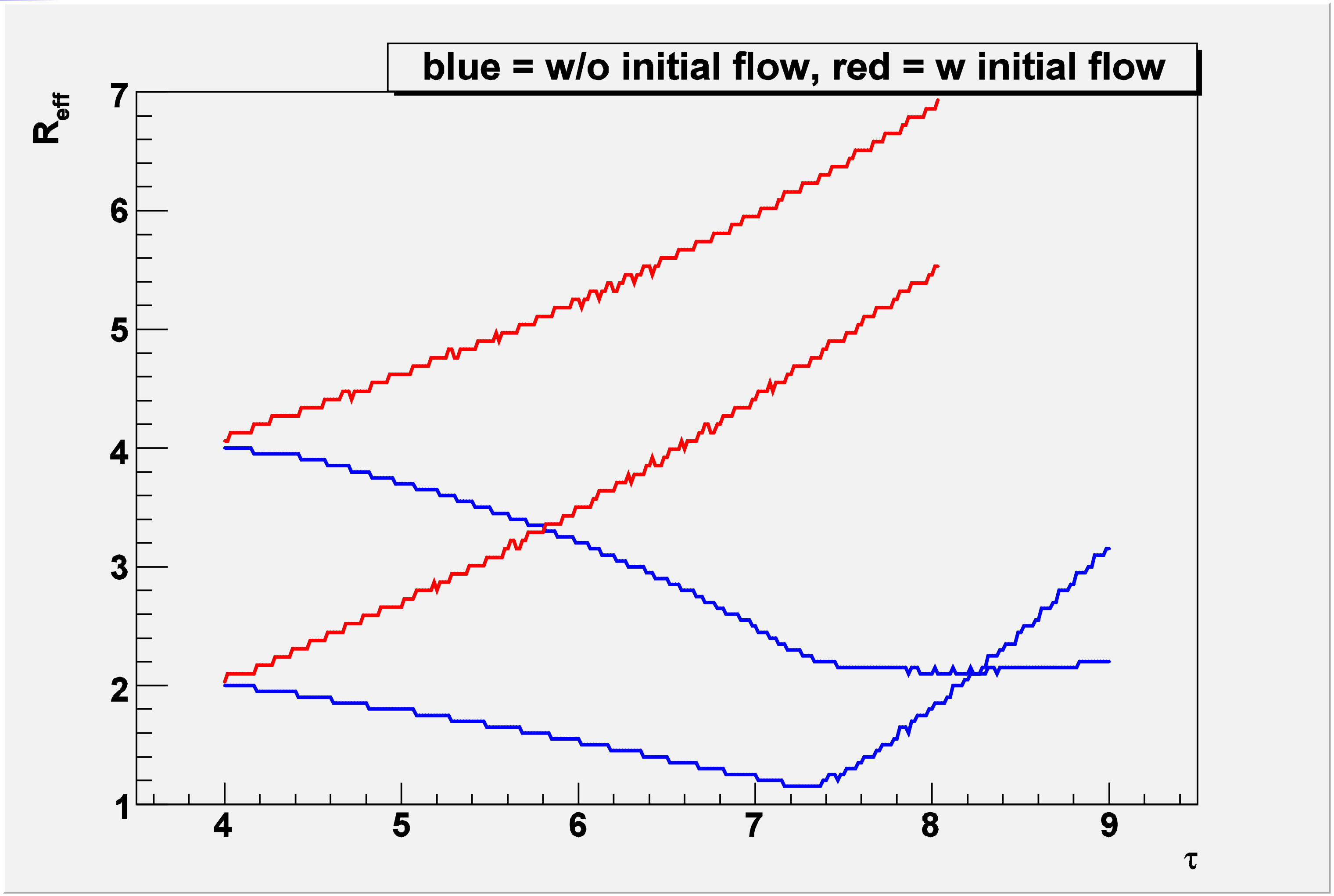
No exact boost invariance!

Numerical 3D anisotropic solutions of relativistic hydro with boost-invariance: freeze-out hypersurface




blue = w/o initial flow, red+green = w initial flow
τ, fm/c
10
9
8
7
6
5
4
3
2
1
2
4
6
8
10
r_T, fm

Numerical 3D anisotropic solutions of relativistic hydro with boost-invariance: evolution of the effective radii



Developing of collective velocities in partonic matter at pre-thermal stage

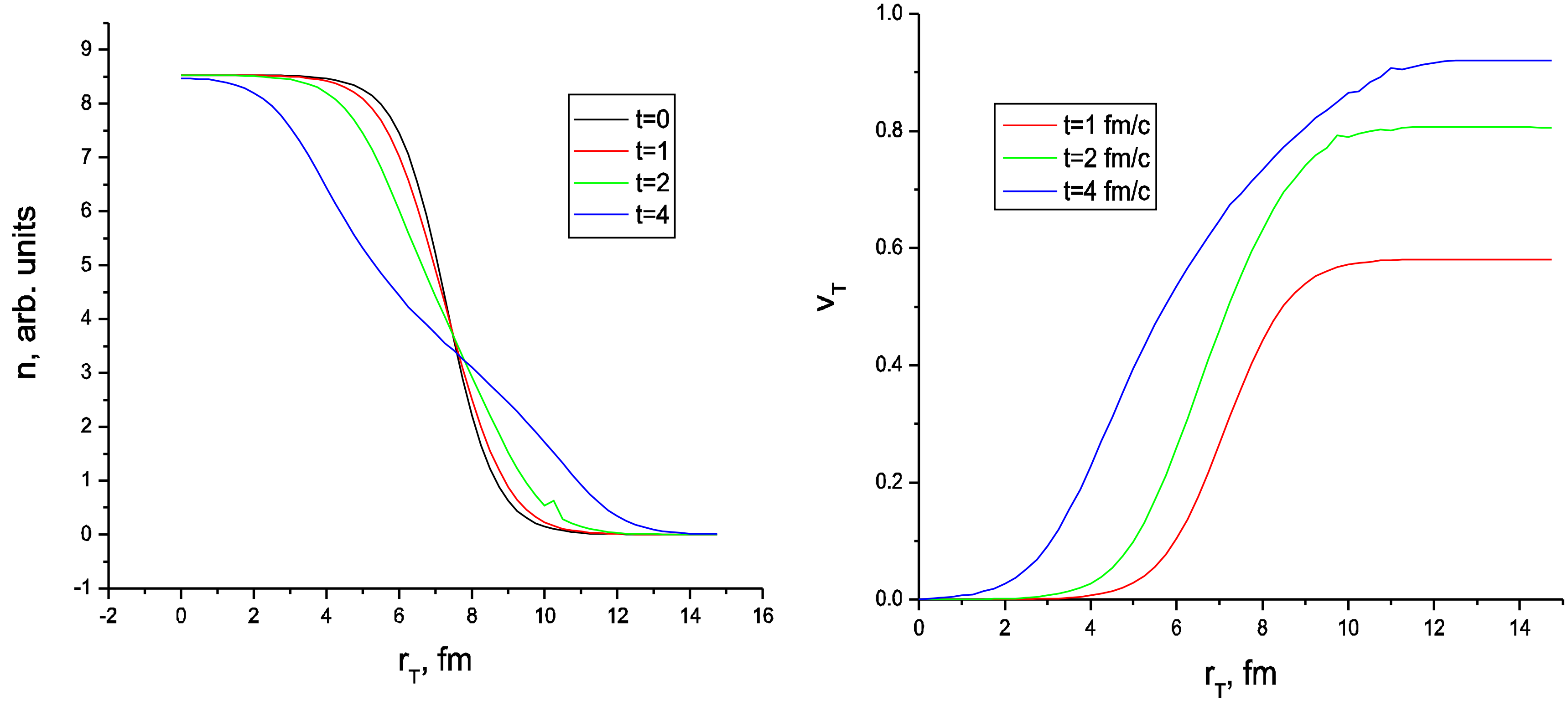
The transverse momentum distribution of partons in CGC [Venugopalan et al., 2003,2005.]

$$f(x,p) = \frac{1}{\exp(\frac{\sqrt{p_t^2+m_0^2}}{T}) - 1} \cdot \frac{1}{\exp(\frac{r-R}{\delta}) + 1}$$

Paramiters: $m_0 = 0.0358\Lambda_s$, $T = 0.465\Lambda_s$, $R = 7.3 fm$, $\delta = 0.67 fm$.

Free streaming at $t > t_0$: $r_x \rightarrow r_x - p_x t/E$, $r_y \rightarrow r_y - p_y t/E$, where $E = \sqrt{p_t^2 + m^2}$.

We calculate at $t \neq 0$ at each radial point the average (collective) velocity $v_x = \langle p_x/E \rangle$.


n, arb. units
r_T, fm
t=0
t=1
t=2
t=4
v_T
r_T, fm
t=1 fm/c
t=2 fm/c
t=4 fm/c

Conclusions

- **A method allowing studies the hadronic matter at the early evolution stage in A+A collisions is developed. It is based on an interferometry analysis of approximately conserved values such as the averaged phase-space density (APSD) and the specific entropy of thermal pions.**

- **An anomalously high rise of the entropy at the SPS energies can be interpreted as a manifestation of the QCD critical end point, while at the RHIC energies the entropy behavior supports hypothesis of crossover.**

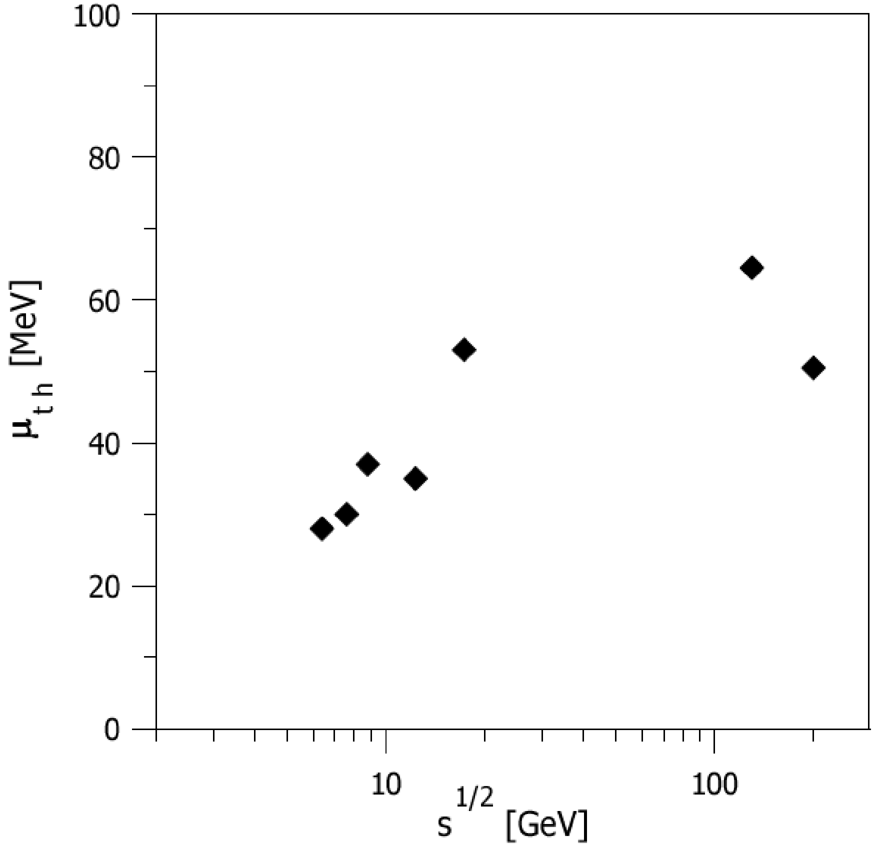
- **The plateau founded in the APSD behavior vs collision energy at SPS is associated, apparently, with the deconfinement phase transition at low SPS energies; a saturation of this quantity at the RHIC energies indicates the limiting Hagedorn temperature for hadronic matter.**

- **It is shown that if the cubic power of effective temperature of pion transverse spectra grows with energy similarly to the rapidity density (that is roughly consistent with experimental data), then the interferometry volume is only slightly increase with collision energy.**

- **An increase of initial of transverse flow with energy as well as isotropization of local spectra at pre-thermal stage could get explanation within partonic CGC picture.**

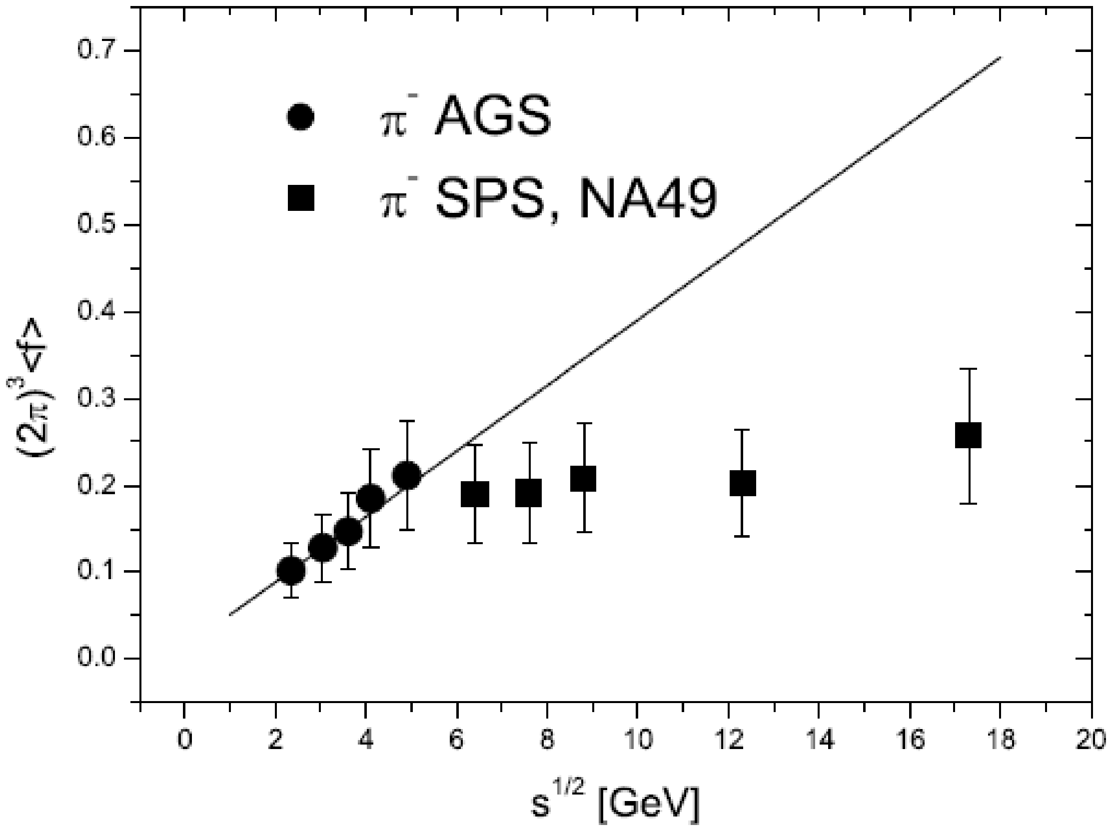
- EXTRA SLIDES

The chemical potential



The chemical potentials of thermal ("direct") negative pions, μ_{th} (rhombus) as functions of c.m. energies per nucleon in heavy ion central collisions.

The statistical errors



The statistical uncertainties caused by the experimental errors in the interferometry radii in the AGS-SPS energy domain. The results demonstrate the range of statistical signicance of nonmonotonic structures found for a behavior of pion averaged phase-space densities as function of c.m. energy per nucleon in heavy ion collisions.