Cross-fertilization of QCD and statistical physics

High energy scattering, reaction-diffusion, selective evolution, spin glasses and their connections

PART II

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Zakopane, June 5, 2006

Previous lecture...

We have shown the relevance of the sF-KPP equation for QCD

$$\partial_t u = \partial_x^2 u + u - u^2 + \sqrt{\frac{u}{N}(1-u)}\,\nu$$

diffusive growth

$\chi(-\partial_x)u$

$\chi(\gamma)=\gamma^2+1$ in the F-KPP case

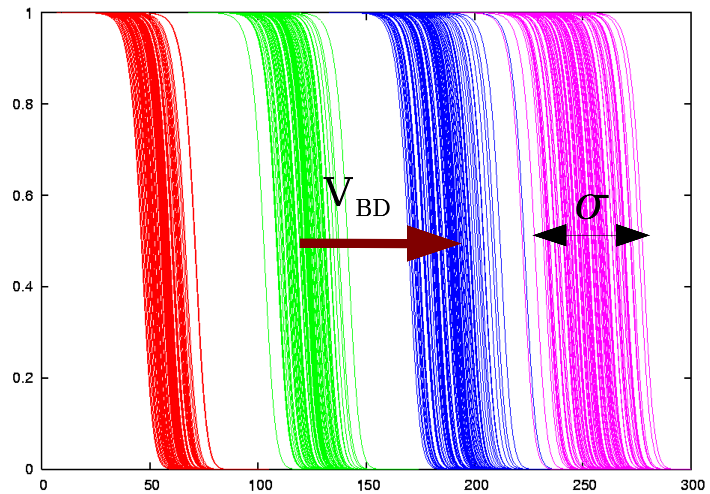
nonlinear damping

statistical noise

This equation describes reaction-diffusion processes of a discrete system of N particles.

The solutions are ***traveling waves:*** we have quoted some universal features

average velocity	$V_{BD}=\frac{\chi(\gamma_0)}{\gamma_0}-\frac{\pi^2\gamma_0\chi''(\gamma_0)}{2\ln^2 N}$
shape	$u \sim e^{-\gamma_0(x-V_{BD}t)}$
dispersion in the position	$\sigma \propto \sqrt{\frac{t}{\ln^3 N}}$



What is known on the sFKPP equation

$$\partial_t u = \partial_x^2 u + u - u^2 + \sqrt{\frac{2}{N} u(1-u)}\,\nu$$

- **1937:** Fisher; Kolmogorov, Petrovsky, Piscounov (deterministic part)

...

- **1983:** Mathematical proof that its deterministic version admits traveling wave solutions by Bramson; computation of the front velocity

...

- **1997:** Computation of the first correction to the front velocity due to fluctuations by Brunet and Derrida (weak noise)
- **1999:** Brunet and Derrida noticed *numerically* that the variance of the front position scales like $t/\ln^3 N$

...

- **2005:** Phenomenological understanding of the effect of the fluctuations on the front position; computation of all its cumulants.
- **2006:** Genealogies, relation to spin glasses

Brunet, Derrida, Mueller, SM (cond-mat)

Outline

Lecture 1

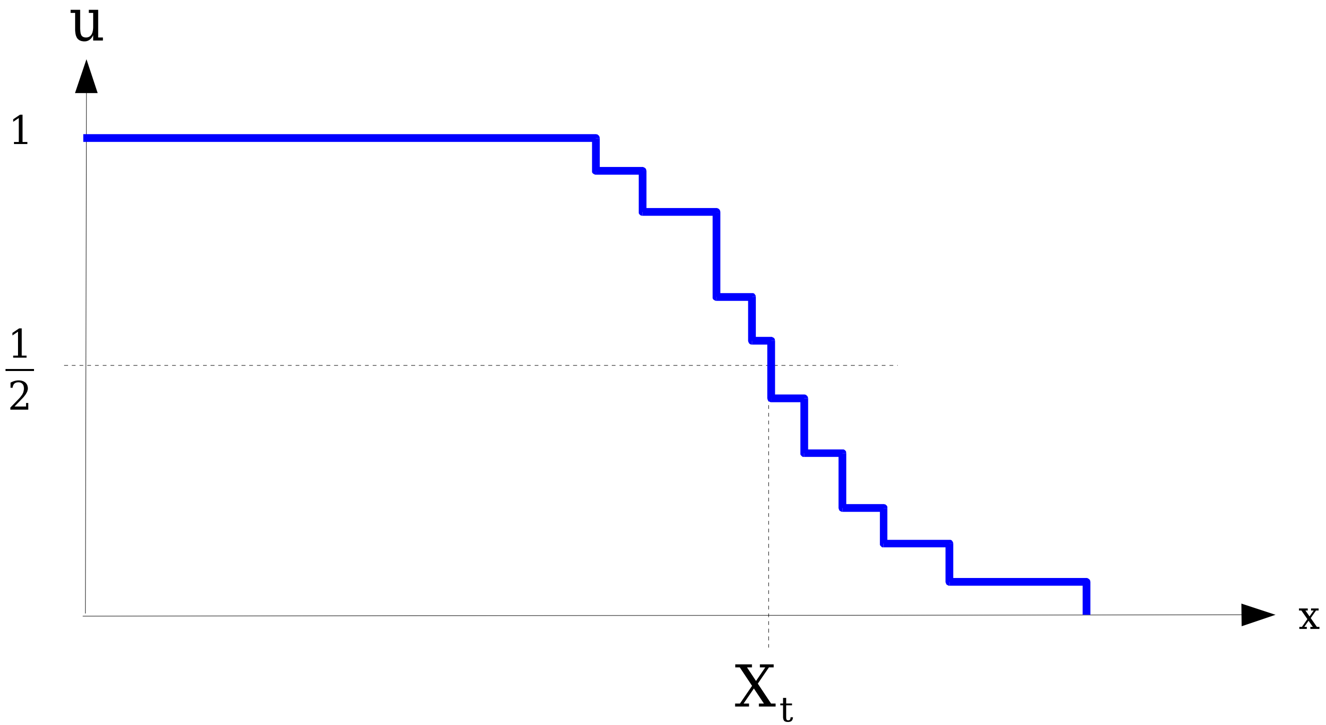
- ✭ Universality: lessons from condensed matter
- ✭ Stochastic processes: simple examples
- ✭ Reaction-diffusion and traveling wave equations
- ✭ High energy scattering as a reaction-diffusion process

Lecture 2

- ✭ Results on noisy traveling waves
- ✭ Genealogies in selective evolution models
- ✭ A connection to the Parisi theory of spin glasses?

The infinite particle number limit

$$\partial_t u = \partial_x^2 u + u - u^2 + \sqrt{\frac{2}{N} u(1-u)}\,\nu$$



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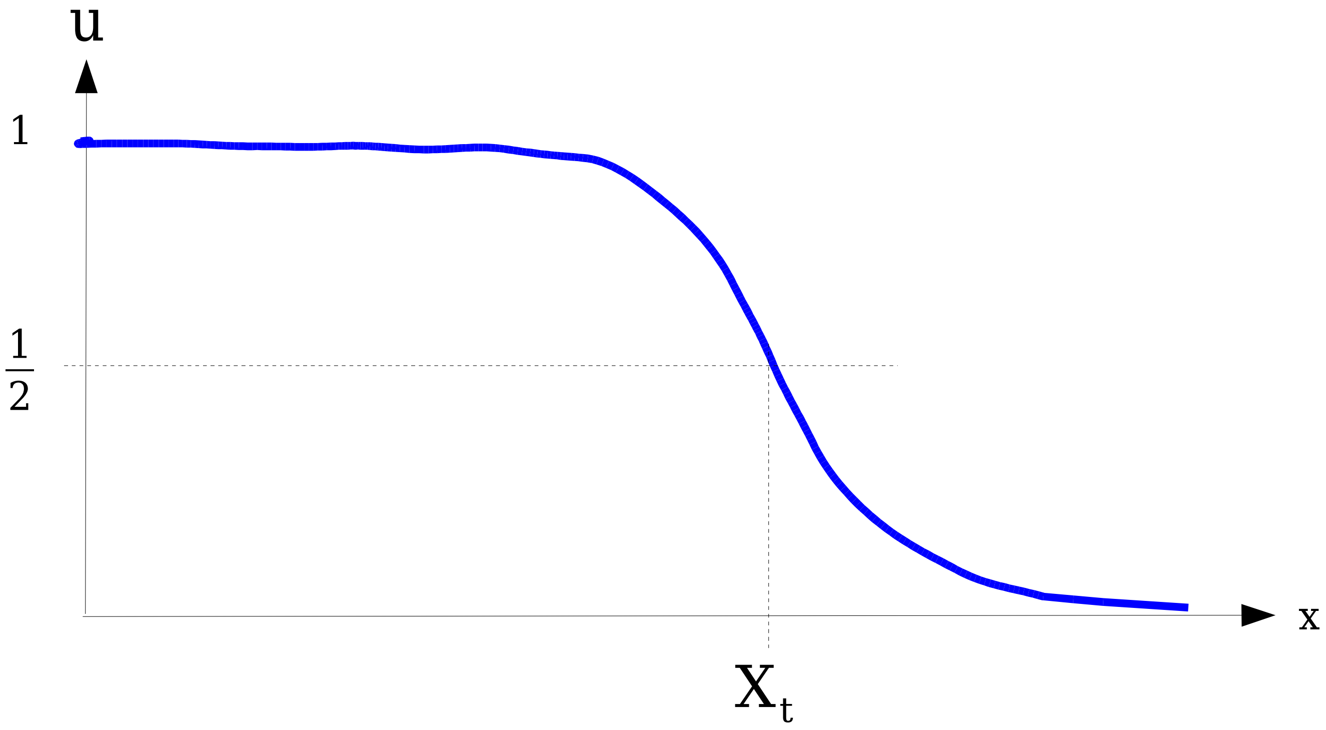

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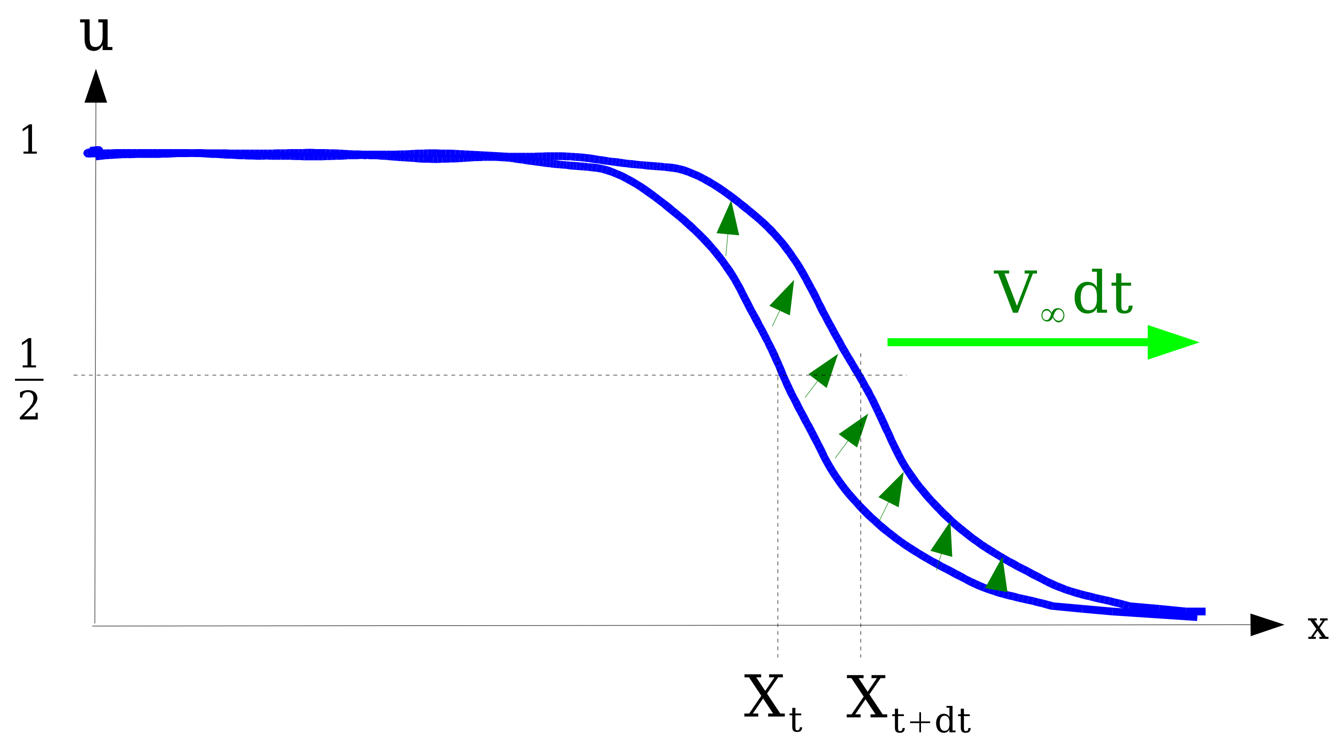
The large time asymptotics are exact traveling waves.

Mathematical result by Bramson

The evolution of *u* is driven by the (linear) growing diffusion part.
The nonlinearity only tames the growth when $u \sim 1$



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$$\partial_t u = \partial_x^2 u + u - u^2 + \sqrt{\frac{2}{N} u(1-u)}\, \nu$$

$\chi(-\partial_x) u$

$\chi(\gamma) = \gamma^2 + 1$ characteristic function of the diffusion kernel

Look for solutions of the form $u_\gamma = \exp(-\gamma(x - v(\gamma) t))$

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Solution: $v(\gamma) = \frac{\chi(\gamma)}{\gamma}$

$v(\gamma) = \gamma + \frac{1}{\gamma}$ in the F-KPP case

General solution: arbitrary superposition of different wave numbers

$$u = \int d\gamma\, f(\gamma)\, u_\gamma = \int d\gamma\, f(\gamma) \exp(-\gamma(x - v(\gamma) t))$$

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General solution: arbitrary superposition of different wave numbers

$$u=\int d\gamma\, f(\gamma) u_\gamma = \int d\gamma\, f(\gamma) \exp\left(-\gamma\left(x - v(\gamma)t\right)\right)$$

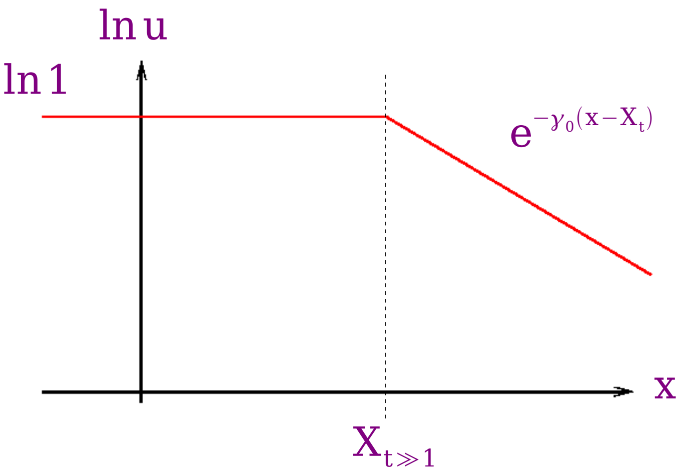
Large times (saddle point ***at constant*** u), select the wave that travels with minimum velocity:

$$v'(\gamma_0)=0 \;\Rightarrow\; \chi'(\gamma_0)=\frac{\chi(\gamma_0)}{\gamma_0}$$

$$\begin{cases} V_\infty = \frac{dX_t}{dt} = v(\gamma_0) = \frac{\chi(\gamma_0)}{\gamma_0} \\ u(x,t) \sim e^{-\gamma_0 (x - X_t)} \end{cases}$$

$\gamma_0=1, V_\infty=2$ in the F-KPP case

Transition to the asymptotics



traveling wave, asymptotic speed:

$$V_\infty = \frac{\chi(\gamma_0)}{\gamma_0}$$

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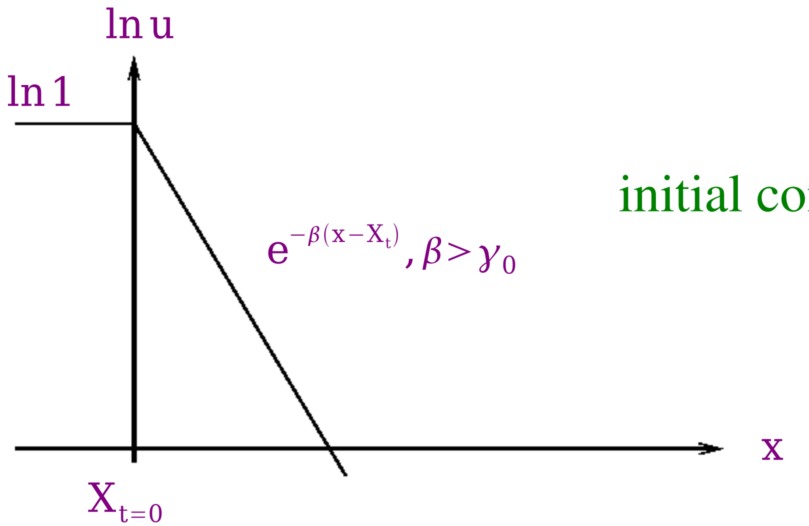
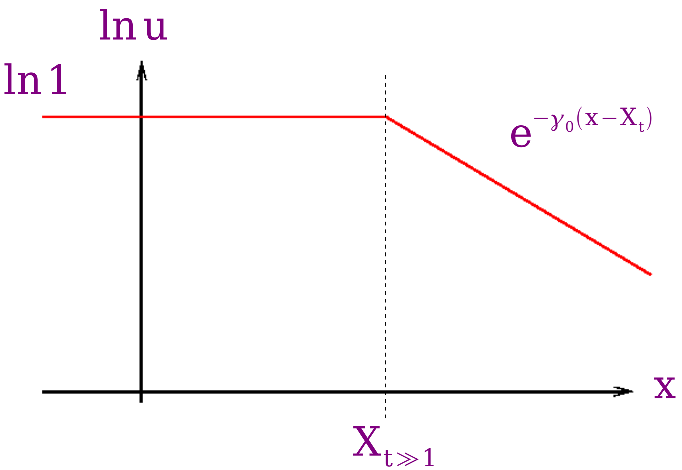


initial condition



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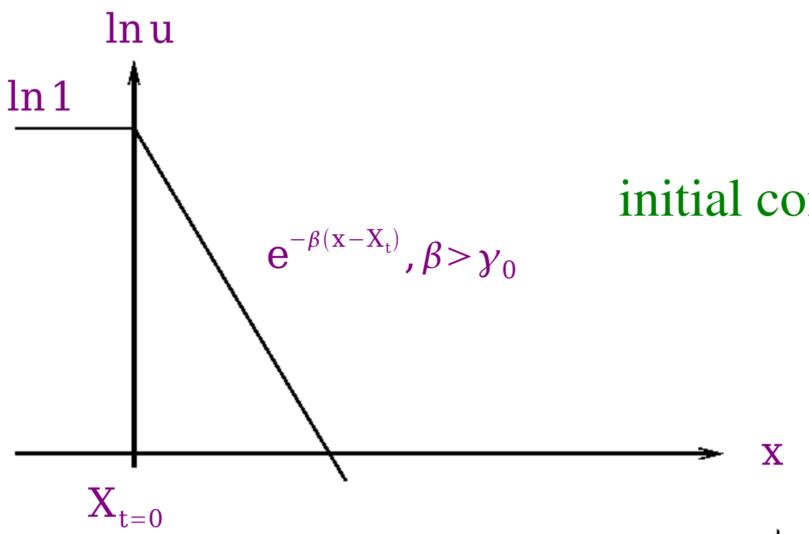

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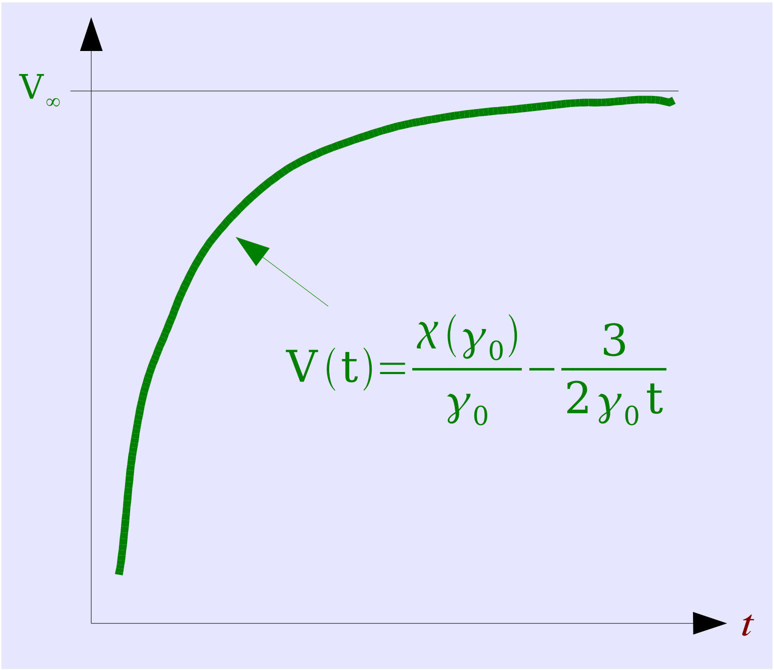
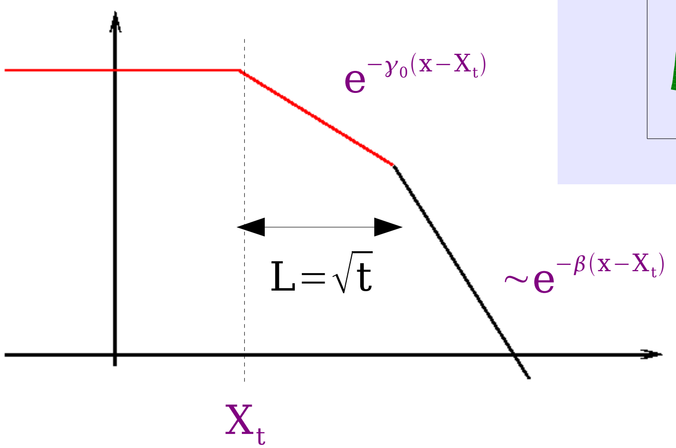


transients:



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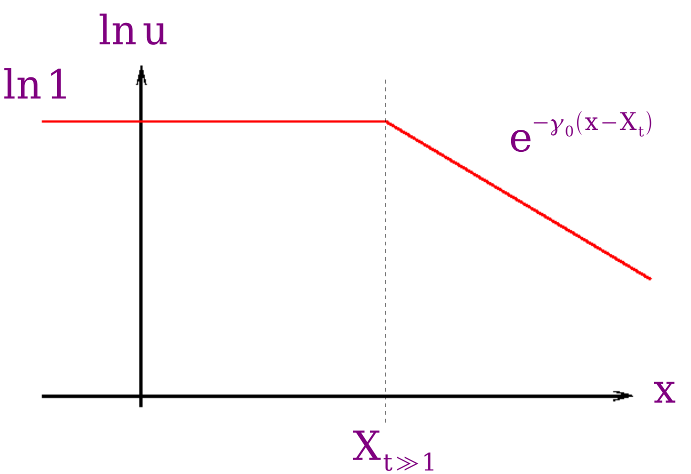
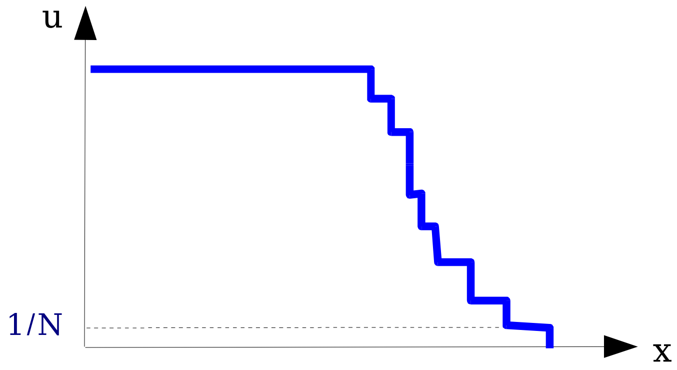


Accounting for discreteness



Observation: u is either 0 or larger than $1/N$

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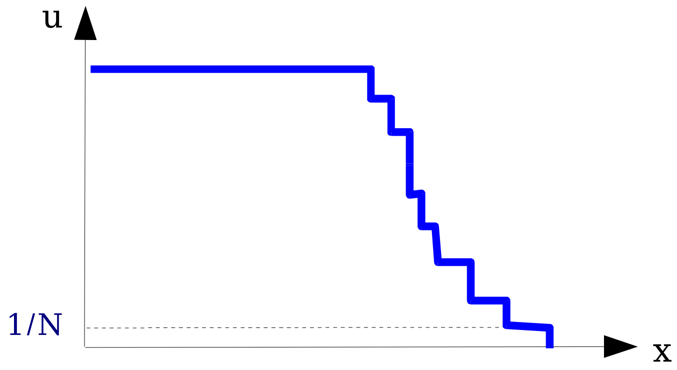
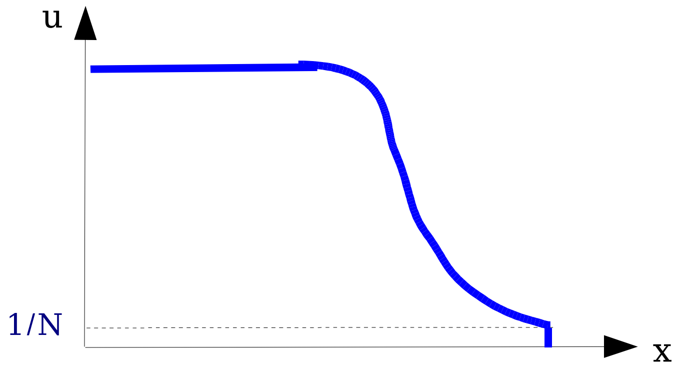


Observation: u is either 0 or larger than $1/N$

Recipe: Whenever there is more than 1 particle on a site apply the mean field evolution

Brunet, Derrida (1997)

Accounting for discreteness



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Infinite N equation + cut-off

(still deterministic)

$$\partial_t u = (\partial_x^2 u + u - u^2)\,\Theta(u - 1/N)$$

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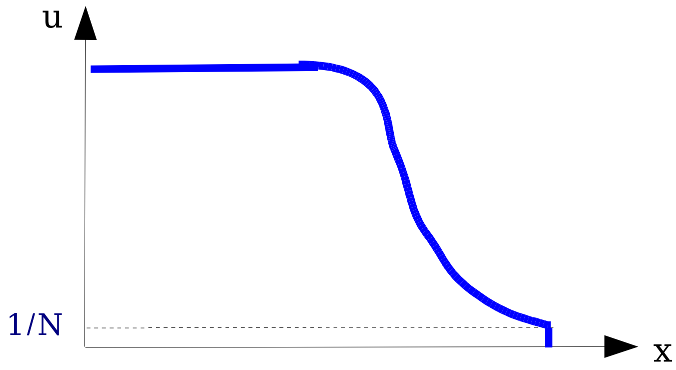

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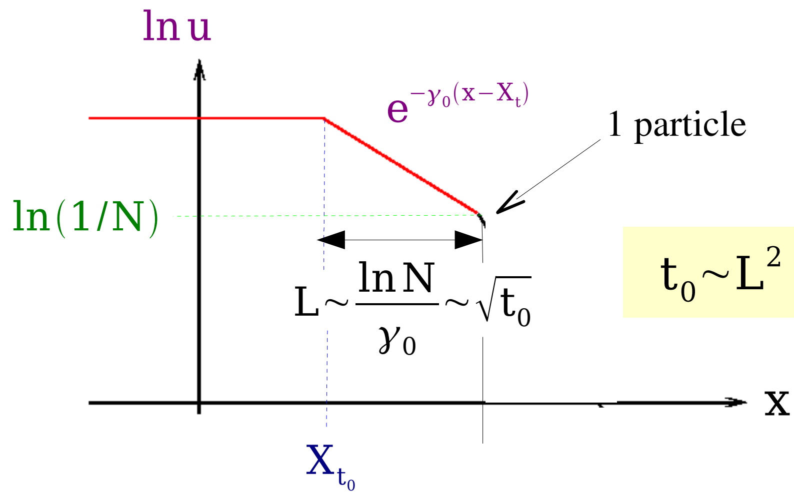

$$t_0 \sim L^2$$

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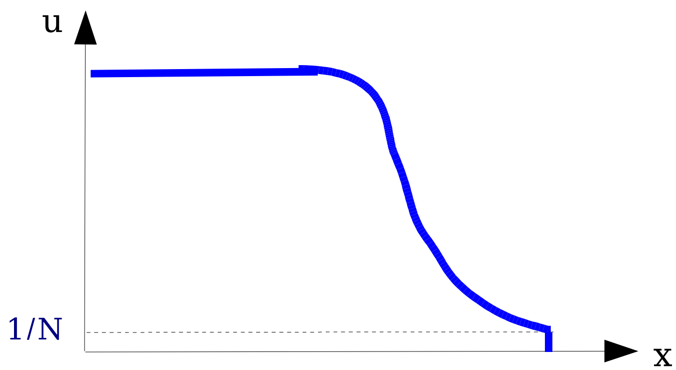

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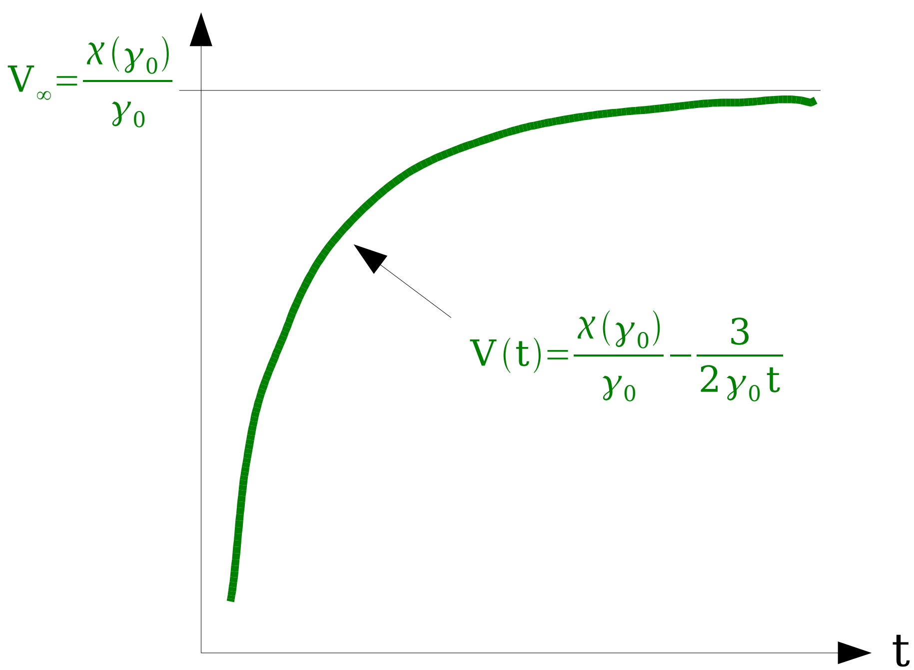

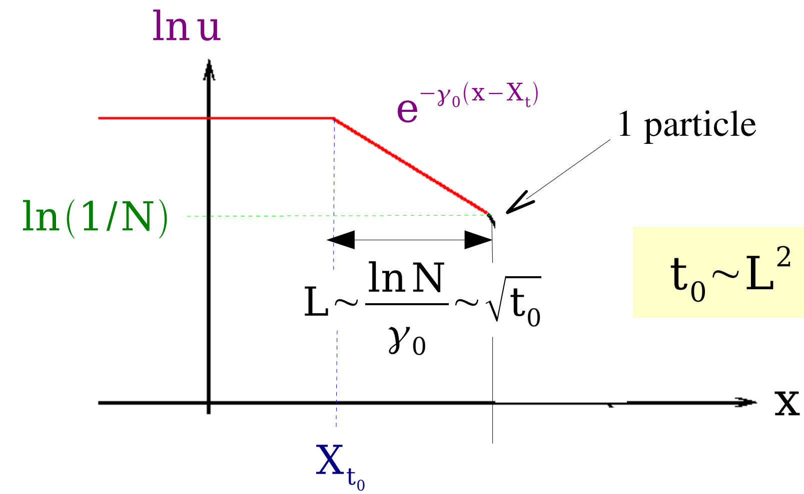

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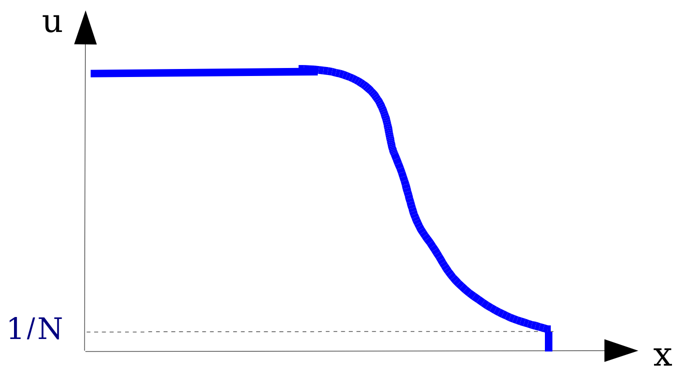

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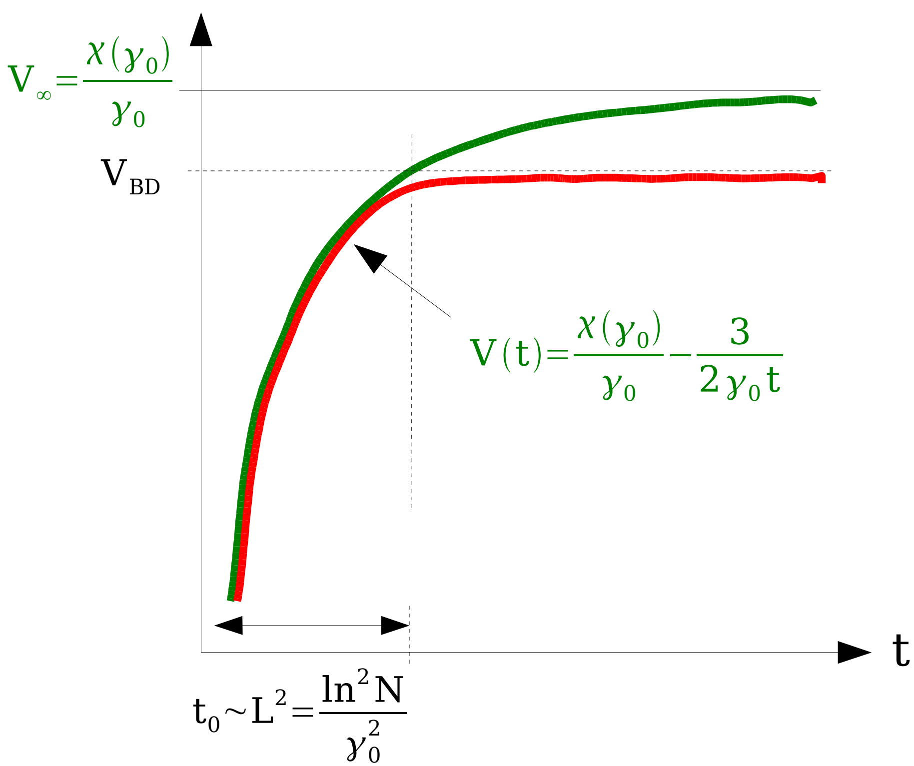

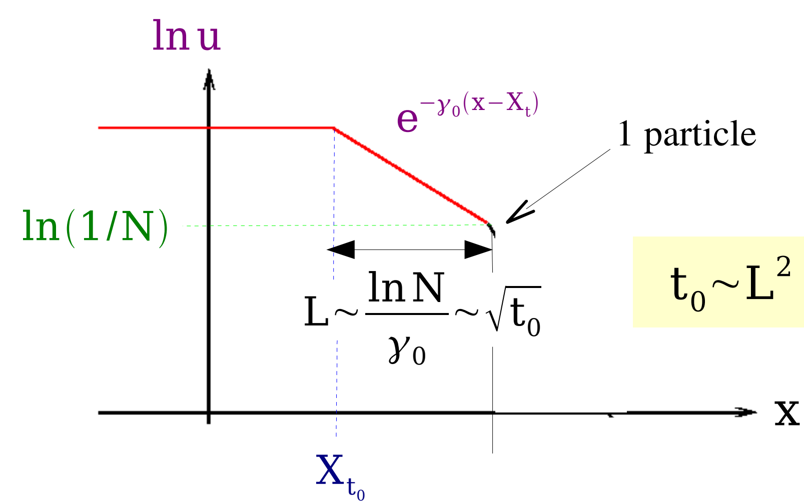

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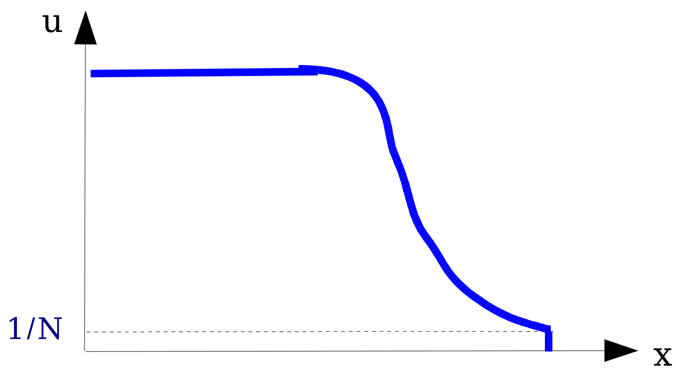

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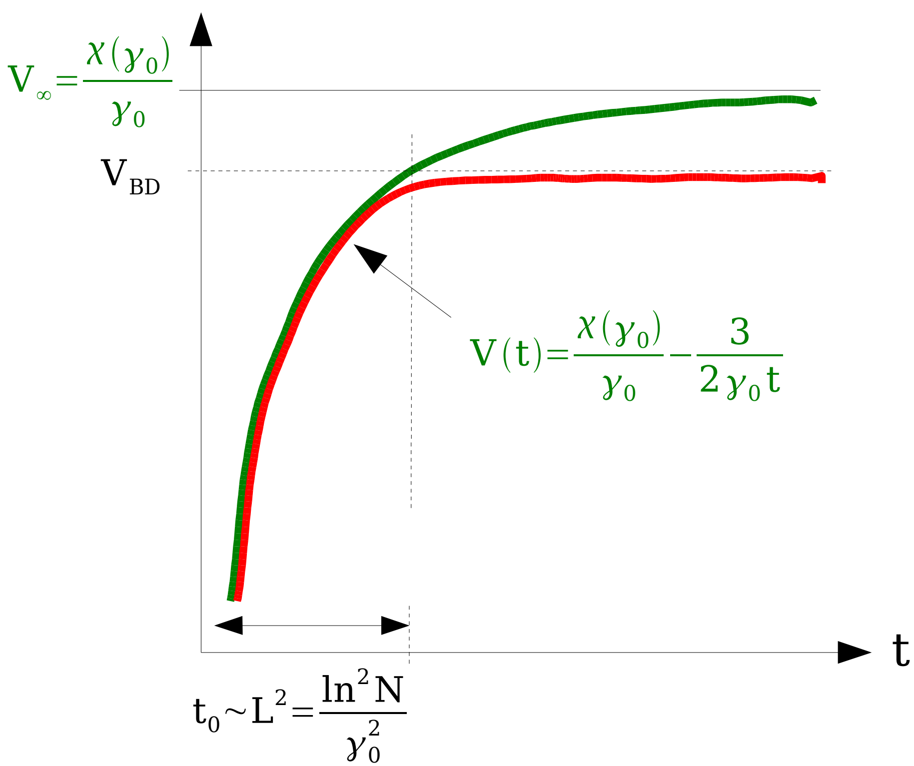
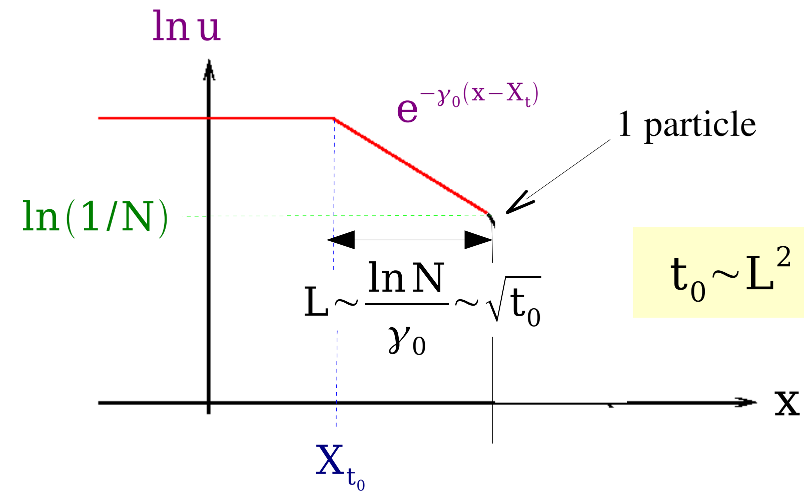




$$t_0 \sim L^2$$

$$V_{BD} = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{\pi^2 \gamma_0 \chi''(\gamma_0)}{2 \ln^2 N}$$

Velocity of a front of size $L = \frac{\ln N}{\gamma_0}$

Summary of the mean field approach

The FKPP equation $\partial_t u = \partial_x^2 u + u - u^2$

admits asymptotic traveling wave solutions, of shape $e^{-\gamma_0 (x - X_t)}$

and velocity $V_\infty = \frac{dX_t}{dt} = \frac{\chi(\gamma_0)}{\gamma_0}$ where $\chi(\gamma) = \gamma^2 + 1$ and γ_0 minimizes $v(\gamma) = \frac{\chi(\gamma)}{\gamma}$

in the F-KPP case

The traveling wave builds up diffusively from a given initial condition

and its velocity during that phase reads $V(t) = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{3}{2\gamma_0 t}$

The FKPP equation may be modified to take into account the fact that in real particle models, occupation numbers are discrete, 0,1,2... : $\partial_t u = (\partial_x^2 u + u - u^2)\Theta(u - 1/N)$

The front reaches its asymptotic shape of width $L = \frac{\ln N}{\gamma_0}$

after a time L^2 and the corresponding velocity is $V_{BD} = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{\pi^2 \gamma_0 \chi''(\gamma_0)}{2\ln^2 N}$

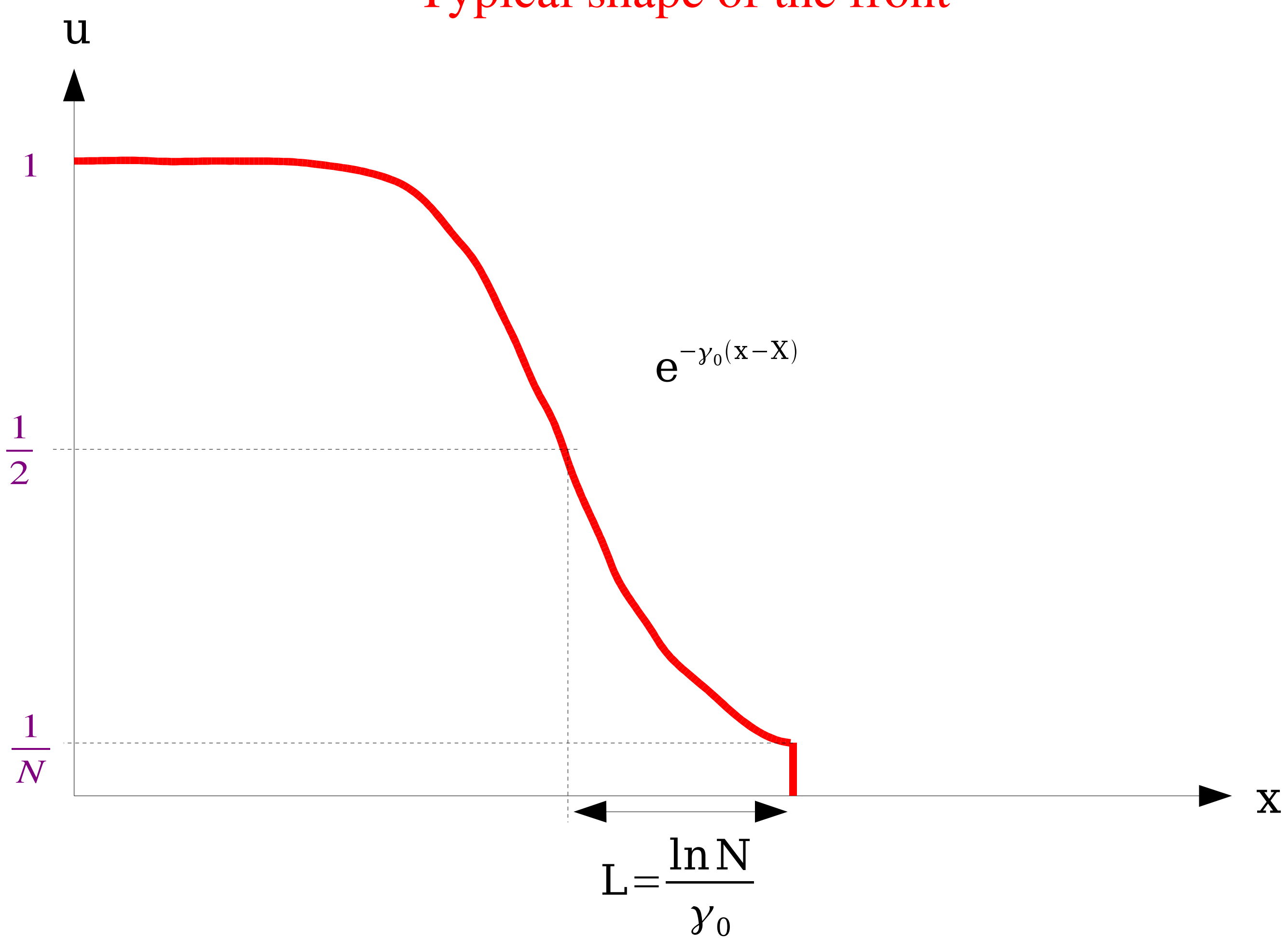
Confirmed to be the right average front velocity
in numerical simulations of fully stochastic models!

Brunet, Derrida; Moro;
Pechenik, Levine; Panja...

Accounting for fluctuations

Assumption #1: the evolution of the stochastic front is essentially deterministic

Typical shape of the front



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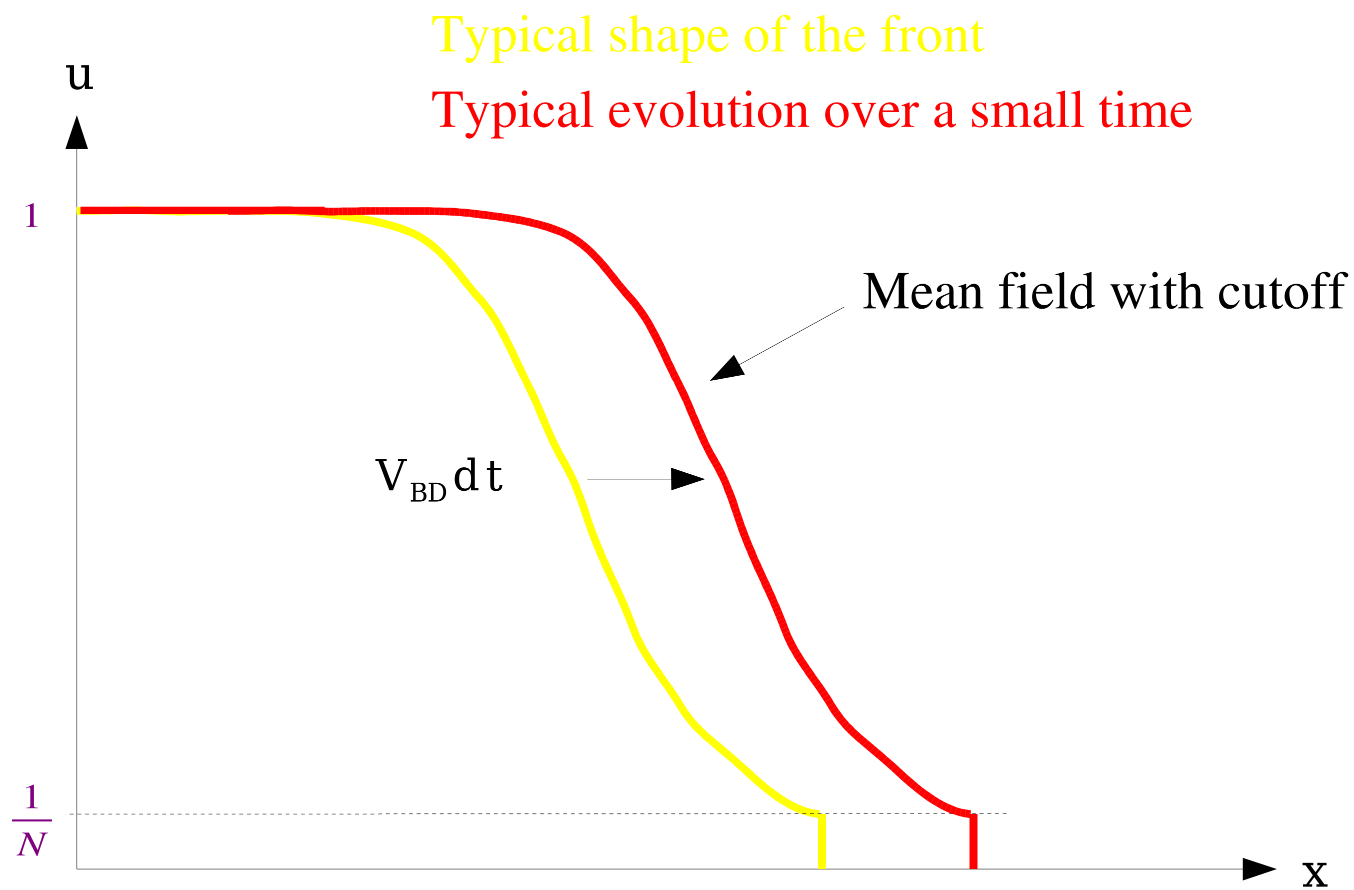
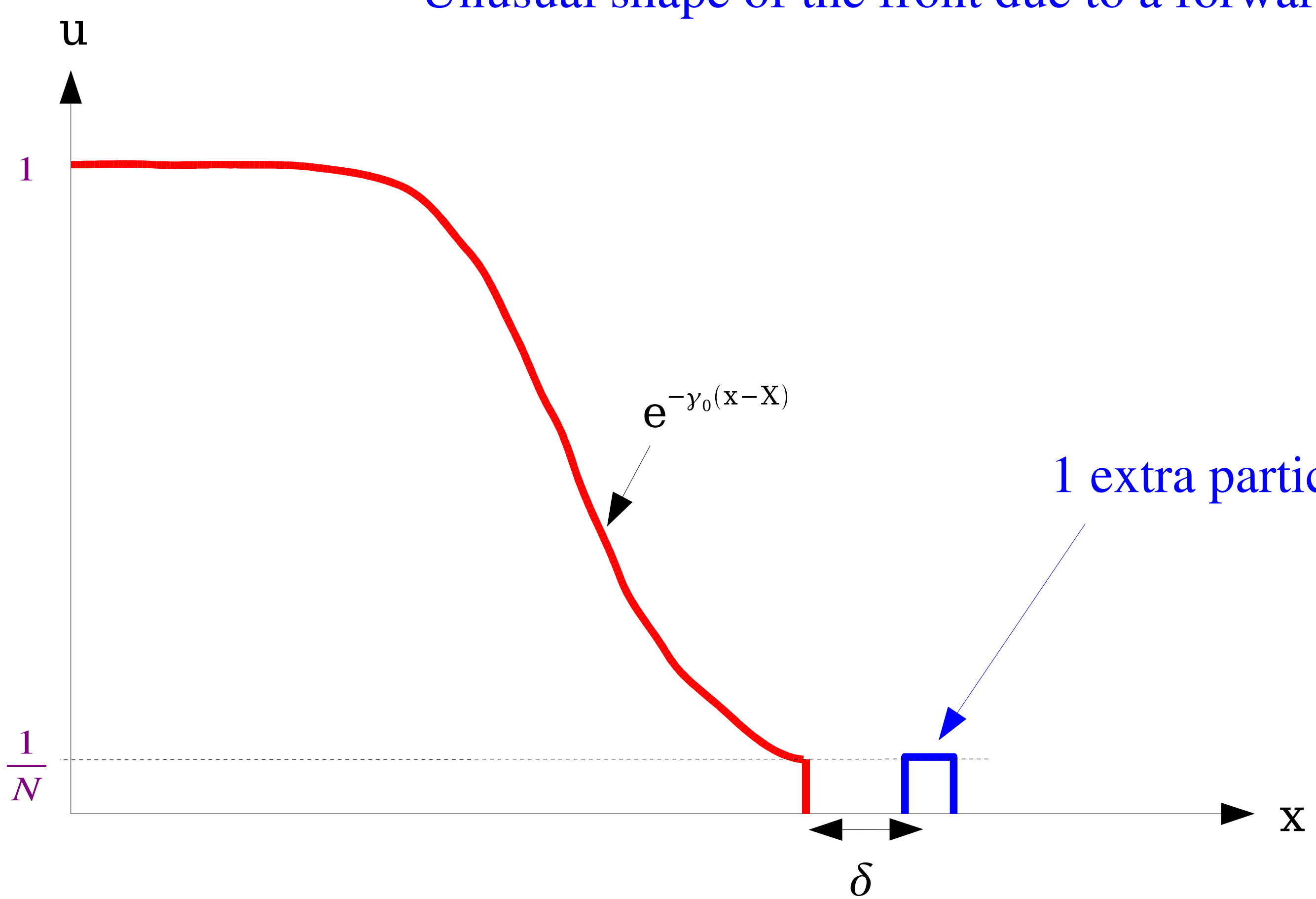


Accounting for fluctuations

Assumption #1: the evolution of the stochastic front is essentially deterministic, ***except for some occasional extra-particles in the tail***

Unusual shape of the front due to a forward extra particle



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Assumption #1: the evolution of the stochastic front is essentially deterministic,

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Assumption #2: the probability for such extra-particles is $p(\delta)\,d\delta\,dt = C_1 e^{-\gamma_0 \delta}\,d\delta\,dt$

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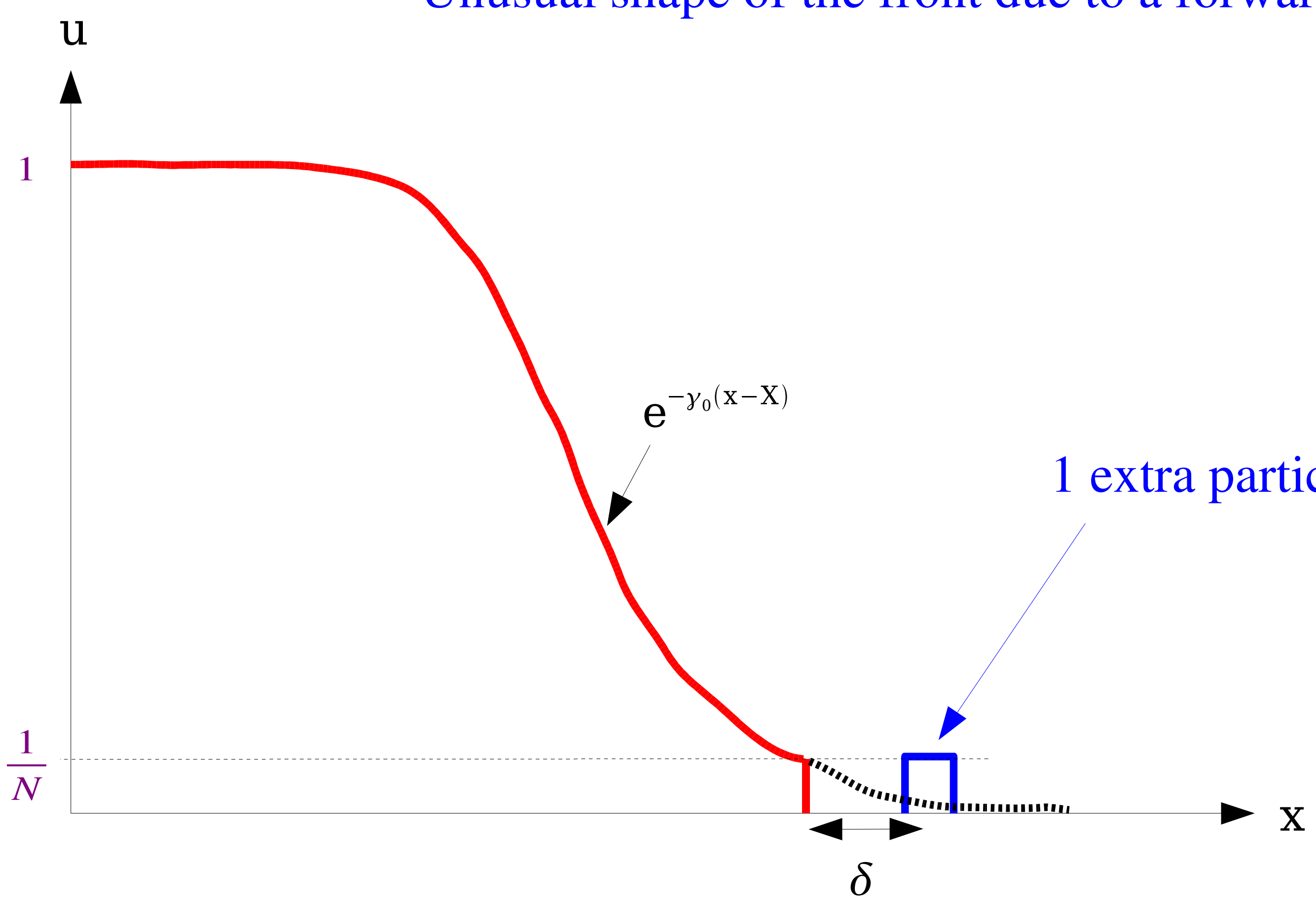

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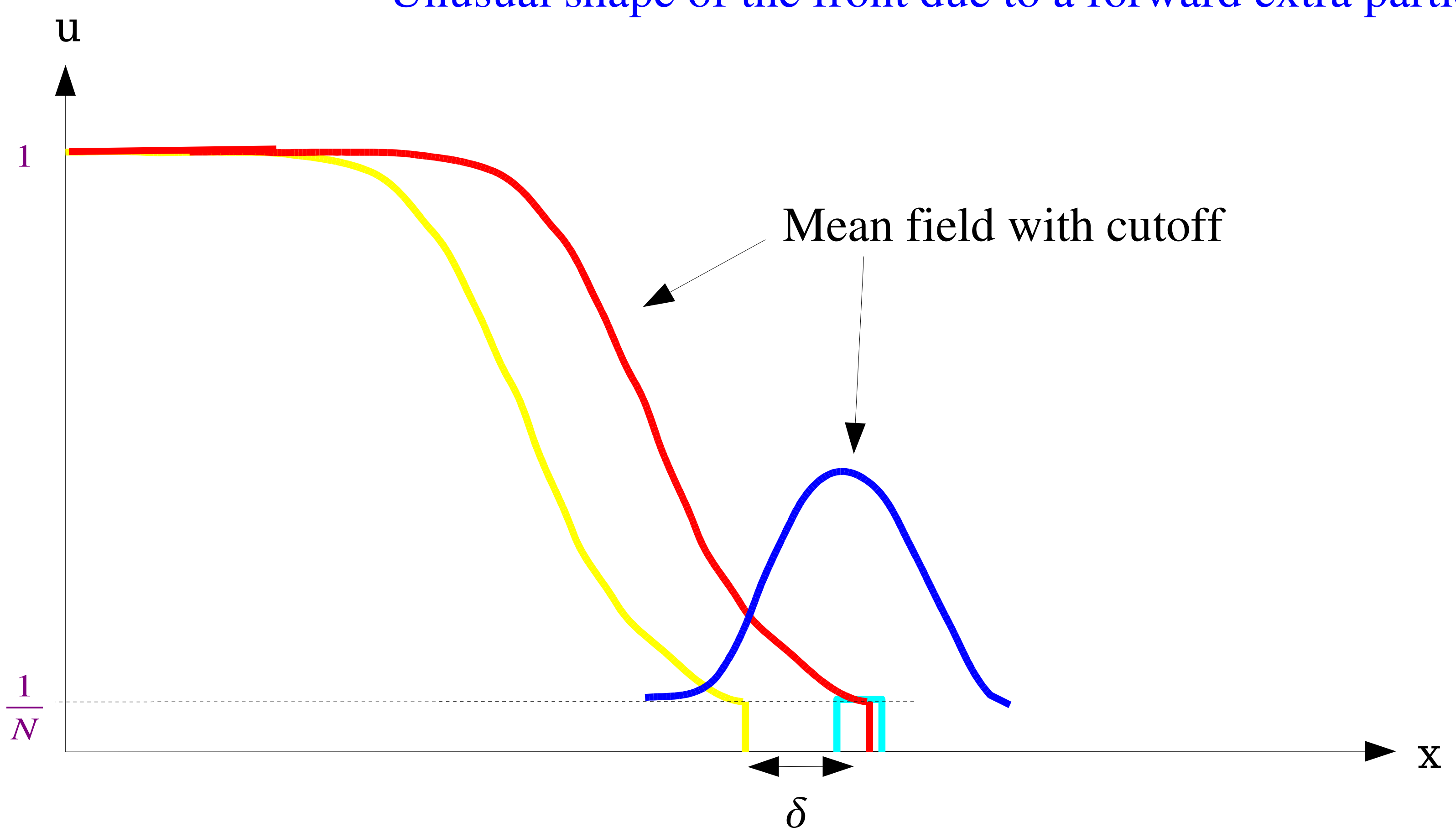

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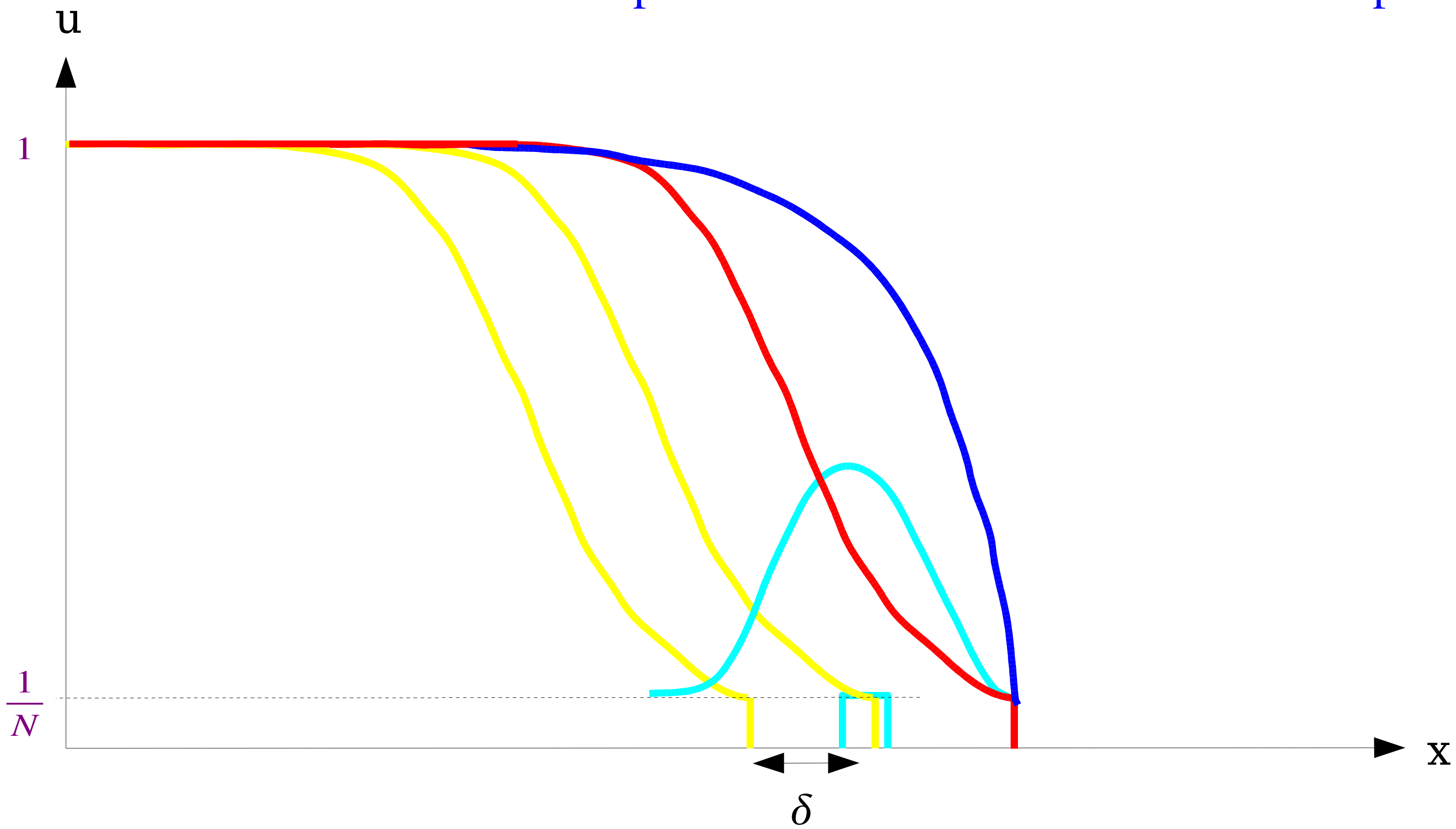

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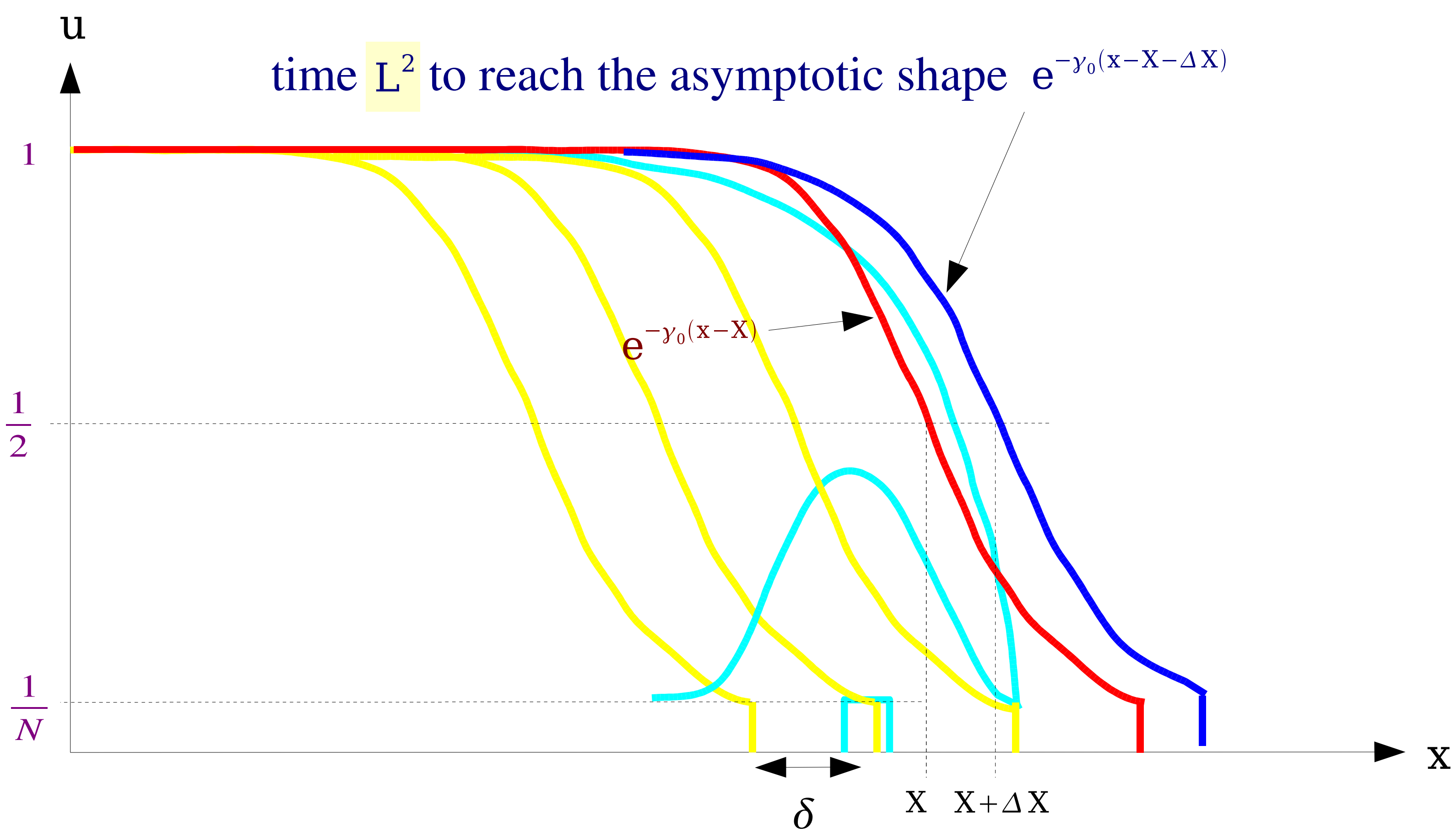

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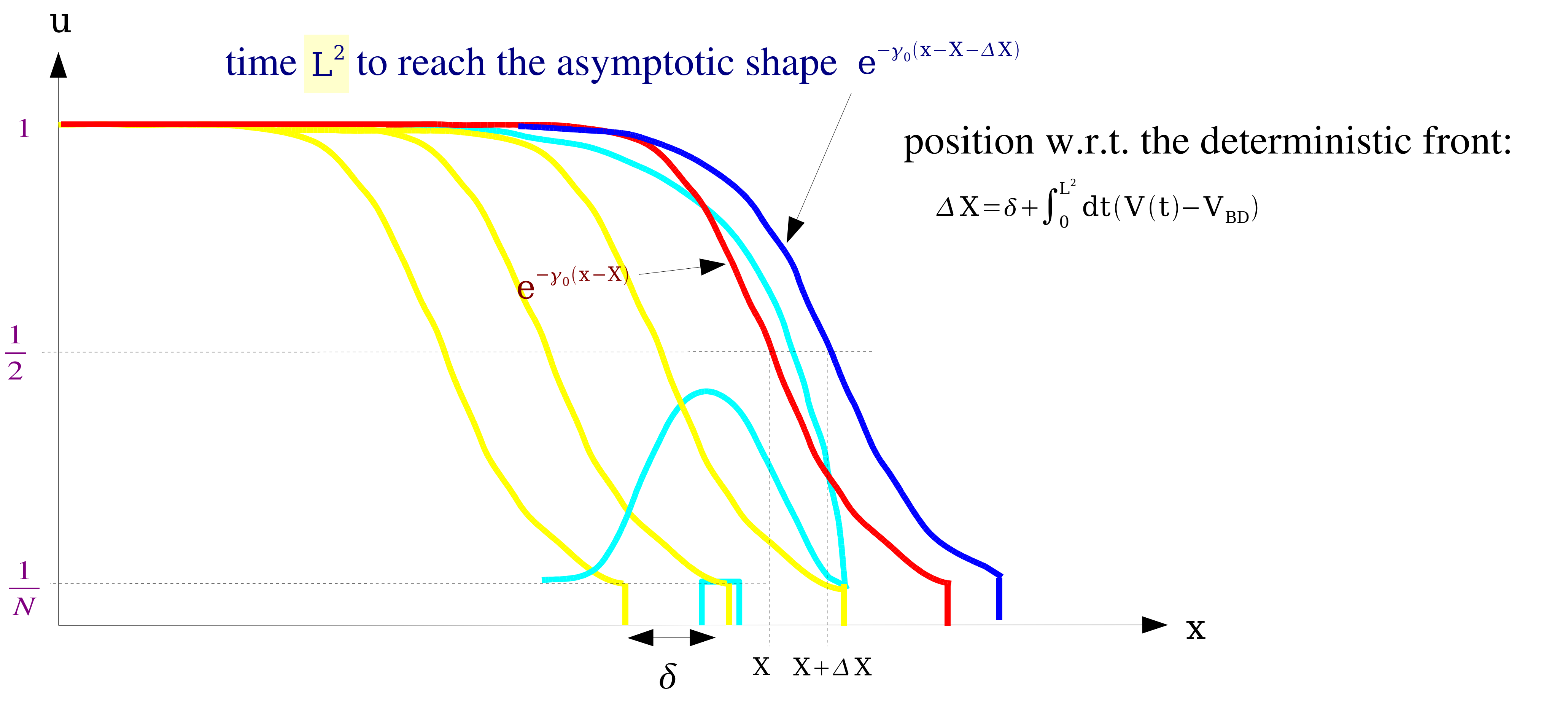

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$$V(t) = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{3}{2\gamma_0 t}$$

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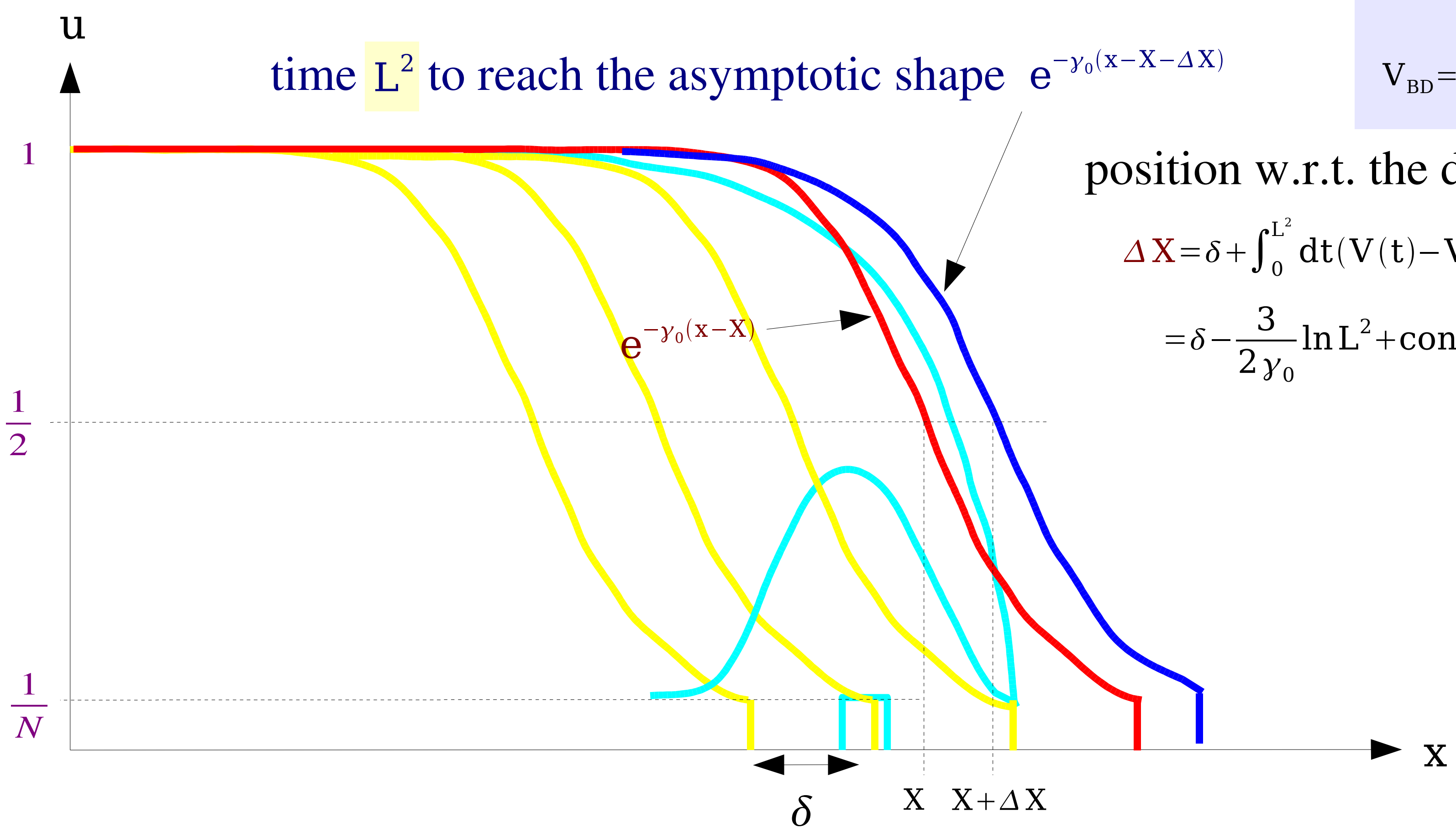

position w.r.t. the deterministic front:

$$\Delta X = \delta + \int_0^{L^2} dt\,(V(t) - V_{BD}) = \delta - \int^{L^2} dt\,\frac{3}{2\gamma_0 t} + \text{const}$$

$$= \delta - \frac{3}{2\gamma_0}\ln L^2 + \text{const} = \frac{1}{\gamma_0}(\gamma_0\delta - \ln L^3) + \text{const}$$

Accounting for fluctuations

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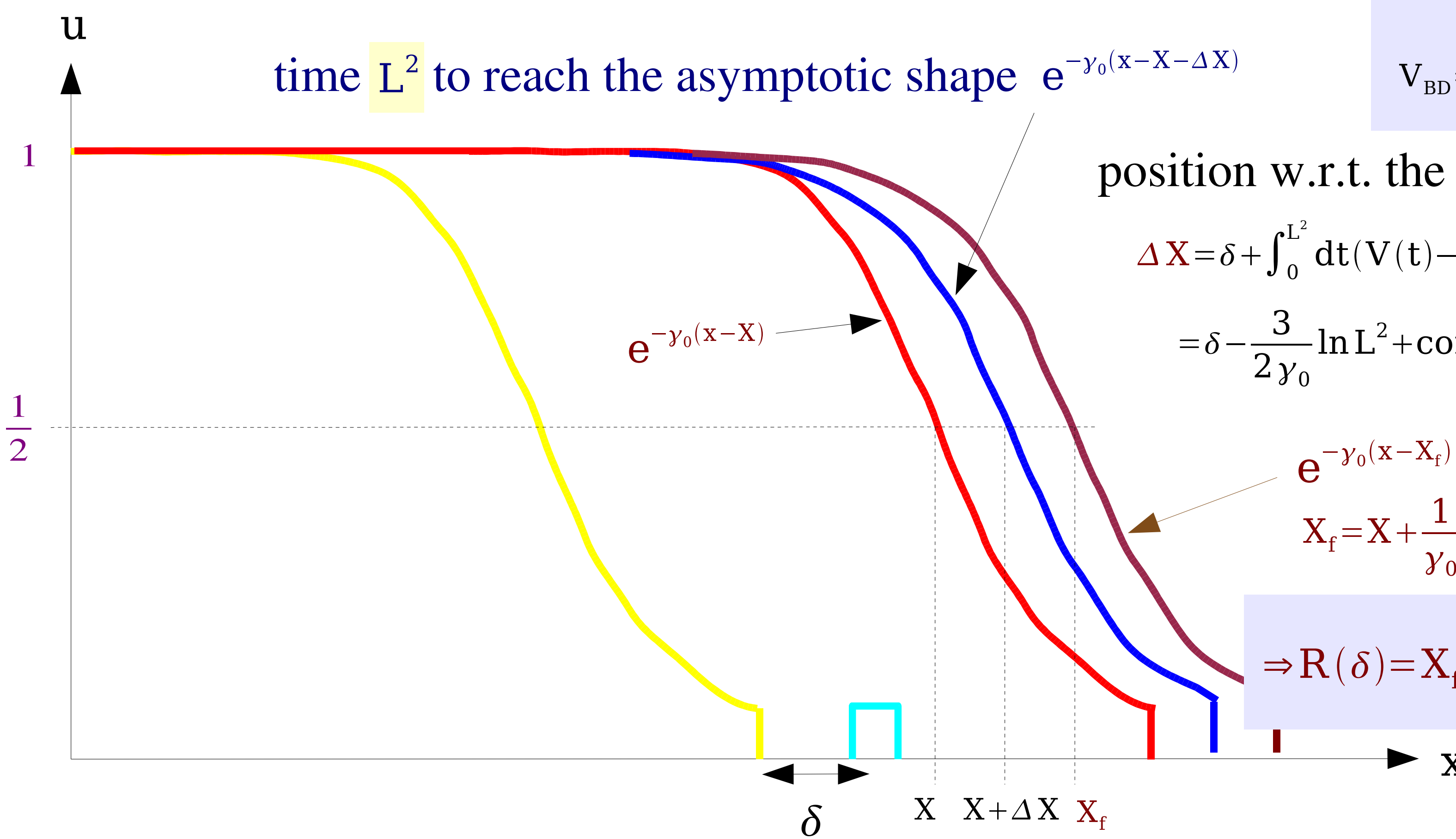

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$$= \delta - \frac{3}{2\gamma_0} \ln L^2 + \text{const} = \frac{1}{\gamma_0}(\gamma_0 \delta - \ln L^3) + \text{const}$$

$$e^{-\gamma_0(x-X_f)} = e^{-\gamma_0(x-X)} + e^{-\gamma_0(x-X-\Delta X)}$$

$$X_f = X + \frac{1}{\gamma_0} \ln\left(1 + e^{\gamma_0 \Delta X}\right)$$

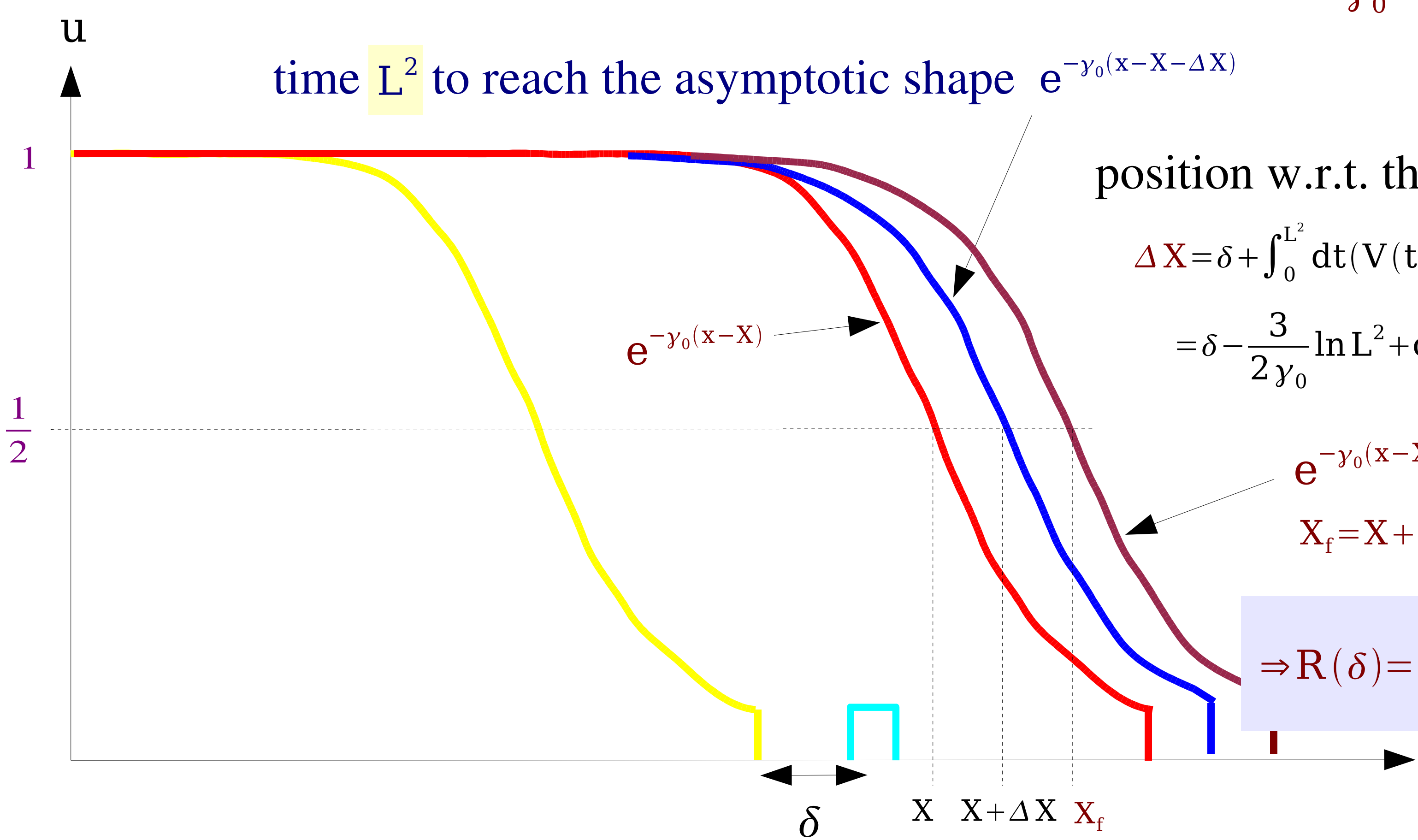
$$\Rightarrow R(\delta) = X_f - X = \frac{1}{\gamma_0} \ln\left(1 + C_2 \frac{e^{\gamma_0 \delta}}{L^3}\right)$$

Accounting for fluctuations

Assumption #1: the evolution of the stochastic front is essentially deterministic,
except for some occasional extra-particles in the tail

Assumption #2: the probability for such extra-particles is $p(\delta)\,d\delta\,dt = C_1 e^{-\gamma_0 \delta}\, d\delta\, dt$

Assumption #3: their effect on the front position is $R(\delta) = X_f - X = \frac{1}{\gamma_0} \ln\left(1 + C_2 \frac{e^{\gamma_0 \delta}}{L^3}\right)$



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$$e^{-\gamma_0(x-X_f)} = e^{-\gamma_0(x-X)} + e^{-\gamma_0(x-X-\Delta X)}$$

$$X_f = X + \frac{1}{\gamma_0} \ln\left(1 + e^{\gamma_0 \Delta X}\right)$$

$$\Rightarrow R(\delta) = X_f - X = \frac{1}{\gamma_0} \ln\left(1 + C_2 \frac{e^{\gamma_0 \delta}}{L^3}\right)$$

Accounting for fluctuations

Assumption #1: the evolution of the stochastic front is essentially deterministic, ***except for some occasional extra-particles in the tail***

Assumption #2: the probability for such extra-particles is $p(\delta)\,d\delta\,dt = C_1 e^{-\gamma_0 \delta}\,d\delta\,dt$

Assumption #3: their effect on the front position is $R(\delta) = X_f - X = \frac{1}{\gamma_0} \ln\left(1 + C_2 \frac{e^{\gamma_0 \delta}}{L^3}\right)$

Stochastic rules for the effective evolution of the position of the front:

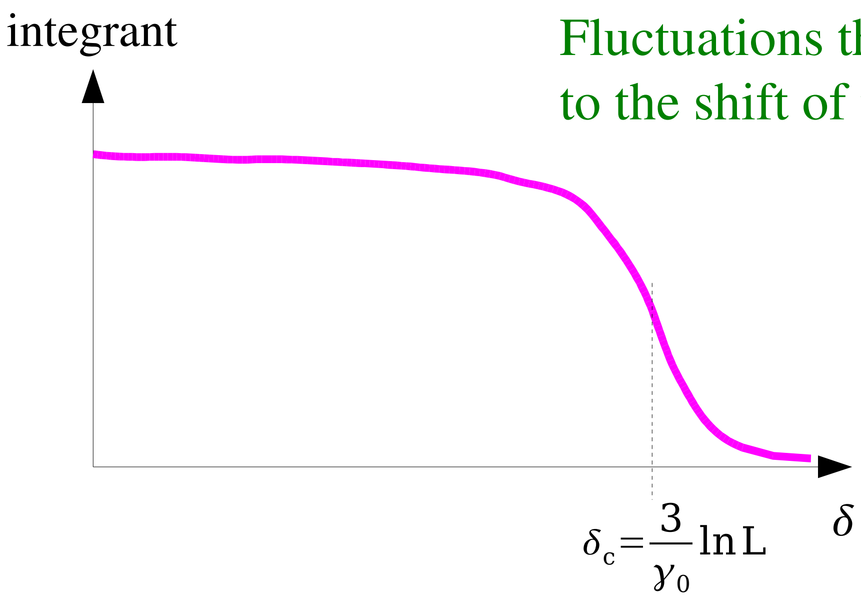
$$X_{t+dt} = \begin{cases} X_t + V_{BD}\,dt, & \text{if no fluctuation occurs} \\ X_t + V_{BD}\,dt + R(\delta), & \text{with proba } p(\delta)\,d\delta\,dt \end{cases}$$

$$V - V_{BD} = \int d\delta\, p(\delta) R(\delta) = \frac{C_1 C_2}{\gamma_0} \frac{3 \ln L}{\gamma_0 L^3}$$

$$\frac{[n\text{-th cumulant}]}{t} = \int d\delta\, p(\delta) R^n(\delta) = \frac{C_1 C_2}{\gamma_0} \frac{n!\,\zeta(n)}{\gamma_0^n L^3}$$

Fixing the constants...

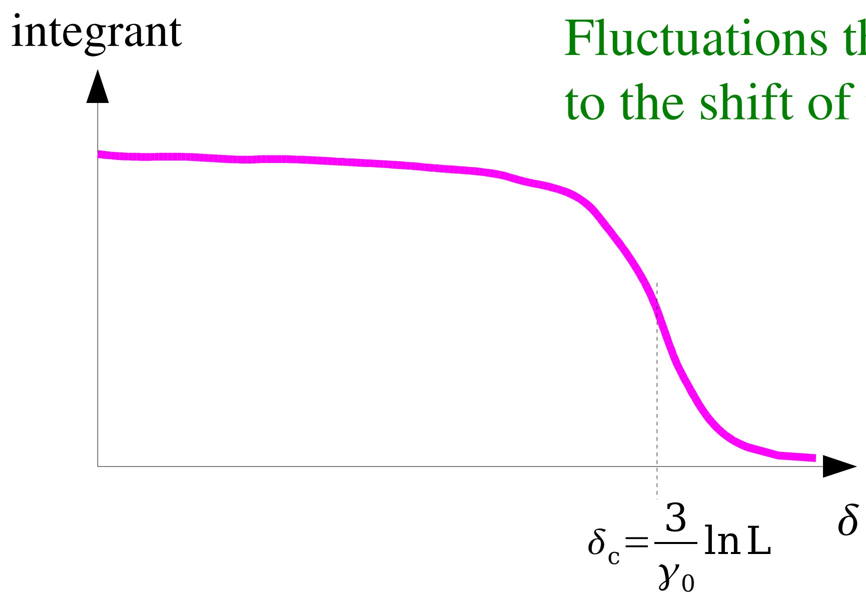
$$V - V_{BD} = \int d\delta\, p(\delta) R(\delta) = \frac{C_1}{\gamma_0} \int d\delta\, e^{-\gamma_0 \delta} \ln\left(1 + C_2 \frac{e^{\gamma_0 \delta}}{L^3}\right) = \frac{C_1 C_2}{\gamma_0} \frac{3 \ln L}{\gamma_0 L^3}$$



Fluctuations that contribute to the shift of the front extend up to $\delta_c = \frac{3}{\gamma_0} \ln L$

Fixing the constants...

$$V - V_{BD} = \int d\delta\, p(\delta) R(\delta) \;=\; \frac{C_1}{\gamma_0}\int d\delta\, e^{-\gamma_0 \delta} \ln\left(1 + C_2 \frac{e^{\gamma_0 \delta}}{L^3}\right) \qquad = \frac{C_1 C_2}{\gamma_0}\frac{3 \ln L}{\gamma_0 L^3}$$



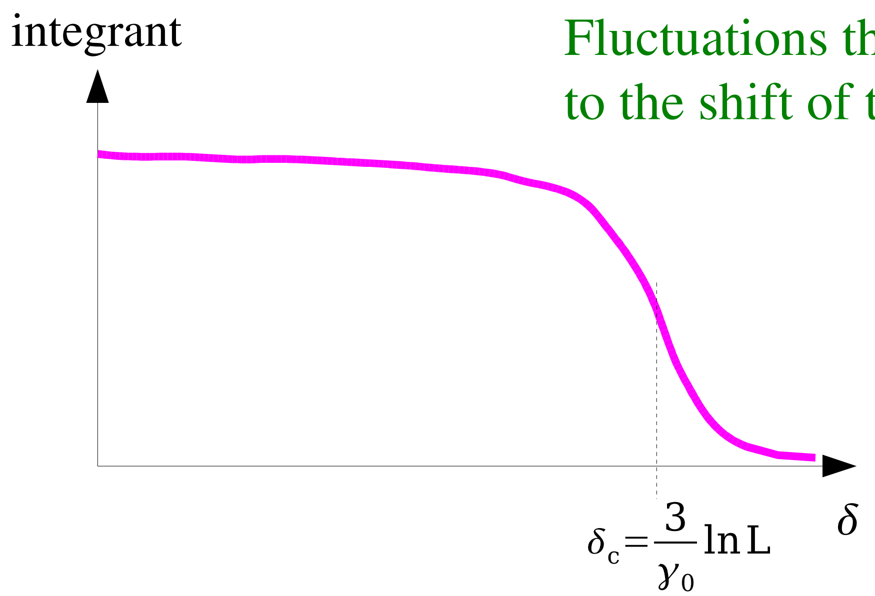
Fluctuations that contribute
to the shift of the front extend up to $\delta_c = \frac{3}{\gamma_0} \ln L$

Assumption #4: $V = V_{BD}$ with the substitution $L \rightarrow L + \delta_c$

$$V = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{\pi^2 \gamma_0 \chi''(\gamma_0)}{2\gamma_0 \left(L \qquad\qquad \right)^2}$$

Fixing the constants...

$$V - V_{BD} = \int d\delta\, p(\delta) R(\delta) = \frac{C_1}{\gamma_0} \int d\delta\, e^{-\gamma_0 \delta} \ln\left(1 + C_2 \frac{e^{\gamma_0 \delta}}{L^3}\right) \qquad = \frac{C_1 C_2}{\gamma_0} \frac{3 \ln L}{\gamma_0 L^3}$$



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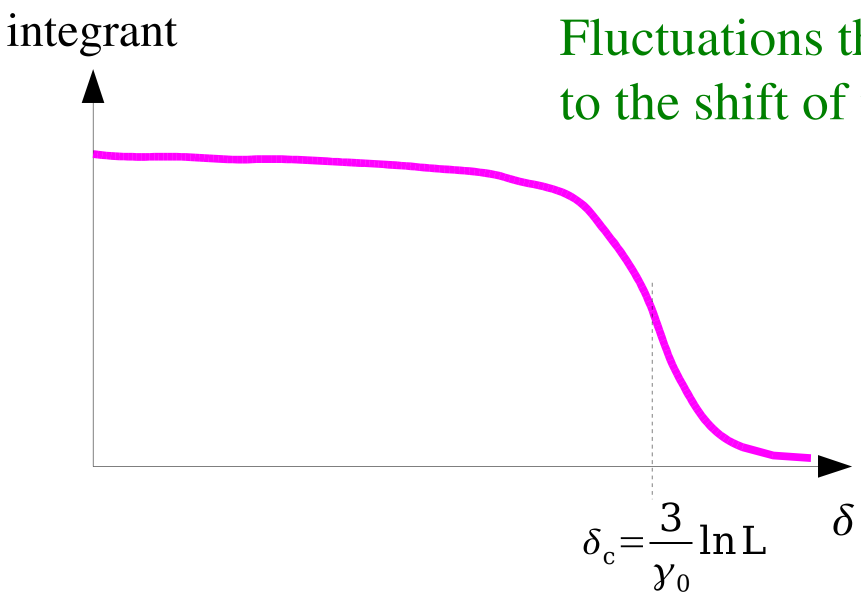

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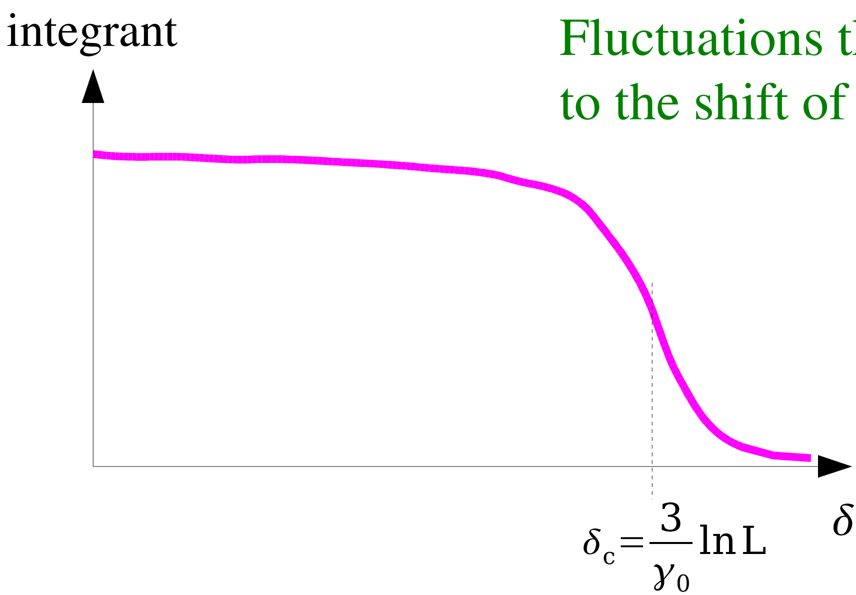
Fixing the constants...

$$V - V_{BD} = \int d\delta\, p(\delta) R(\delta) \;=\; \frac{C_1}{\gamma_0}\int d\delta\, e^{-\gamma_0\delta}\ln\left(1 + C_2\frac{e^{\gamma_0\delta}}{L^3}\right) \qquad = \frac{C_1 C_2}{\gamma_0}\frac{3\ln L}{\gamma_0 L^3}$$



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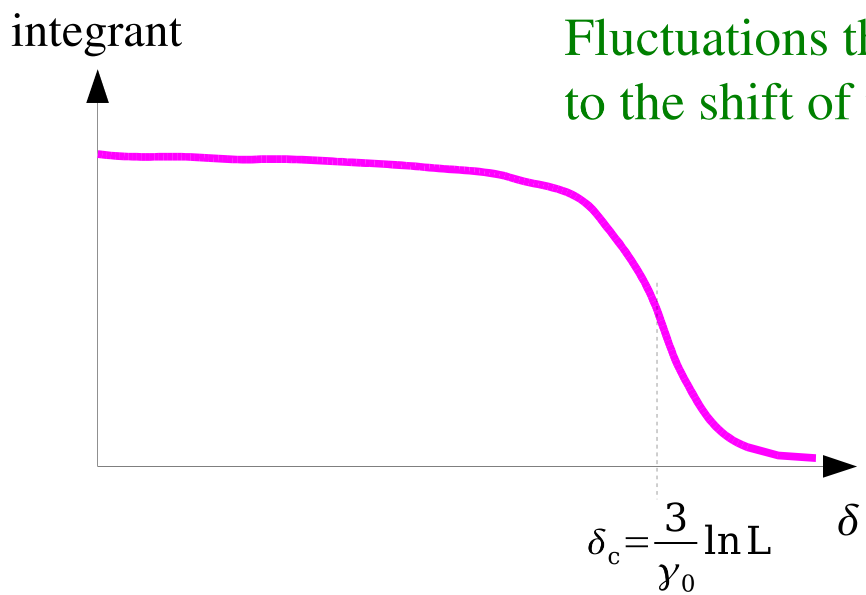


Fluctuations that contribute
to the shift of the front extend up to $\delta_c = \frac{3}{\gamma_0} \ln L$

NB: when $\delta \sim \delta_c$, the front due to the fluctuation is at the same position as the old deterministic front i.e. most of the particles are replaced. This happens once in $1/p(\delta_c) \sim e^{\gamma_0 \delta_c} \sim \ln^3 N$ steps of time.

Assumption #4: $V = V_{BD}$ with the substitution $L \to L + \delta_c$

$$V = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{\pi^2 \gamma_0 \chi''(\gamma_0)}{2\gamma_0 \left(L + \frac{3}{\gamma_0} \ln L\right)^2} \sim V_{BD} + \frac{\pi^2 \chi''(\gamma_0)}{2\gamma_0} \frac{3 \ln L}{\gamma_0 L^3}$$

Summary of the effect of fluctuations

We proposed a phenomenological model for the propagation stochastic fronts, that we expect to be valid in the weak noise limit (for a large enough number of particles). This model is summarized in the following assumptions:

Assumption #1: the evolution of the stochastic front is essentially deterministic, except for some occasional extra-particles in the tail

Assumption #2: the probability for such extra-particles is $p(\delta)\,d\delta\,dt = C_1 e^{-\gamma_0 \delta}\,d\delta\,dt$

Assumption #3: their effect on the front position is $R(\delta) = \frac{1}{\gamma_0} \ln\left(1 + C_2 \frac{e^{\gamma_0 \delta}}{L^3}\right)$

Assumption #4: $V = V_{BD}$ with the substitution $L \to L + \delta_c$

It leads to ***quantitative*** predictions for the position of the front:

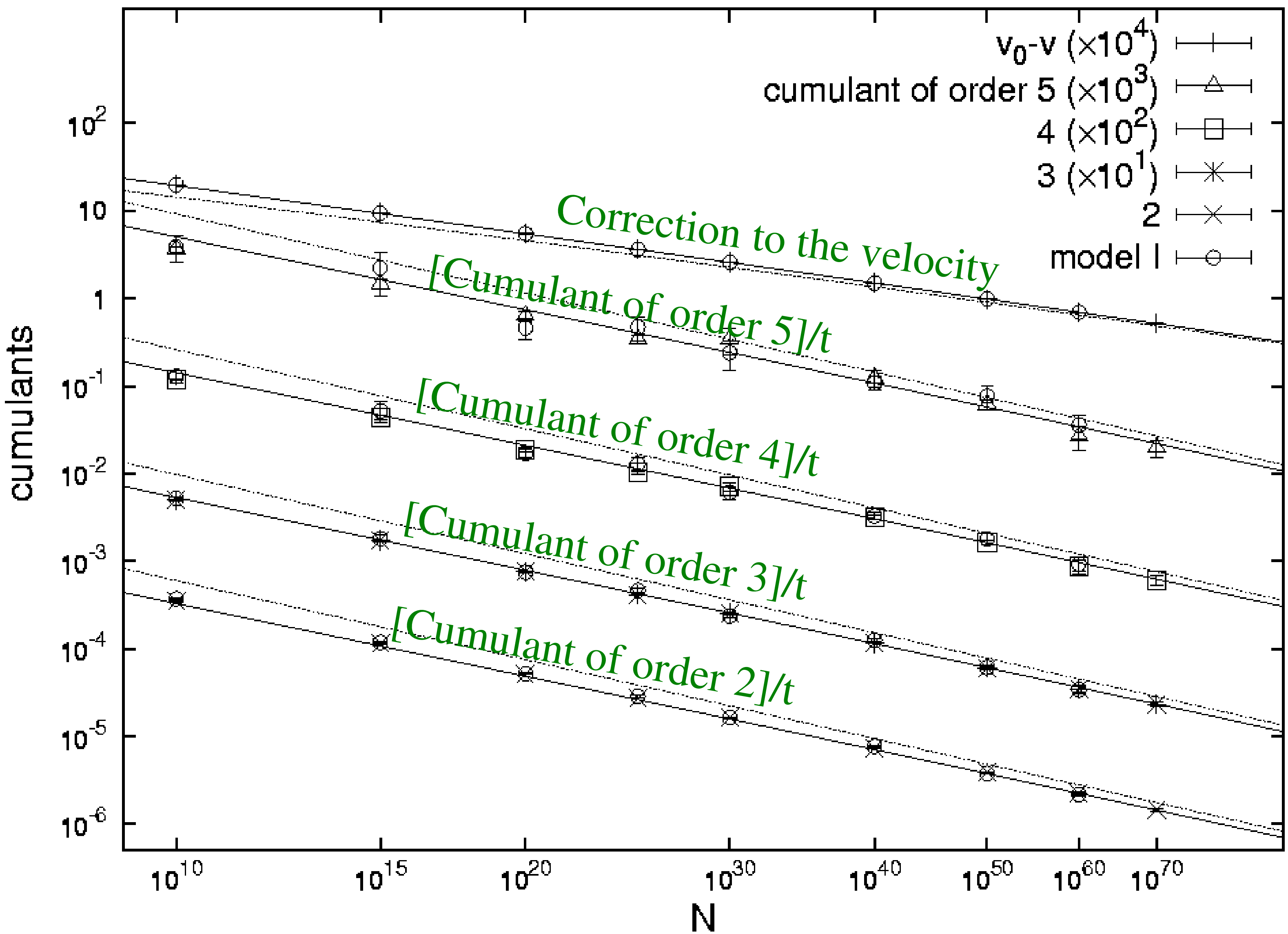
$$V = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{\pi^2 \gamma_0 \chi''(\gamma_0)}{2\ln^2 N} + \pi^2 \gamma_0^2 \chi''(\gamma_0) \frac{3 \ln\ln N}{\gamma_0 \ln^3 N}$$

$$\frac{[n-\text{th cumulant}]}{t} = \pi^2 \gamma_0^2 \chi''(\gamma_0) \frac{n!\,\zeta(n)}{\gamma_0^n \ln^3 N}$$

$\ln N \gg 1$

distribution very wide!
very far from a Gaussian!

Numerical checks



Reaction-diffusion model, discrete in space and time

Use the dictionary...

Position x	$\ln(k^2/k_0^2)$
Time t	$\bar{\alpha}\,Y$
Particle density/fraction u ⟷	Partonic amplitude T
Maximum/equilibrium number of particles N	$\frac{1}{\alpha_s^2}$
Position of the wave front X	Saturation scale $\ln(Q_s^2/k_0^2)$

...to get predictions for QCD!

Outline

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- Universality: lessons from condensed matter
- Stochastic processes: simple examples
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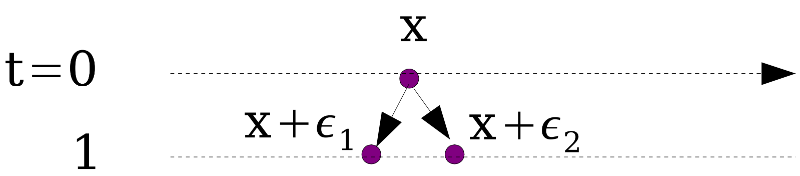
Lecture 2

- Results on noisy traveling waves
- Genealogies in selective evolution models
- A connection to the Parisi theory of spin glasses?

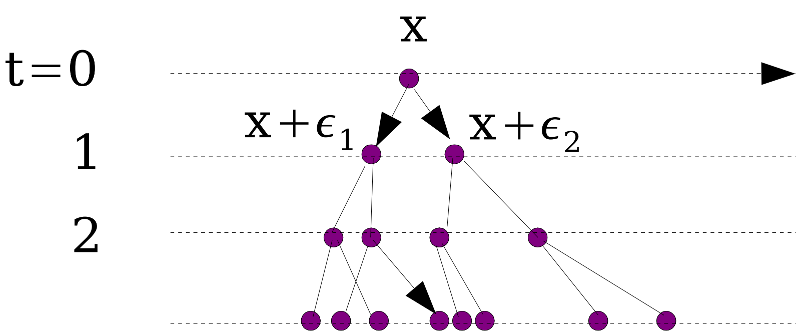
Selective evolution



The individual say at position x has
2 descendants at positions $x+\epsilon_1$ and $x+\epsilon_2$

distribution $\rho(\epsilon)$

Selective evolution



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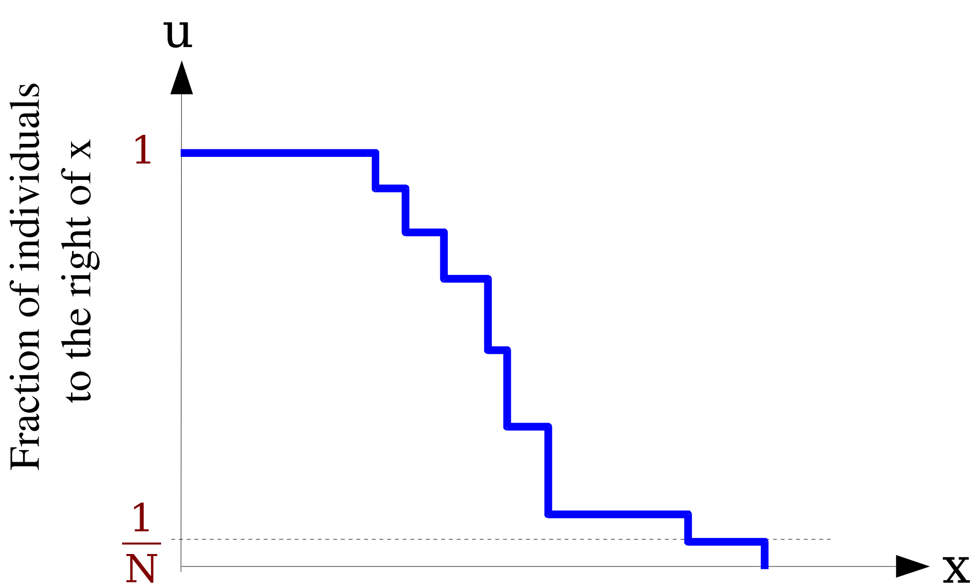
Keep only the ***N rightmost*** individuals



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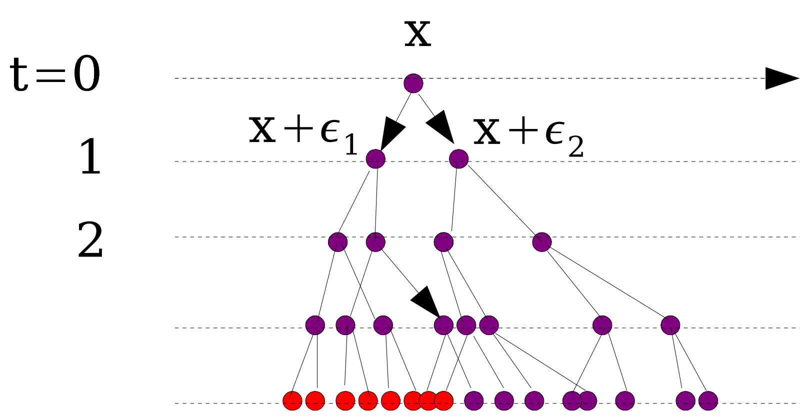

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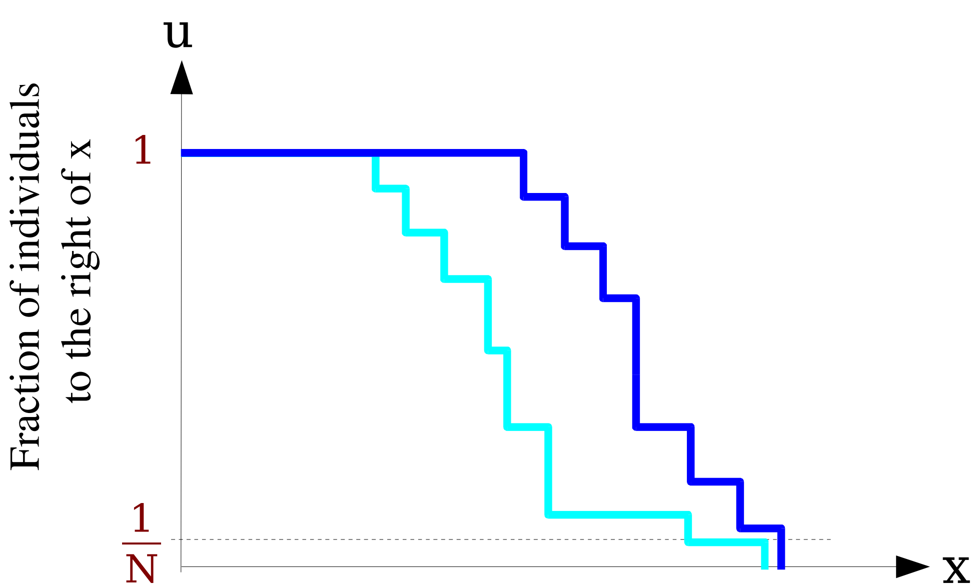

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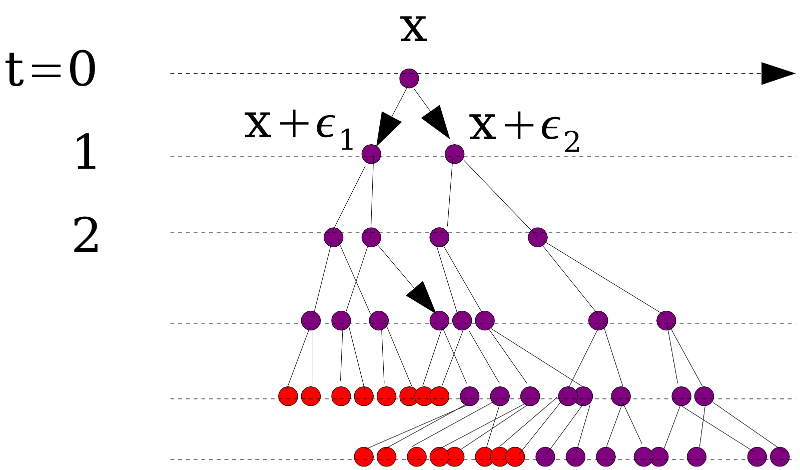

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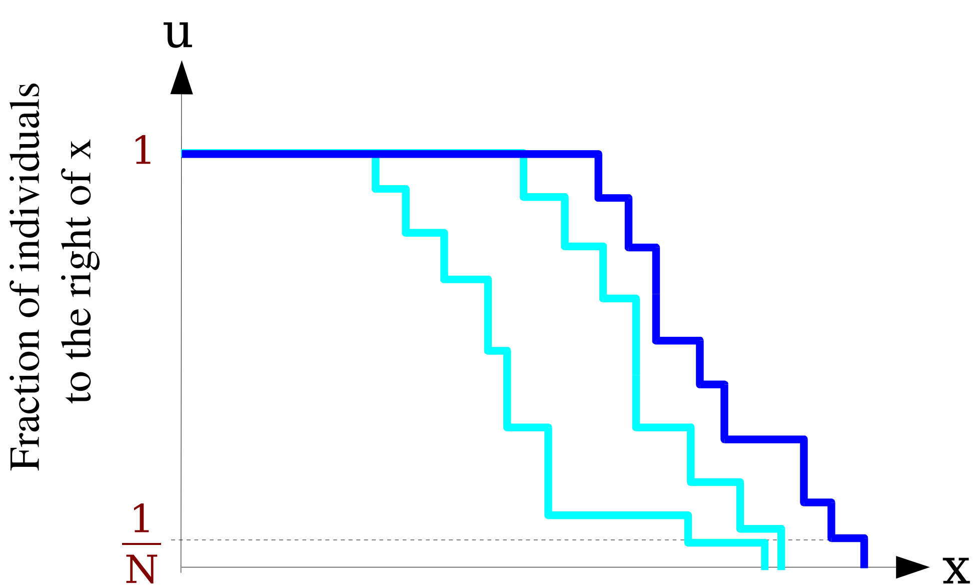

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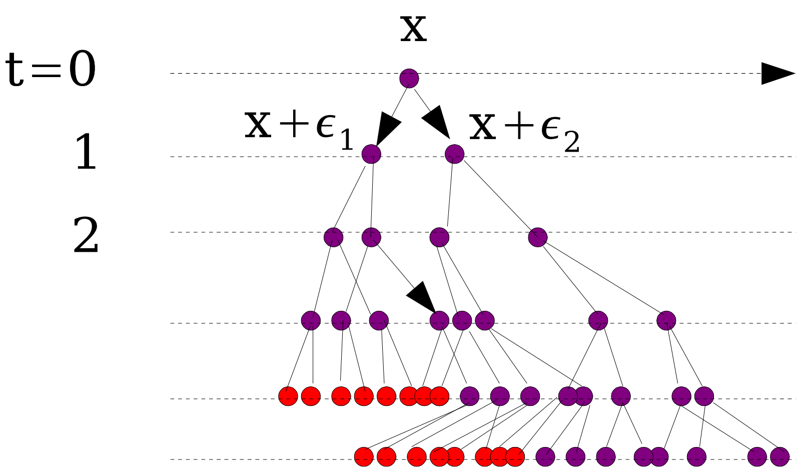

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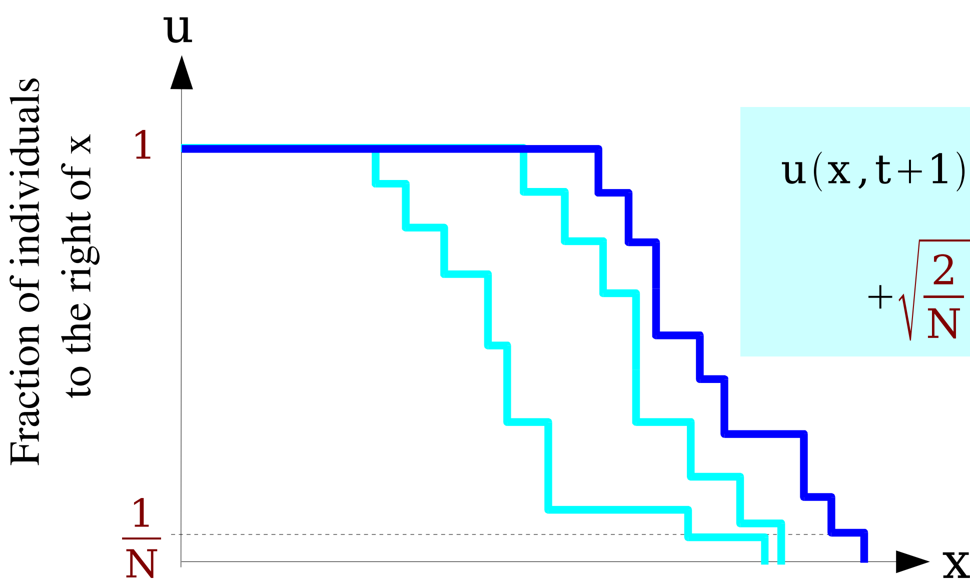
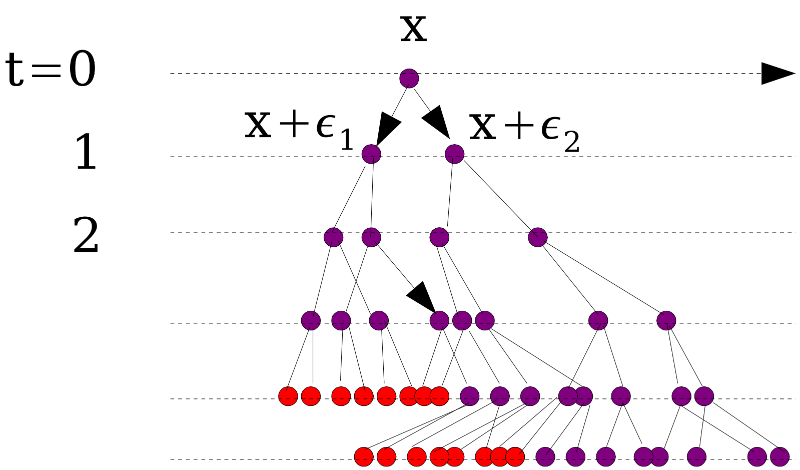


$$u(x,t+1) = \mathrm{Min}\begin{pmatrix} 1 \\ 2\int d\epsilon\,\rho(\epsilon)\,u(x-\epsilon,t) \end{pmatrix} + \sqrt{\frac{2}{N}\int d\epsilon\,\rho(\epsilon)\,u(x-\epsilon,t)\left(1-2\int d\epsilon\,\rho(\epsilon)\,u(x-\epsilon,t)\right)}\;\nu(x,t+1)$$

$$\begin{cases} \langle \nu \rangle = 0 \\ \langle \nu^2 \rangle = 1 \end{cases}$$

Selective evolution



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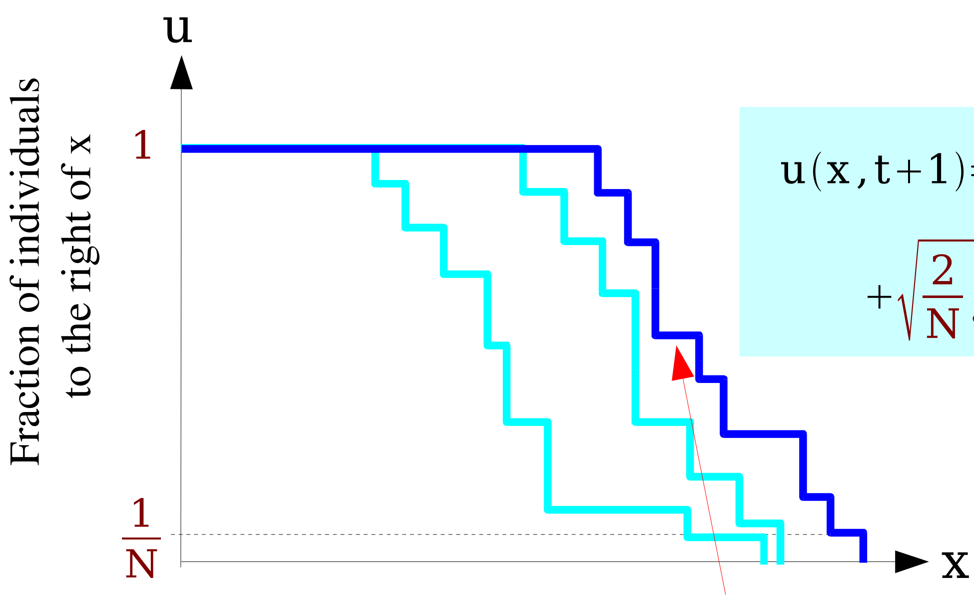

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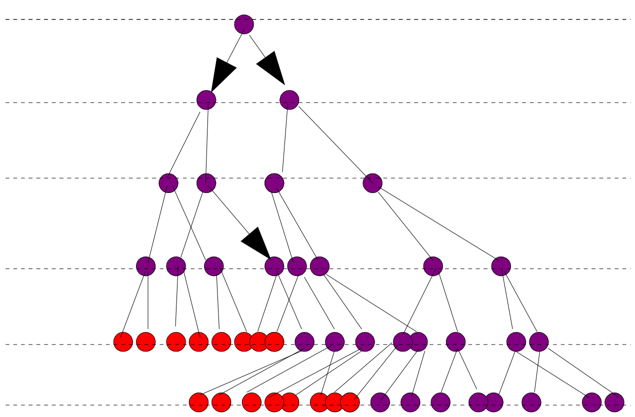
$$\begin{cases}\langle\nu\rangle=0\\ \langle\nu^2\rangle=1\end{cases}$$

Traveling wave: all universal results on the front are applicable

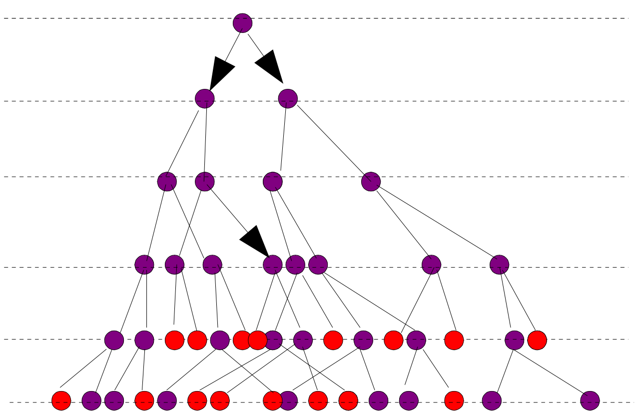
with $\chi(\gamma)=\ln\left(2\int d\epsilon\,\rho(\epsilon)\,e^{\gamma\epsilon}\right)$

What else can we learn on selective evolution?

With selection



Without selection



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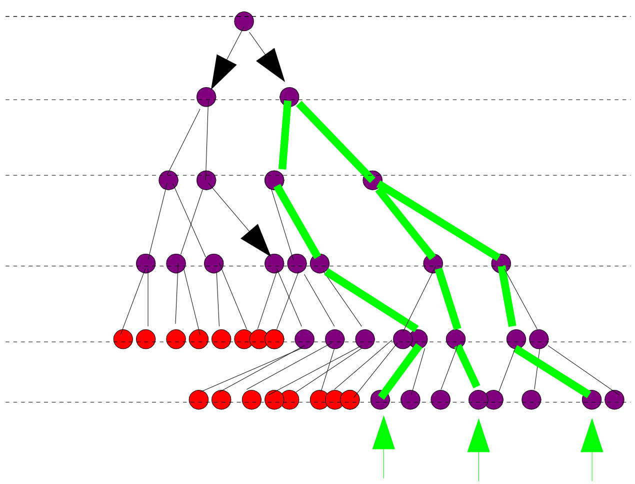
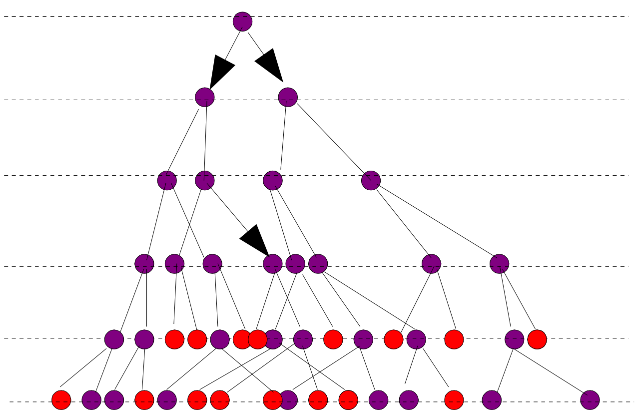


Without selection



Average number of generations to the first common ancestor of k=2, 3... randomly chosen individuals?

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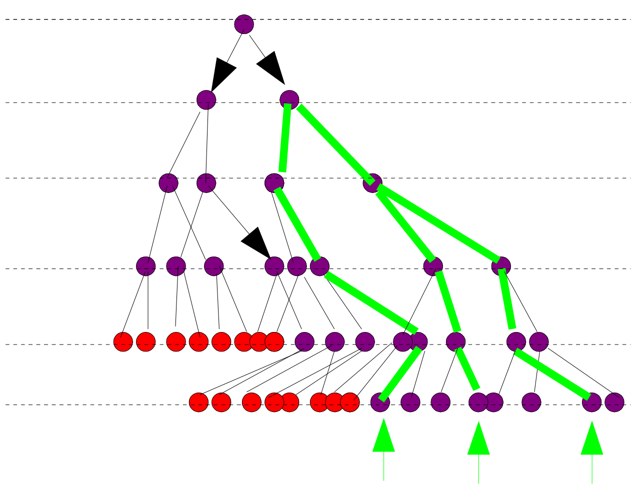
With selection



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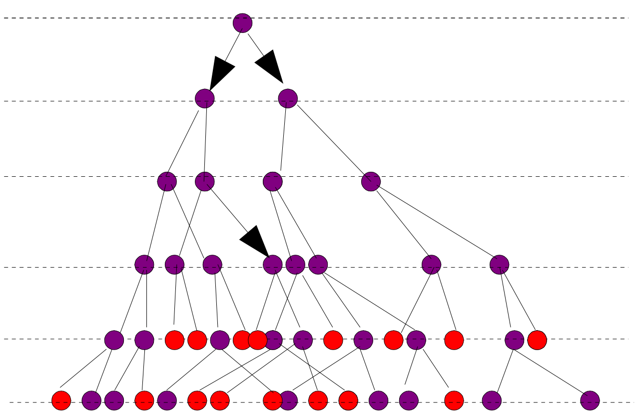


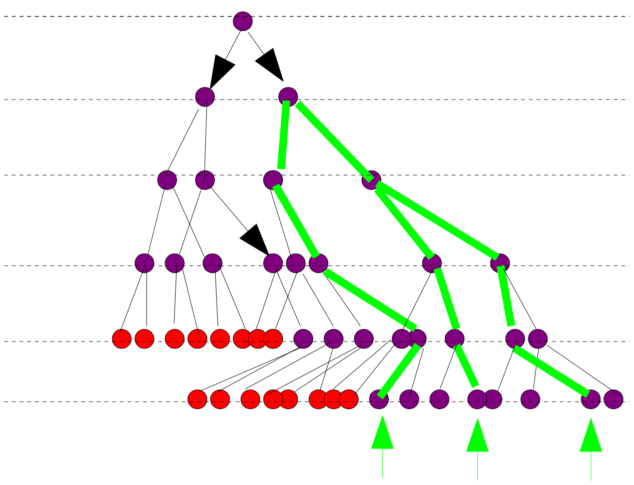
Average number of generations to the first common ancestor of k=2, 3... randomly chosen individuals?

$$\langle T_k \rangle \sim N$$

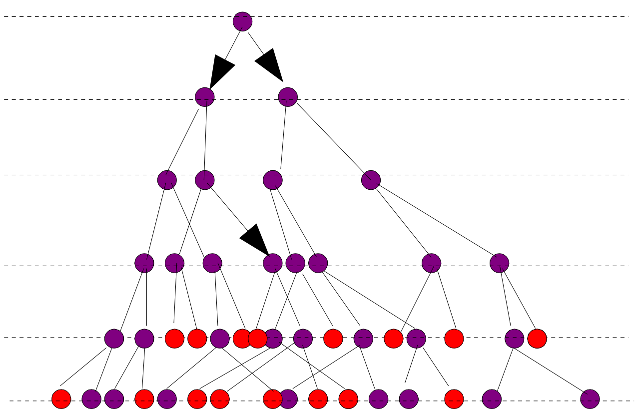
$$\frac{\langle T_3 \rangle}{\langle T_2 \rangle} = \frac{4}{3} \qquad \frac{\langle T_4 \rangle}{\langle T_2 \rangle} = \frac{3}{2}$$

What else can we learn on selective evolution?

With selection



Without selection



Average number of generations to the first common ancestor of k=2, 3... randomly chosen individuals?

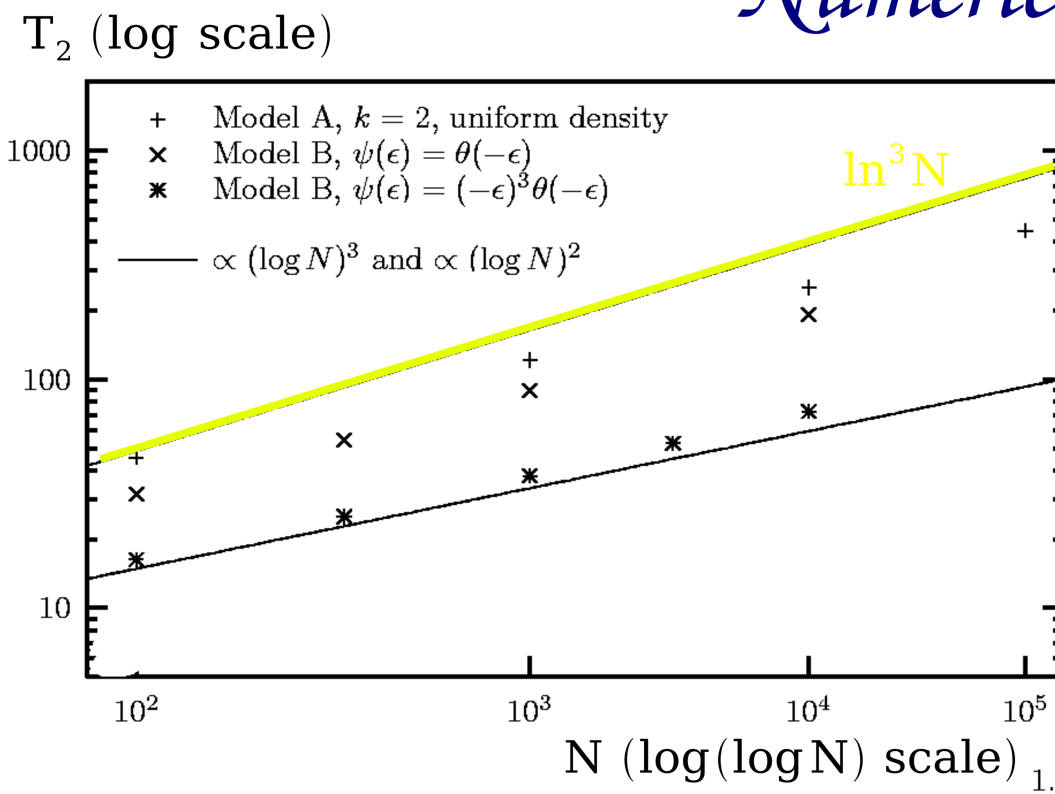
$$\langle T_k \rangle \sim \ln^3 N$$

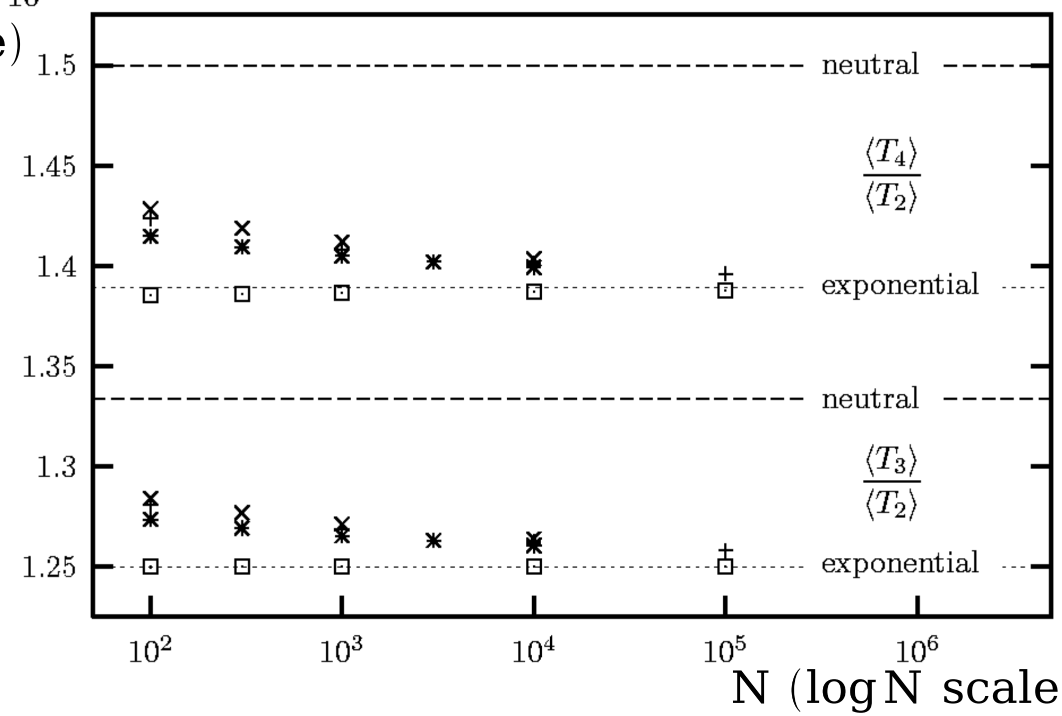
$$\frac{\langle T_3 \rangle}{\langle T_2 \rangle} = \frac{5}{4} \qquad \frac{\langle T_4 \rangle}{\langle T_2 \rangle} = \frac{25}{18}$$

$$\langle T_k \rangle \sim N$$

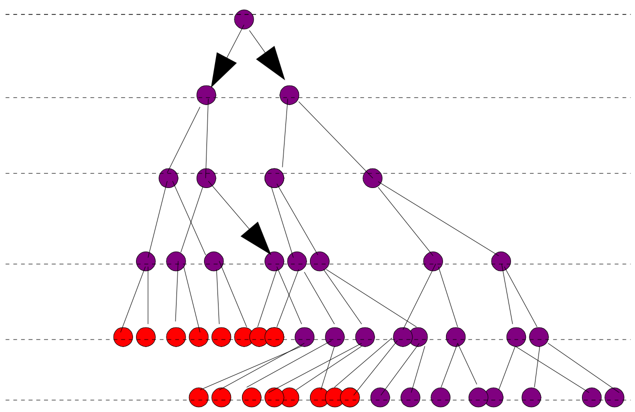
$$\frac{\langle T_3 \rangle}{\langle T_2 \rangle} = \frac{4}{3} \qquad \frac{\langle T_4 \rangle}{\langle T_2 \rangle} = \frac{3}{2}$$

Numerical checks

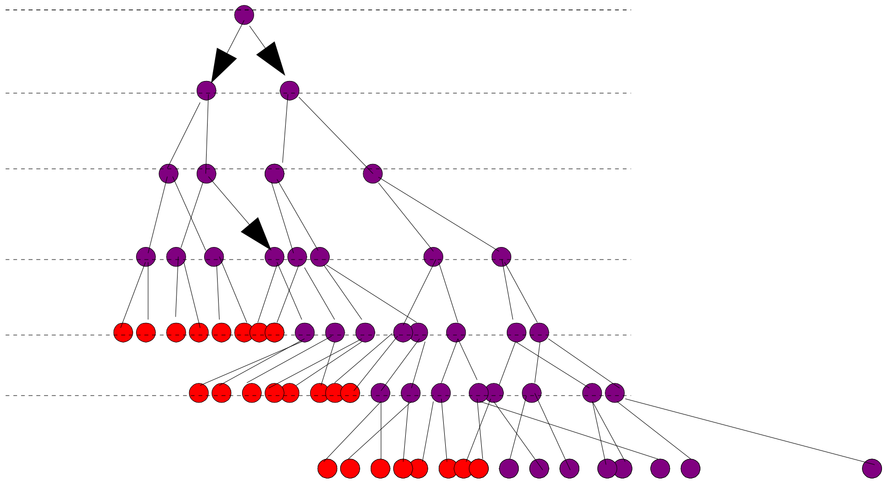




Relation to the phenomenological model



Relation to the phenomenological model



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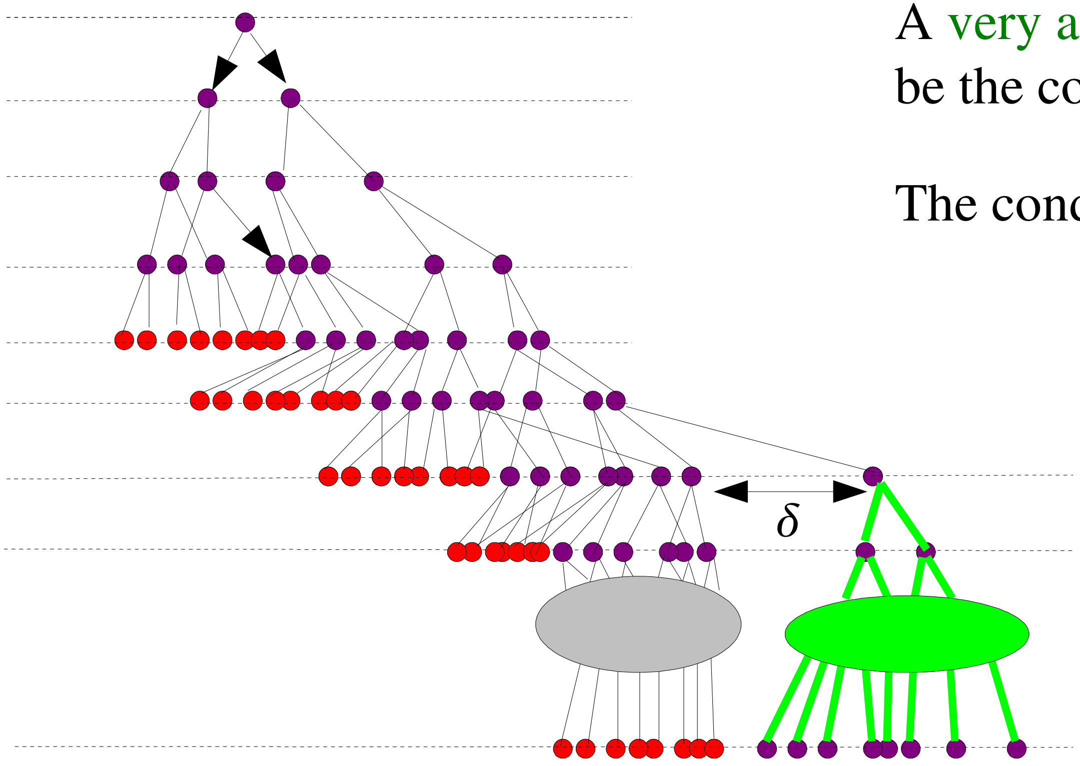

A very advanced individual will, at a later time, be the common ancestor of all particles

The condition for that is $\delta > \delta_c = \frac{3}{\gamma_0} \ln\left(\frac{\ln N}{\gamma_0}\right)$

This happens precisely once in $\frac{1}{p(\delta_c)} \sim \ln^3 N$ timesteps!

Our assumptions on the mechanism for stochastic front propagation also lead to the expressions for $\langle T_k \rangle / \langle T_2 \rangle$

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Spin glasses

A spin glass is a system of Ising spins, for example, with random interactions.

Hamiltonian: $H_J[\{S_i\}] = -\sum_{i,j=1}^{N} J_{i,j} S_i S_j \qquad J_{i,j} = \pm 1 \,(\text{random})$

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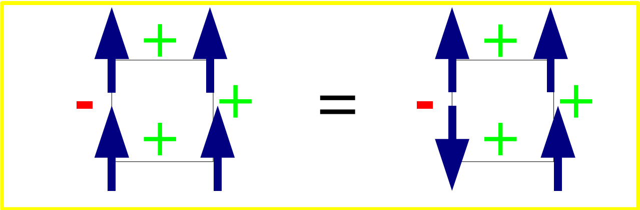
Due to frustration (all interactions cannot be satisfied), there are many different configurations that are local minima of the free energy F.



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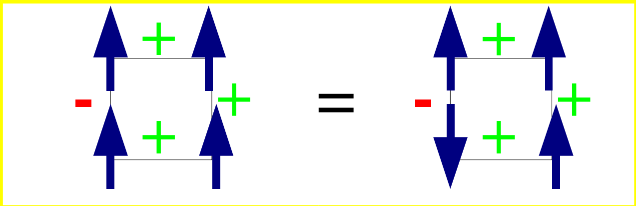

A distance between configs a and b may be defined with the help of the **overlap function**

$$q^{ab} = \frac{1}{N} \sum_i S_i^a S_i^b$$

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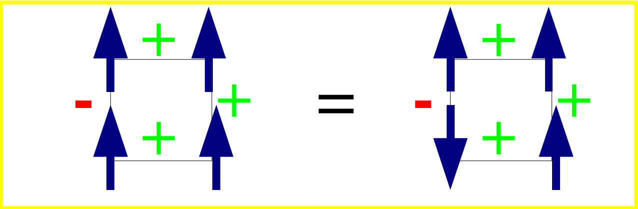
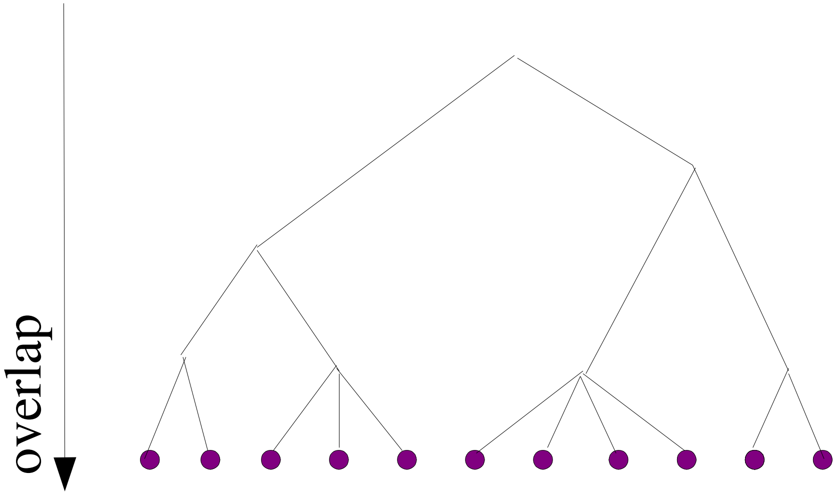


A distance between configs a and b may be defined with the help of the **overlap function**

$$q^{ab} = \frac{1}{N} \sum_i S_i^a S_i^b$$

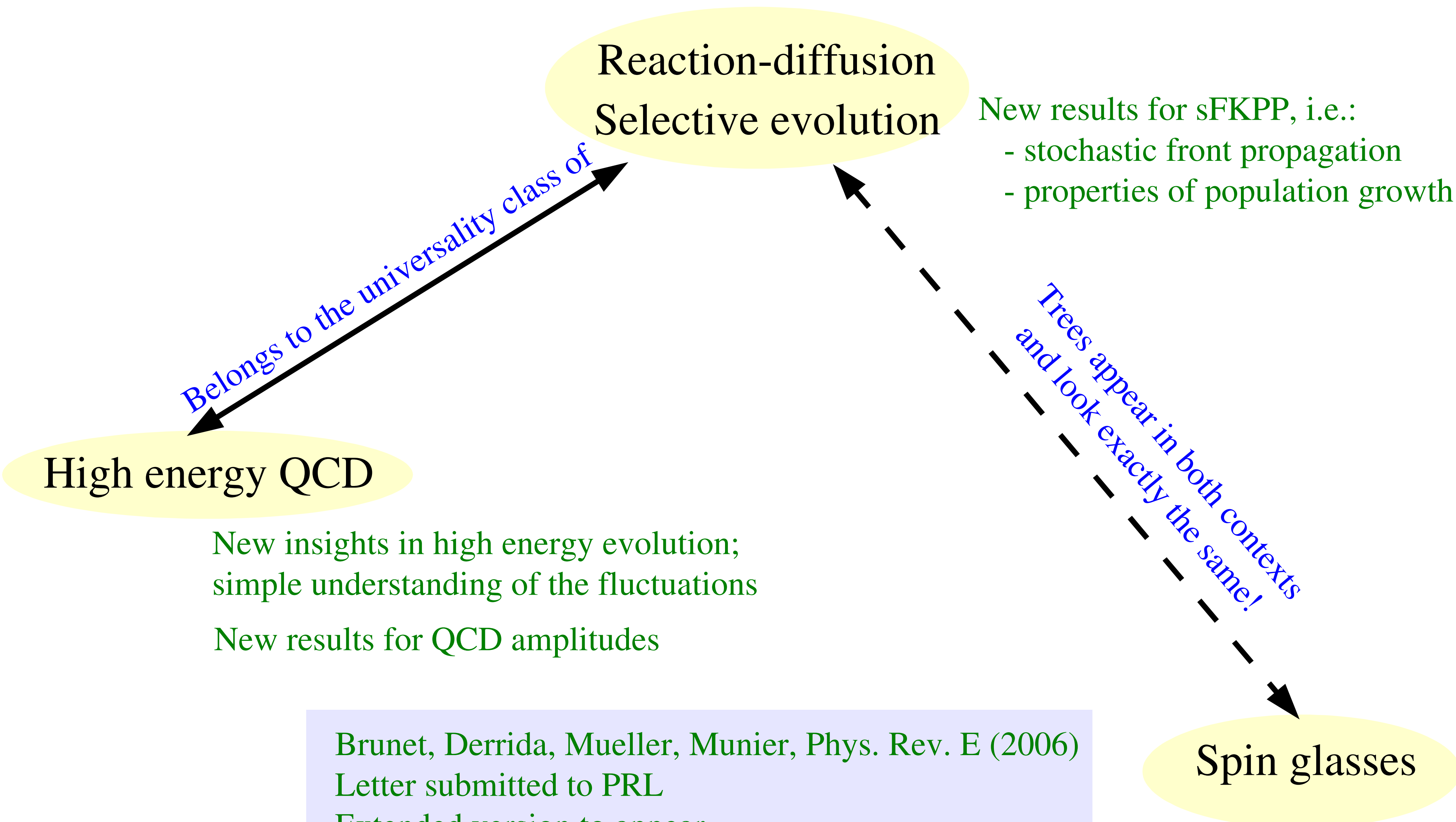
This distance is **ultrametric:** the configurations may be represented by a (random) tree

Parisi; Mezard, Virasoro... (1980...)



We observed that the structure of the tree is the same as in the growth model!

Summary



Brunet, Derrida, Mueller, Munier, Phys. Rev. E (2006)
Letter submitted to PRL
Extended version to appear

Outlook

The correspondence high energy QCD/ reaction-diffusion has provided new and fruitful insights. Lots of points deserve more studies:

- does the impact parameter dependence change the picture?
- can one get phenomenological predictions for e.g. LHC?
- how does the picture fit the traditional approach to QCD through Feynman diagrams?
- does statistical physics have anything to say beyond the energy dependence of total cross sections?

There is an intriguing universality beyond this correspondence, between some evolution processes with selection related to traveling waves and the theory of spin glasses.

- is this deep or accidental?
- what insight can be gained for spin glasses?
- do our results on genealogies have a practical use in QCD/chemistry/biology?

Thank you to the organizers and to all participants

Happy birthday to Prof Bialas!