Lattice Flavourdynamics

Chris Sachrajda

School of Physics and Astronomy
University of Southampton
Southampton
UK

Krakòw School of Theoretical Physics
May 27th – June 5th 2006

Table of Contents

The Cabibbo-Kobayashi-Maskawa Matrix

Lattice QCD contributes very significantly to the phenomenology of the Unitarity Triangle.

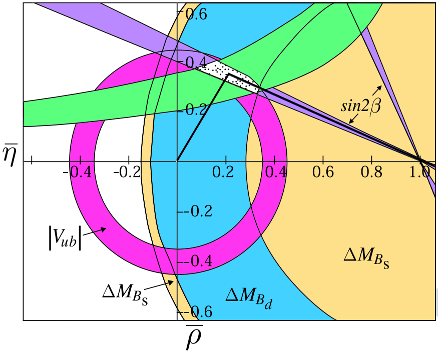
PDG Values for the moduli of the CKM Matrix elements:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9739 - 0.9751 & 0.221 - 0.227 & 0.0029 - 0.0045 \\ 0.221 - 0.227 & 0.9730 - 0.9744 & 0.039 - 0.044 \\ 0.0048 - 0.014 & 0.037 - 0.043 & 0.9990 - 0.9992 \end{pmatrix} .$$

Wolfenstein Parametrization of the CKM Matrix:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} .$$

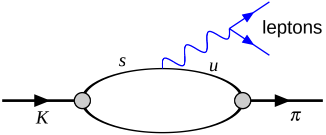
Unitarity Triangle



The status of the Unitarity Triangle currently on PDG Web-Site.

New CDF (2006) measurement of Δm_S changes the picture significantly (see below).

Kaon Physics - $K_{\ell 3}$ Decays



$\Rightarrow \quad V_{us}$

$$\langle \pi(p_\pi) | \bar{s}\gamma_\mu l | K(p_K) \rangle = f^0(q^2)\, \frac{M_K^2 - M_\pi^2}{q^2} q_\mu + f^+(q^2) \left[(p_\pi + p_K)_\mu - \frac{M_K^2 - M_\pi^2}{q^2} q_\mu \right]$$

where $q \equiv p_K - p_\pi$ and $l = u, d$.

To be useful in extracting V_{us} we require $f^0(0) = f^+(0)$ to better than about 1% precision.

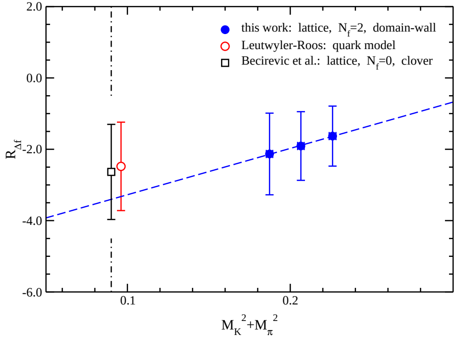
$$\chi\text{PT} \Rightarrow f^+(0) = 1 + f_2 + f_4 + \cdots \quad \text{where} \quad f_n = O(M^n_{K,\pi,\eta}).$$

Reference value $f^+(0) = 0.961 \pm 0.008$ where $f_2 = -0.023$ is relatively well known from χPT and $f_4, f_6, \cdots$ are obtained from models. Leutwyler & Roos (1984)

1% precision of $f^+(0)$ is conceivable because it is actually $1 - f^+(0)$ which is computed using *double ratios* such as: S.Hashimoto et al.

$$\frac{\langle\pi|\bar{s}\gamma_0 l|K\rangle\langle K|\bar{l}\gamma_0 s|\pi\rangle}{\langle\pi|\bar{l}\gamma_0 l|\pi\rangle\langle K|\bar{s}\gamma_0 s|K\rangle} = \left[f^0(q^2_{\rm max})\right]^2 \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}\,.$$

q^2 and m^2 extrapolations still have to be done.



For example, the RBC collaboration show the (valence) mass behaviour of

$$R_{\Delta f} \equiv \frac{\sum_2^\infty f_{2k}}{(M_K^2 - M_\pi^2)^2}\,.$$

Results are in units of a ($a^{-1} \simeq 1.7\,$GeV).

1% precision of $f^+(0)$ is conceivable because it is actually $1 - f^+(0)$ which is computed using *double ratios* such as: S.Hashimoto et al.

$$\frac{\langle\pi|\bar{s}\gamma_0 l|K\rangle\langle K|\bar{l}\gamma_0 s|\pi\rangle}{\langle\pi|\bar{l}\gamma_0 l|\pi\rangle\langle K|\bar{s}\gamma_0 s|K\rangle} = \left[f_0(q^2_{\rm max})\right]^2 \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}\,.$$

q^2 and m^2 extrapolations still have to be done.

Three *preliminary* new (unquenched) results for $f^+(0)$:

RBC (2005)	0.955(12)
JLQCD (2005)	0.952(6)
FNAL/MILC/HPQCD (2004)	0.962(6)(9)

in good agreement with the Leutwyler-Roos result of 0.961(8).

Systematic Errors (continuum, chiral, q^2 extrapolations) will be reduced over the next year or two.

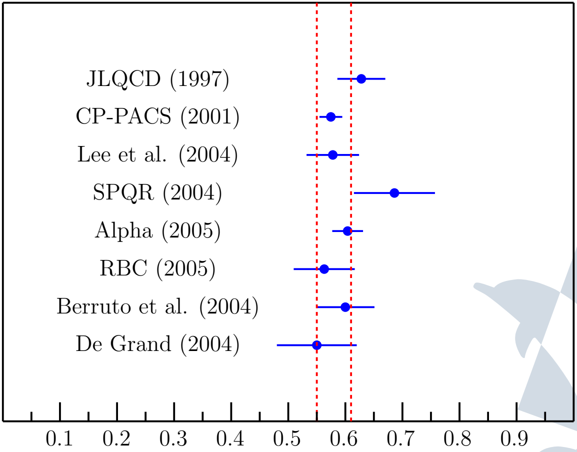
Kaon Physics - B_K

The parameter B_K contains the non-perturbative QCD effects in $K - \bar{K}$ mixing and has been computed in lattice simulations for a long time.

$$\langle \bar{K}^0 \,|\, (\bar{s}\gamma^\mu(1-\gamma^5)d)\,(\bar{s}\gamma_\mu(1-\gamma^5)d) |K^0\rangle = \frac{8}{3} M_K^2 f_K^2 B_K \,.$$

B_K depends on the renormalization scheme and scale and is conventionally given in the NDR, $\overline{\mathrm{MS}}$ scheme at $\mu = 2\,\mathrm{GeV}$ or as the RGI parameter $\hat{B}_K$ ($\hat{B}_K \simeq 1.4\, B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$.)

Selected Quenched Results for $B_K^{\overline{MS}}(2\,\mathrm{GeV})$



- Recent summaries of the quenched value of B_K include:

$$\begin{aligned} B_K^{\overline{\rm MS}}(2\,{\rm GeV}) &= 0.58(4) \qquad \text{S.Hashimoto (ICHEP 2004)} \\ B_K^{\overline{\rm MS}}(2\,{\rm GeV}) &= 0.58(3) \qquad \text{C.Dawson (Lattice 2005)}\,. \end{aligned}$$

- Dynamical computations of B_K are underway by a number of collaborations, but so far the results are very preliminary.

  C.Dawson's guesstimate (from comparison of unquenched & quenched results at similar masses and lattice spacings)

$$B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.58(3)(6) \qquad \text{C.Dawson (Lattice 2005)}\,.$$

  We need to wait until reliable dynamical results are available in the next year or two.

Lattice QCD & Heavy Quark Physics

- Quantities computed in lattice simulations include:
  - Quark masses;
  - Leptonic Decay Constants & B-parameters;
  - Semileptonic Form Factors (and $B\to K^*\gamma$);
  - Beauty lifetimes;
  - $g_{BB^*\pi}$ and $g_{DD^*\pi}$ couplings.
- QCD effects in decays can be computed in both the SM and in BSM.
- The prospects for a successful future depend on being able to control and reduce the systematic errors.

Tremendous progress is being made!

- Quantities which we have been unable to formulate for (direct) lattice computations include
  - Inclusive semileptonic decays;
  - $B\to M_1M_2$ decays ($M_{1,2}$ - light mesons).

Improving the Precision - Choice of Heavy Quark Action

- Since typical lattice spacings are in the range $1.5\,\mathrm{GeV} < a^{-1} < 3$ GeV, it is not possible to perform simulations directly with the b-quark and it is questionable whether simulations can be performed with the c-quark.
- Actions which are used in heavy-quark physics include:
  - QCD with *lightish* heavy quarks and extrapolations in the mass towards the physical values;
  - HQET;
  - NRQCD;
  - Fermilab/Tsukuba action.

HQET

- Lattice simulations using leading-order HQET have been calculated for over 15 years. The difficulty is to go to NLO, i.e. to compute $\Lambda_{\rm QCD}/m_b$ corrections and to perform the non-perturbative renormalization.
- A nice example is the computation of m_b.

$$\sum_{\vec{x}} \langle A_4(x) A_4(0) \rangle \simeq Z^2 e^{-\xi t}\,.$$

From Z we obtain $f_B^{\rm stat}$ (Eichten (1987)) and from ξ (with perturbative subtraction of the $1/a$ terms) we obtain m_b (Crisafulli, Gimenez, Martinelli & CTS (1995)).

$$m_b^{\overline{MS}}(m_b^{\overline{MS}}) = 4.2 \pm 0.1 \pm 0.1\,\mathrm{GeV}\,.$$

The perturbative subtractions have been performed to three-loop order in the lattice theory using stochastic methods (di Renzo & Scorzato (2004)) .

Working to leading order there are unavoidable $O(\Lambda_{\rm QCD}^2/m_b) \sim 20\,$MeV errors in m_b.

HQET (cont.)

- The α-collaboration are undertaking a major program of simulations using the HQET in order to:
  - perform the renormalization non-perturbatively;
  - include the $1/m_b$ corrections.
- The key technique in this programme is *step-scaling*; the numerical matching of results obtained on large (but course) lattices on which hadronic quantities are computed to those on fine (but small) lattices. On the fine lattices HQET can be matched to QCD avoiding the need for (lattice) perturbation theory

  (Della Morte, Garron, Papinutto & Sommer, hep-lat/0509084).
- To date, DGPS have only published a quenched result:

$$\bar{m}_b(\bar{m}_b) = 4.350(64)\,\mathrm{GeV}\,,$$

  with the $O(\Lambda^2_{\mathrm{QCD}})/m_b$ contribution being (-50 ± 30) MeV .

  (Della Morte, Garron, Papinutto & Sommer, hep-lat/0509173)
- Last month, preliminary unquenched results were presented at the Ringberg workshop.

Improving the Precision - Choice of Heavy Quark Action (Cont.)

NRQCD

- NRQCD is an expansion in the velocity of the heavy quarks and is particularly applicable to quarkonium physics.
- NRQCD is also used for the physics of heavy-light hadrons.
- Perturbation theory is used to compute the coefficients of the higher dimensional operators.
- In NRQCD the continuum limit cannot be taken (there are errors of $O(1/(m_b a)^n)$).

Fermilab/Tsukuba Action

El-Khadra, Kronfeld, Mackenzie (1996)

- Symanzik Improvement is the addition of *irrelevant* operators to the lattice action such that the errors due to the discretization of space-time is formally reduced.

Fermilab (Tsukuba) Action (Cont)

$$\begin{aligned} S \quad = \quad & \sum_x \Big\{ m_0 \bar{q}(x) q(x) + \bar{q}(x)\gamma_0 D_0 q(x) + \nu \bar{q}(x)\vec{\gamma}\cdot\vec{D} q(x) - \frac{r_t a}{2}\bar{q}(x) D_0^2 q(x) \\ & - \frac{r_s a}{2}\bar{q}(x) D_i^2 q(x) - \frac{iga}{2} c_E \sum_i \bar{q}(x)\sigma_{0i} F_{0i} q(x) - \frac{iga}{2} c_B \sum_{ij} \bar{q}(x)\sigma_{i,j} F_{ij} q(x) \Big\}\,. \end{aligned}$$

- The parameters ν, $r_{s,t}$, $c_{E,B}$ are functions of ma. Some of these are *redundant* but ν, $c_{E,B}$ as well as the quark mass must be determined.
- For each operator whose matrix element is being determined, there will be corresponding parameters to be fixed.
- Since all the coefficients are functions of ma, this formalism applies to charm physics rather than to B-physics. Otherwise we cannot be sure that neglected terms of the form:

$$C_n(ma)\,\left(a\Lambda_{\rm QCD}\right)^n$$

  are indeed negligible.
- In NRQCD applications to heavy-light physics we neglect terms of order $1/(ma)^n$ and hence ma should be large.

Fermilab (Tsukuba) Action (Cont)

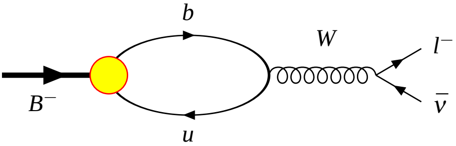
- (In my view) the next stage of this programme should be the reliable determination of the coefficients in the action and operators.
  A very nice start was presented at Lattice 2005, where an exploratory quenched determination of the parameters of the action (using step scaling) was presented. Lin and Christ found a 20% error on the mass and

$$c_B = 1.725(10),\ c_E = 1.3(5) \quad \text{and} \quad \nu = 1.036(17)\,.$$

Lin & Christ hep-lat/0510111

Leptonic Decays of the B-Meson (f_B)

Consider the leptonic decays of pseudoscalar mesons in general and of the B-meson in particular.



Non-perturbative QCD effects are contained in the matrix element

$$\langle 0|\,\bar{b}\gamma^\mu(1-\gamma^5)u\,|B(p)\rangle\ .$$

- Lorentz Inv. + Parity $\Rightarrow\ \langle 0|\,\bar{b}\gamma^\mu u\,|B(p)\rangle = 0.$
- Similarly $\langle 0|\,\bar{b}\gamma^\mu\gamma^5 u\,|B(p)\rangle = if_B p^\mu\ .$
- All QCD effects are contained in a single constant, f_B, the B-meson's *(leptonic) decay constant.* ($f_\pi \simeq 132$ MeV)

Old and New Compilations of f_B and ξ

- “B-Decays from Lattice QCD”, CTS in *B-Decays* (1994, ed. S.Stone):
  $f_B = 180\pm 40\,\mathrm{MeV}$ and $f_D = 200\pm 30\,\mathrm{MeV}$) .

- ”Heavy Quark Physics from Lattice QCD” J.M.Flynn & CTS in *Heavy Flavours II* (1998, ed. A.J.Buras and M.Lindner):
  - f_B ($f_B = 170\pm 35\,\mathrm{MeV}$ and $f_D = 200\pm 30\,\mathrm{MeV}$) .
  - $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}} = 1.14(8)$.

- S.Hashimoto (ICHEP - 2004)

$$f_B = 189\pm 27\,\mathrm{MeV} \quad \text{and} \quad \xi = 1.23(6).$$

- C.Davies (EPS - 2005)

$$f_B = 216\pm 22\,\mathrm{MeV} \qquad \frac{f_{B_s}}{f_{B_d}} = 1.20(3)\,.$$

The Davies’ results are from a single calculation .

New Experimental Results and Lattice QCD

G.Martinelli (for UTfit Collaboration) - Ringberg April 2006

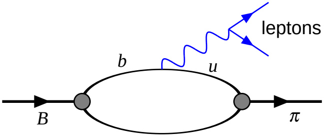
- Belle(2006) $\quad B(B^- \to \tau^- \overline{\nu}_\tau) \;=\; (1.06^{+0.34}_{-0.28}(\text{syst})^{+0.18}_{-0.16}(\text{stat})) \times 10^{-4}$

$$\Rightarrow f_B = 201 \pm 39\,\text{MeV} \quad (V_{ub} = (38.0 \pm 2.7 \pm 4.7)10^{-4} \text{ from exclusive decays})$$
$$\text{or } \; f_B = 173 \pm 30\,\text{MeV} \quad (V_{ub} = (43.9 \pm 2.0 \pm 2.7)10^{-4} \text{ from inclusive decays})$$
$$\text{or } \; f_B = 180 \pm 31\,\text{MeV} \quad (\text{combined inclusive + exclusive})\,.$$

- CDF (2006) $\quad \Delta m_S \;=\; (17.33^{+.42}_{-.21}(\text{stat}) \pm 0.07\text{syst})\,\text{ps}^{-1}$

$$\Rightarrow \quad \xi = 1.15 \pm 0.08 \quad (V_{ub} \text{ exclusive})$$
$$\text{or} \quad \xi = 1.05 \pm 0.10 \quad (V_{ub} \text{ inclusive})$$
$$\text{or} \quad \xi = 1.06 \pm 0.09 \quad (V_{ub} \text{ combined})$$

$B \to \pi$ Semileptonic Decays



$\Rightarrow \quad V_{ub}$

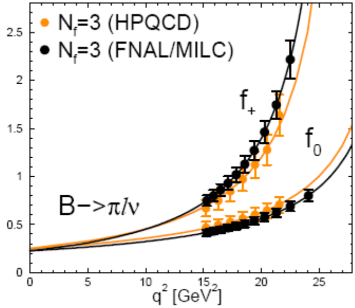
- Small lattice artefacts $\Rightarrow$ momentum of the pion must be small $\Rightarrow$ we obtain form factors at large q^2.

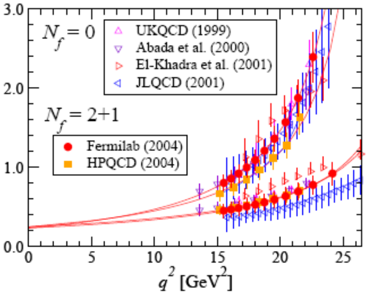
  There is a proposal to eliminate this constraint by using a formulation in which the B-meson is moving. A.Dougall et al., hep-lat/0509108

- Experimental results in q^2 bins together with theoretical constraints, helps one use the lattice data to obtain V_{ub} precisely.

  I.Stewart LP2005, T.Becher & R.Hill hep-lat/0509090

Recent Results for $B \to \pi$ Form Factors

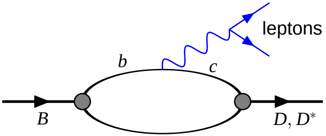




Courtesy of T.Onogi, Chamonix Flavour Dynamics Workshop, October 2005

HPQCD	Staggered Light & NRQCD Heavy	$\lvert V_{ub}\rvert = 4.04(20)(44)(53) \times 10^{-3}$
FNAL/MILC	Staggered Light & Fermilab Heavy	$\lvert V_{ub}\rvert = 3.48(29)(38)(47) \times 10^{-3}$

$B \to D^{(*)}$ Semileptonic Decays



$\Rightarrow \quad V_{cb}$

- For $B \to D^*$ decays

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3}(m_B - m_{D^*})^2 m_{D^*}^3 \sqrt{\omega^2-1}\,(\omega+1)^2 \times \left[1 + \frac{4\omega}{\omega+1}\frac{m_B^2 - 2\omega m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right] |V_{cb}|^2\, \mathscr{F}^2(\omega)\,,$$

where $\mathscr{F}(\omega)$ is the IW-function combined with perturbative and power corrections. $(\omega = v_B \cdot v_{D^*})$

- $\mathscr{F}(1) = 1$ up to power corrections and calculable perturbative corrections.

- To determine the difference of $\mathscr{F}(1)$ from 1, the method of double ratios is used. Hashimoto, Kronfeld, Mackenzie, Ryan & Simone (1998)

  For example

$$\mathscr{R}_+ = \frac{\langle D|\bar{c}\gamma^4 b|\bar{B}\rangle\,\langle \bar{B}|\bar{b}\gamma^4 c|D\rangle}{\langle D|\bar{c}\gamma^4 c|D\rangle\,\langle \bar{B}|\bar{b}\gamma^4 b|\bar{B}\rangle} = |h_+(1)|^2$$

  with

$$h_+(1) = \eta_V\left\{1-\ell_P\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)^2\right\}.$$

  By calculating $\mathscr{R}_+$ and similar ratios of $V\leftrightarrow P$ and $V\leftrightarrow V$ matrix elements all three ℓ's can be determined.

- A recent result from the FNAL/MILC/HPQCD Collaborations gives

$$|V_{cb}| = 3.9(1)(3)\times 10^{-2}.$$

  M.Okamoto, `hep-lat/0412044`

- Form Factors for $D\to\pi, K$ semileptonic decays are also being evaluated.

$K \to \pi\pi$ Decays

- A quantitative understanding of the non-perturbative QCD effects in $K \to \pi\pi$ decays is an important future milestone for lattice QCD, e.g.:
  - the empirical $\Delta I = 1/2$ rule, which states that amplitudes for decays with an $I = 0$ final state are enhanced by a factor of about 22 w.r.t. amplitudes for decays with an $I = 2$ final state.
  - the quantity ε'/ε, whose measurement with a non-zero value, $(17.2 \pm 1.8) \times 10^{-4}$, was the first observation of direct CP-violation.

In 2001, two collaborations published some very interesting results on these quantities:

Collaboration(s)	Re A_0/Re A_2	ε'/ε
RBC*	25.3 ± 1.8	$-(4.0 \pm 2.3) \times 10^{-4}$
CP-PACS	9÷12	(-7 ÷ -2)$\times 10^{-4}$
Experiments	22.2	$(17.2 \pm 1.8) \times 10^{-4}$

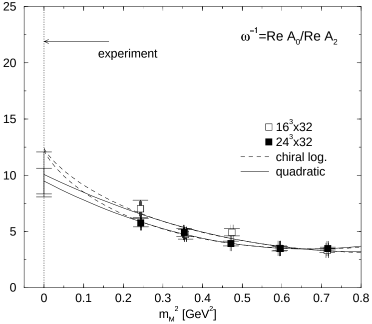
*updated results from July 2002 version of the paper.

- The lattice calculation is of the $K \to \pi\pi$ matrix elements of the $\Delta S = 1$ operators which appear in the Effective Hamiltonian.
- The two collaborations evaluated matrix elements of the form

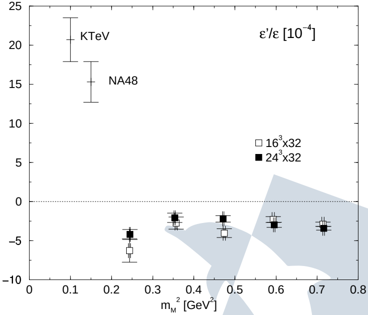
$$\langle M|\mathscr{O}|M\rangle$$

(where M is a pseudoscalar meson) and used LO χPT to determine the corresponding $K \to \pi\pi$ matrix elements.

CP-PACS



ReA_0**/Re**A_2



ε'/ε

- Is χPT applicable/reliable in the accessible range (400-800 MeV)?
- Results from RBC and CP-PACS are very interesting and will provide valuable benchmarks for future calculations.
- One suggestion for the next stage is to improve the precision to NLO in the chiral expansion.

  Lin, Pallante, Martinelli, CTS & Villadoro

  Laiho & Soni
- In general we need to evaluate $K \to \pi\pi$ decay amplitudes directly.
- With two hadrons in the final state the finite-volume corrections decrease only as $1/L^n$ and not exponentially.

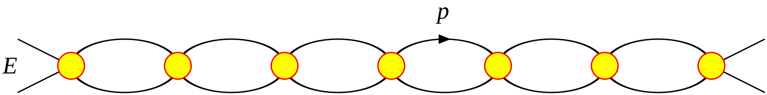
  The theory of finite-volume effects for two-hadron states in the elastic region is now fully understood $\Rightarrow \pi\pi$ phase-shifts.

  Lüscher (1986-91); Lellouch and Lüscher(2000); Lin, Martinelli, CTS and Testa (2001) Rummukainen & Gottlieb (1995); Kim, CTS and Sharpe (2005);

  Christ, Kim and Yamazaki (2005)

Finite-Volume Corrections for Two-Pion States

For two-particle states the finite-volume corrections decrease as powers of the volume and not exponentially. They are numerically significant and hence need to be controlled.



where $E^2 = 4(k^2 + m^2)$.

Performing the p_0 integration by contours we obtain summations over loop-momenta of the form:

$$\frac{1}{L^3} \sum_{\vec{p}} \frac{f(p^2)}{p^2 - k^2}$$

where $f(p^2)$ is non-singular.

For simplicity I am assuming here that only the s-wave $\pi\pi$ phase-shift is significant and that we are in the centre-of-mass frame.

$$\frac{1}{L^3}\sum_{\vec{p}}\frac{f(p^2)}{p^2-k^2}$$

The required relation between the FV sums and infinite-volume integrals is the **Poisson Summation Formula**, which in 1-dimension is:

$$\frac{1}{L}\sum_{p} g(p) = \sum_{l=-\infty}^{\infty}\int\frac{dp}{2\pi}e^{ilLp}g(p)$$

If $g(p)$ is non-singular then only the term with $l=0$ on the rhs contributes, up to exponentially small in L.

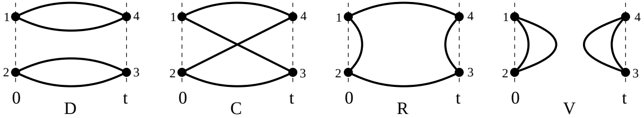
From the above it follows that this is not the case for two-hadron final states $\Rightarrow$ finite-volume corrections $\sim 1/L^n$.

In 2005 we understood these effects also in frames with $\vec{P}\neq\vec{0}$.

Similar issues and results also hold for other two-hadron states (e.g. $\pi - N$ and $N-N$).

For $I = 2$ final states, there is now no barrier to calculating the matrix elements precise, and these are underway.

For $I = 0$ $\pi\pi$ states we need to learn how to calculate the disconnected diagrams with sufficient precision.



Summary and Conclusions

- Lattice Simulations of QCD, in partnership with Experiments and Theory, play a central rôle in
  - the determination of the fundamental parameters of the Standard Model (e.g. CKM matrix elements, quark masses);
  - in searches of signatures of *New Physics*
  - and potentially in understanding the structure of the new physics.
- With the advent of unquenched simulations, a major source of uncontrolled systematic uncertainty has been eliminated, and the main aim now is to control the chiral extrapolation and reduce other sources of error.
- Much work continues to be done to extend the range of applicability of lattice simulations to more processes and physical quantities.
- I have given a selection of recent results and developments.

  A more complete set can be found on the web-site of the Lattice 2005 symposium, `www.maths.tcd.ie/lat05` .