Heavy flavours in DGLAP improved Saturation Model

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EXPLANATION

DGLAP Improved Saturation Model
= Saturation model of Bartels, Golec and Kowalski (BGK)

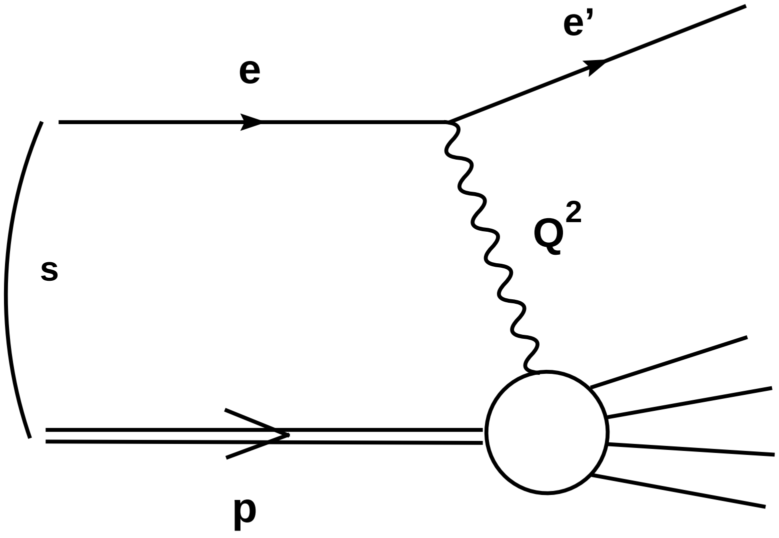
MOTIVATION

Consistent description of deep inelastic scattering at HERA in the limit of low x - heavy quarks cannot be neglected

OUTLINE

1. DIS - introduction, dipole representation
2. Saturation Models
    - Golec–Biernat Wüsthoff Model (GBW)
    - Bartels–Golec–Kowalski Model (BGK)
3. BGK Model with Heavy Quarks (this results)
4. Summary

DEEP INELASTIC SCATTERING



photon virtuality $Q^2 = -q^2 = -(e-e')^2$

Bjorken variable $x = \frac{Q^2}{2p \cdot q}$

ep CMS energy $\sqrt{s} = \sqrt{(e+p)^2}$

inelasticity $y = \frac{Q^2}{xs}$

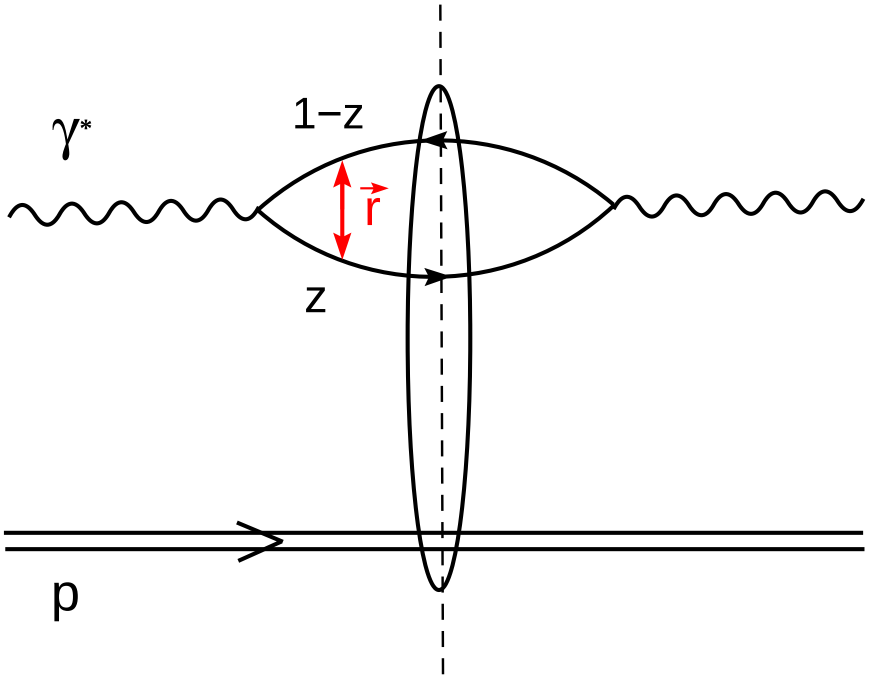
$$\frac{d\sigma_{ep}}{dx dQ^2} = \frac{2\pi\alpha_{em}^2}{x\, Q^4}\left[(1+(1-y)^2)F_2(x,Q^2) - y^2 F_L(x,Q^2)\right]$$

In the infinite momentum frame, in the Bjorken limit:

Bjorken variable x - proton momentum fraction carried by the struck parton

$$F_2(x) = \sum_i e_i^2\, x f_i(x), \qquad f_i(x) - \text{parton distribution}$$

DIPOLE REPRESENTATION AT LOW x



$$F_2 = \frac{Q^2}{4\pi^2\,\alpha_{em}}\left(\sigma_T^{\gamma^* p} + \sigma_L^{\gamma^* p}\right)$$

$$\sigma_{T,L}^{\gamma^* p}(x,Q^2) = \sum_f \int d^{\,2}\vec{r}\int_0^1 dz\,|\Psi_{T,L}^{f}\,(\vec{r},z,Q^2)|^2\,\hat{\sigma}\,(x,\vec{r})$$

f - active flavours

$|\Psi_{T,L}^{f}\,(\vec{r},z,Q^2)|^2$ – describes $\gamma^* \to q\bar{q}$ splitting , known from QED

$\hat{\sigma}(x,r)$ – dipole-proton cross section, preserves unitarity

small r: $\hat{\sigma}(x,r) \sim r^2$ - color transparency

large r: $\hat{\sigma}(x,r) \simeq \sigma_0$ - saturation

GOLEC–BIERNAT WÜSTHOFF SATURATION MODEL

$$\hat{\sigma}(x,r) = \sigma_0 \left\{ 1 - \exp\left(-\frac{r^2}{4R_0^2(x)} \right) \right\}$$

saturation scale

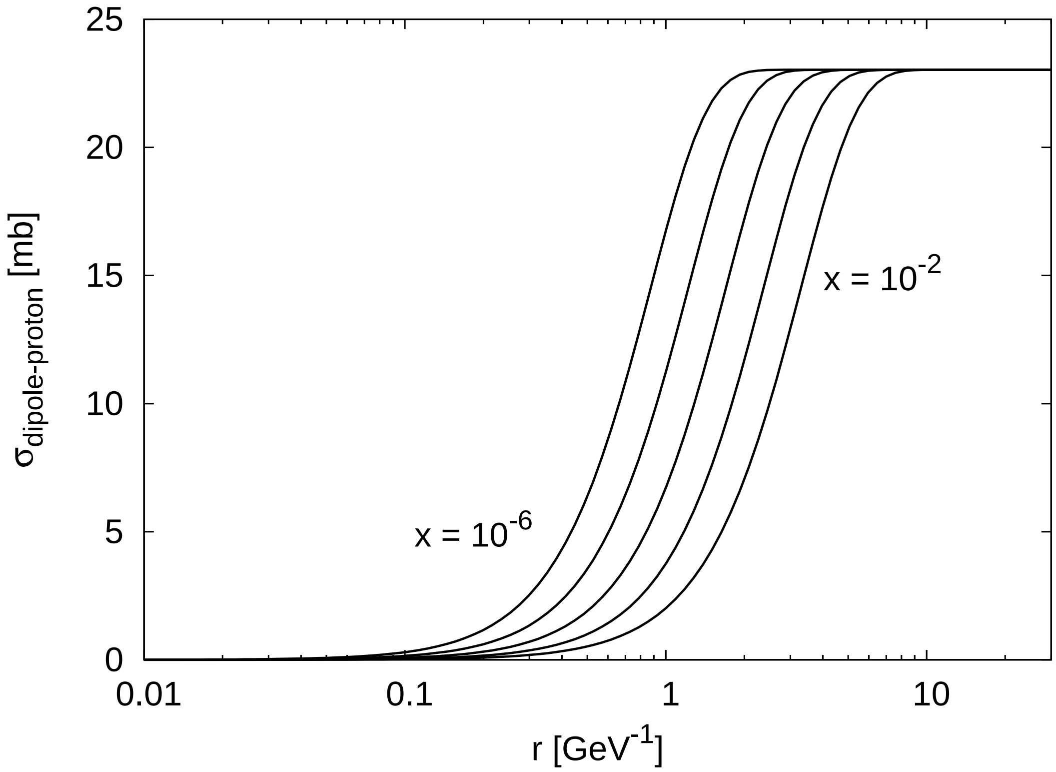
$$Q_s^2(x) = \frac{1}{R_0^2(x)} = \left(\frac{x_0}{x} \right)^{\lambda}$$

fit results to F_2 old data

H1/ZEUS 94

FIT	χ^2/ndf
light	1.18
light + c	1.50

at HERA: $Q_s^2(x) \simeq 1\mathrm{GeV}^2$



SATURATION MODEL WITH DGLAP EVOLUTION

Standard pQCD formula for the dipole cross section, valid at small r

$$\hat{\sigma}(x,r) \simeq \frac{\pi^2}{3} \alpha_s(\mu^2)\, r^2\, xg(x,\mu^2) \qquad \text{with} \quad \mu^2 \propto \frac{1}{r^2}$$

Bartels Golec Kowalski model

$$\hat{\sigma}(x,r) = \sigma_0 \left\{ 1 - \exp\left(- \frac{r^2\, \pi^2\, \alpha_s\, xg(x,\mu^2)}{3\, \sigma_0} \right) \right\}$$

with

$$\mu^2 = \frac{C}{r^2} + \mu_0^2$$

Gluon density $xg(x,\mu^2)$ evolves with μ^2 according to DGLAP equations with the initial condition

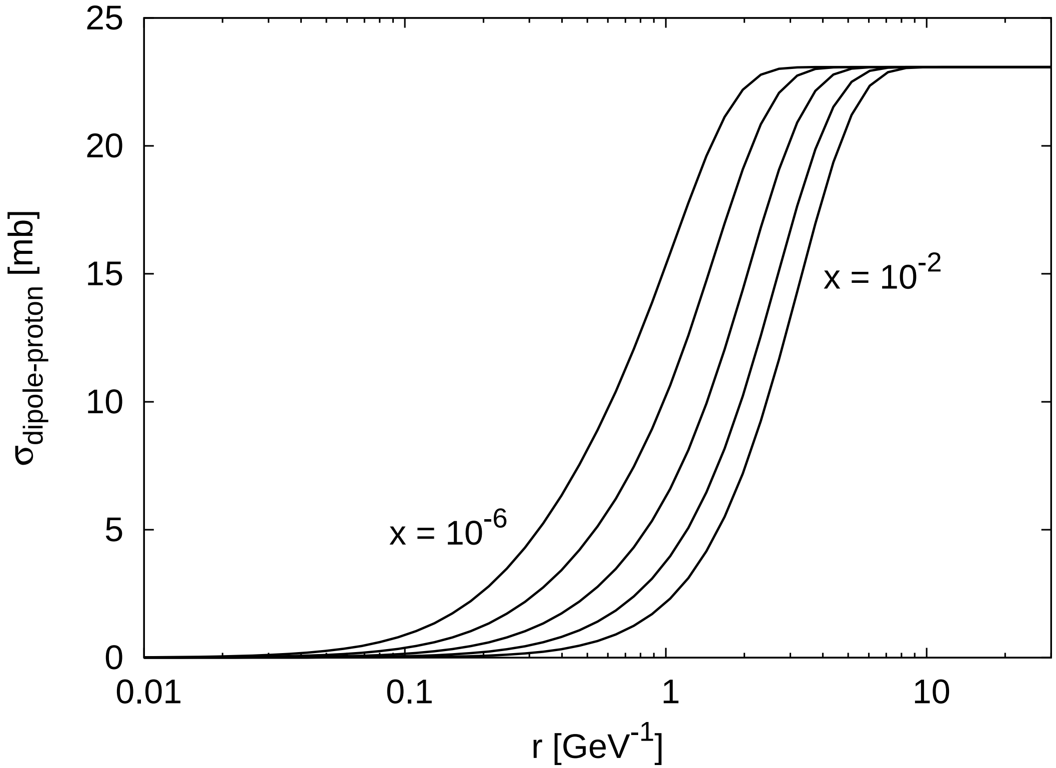
$$xg(x,Q_0^2) = A_g\, x^{-\lambda_g} (1-x)^{5.6} \qquad \text{at} \quad Q_0^2 = 1\ \text{GeV}^2$$

SATURATION MODEL WITH DGLAP EVOLUTION

fit results to F_2 new data

H1/ZEUS 96-97

FIT	χ^2/ndf
light	0.97–1.18
light + c + b	???



dipole cross section

small r: improved by DGLAP evolution of gluon density

large r: saturates identically with GBW model

MOTIVATION FOR HEAVY QUARKS

Universal form of the dipole cross section

$$F_2 = F_2^{\text{light}} + F_2^{c\bar{c}} + F_2^{b\bar{b}}$$

$$F_2 = \int |\Psi_{\text{light}}|^2\, \hat{\sigma}_{\text{dipole}} + \int |\Psi_{\text{charm}}|^2\, \hat{\sigma}_{\text{dipole}} + \int |\Psi_{\text{beauty}}|^2\, \hat{\sigma}_{\text{dipole}}$$

HERA data

$F_2^{c\bar{c}}/F_2$	from 10% to 30%
$F_2^{b\bar{b}}/F_2$	up to 3%

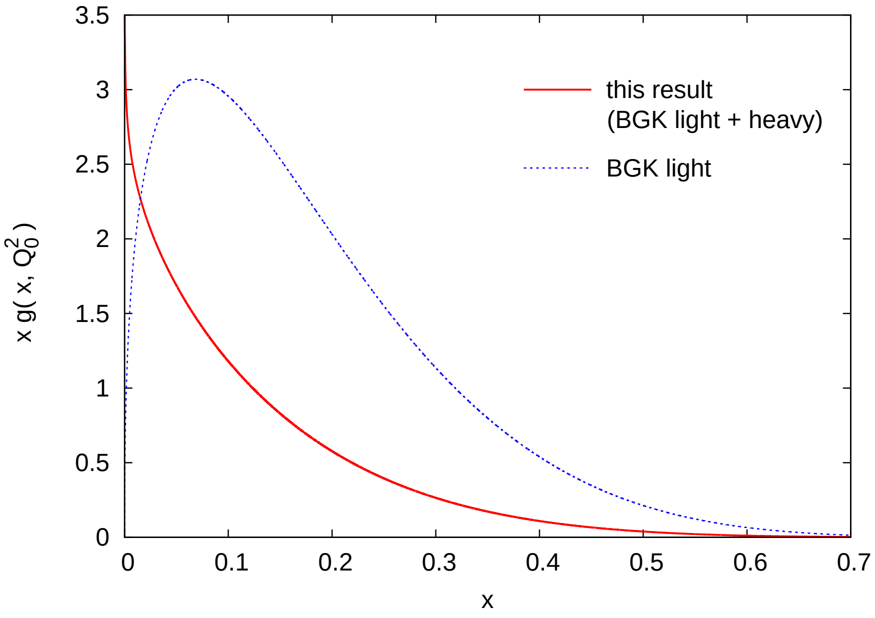
Goal: to include heavy quarks in DGLAP improved saturation model and see its impact on saturation properties

THE FIT TO DIS DATA

$m_{\text{light}} = 0.0$ MeV $\quad m_{\text{charm}} = 1.3$ GeV $\quad m_{\text{beauty}} = 5.0$ GeV

	σ_0 [mb]	A_g	λ_g	C	μ_0^2	χ^2/ndf
light	23.8	13.71	- 0.41	11.10	0.52	0.97
light + c	22.4	1.78	0.0803	0.679	2.11	1.15
light + c + b	22.5	1.77	0.0793	0.761	2.08	1.25

- BGK model with heavy quarks gives still very good fit
- parameters significantly different than in the case of light quarks only
- very small and positive λ_g, initial distribution is gluon-like

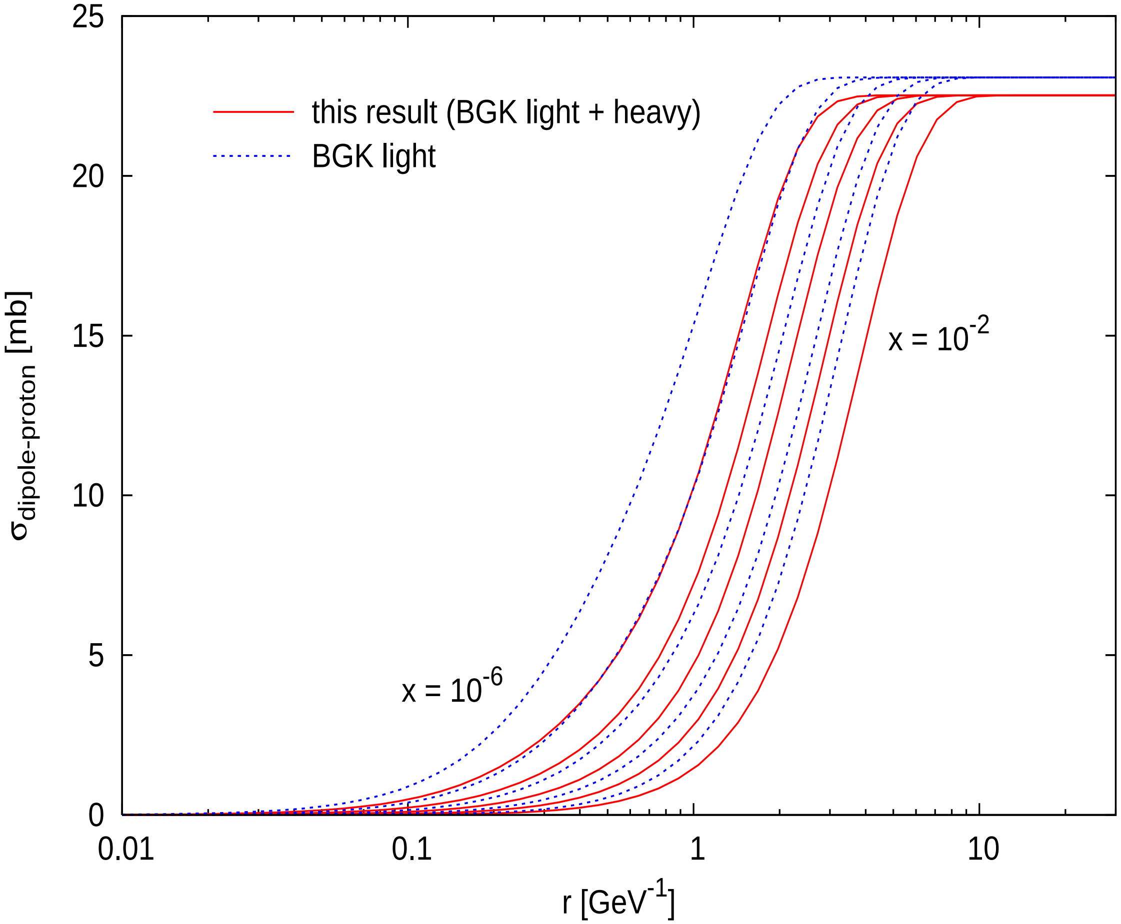


DIPOLE CROSS SECTION

$$F_2 = F_2^{\rm light} + F_2^{\rm c+b}$$

$$F_2 = \int \sum \underbrace{|\Psi|^2 \, \hat{\sigma}}_{\text{suppressed at large r}}$$

In the model with heavy quarks the dipole cross section for a given dipole size saturates at higher energy (smaller x)



CRITICAL LINE

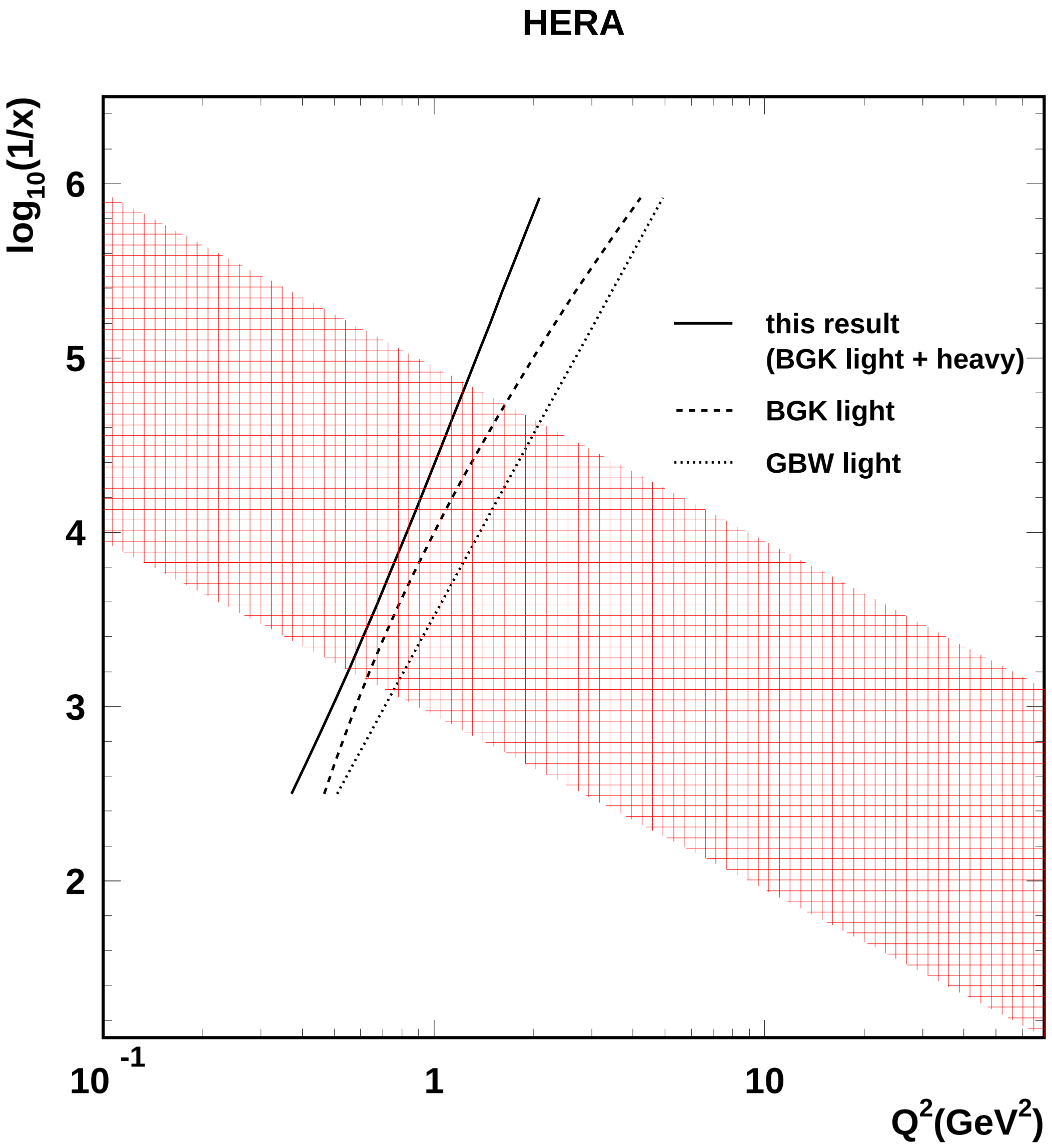
Characteristic dipole size

$$\bar{r} = 2/Q$$

at which

$$\hat{\sigma}(x, \bar{r}) \simeq \sigma_0$$

Shifting of the dipole cross section results in the shift of the critical line towards smaller values of Q^2



CHARM STRUCTURE FUNCTION

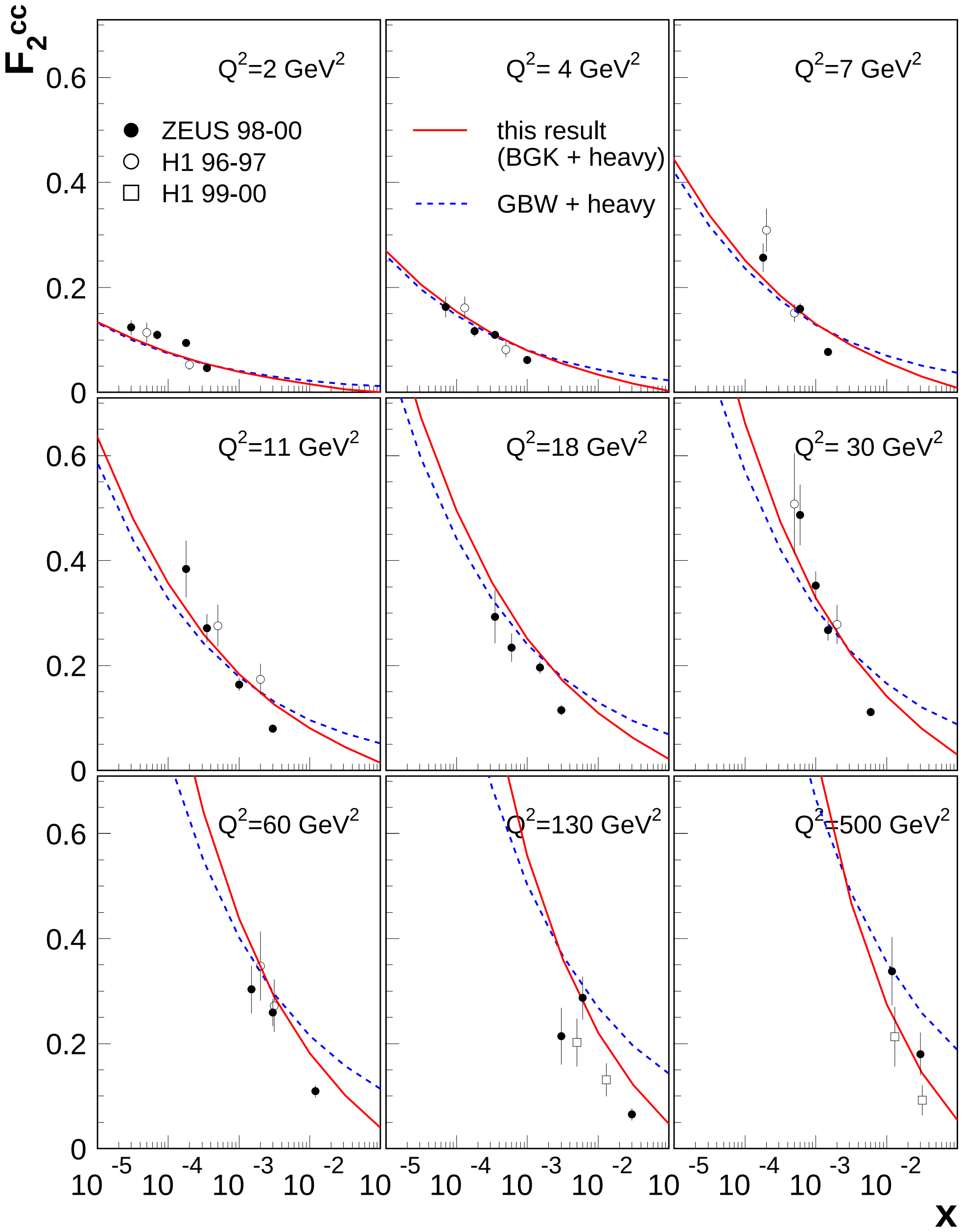
Correct predictions for $F_2^{c\bar{c}}$

- proper normalization
- DGLAP evolution improves the slope of $F_2^{c\bar{c}}$ at large Q^2

$$F_2 = F_2^{\text{light}} + F_2^{c\bar{c}} + F_2^{b\bar{b}}$$

$\uparrow$ fitted

$\uparrow$ predicted



BEAUTY STRUCTURE FUNCTION

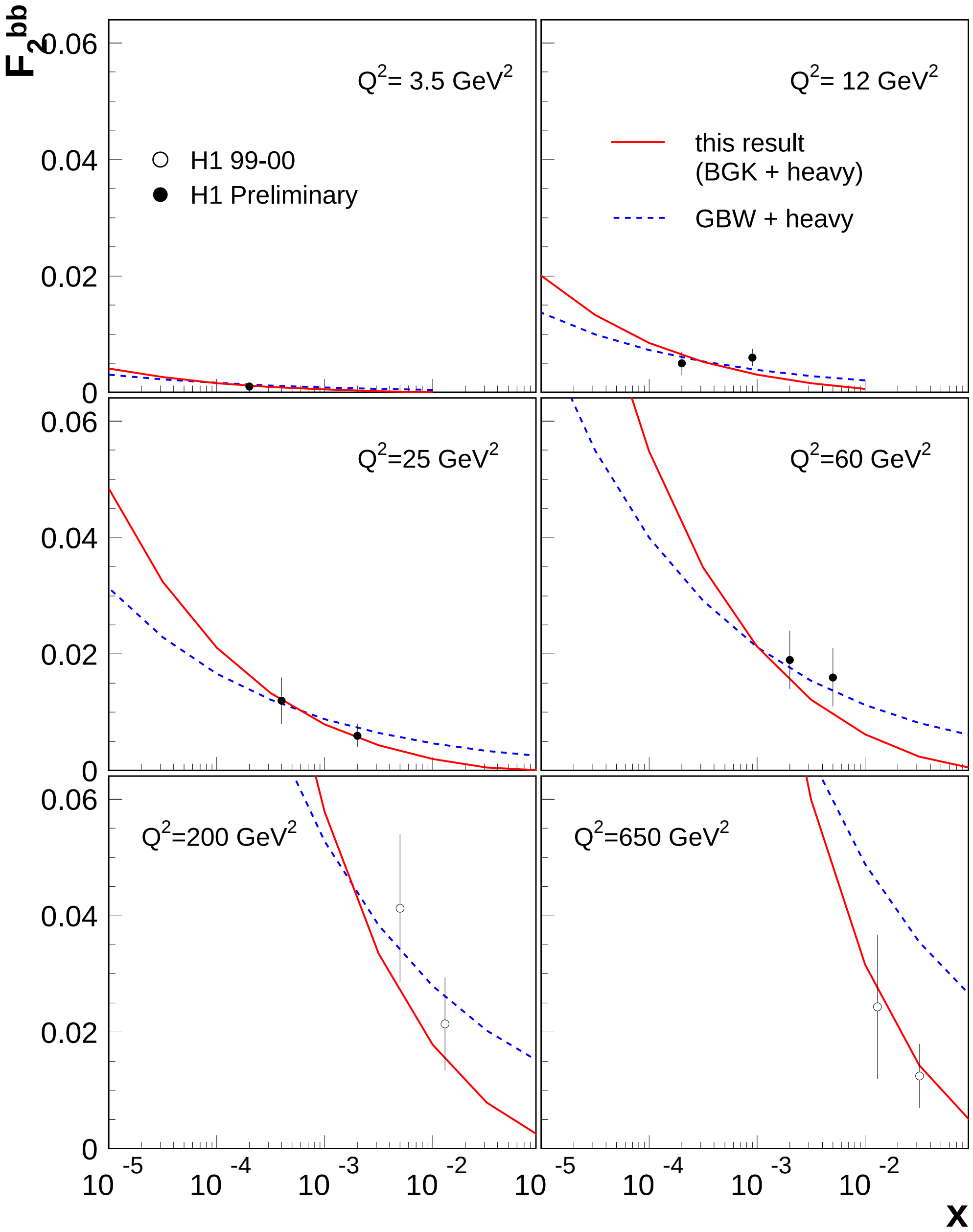
Correct predictions for $F_2^{b\bar{b}}$

- proper normalization !
- DGLAP evolution improvement important at large Q^2 !

$$F_2 = F_2^{\text{light}} + F_2^{c\bar{c}} + F_2^{b\bar{b}}$$

$$\uparrow \qquad\qquad\qquad\qquad \uparrow$$

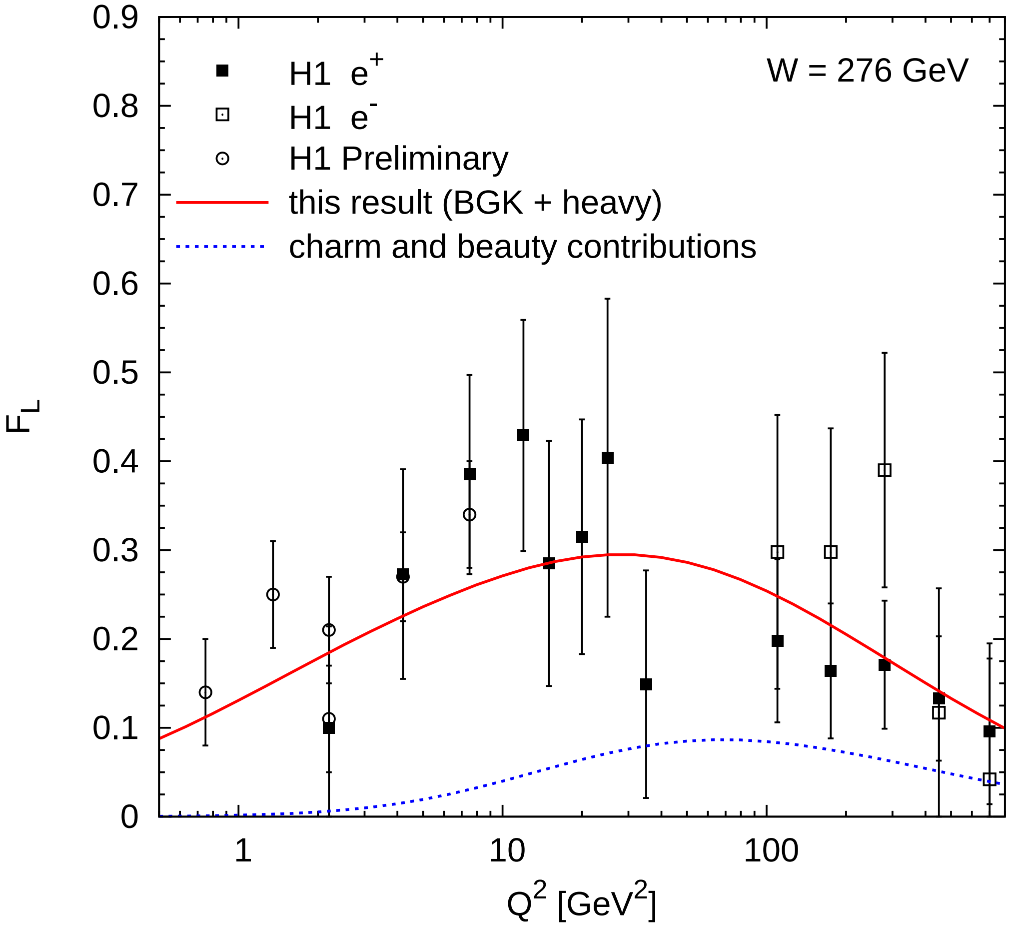
fitted predicted



LONGITUDINAL STRUCTURE FUNCTION

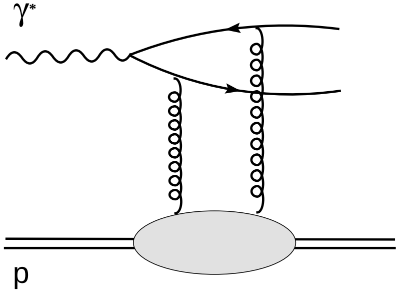
No direct measurements of F_L available

- rough agreement with H1 estimations
- direct measurements planned by H1 and ZEUS

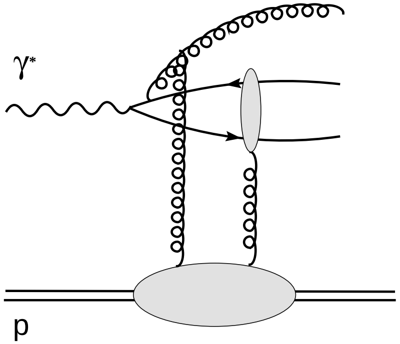


DIFFRACTIVE STRUCTURE FUNCTION

$q\bar{q}$ dipole

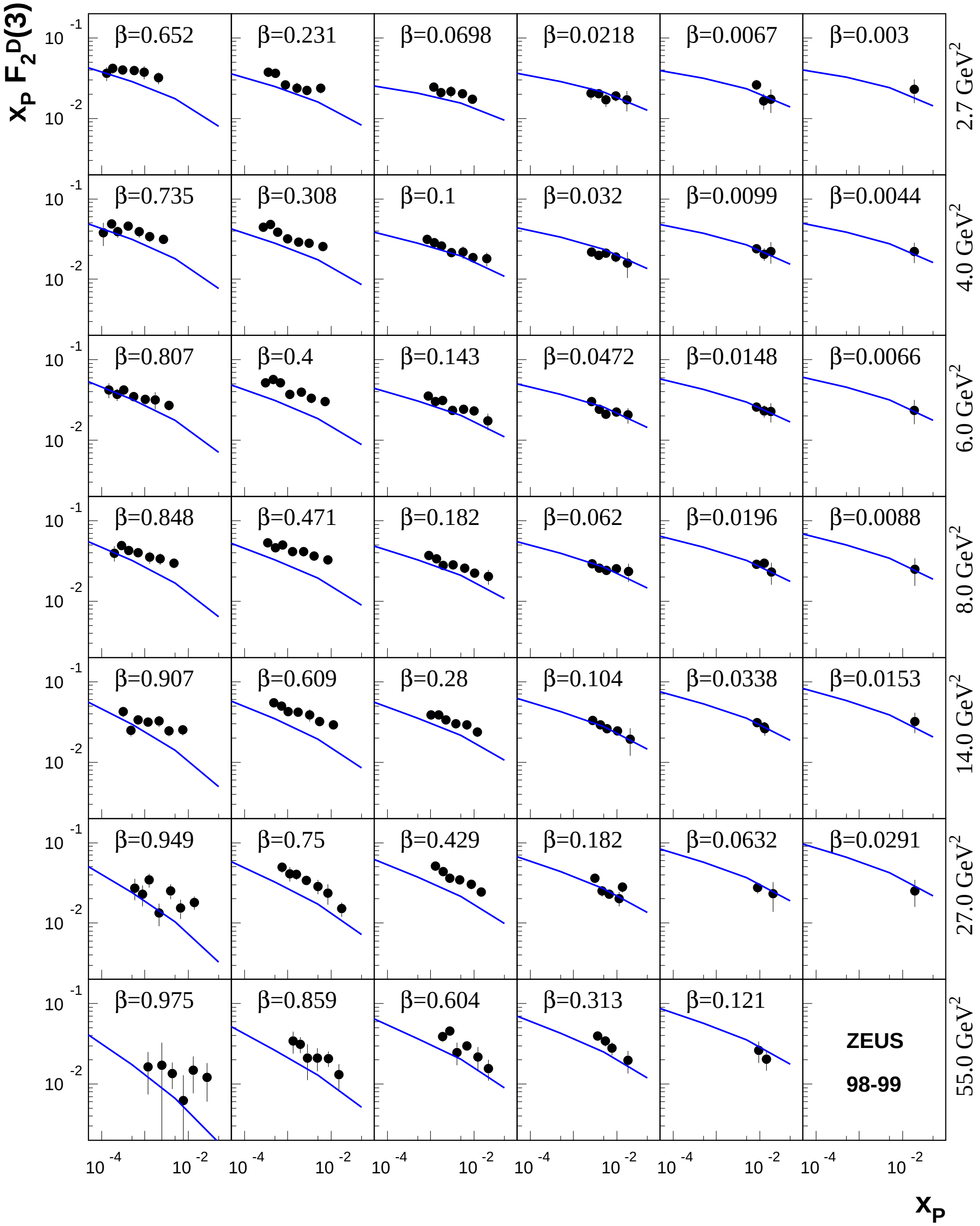


$q\bar{q}g$ dipole



$$F_2^{D(3)} = F_{T,q\bar{q}} + F_{L,q\bar{q}} + F_{q\bar{q}g}$$

diff. slope: $B_D = 6.0\,\mathrm{GeV}^{-2}$



SUMMARY

- Heavy quarks have to be taken into account in order to consistently describe deep inelastic scattering at HERA in the limit of low x
- DGLAP improved saturation model with heavy quarks provides successful description of wide scope of observables for DIS at low x
  - fit to inclusive F_2 data, $\chi^2/ndf = 1.15 - 1.25$
  - correct predictions for $F_2^{c\bar{c}}$ and $F_2^{b\bar{b}}$
  - predictions for F_L
  - reasonable agreement for diffractive structure function $F_2^{D(3)}$
- Adding heavy quarks results in the shift of the critical line towards smaller values of Q^2