

Quantum Chromo-Dynamics at the high energy/density frontier

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BNL

Quarks and the Standard Model

1/2 of all “elementary”
particles of
the Standard Model
are not observable;



they are confined
within hadrons
by “color” interactions

Theoretical notion of color:

“The red-haired man”

by Daniil Kharms (1905-1942)

from “Blue Notebook No.10”

Once there was a red-haired man who had no eyes or ears.

Neither did he have any hair,

so he was called “red-haired” **only theoretically**

...

Therefore there's no knowing whom we are even talking about.

In fact we better not say anything about him...

Outline

- Quantum Chromo-Dynamics -
the theory of strong interactions
- QCD at high energies
- QCD at high temperatures
- A glimpse of RHIC results
- New facets of QCD at RHIC and LHC:
 - Color Glass Condensate & thermalization
 - (strongly coupled) Quark-Gluon Plasma
 - topological effects at finite T

QCD and the origin of mass

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \sum_f \bar{q}_f^a (i\gamma_\mu D_\mu - m_f) q_f^a;$$

$$D_\mu = \partial_\mu - igA_\mu^a t^a$$

gluons

quarks



Invariant under scale ($x \rightarrow \lambda x$) and chiral Left \longleftrightarrow Right

transformations in the limit of massless quarks

Experiment: u,d quarks are almost massless...

... but then... all hadrons must be massless as well!

Where does the “dark mass” of the proton come from?

QCD and quantum anomalies

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \sum_f \bar{q}_f^a (i\gamma_\mu D_\mu - m_f) q_f^a;$$

Classical scale invariance is broken by quantum effects:

scale anomaly

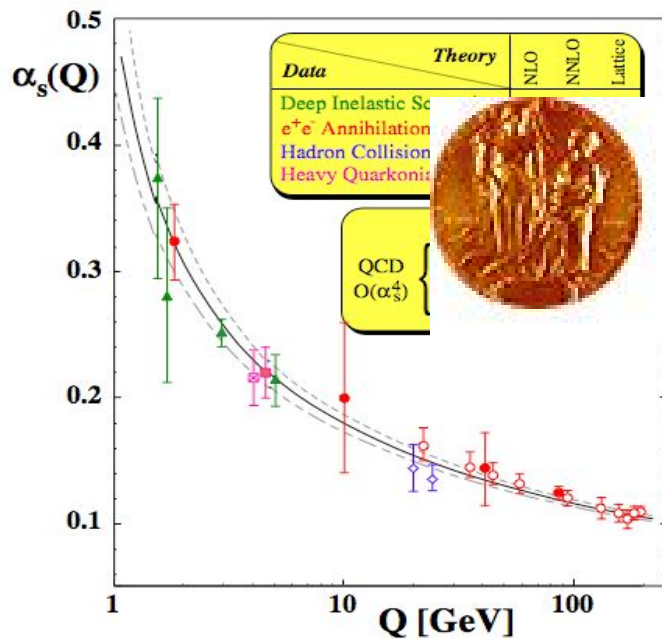
$$\theta_\mu^\mu = \frac{\beta(g)}{2g} G^{\alpha\beta a} G_{\alpha\beta}^a + \sum_q m_q \bar{q}q$$

trace of the energy-momentum tensor

“beta-function”; describes the dependence of coupling on momentum

Hadrons get masses \longleftrightarrow coupling runs with the distance

Asymptotic Freedom



At short distances,
the strong force becomes weak
(**anti**-screening) -
one can access the “asymptotically
free” regime in hard processes

and in super-dense matter
(inter-particle distances $\sim 1/T$)

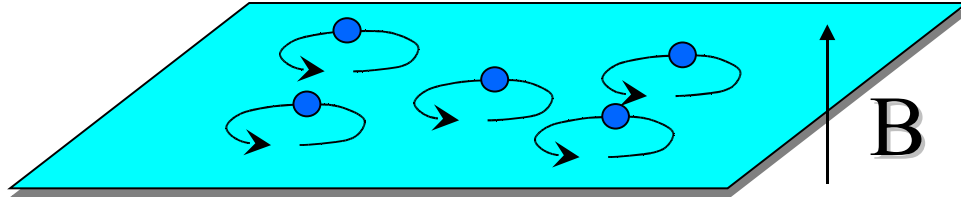
$$\alpha_s(Q) \simeq \frac{4\pi}{b \ln(Q^2/\Lambda^2)}$$

number
of colors

number
of flavors

$$b = (11N_c - 2N_f)/3$$

Asymptotic freedom and Landau levels of 2D parton gas

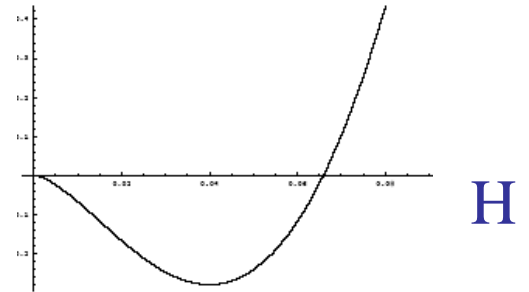


The effective potential: sum over 2D Landau levels

$$V_{\text{pert}}(H) = \frac{g H}{4 \pi^2} \int dp_z \sum_{n=0}^{\infty} \sum_{s_z=\pm 1} \sqrt{2 g H (n + 1/2 - s_z) + p_z^2}.$$

Paramagnetic response of the vacuum: V

$$\text{Re } V_{\text{pert}}(H) = \frac{1}{2} H^2 + (g H)^2 \frac{b}{32 \pi^2} \left(\ln \frac{g H}{\mu^2} - \frac{1}{2} \right)$$



1. The lowest level $n=0$ of radius $\sim (gH)^{-1/2}$ is **unstable!**

2. Strong fields \longleftrightarrow Short distances

QCD and the classical limit

Classical dynamics applies when the action $S = \int d^4x \mathcal{L}(x)$ is large in units of the Planck constant (Bohr-Sommerfeld quantization)

$$\frac{S_{QCD}}{\hbar} \sim \frac{1}{g^2 \hbar} \int d^4x \operatorname{tr} G^{\mu\nu}(x) G_{\mu\nu}(x) \gg 1$$

(equivalent to setting $\hbar \rightarrow 0$)

=> Need weak coupling and strong fields

$$D_\mu = \partial_\mu - ig A_\mu^a t^a$$

$$A^2 \ll \frac{p^2}{g^2}$$

weak
field

$$A^2 \sim \frac{p^2}{g^2}$$

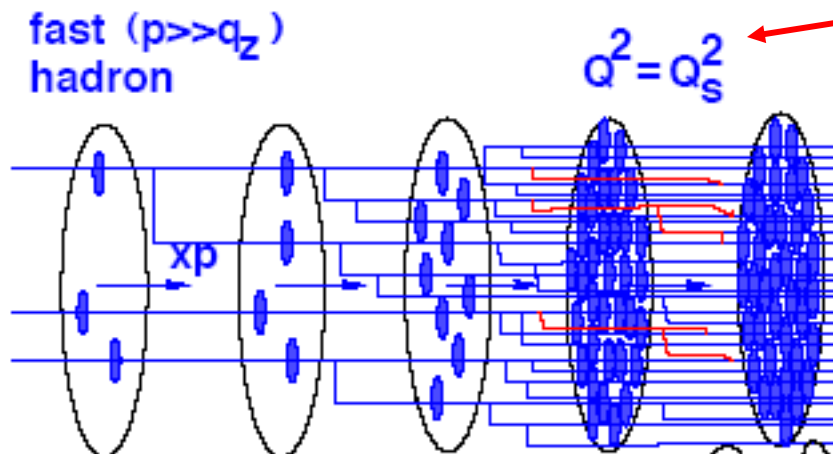
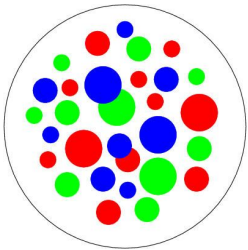
strong
field

Building up strong color fields:

small x (high energy) and large A (heavy nuclei)

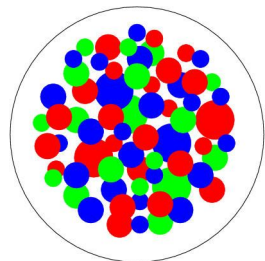
Bjorken x : the fraction of hadron's momentum carried by a parton; high energies s open access to small $x = Q^2/s$

Large x



the boundary of non-linear regime: partons of size $1/Q > 1/Q_s$ overlap

small x



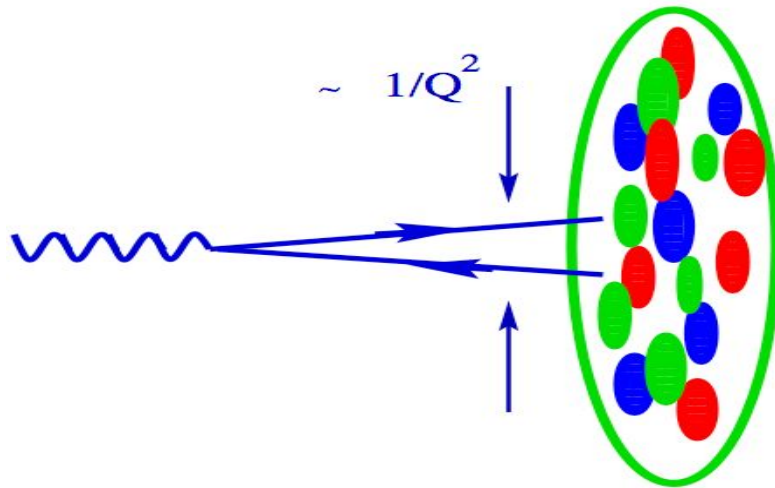
Because the probability to emit an extra gluon is $\sim \alpha_s \ln(1/x) \sim 1$, the number of gluons at small x grows; the transverse area is limited



transverse density becomes large

Strong color fields in heavy nuclei

At small Bjorken x , hard processes develop over large longitudinal distances $l_c \sim \frac{2\nu}{Q^2} = \frac{1}{mx}$



Density of partons in the transverse plane as a **new dimensional parameter** Q_s (“saturation scale”)

Gribov, Levin, Ryskin

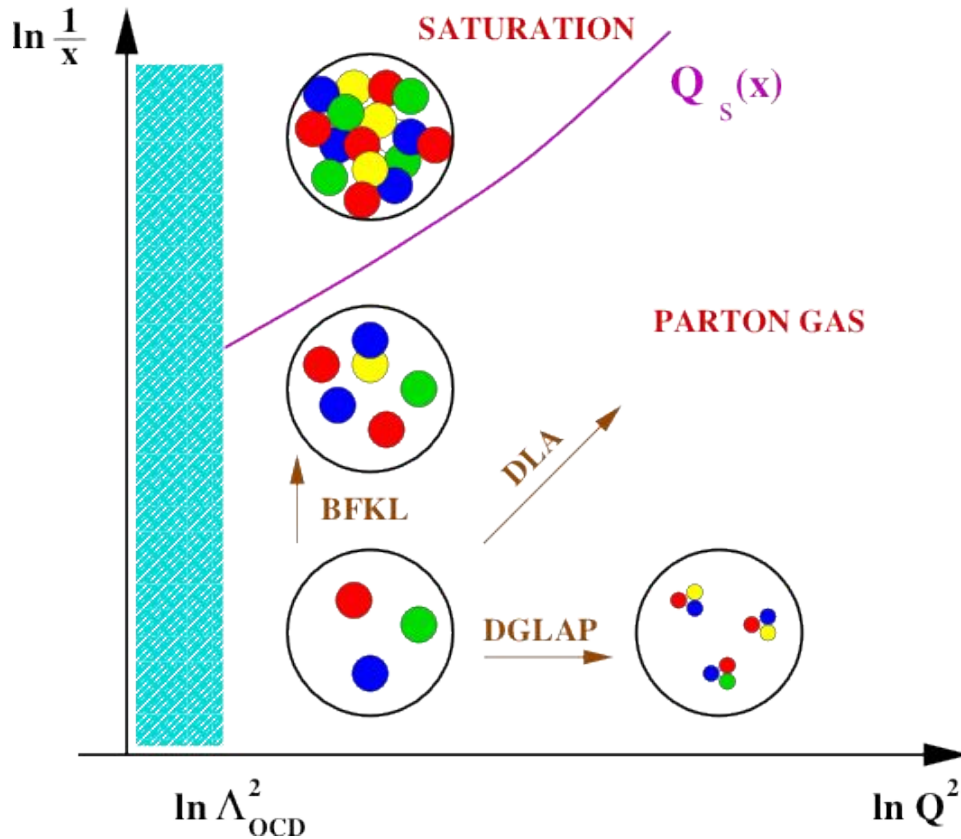
All partons contribute coherently \Rightarrow at sufficiently small x and/or large A strong fields, **weak coupling!**

McLerran, Venugopalan

Non-linear QCD evolution and population growth

Color glass condensate

time $t \rightarrow \ln \frac{1}{x} \equiv y$ rapidity



QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Linear evolution: T. Malthus (1798)

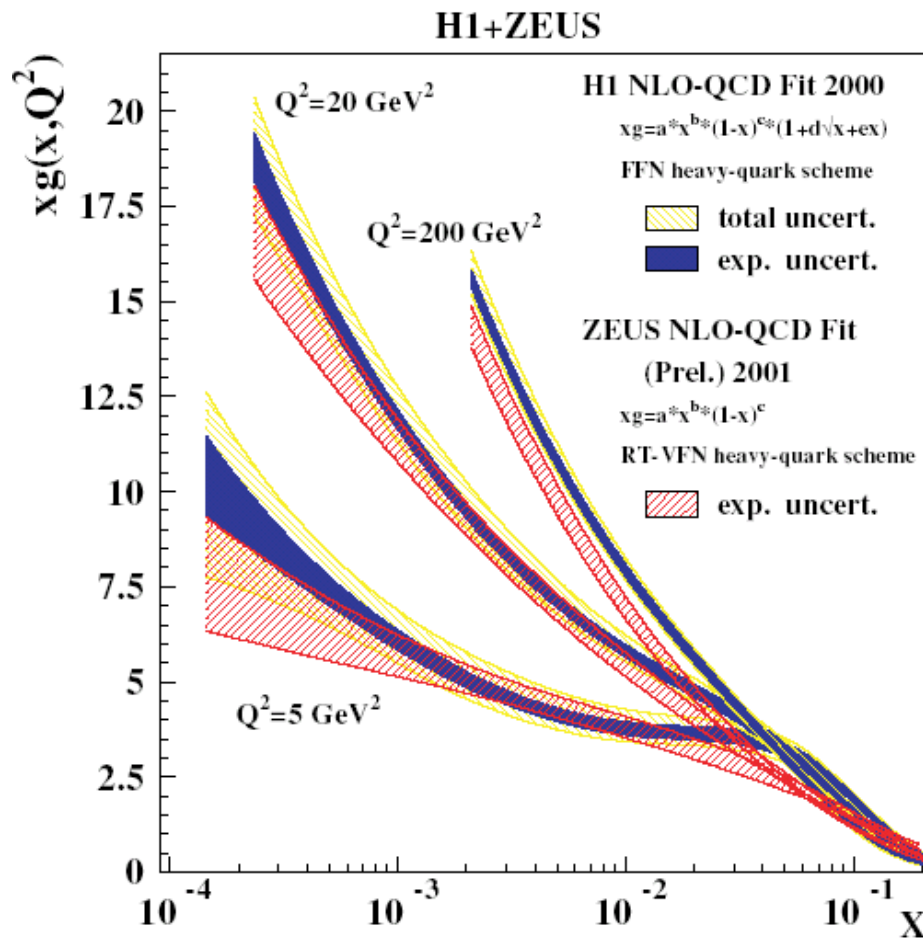
$$\frac{d}{dt} N(t) = r N(t)$$

r - rate of maximum
population growth

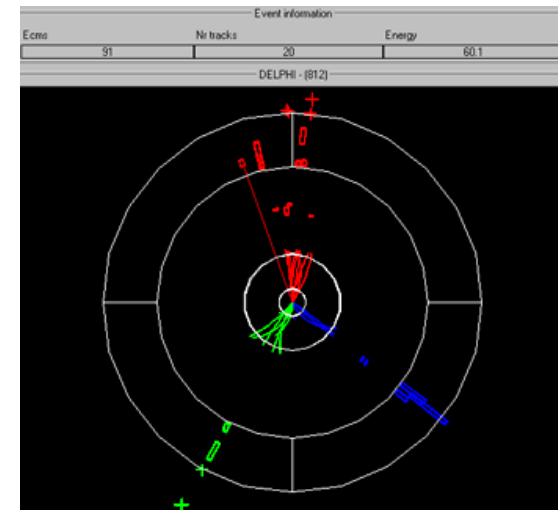
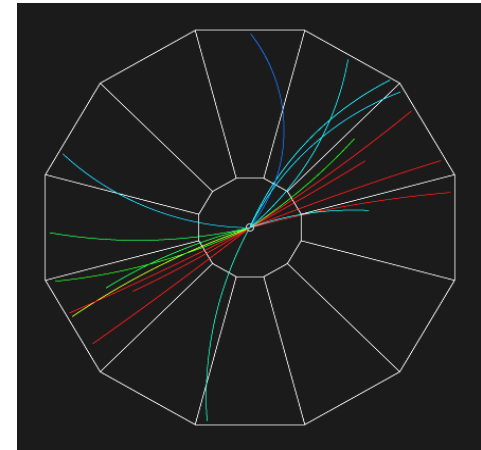
$$N(t) = N_0 \exp(r t)$$

Unlimited growth!

Resolving the gluon cloud at small x and short distances $\sim 1/Q^2$



number of gluons



“jets”: high momentum partons

Population growth in a limited environment

$$\frac{dN(t)}{dt} = \frac{rN(K - N)}{K} \quad \text{“logistic equation”}$$

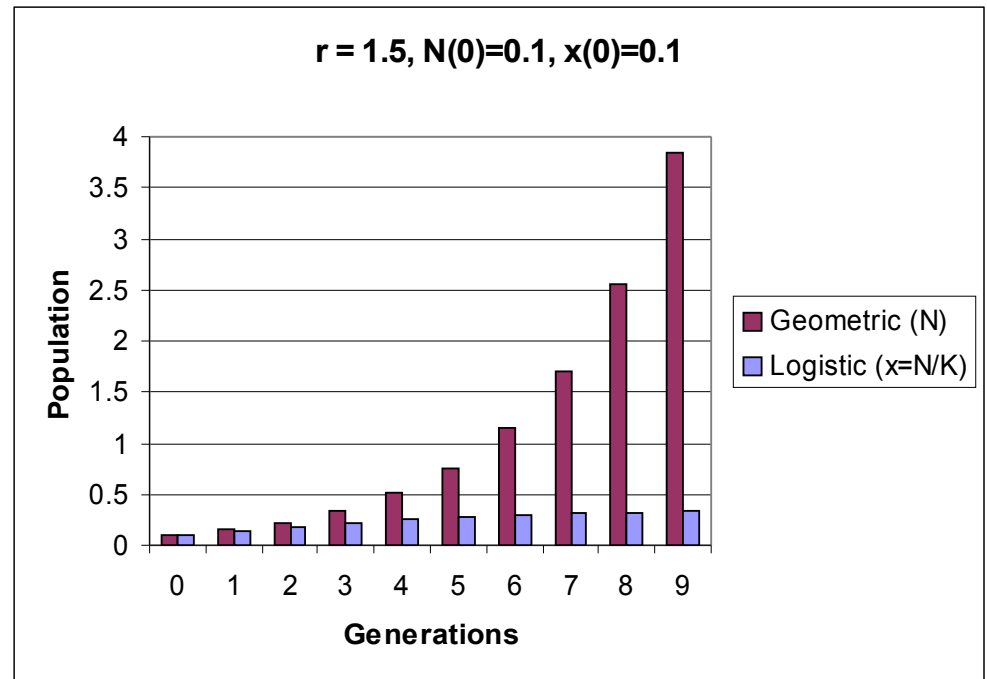
QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

K - maximum sustainable population; define $x = \frac{N}{K}$

Pierre Verhulst (1845)

$$\frac{dx}{dt} = rx(1 - x)$$

$$x(t) = \frac{1}{1 + \left(\frac{1}{x_0} - 1\right) e^{-rt}}$$



Stable population: $N(t) \rightarrow K, \quad \frac{dN}{dt} \rightarrow 0 \quad \text{as } t \rightarrow \infty$

The limit is **universal** (no dependence on the initial condition)

Population growth in a limited environment: a (short) path to chaos

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

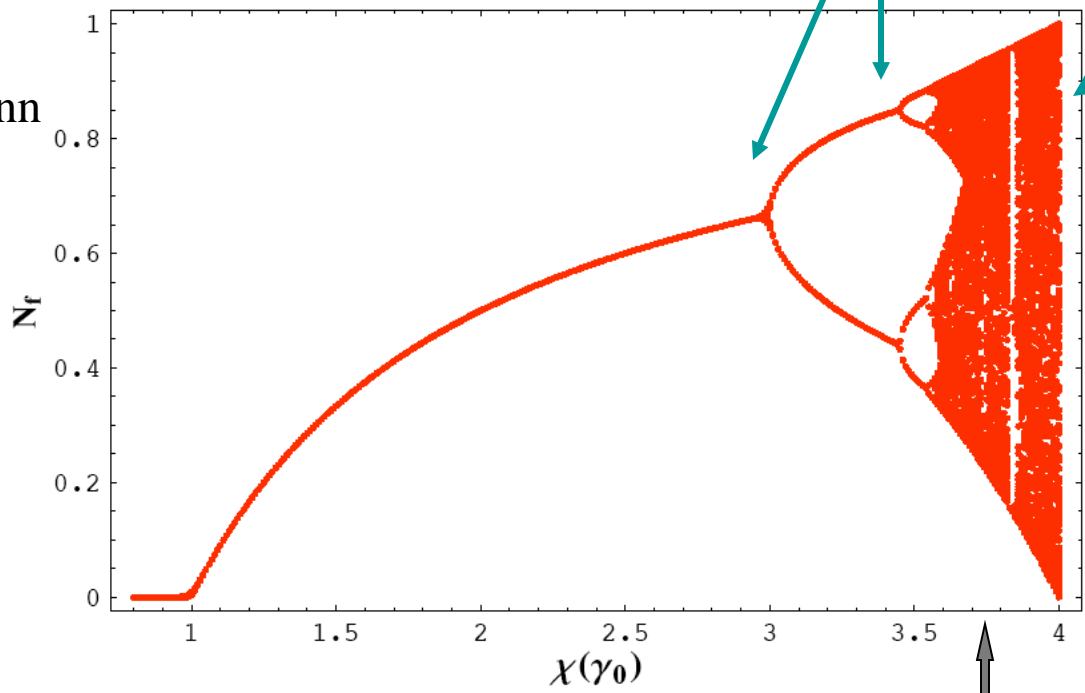
Discrete version: **bifurcations** leading to **chaos**

John von Neumann

Fixed
points

($t \rightarrow \infty$ limit
of the popu-
lation)

$$N_{n+1} = \chi(\gamma_0) N_n (1 - N_n)$$



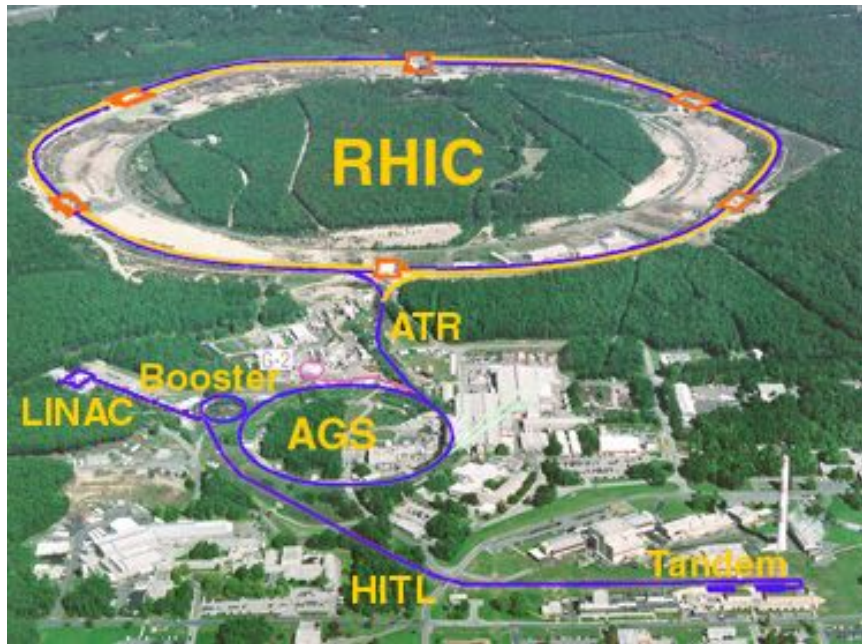
“logistic map”

Rate of growth
(the value of $\alpha_s \ y$)

High energy evolution
starts here (?)

DK, K. Tuchin

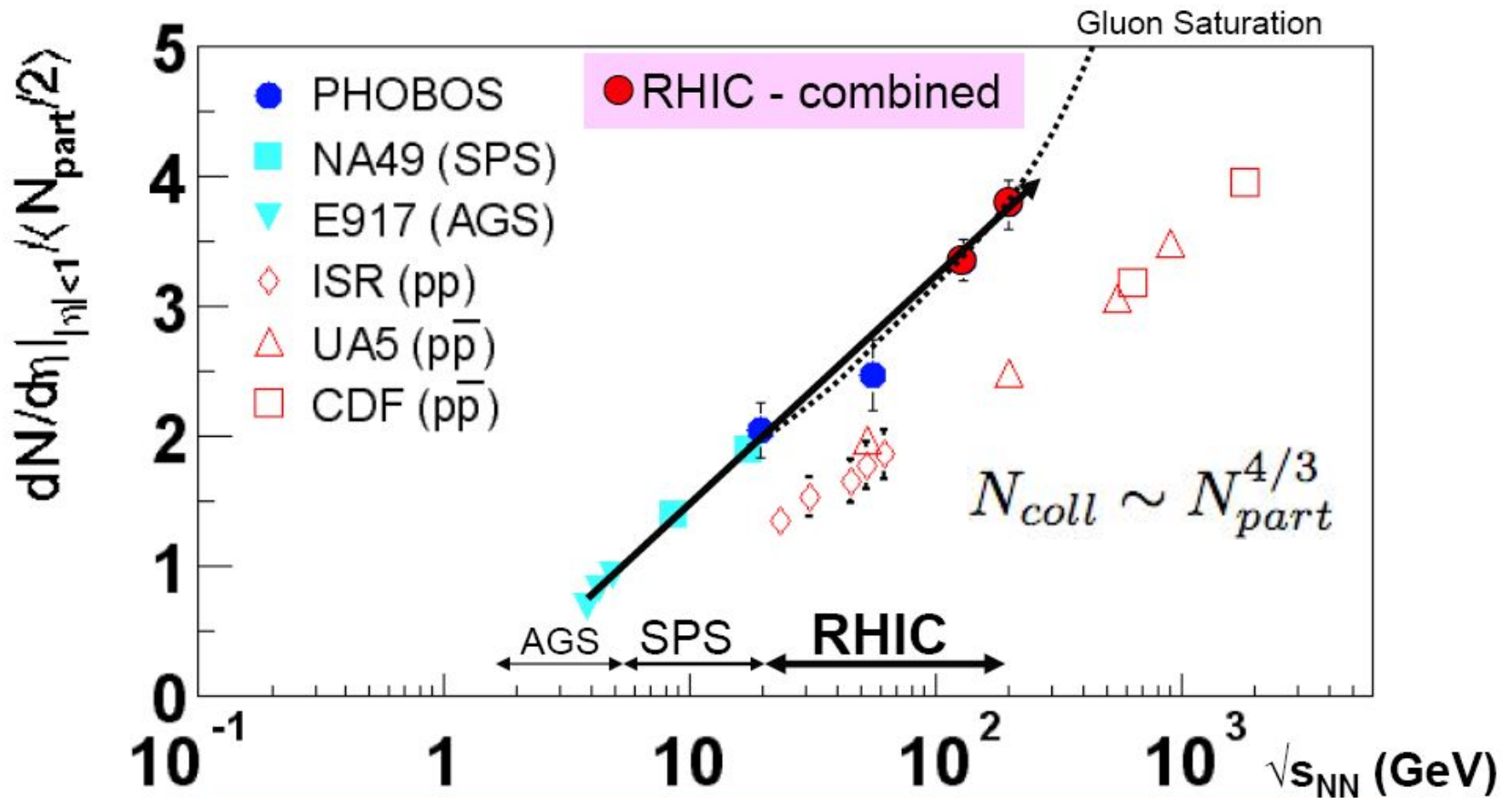
Semi-classical QCD: experimental tests



QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

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Hadron multiplicities: the effect of parton coherence

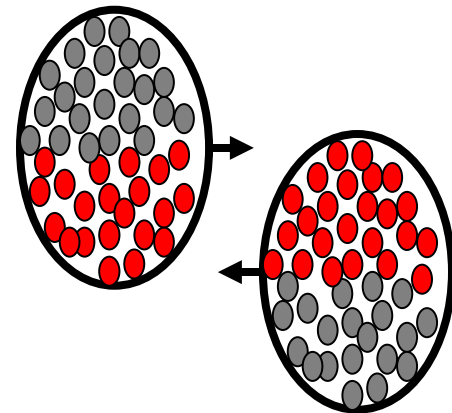


Semi-classical QCD and total multiplicities in heavy ion collisions

Expect very simple dependence of multiplicity on atomic number A / N_{part} :

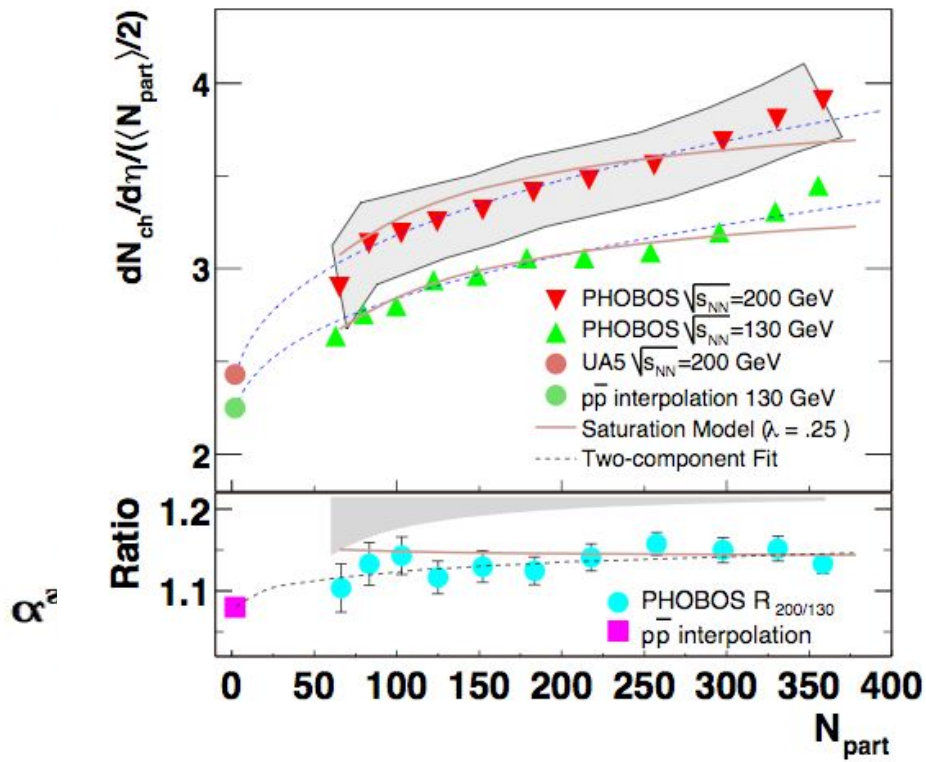
$$n \sim \frac{S_A Q_s^2}{\alpha_s(Q_s^2)} \sim N_{part} \ln N_{part}$$

N_{part} :



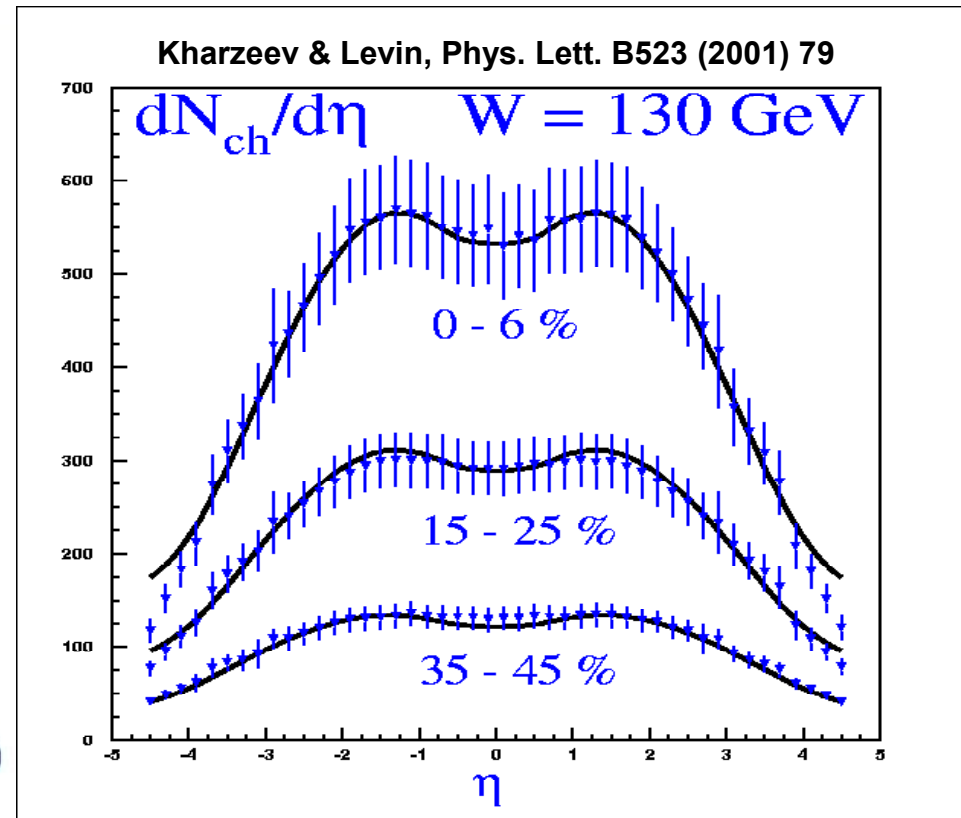
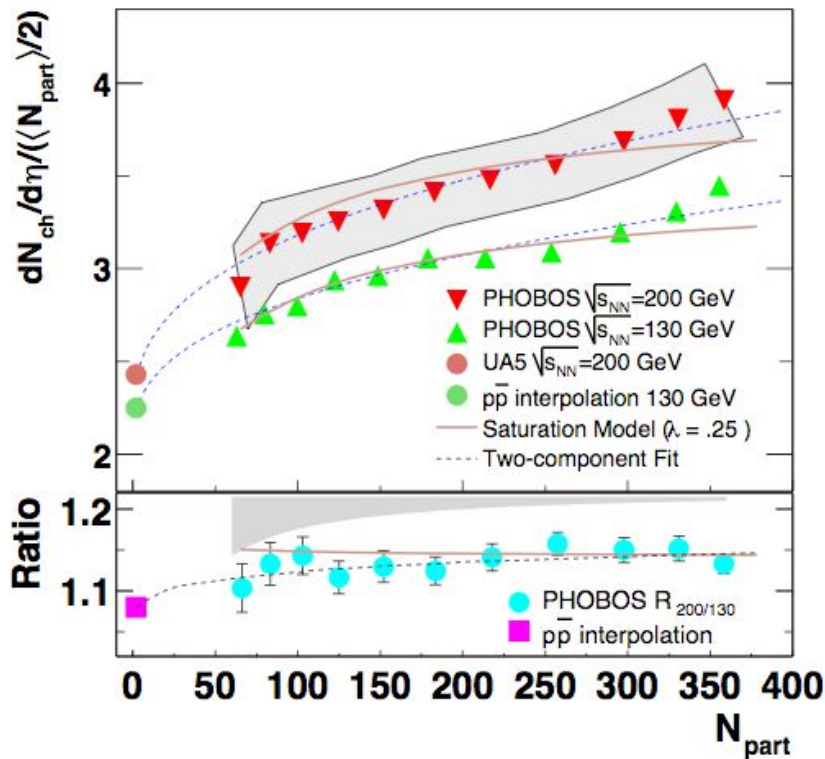
Almost the wounded nucleon model!

$$\frac{1}{N_{part}} \frac{dN}{d\eta} \sim \frac{1}{\alpha_s(Q_s^2)}$$



Classical QCD dynamics in action

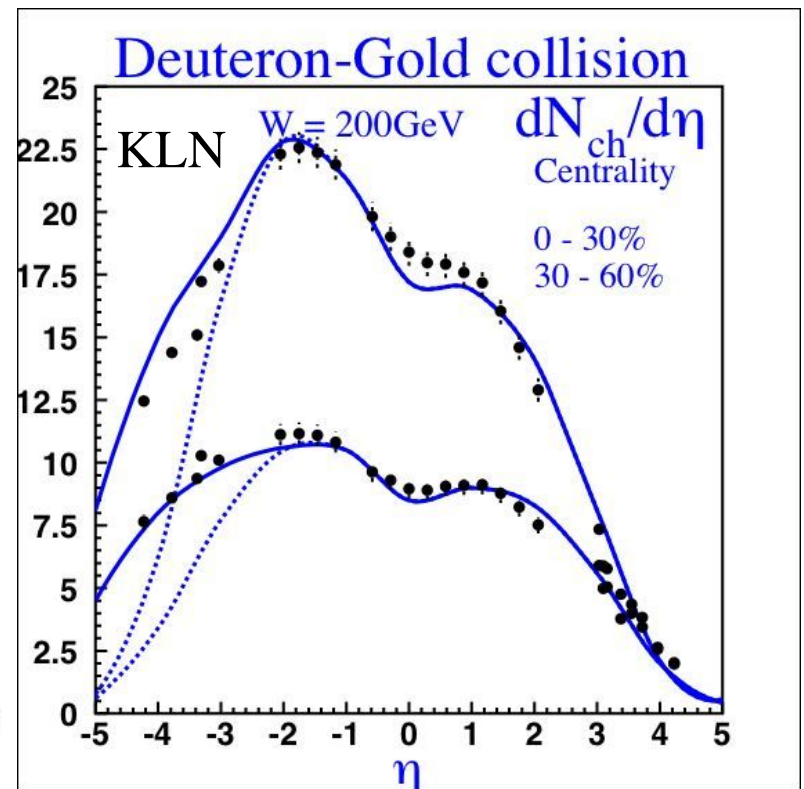
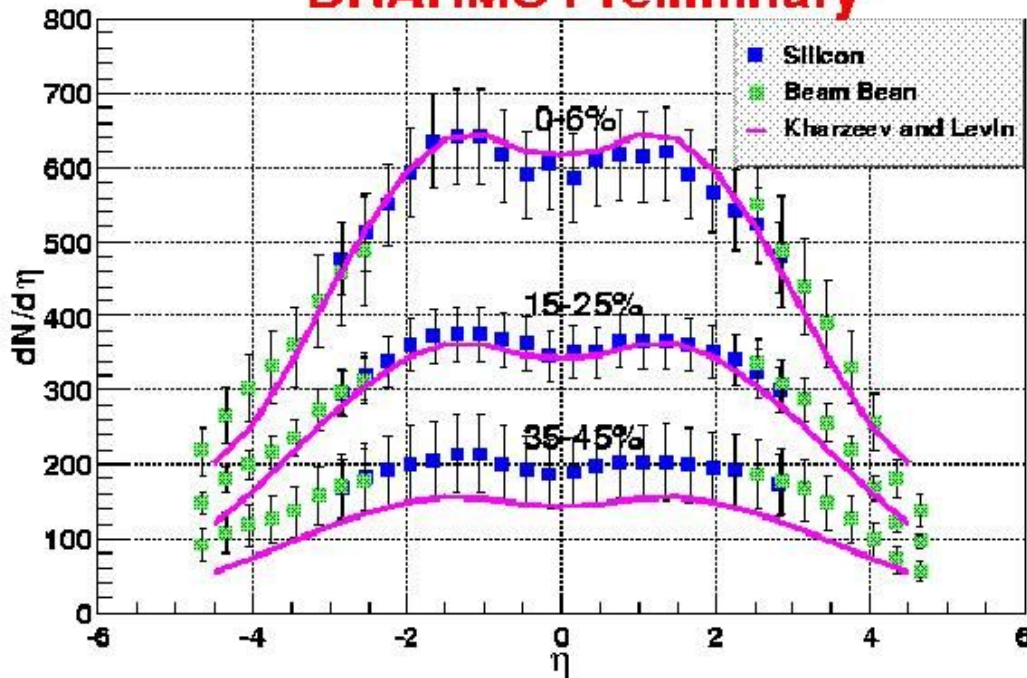
The data on hadron multiplicities in Au-Au and d-Au collisions support the quasi-classical picture



Classical QCD in action

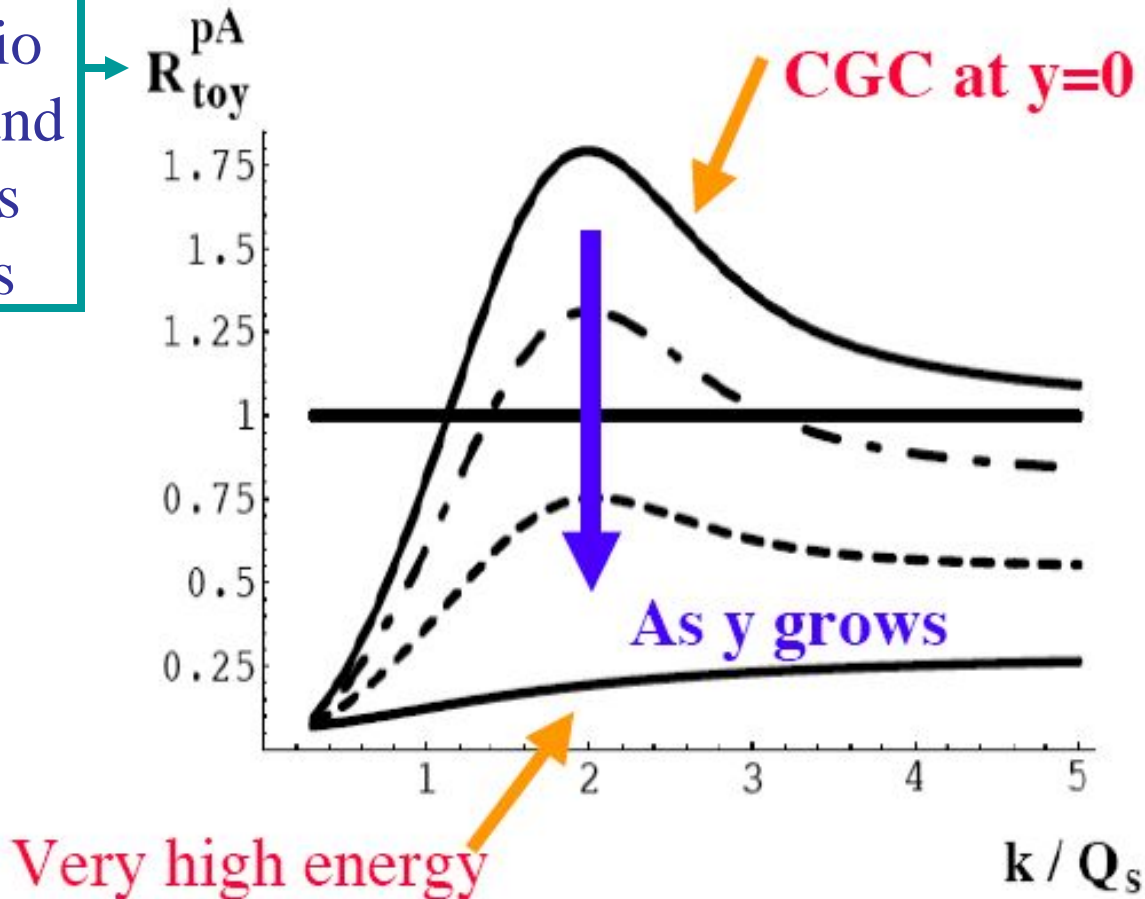
The data on hadron multiplicities in Au-Au and d-Au collisions support the semi-classical picture

BRAHMS Preliminary



Gluon multiplication in a limited (nuclear) environment

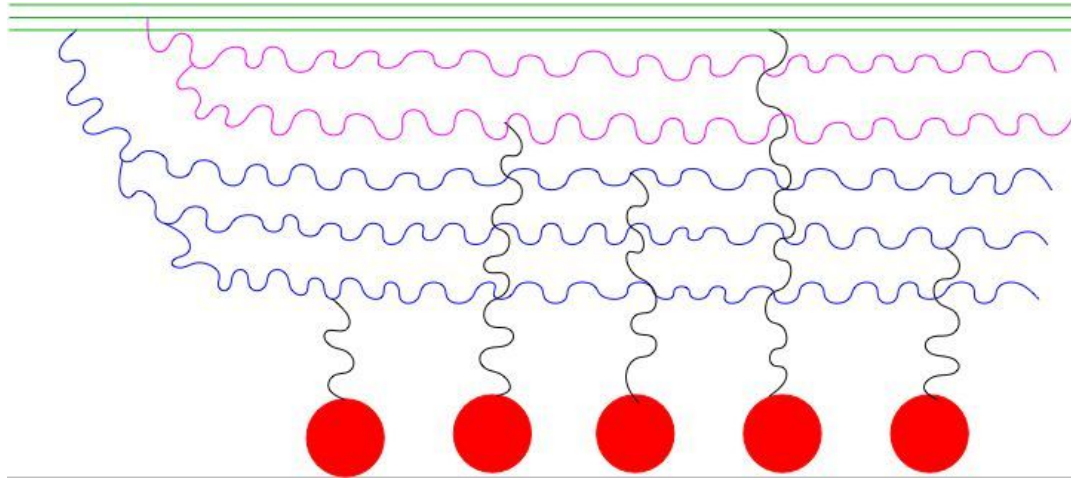
The ratio of pA and pp cross sections



At large rapidity y (small angle) expect
suppression of hard particles!

DK, Levin,
McLerran;
KKT; Albacete et al

Quantum regime at short distances: the effect of the classical background



1) Small x evolution leads to anomalous dimension

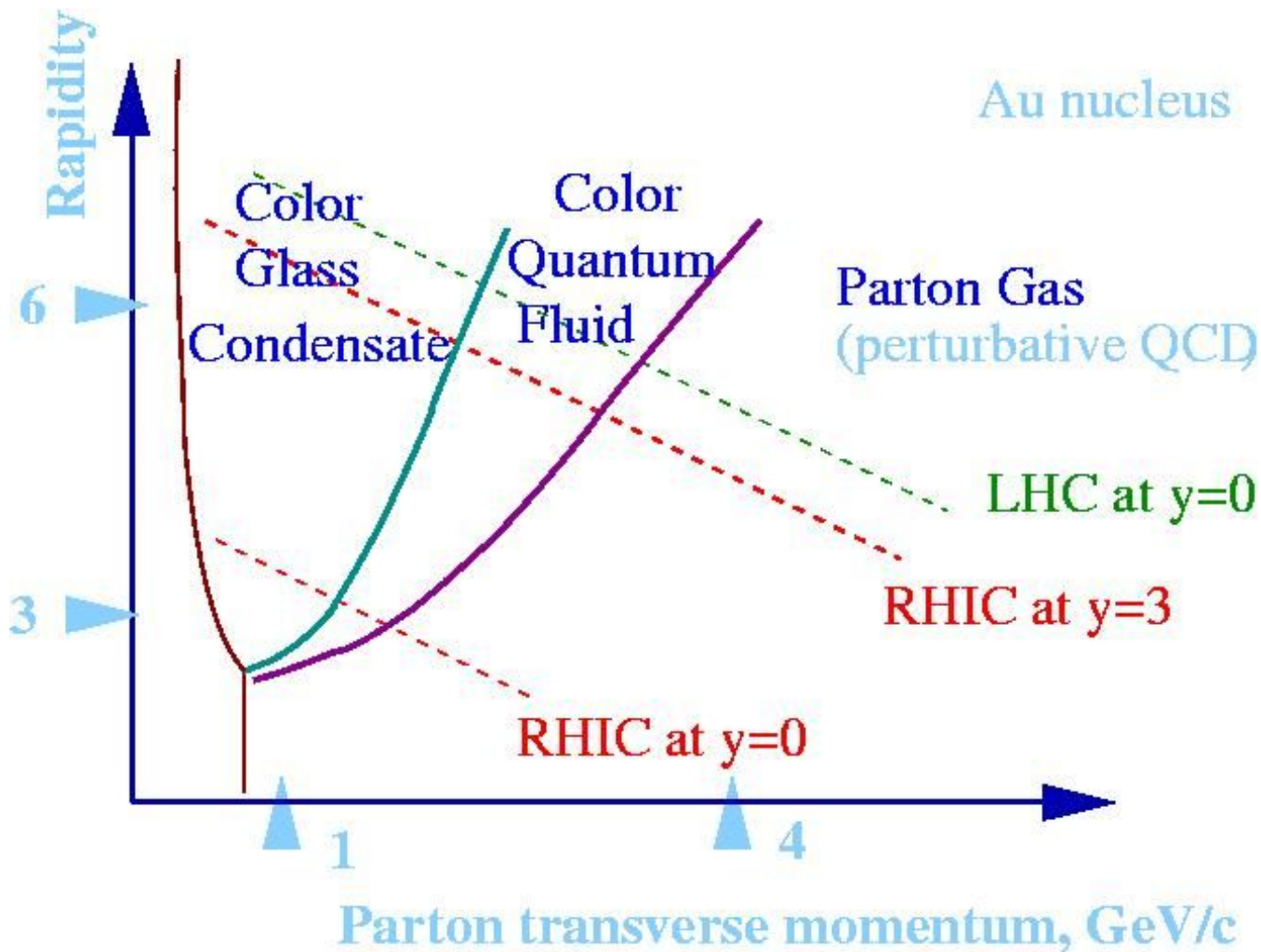
$$\frac{1}{Q^2} \rightarrow \left(\frac{1}{Q^2} \right)^\gamma \quad \gamma \simeq 1/2$$

2) Q_s is the only relevant dimensionful parameter in the CGC;
thus everything scales in the ratio Q_s^2/Q^2

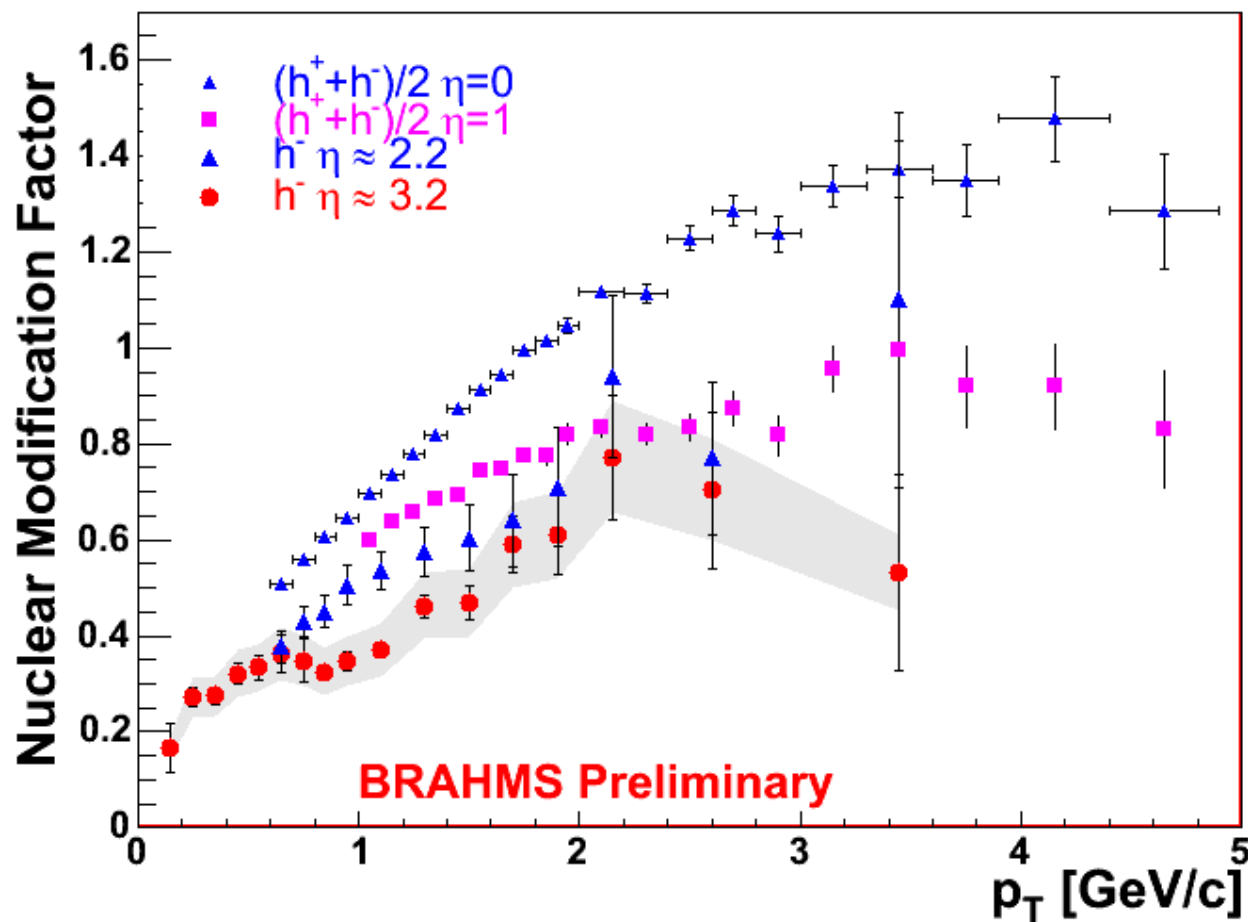
3) Since $Q_s^2 \sim A^{1/3}$ the A -dependence is changed

→ Expect high p_T suppression at sufficiently small x

Phase diagram of high energy QCD

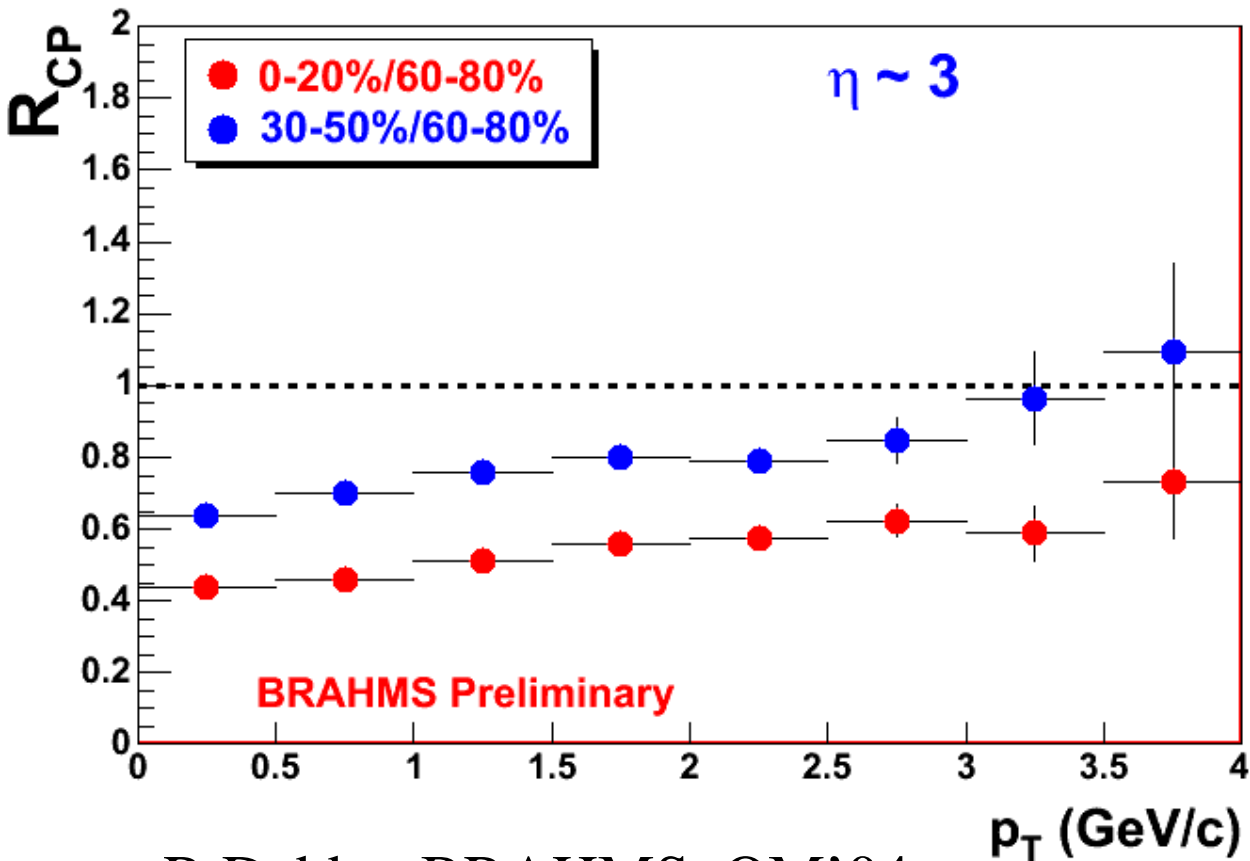


Nuclear Modification of Hard Parton Scattering

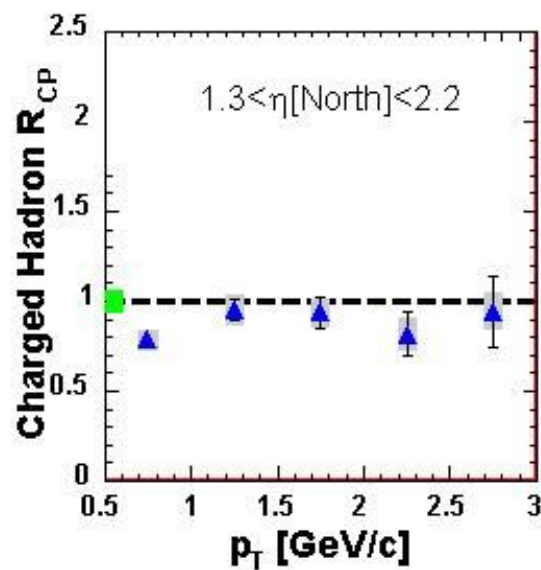
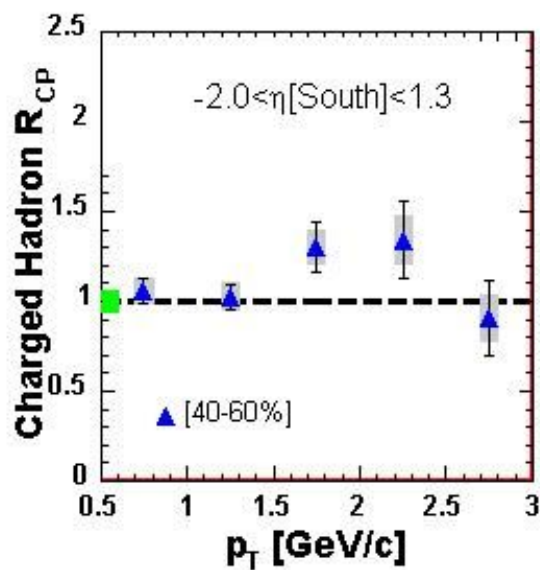
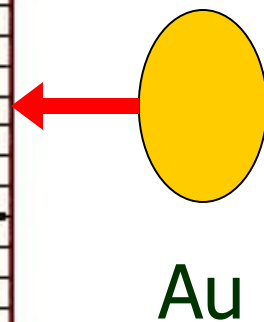
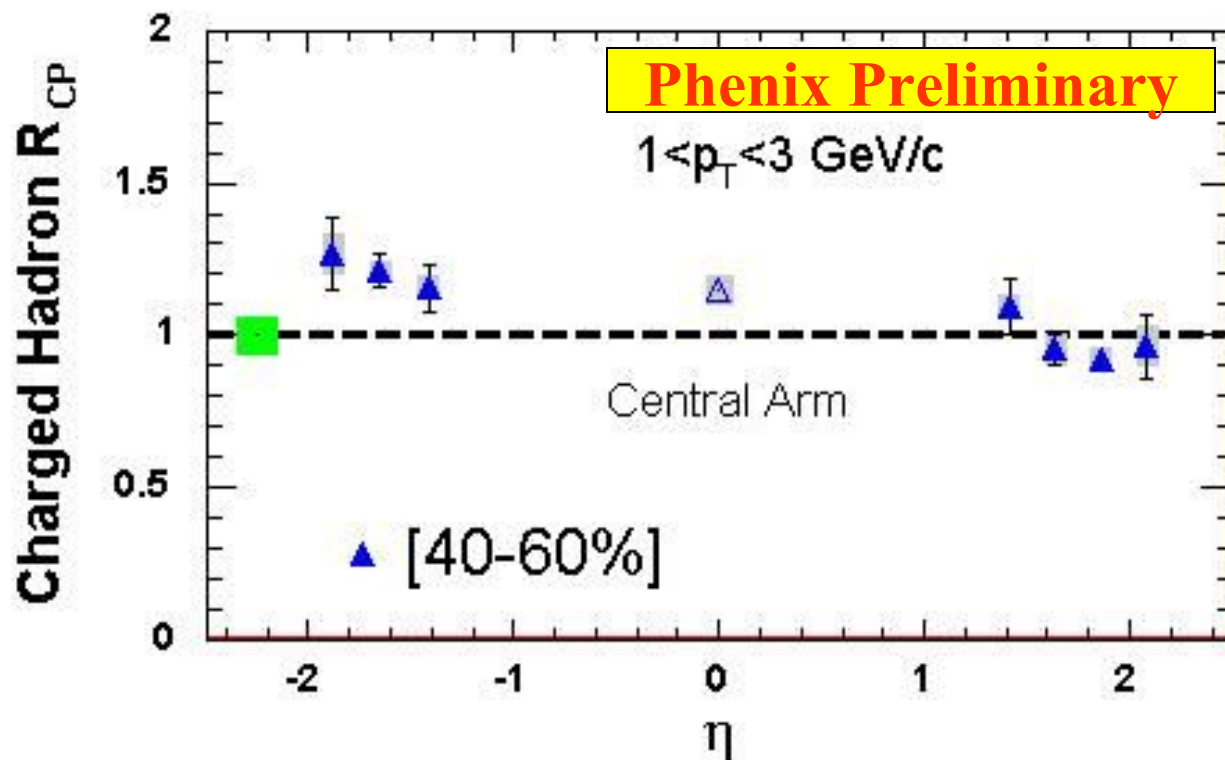
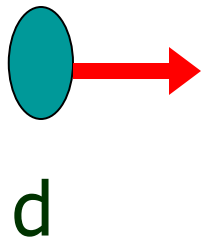


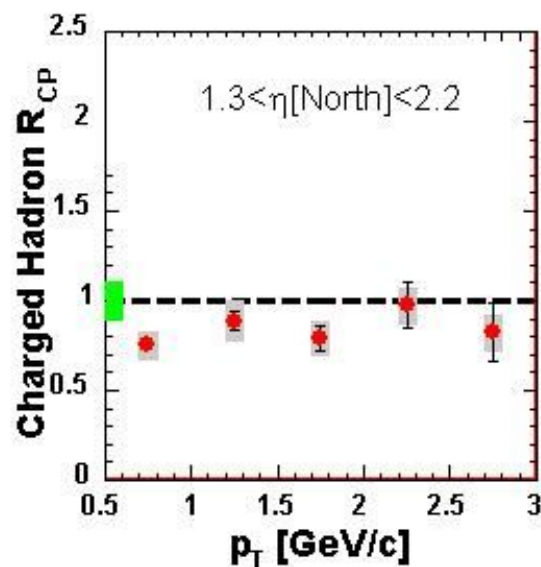
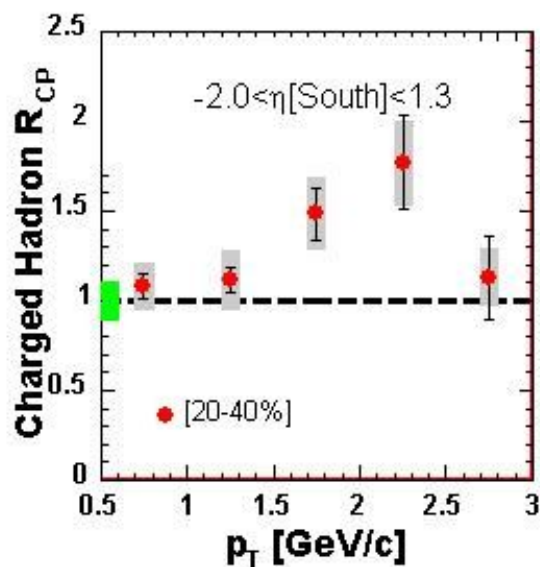
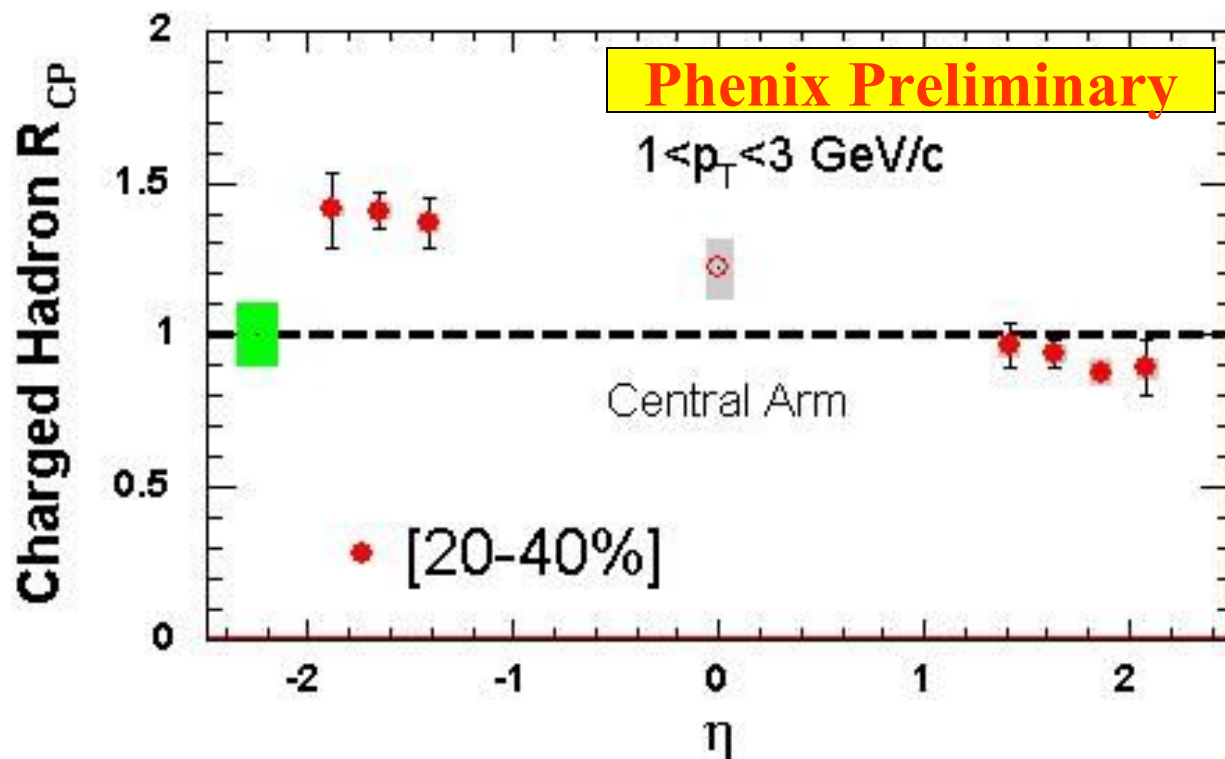
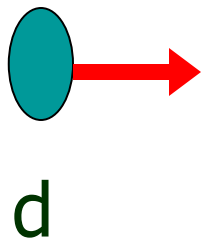
R. Debbé, BRAHMS,
QM'04

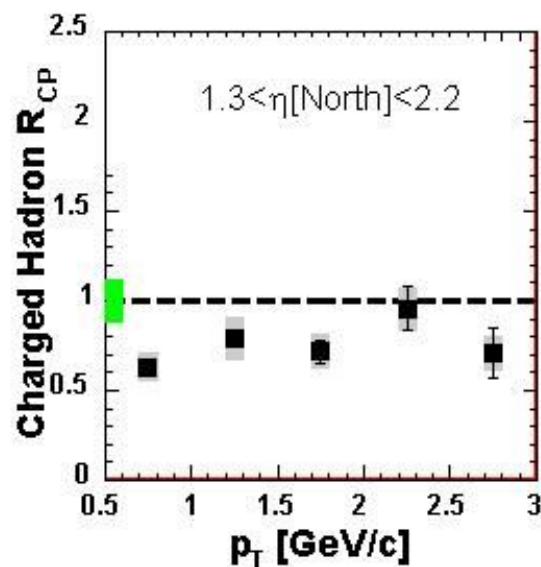
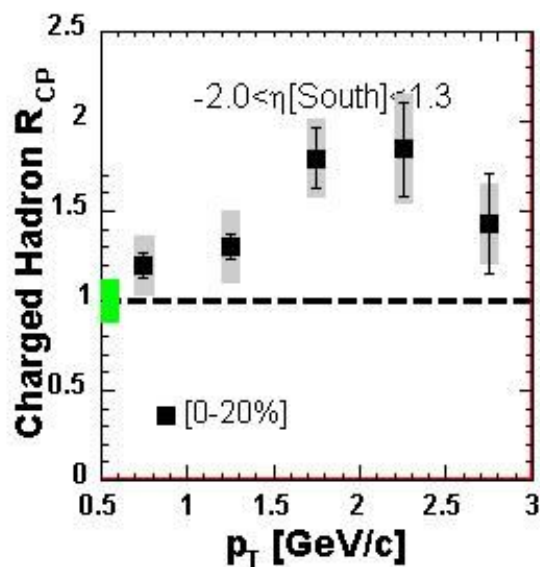
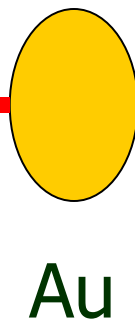
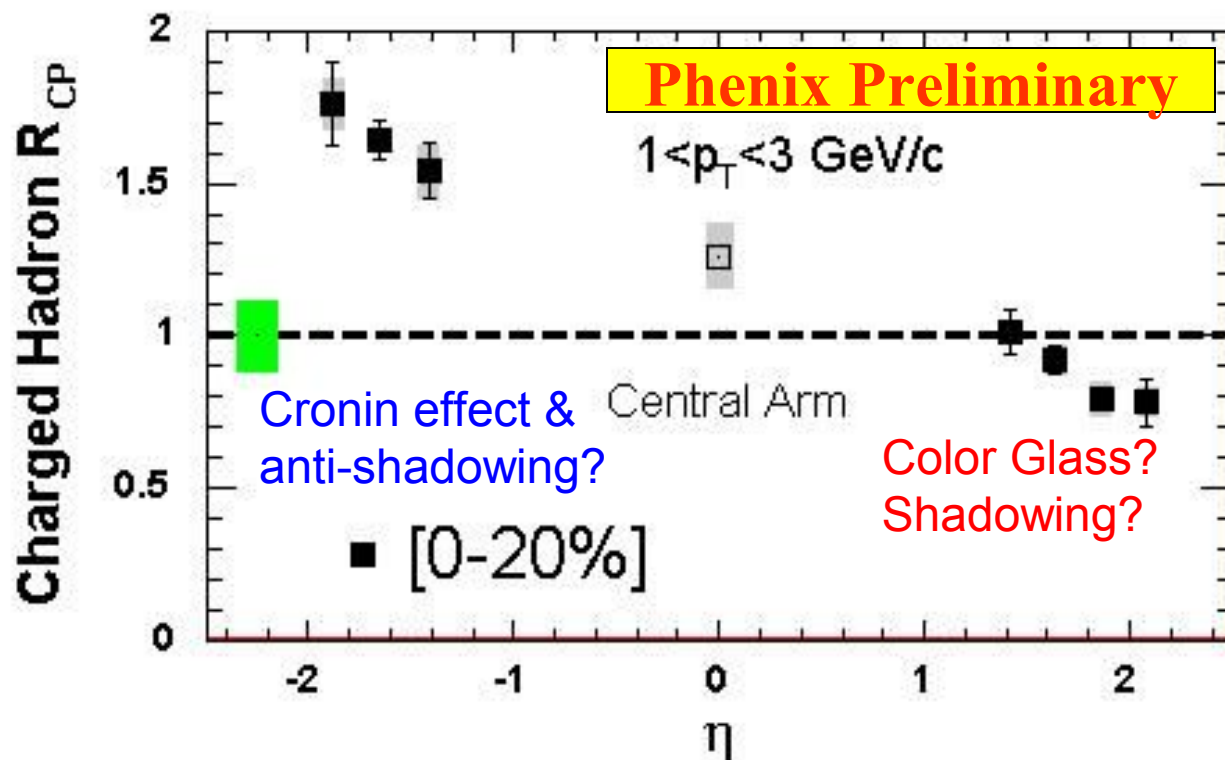
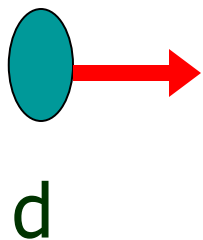
Centrality dependence



R. Debbé, BRAHMS, QM'04

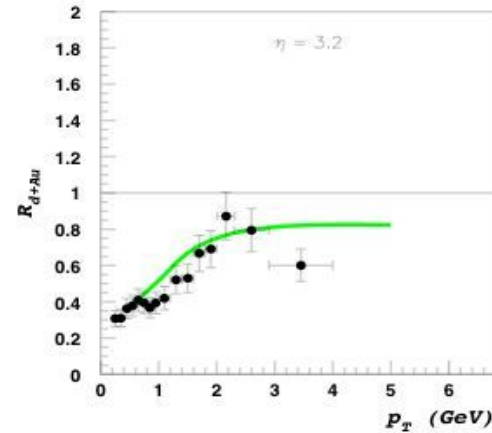
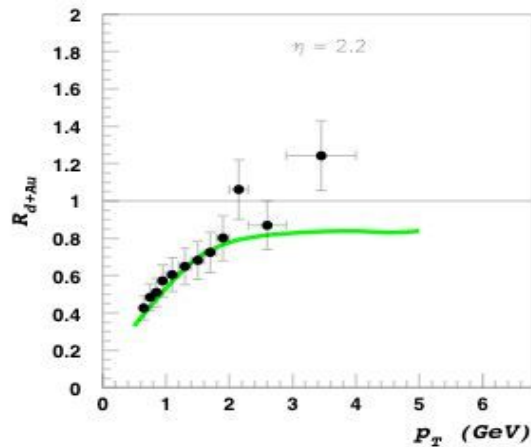
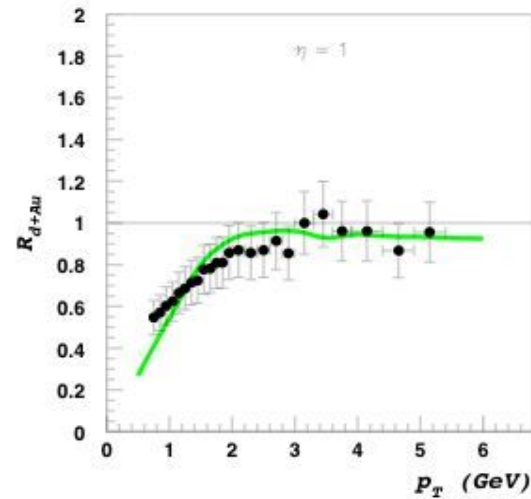
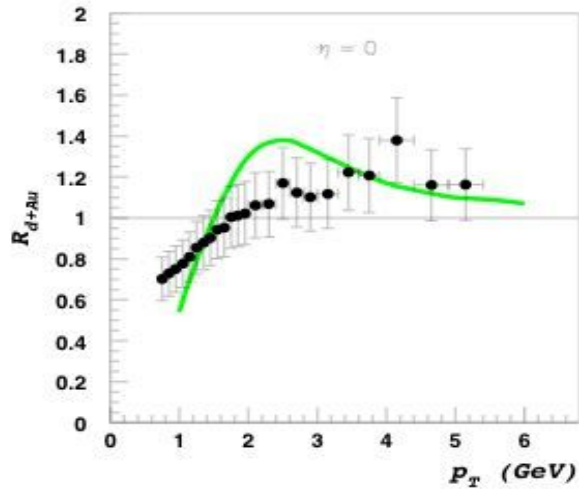




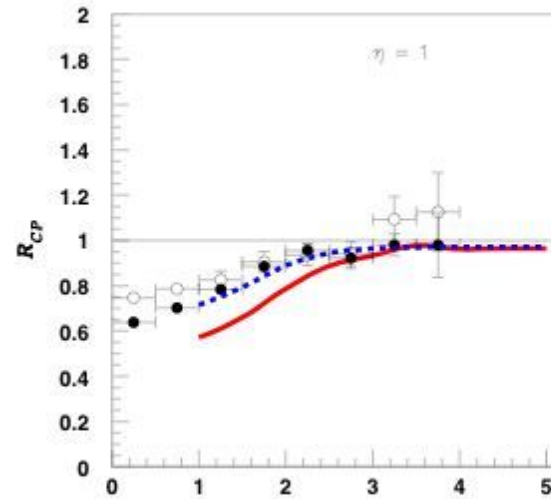
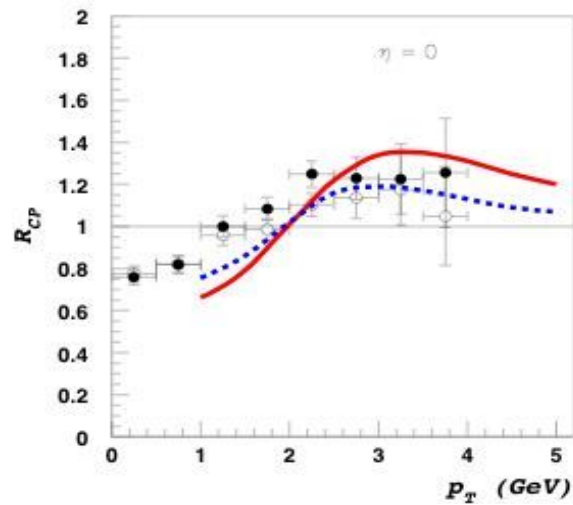


Color Glass Condensate: confronting the data

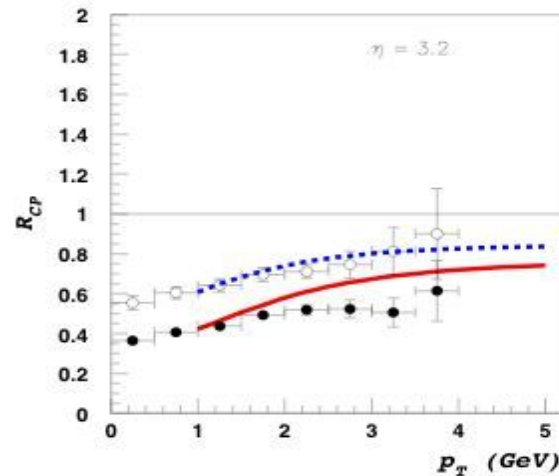
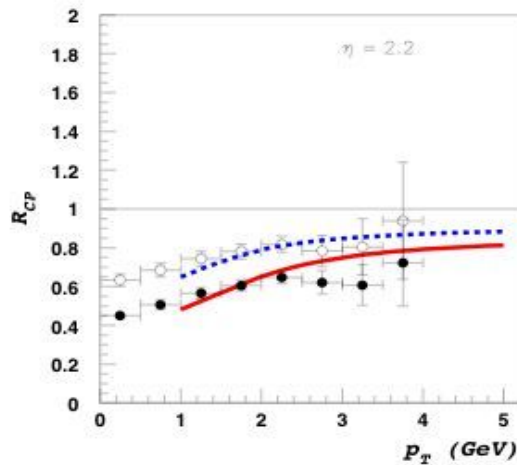
BRAHMS
data,
 $\eta = 0, 1,$
 $2.2, 3.2$



Color Glass Condensate: confronting the data

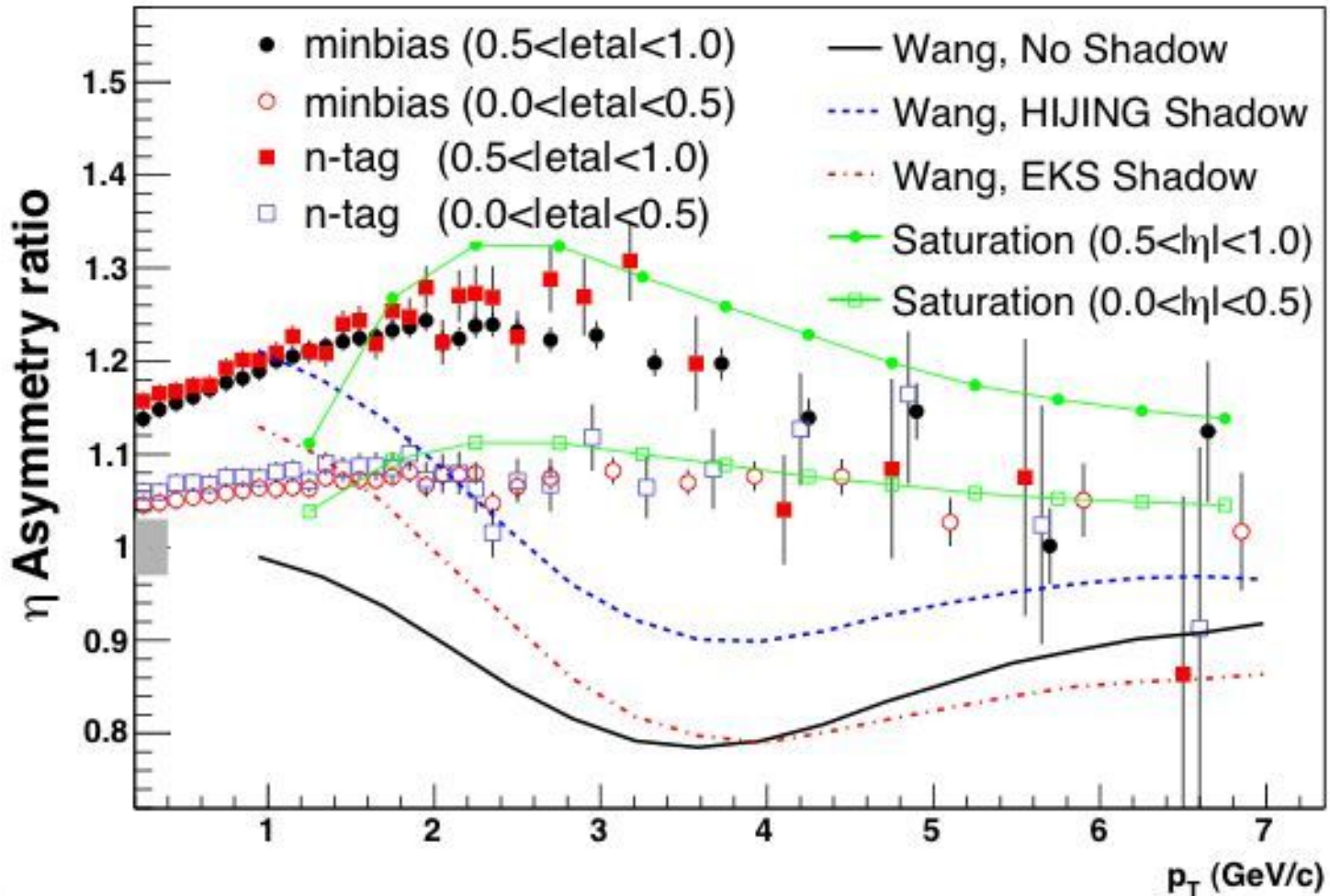


BRAHMS
data, R_{CP}
 $\eta = 0, 1,$
 $2.2, 3.2$



Confronting the RHIC data

STAR Collaboration



How dense is the produced matter?

The initial energy density achieved:

$$\epsilon_{initial} \simeq \frac{\langle k_t \rangle}{\tau_0} \frac{d^2 N}{d^2 b d\eta} \simeq Q_s^2 \frac{d^2 N}{d^2 b d\eta} \simeq 18 \text{ GeV}/\text{fm}^3$$

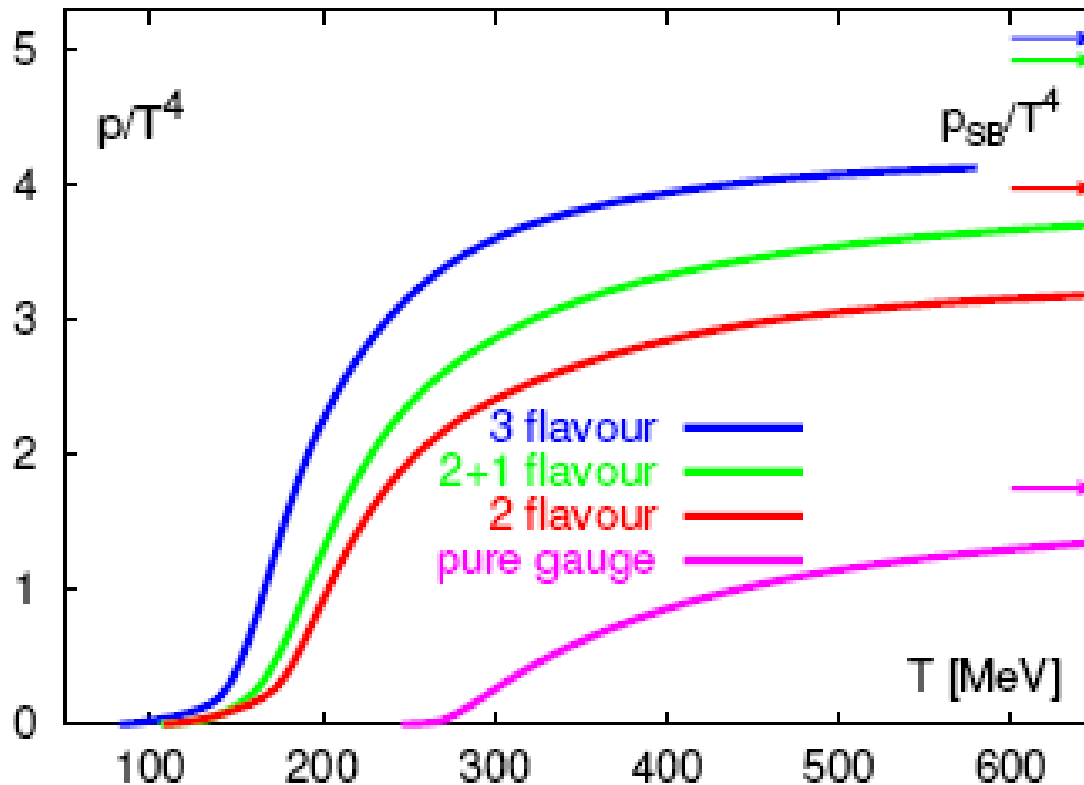
mean
transverse
momentum
of produced
gluons

gluon
formation
time

the density
of the gluons
in the transverse
plane and in
rapidity

about
100 times
nuclear
density !

What happens at such energy densities?



Phase transitions:

deconfinement

Chiral symmetry restoration

$U_A(1)$ restoration

Data from lattice QCD simulations F. Karsch et al

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

critical temperature $\sim 10^{12}$ K; cf temperature inside the Sun $\sim 10^7$ K

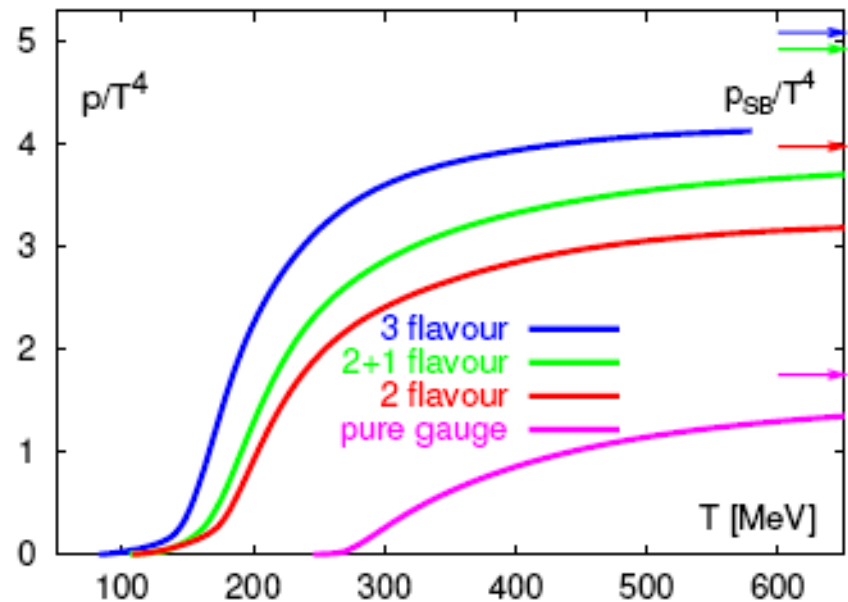
Strongly coupled QCD plasma

$$\langle \theta_{\mu}^{\mu} \rangle = \epsilon - 3P \neq 0$$

Interactions are important!
(strongly coupled
Quark-Gluon Plasma)

C. Bernard, T.Blum, '97

F. Karsch, P.Petreczky,



Coulomb potential in QCD

Spectral representation in the t-channel: $V(R) = \sum_m \sigma(m^2) \frac{\exp(-mR)}{R}$

$$\text{Disc}_t \left(\begin{array}{c} | \text{---} \text{---} \text{---} | \\ \text{---} \text{---} \text{---} \\ | \end{array} \right) = \begin{array}{c} | \text{---} \text{---} \text{---} | \\ \text{---} \text{---} \text{---} \\ | \end{array} \propto \sigma \left(\left(\begin{array}{c} | \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ | \end{array} \right) \right)$$

The diagram on the left shows a vertical line on the left and a vertical line on the right, connected by a dashed line. A diamond-shaped loop is attached to the dashed line, with a vertical dashed line passing through its center. The label 's' is below the loop, and 't' is to the right of the dashed line. The diagram in the middle is identical but with a vertical dashed line through the center of the loop. The diagram on the right is a vertical line on the left and a vertical line on the right, connected by a dashed line, with a triangle pointing to the right attached to the dashed line.

If physical particles can be produced (positive spectral density), then unitarity implies screening

$$\left\{ \frac{d \alpha_s^{-1}(R)}{d \ln R} \right\}^{\text{phys}} \propto \frac{1}{3} N + \frac{2}{3} n_f$$

Gluons Quarks
(transverse)

Coulomb potential in QCD - II

Missing non-Abelian effect: instantaneous Coulomb exchange dressed by (zero modes of) transverse gluons

$$\sum_n \left[0 + \perp \rightarrow 0 \right]^n = \left| \begin{array}{c} 0 \quad 0 \\ \text{---} \bullet \text{---} \\ \text{wavy } \perp \end{array} \right| + \left| \begin{array}{c} 0 \quad 0 \quad 0 \\ \text{---} \bullet \text{---} \bullet \text{---} \\ \text{wavy } \perp \quad \text{wavy } \perp \end{array} \right| + \dots$$

Negative sign

(the shift of the ground level

due to perturbations - **unstable vacuum!**):

$$\delta E \equiv E - E_0 = \sum_n \frac{|\langle 0 | \delta V | n \rangle|^2}{E_0 - E_n} < 0$$

Anti-screening

$$\left\{ \frac{d \alpha_s^{-1}(R)}{d \ln R} \right\}^{\text{stat}} \propto -4N$$

Screening in Quark-Gluon Plasma

These diagrams are enhanced at finite temperature:
(scattering off thermal gluons and quarks)

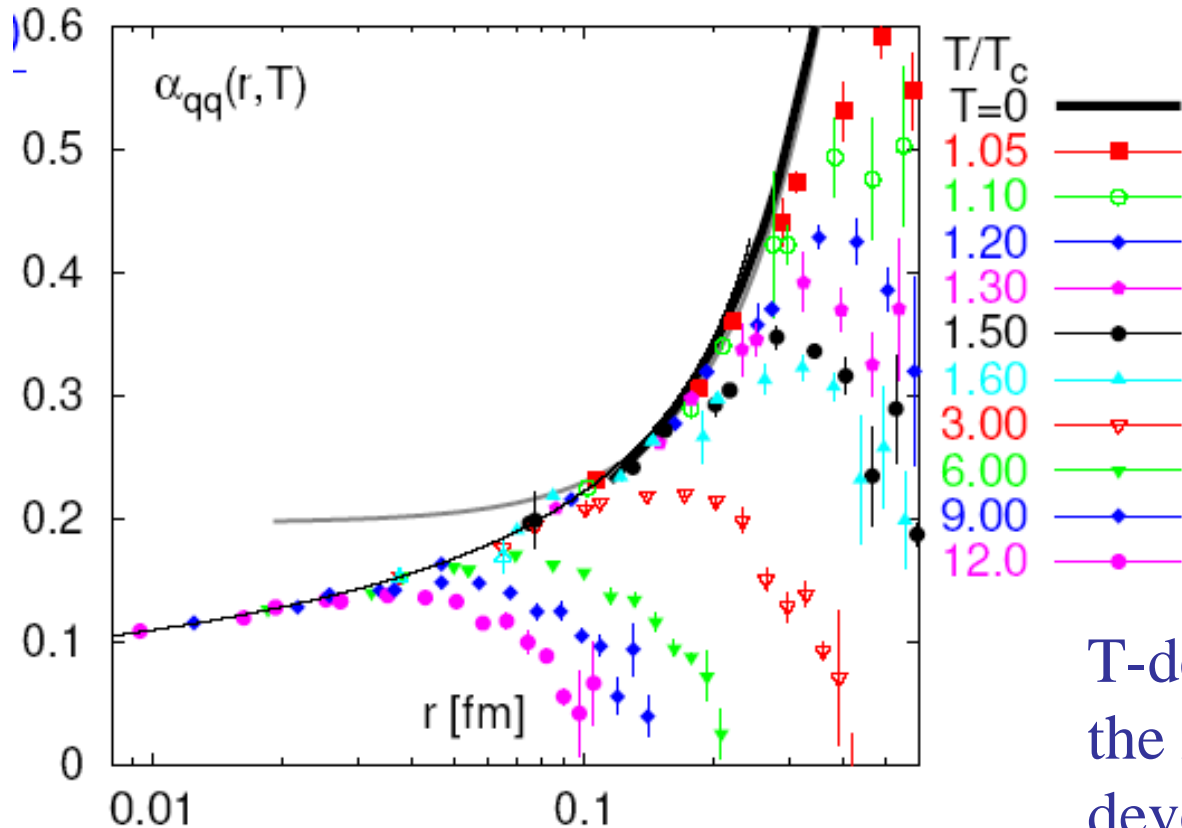
$$\text{Disc}_t \left(\text{---} \text{---} \text{---} \right) = \text{---} \text{---} \text{---} \propto \sigma \left(\text{---} \text{---} \text{---} \right)$$

This diagram is not:

$$\sum_n \left[0 + \perp \rightarrow 0 \right]^n = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots$$

=> At high enough T, screening wins

Screening in QGP



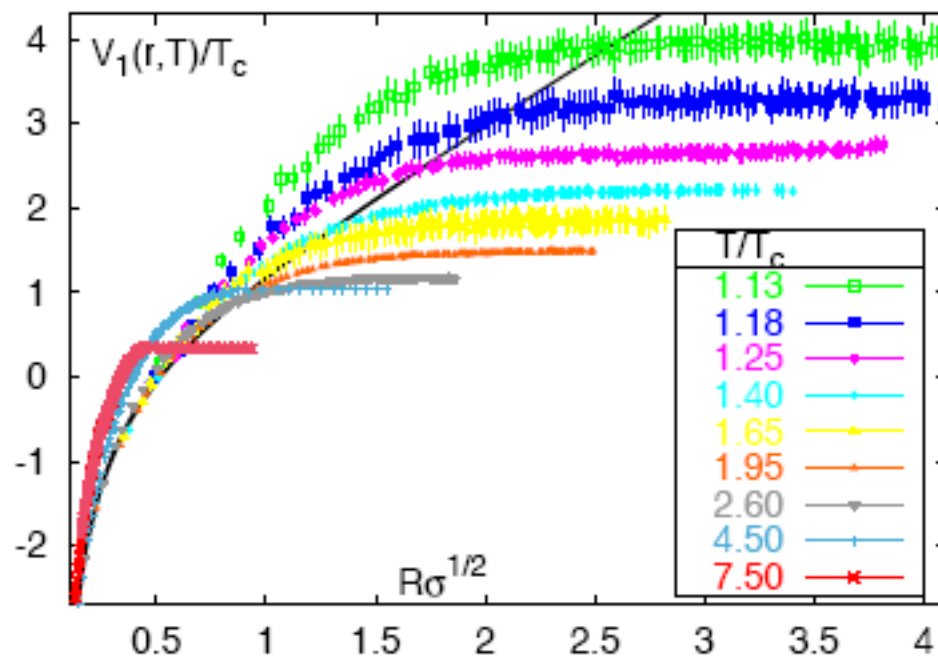
Strong force is
screened by
the presence of
thermal gluons
and quarks

T-dependence of
the running coupling
develops in
the non-perturbative region
at $T < 3 T_c$; $\Delta E/T > 1$

F.Karsch et al

- “cold” plasma

Heavy quark internal energy above T_c



O.Kaczmarek, F. Karsch, P.Petreczky,
F. Zantow, hep-lat/0309121

J/Ψ as a probe:

Volume 178, number 4

PHYSICS LETTERS B

9 October 1986

J/ψ SUPPRESSION BY QUARK–GLUON PLASMA FORMATION ☆

T. MATSUI

*Center for Theoretical Physics, Laboratory for Nuclear Science, Massachusetts Institute of Technology,
Cambridge, MA 02139, USA*

and

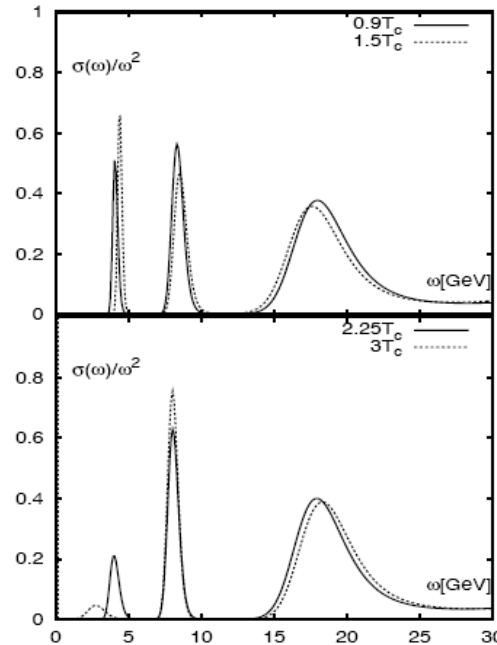
H. SATZ

*Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld, Fed. Rep. Germany
and Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

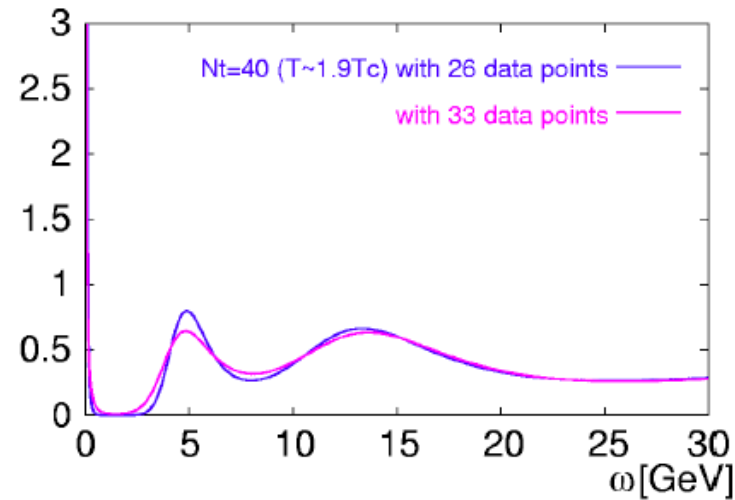
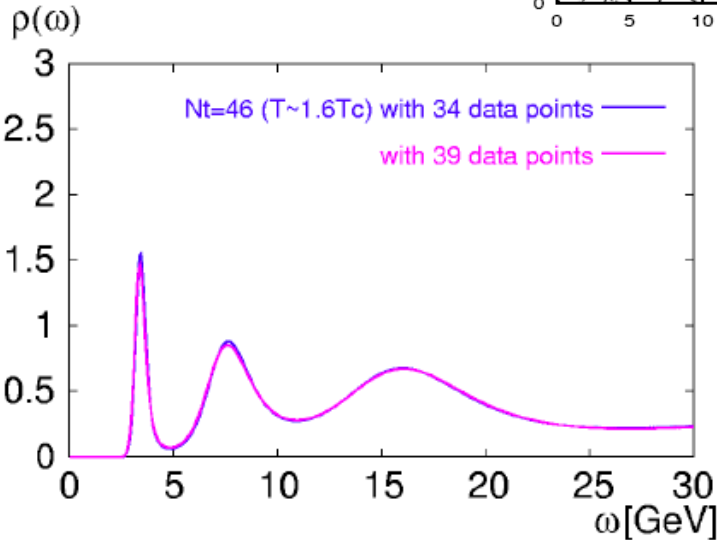
Received 17 July 1986

If high energy heavy ion collisions lead to the formation of a hot quark–gluon plasma, then colour screening prevents $c\bar{c}$ binding in the deconfined interior of the interaction region. To study this effect, the temperature dependence of the screening radius, as obtained from lattice QCD, is compared with the J/ψ radius calculated in charmonium models. The feasibility to detect this effect clearly in the dilepton mass spectrum is examined. It is concluded that J/ψ suppression in nuclear collisions should provide an unambiguous signature of quark–gluon plasma formation.

J/Ψ above T_c: alive and well?



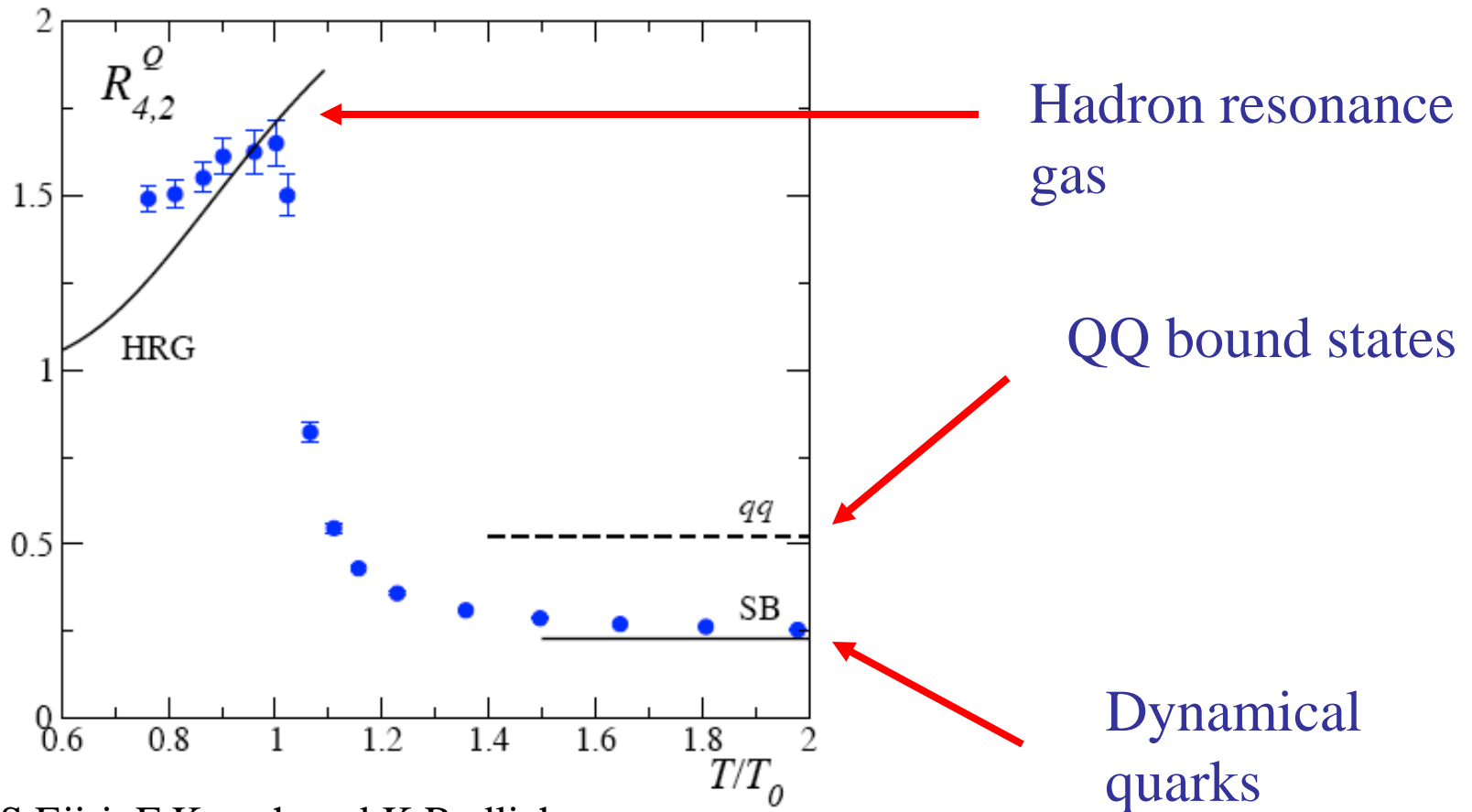
S.Datta,
F.Karsch,
P.Petreczky,
I.Wetzerke



M.Asakawa,
T.Hatsuda

What are the dynamical degrees of freedom in the plasma?

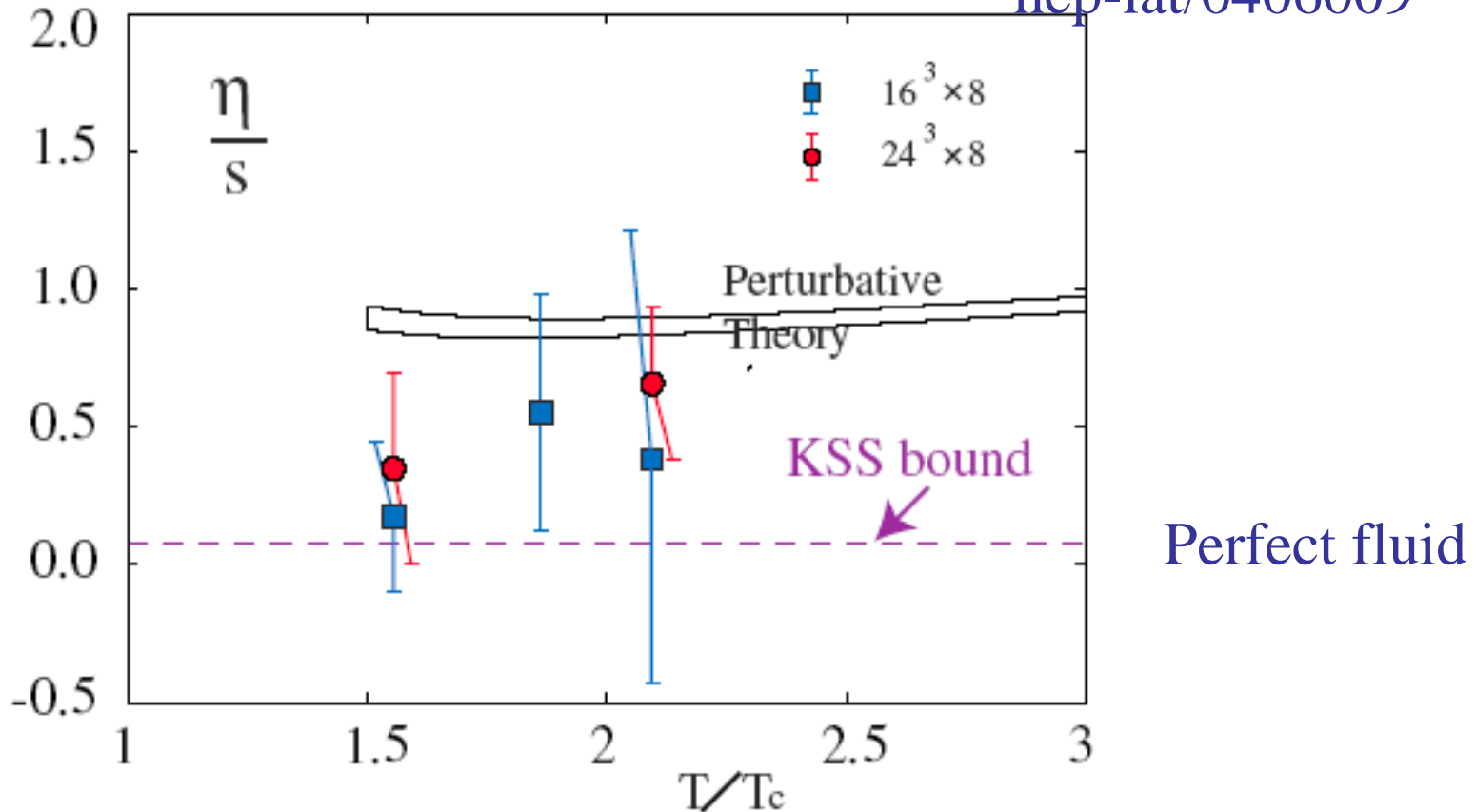
Let us look at the charge fluctuations in sQGP:



S.Ejiri, F.Karsch and K.Redlich

Viscosity of sQGP

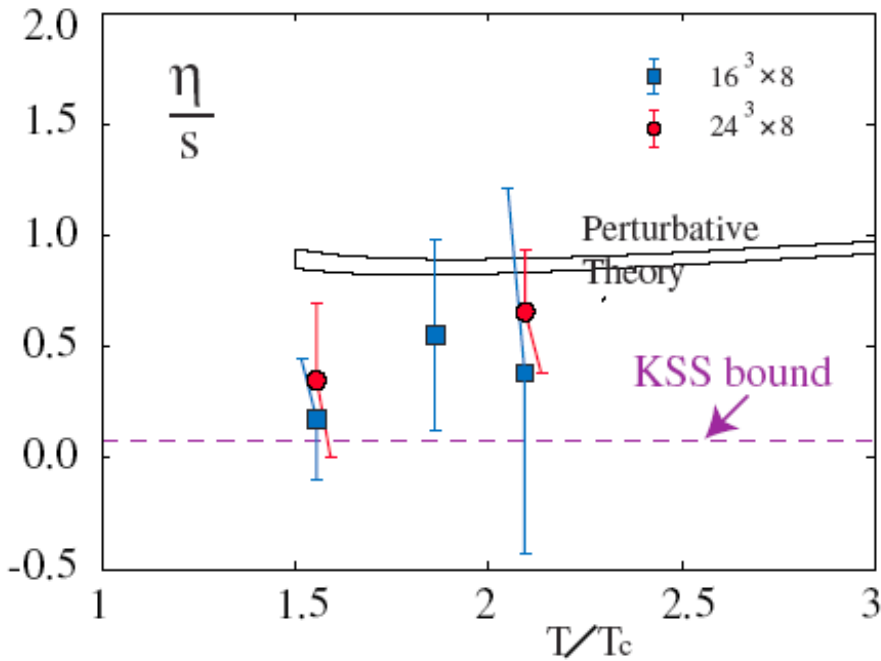
A.Nakamura and S.Sakai,
hep-lat/0406009



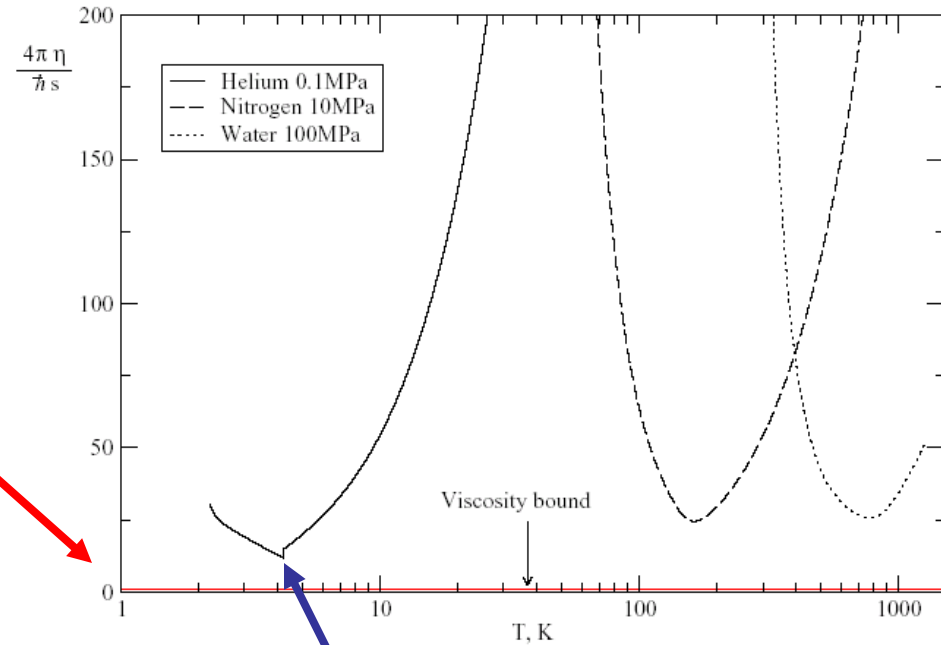
KSS bound:

strongly coupled SUSY QCD = classical supergravity

Quark-gluon plasma: more fluid than water?



A.Nakamura and S.Sakai,
hep-lat/0406009



Superfluid
helium

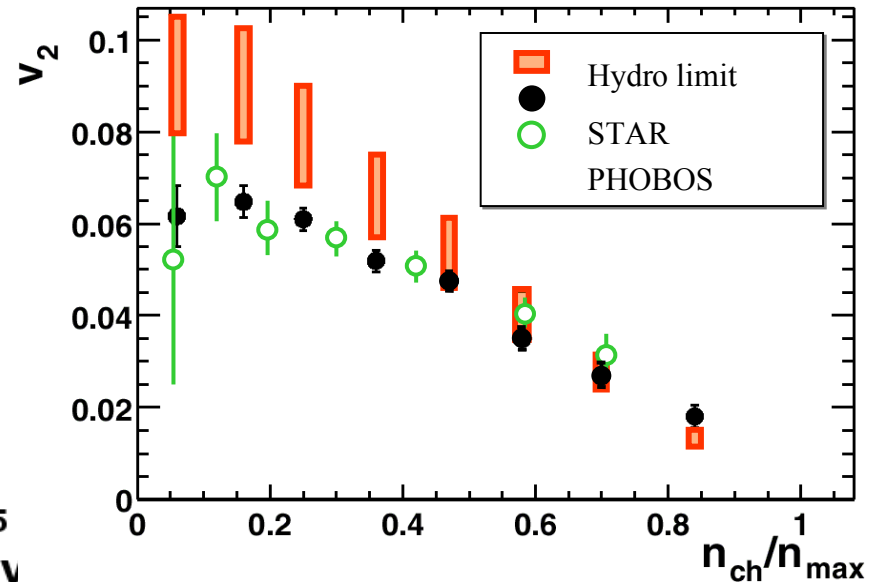
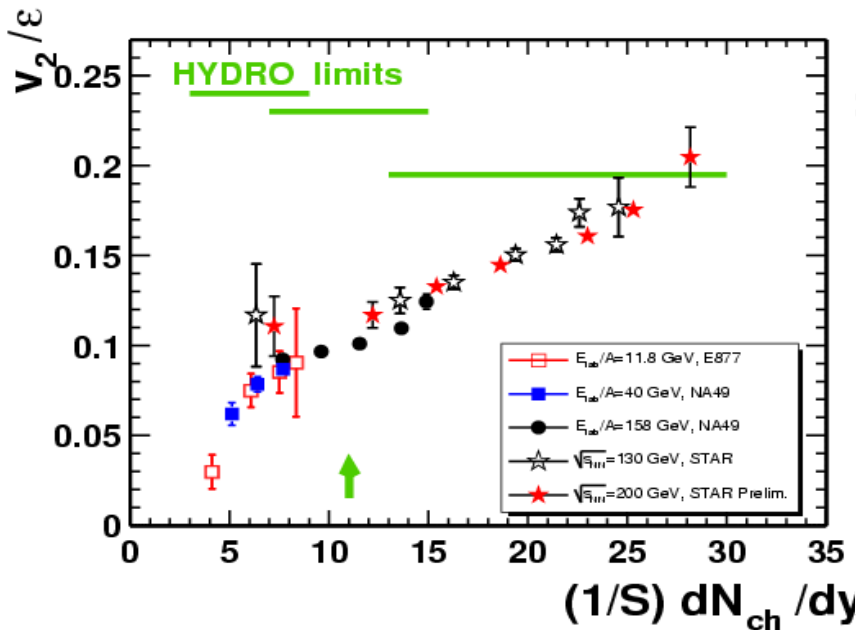
Do we see the QCD matter at RHIC ?

I. Collective flow =>

Au-Au collisions at RHIC produce strongly interacting matter

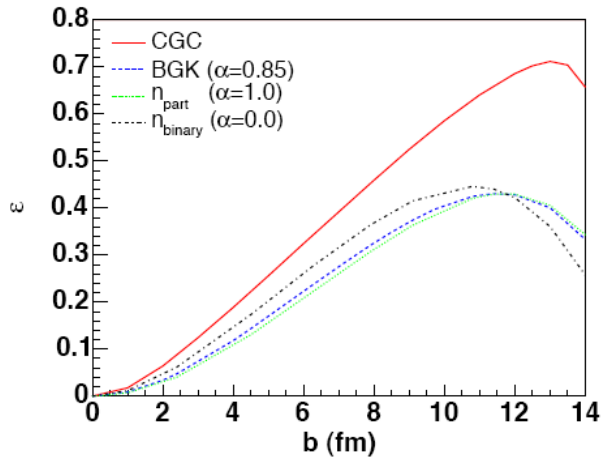
shear viscosity - to - entropy ratio

hydrodynamics: QCD liquid is **more fluid than water**

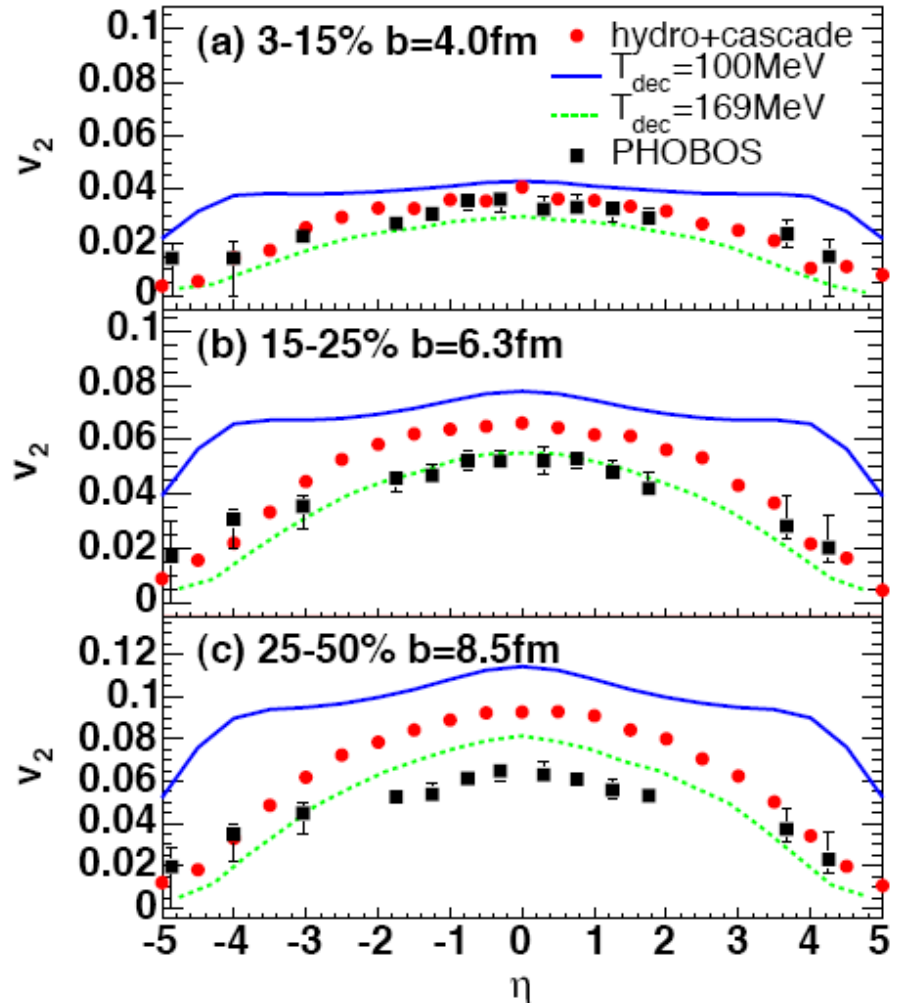
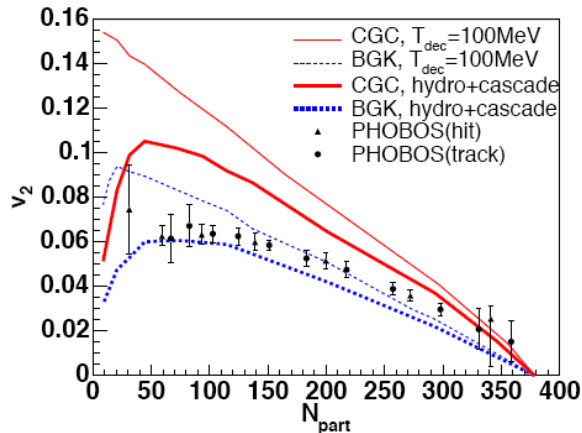


How small is really the viscosity?

CGC initial conditions lead to larger ellipticity,



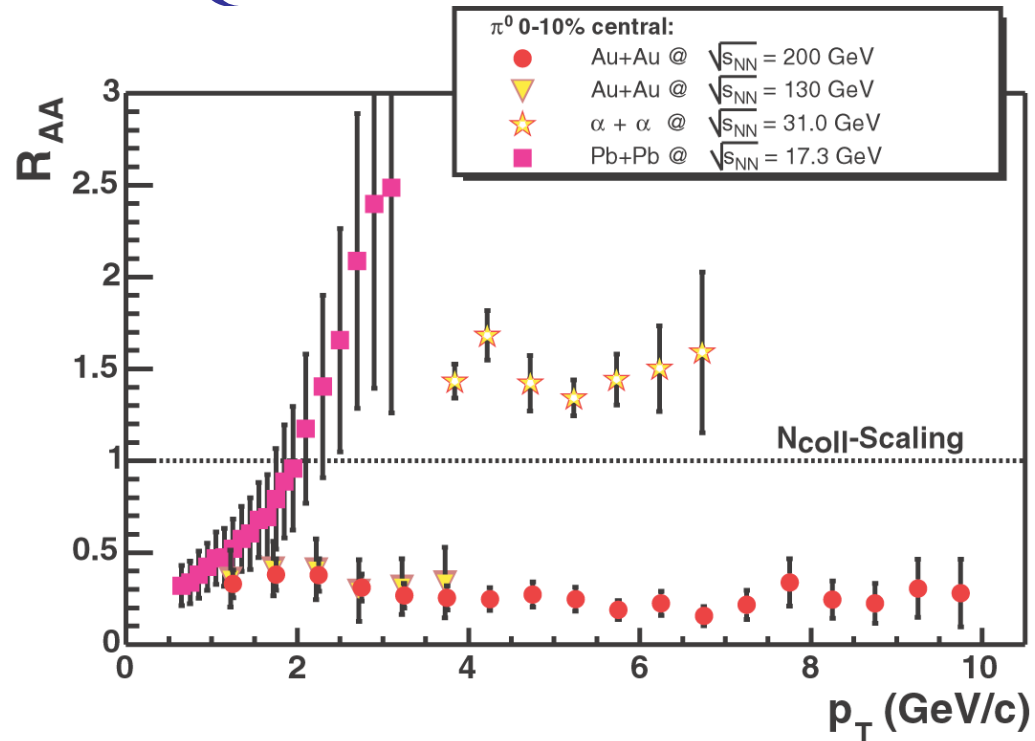
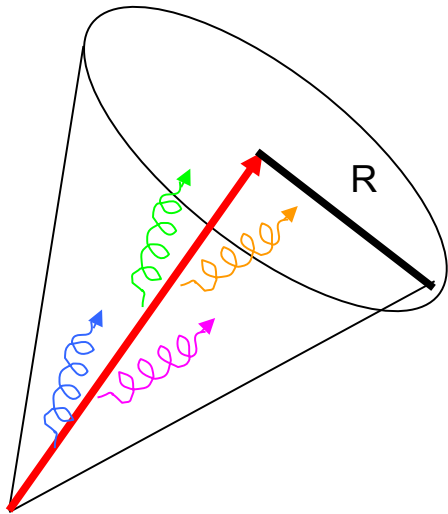
require some viscous effects:



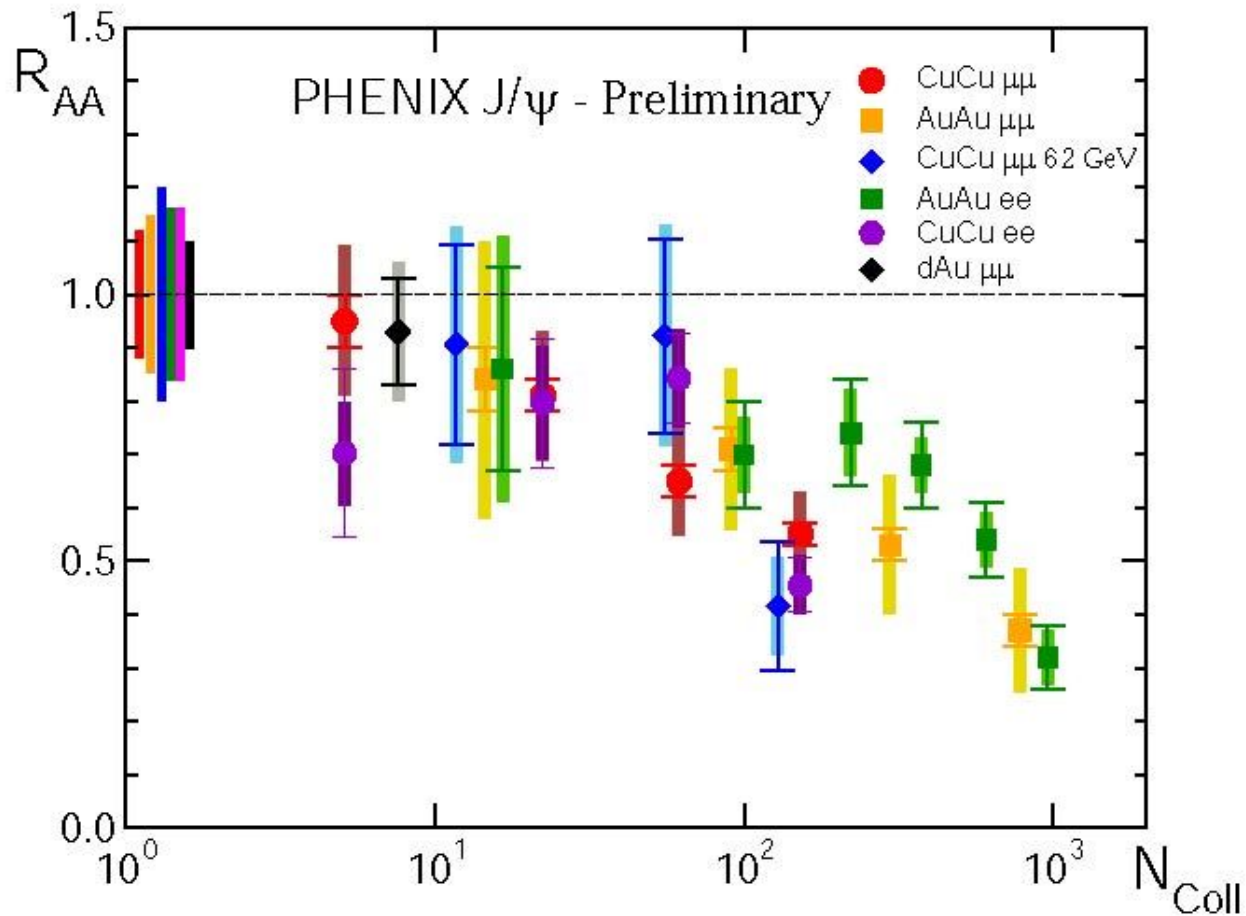
Do we see the QCD matter at RHIC?

II. Suppression of high p_T particles =>

consistent with the predicted parton energy loss from induced gluon radiation in dense QCD matter



J/ψ suppression at RHIC



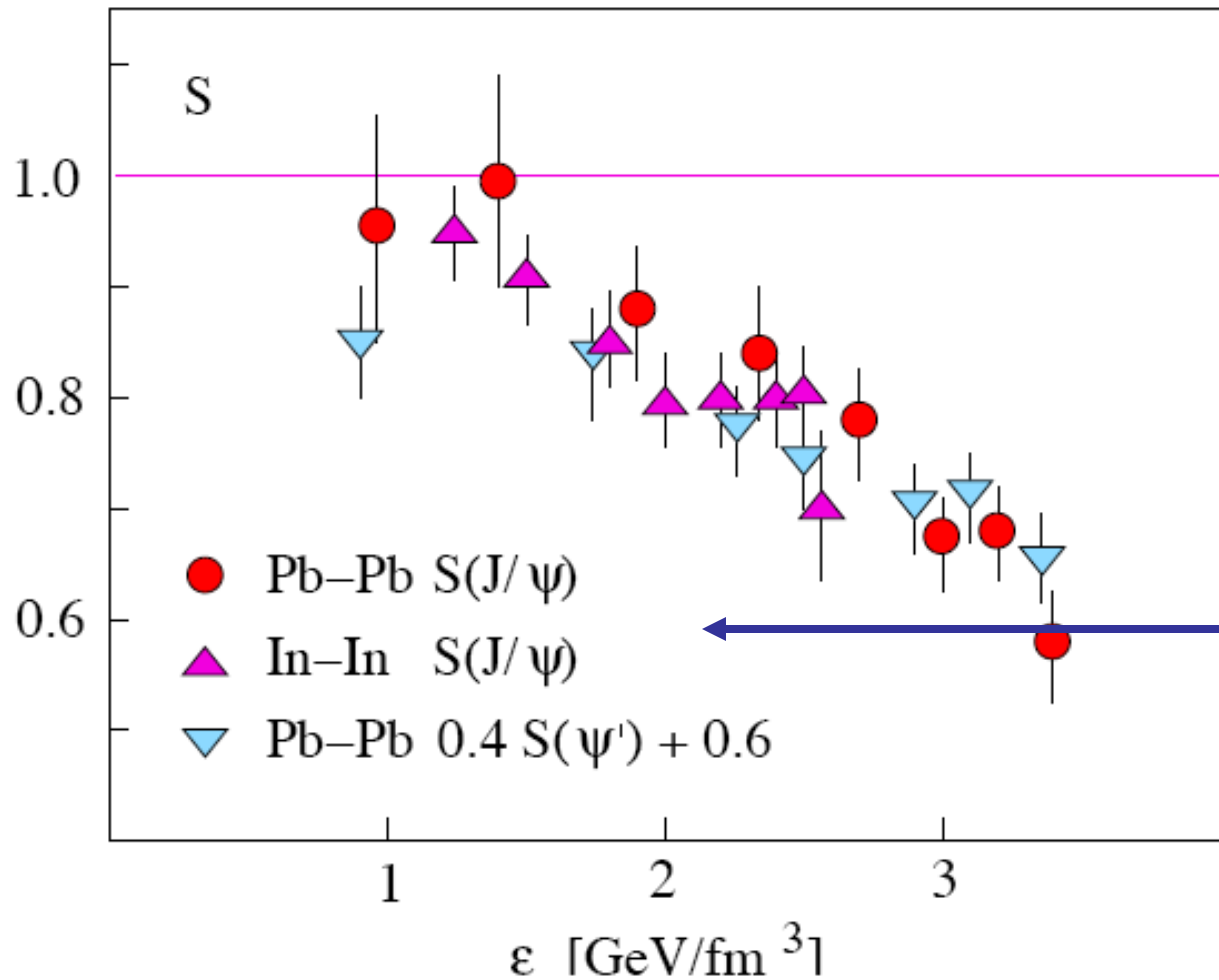
“same as at SPS”?

Sequential charmonium dissociation?

Both the absence of J/ψ suppression up to $\sim 2 T_c$ in the lattice QCD data and the apparent similarity of the magnitude of suppression at RHIC and SPS are puzzling;

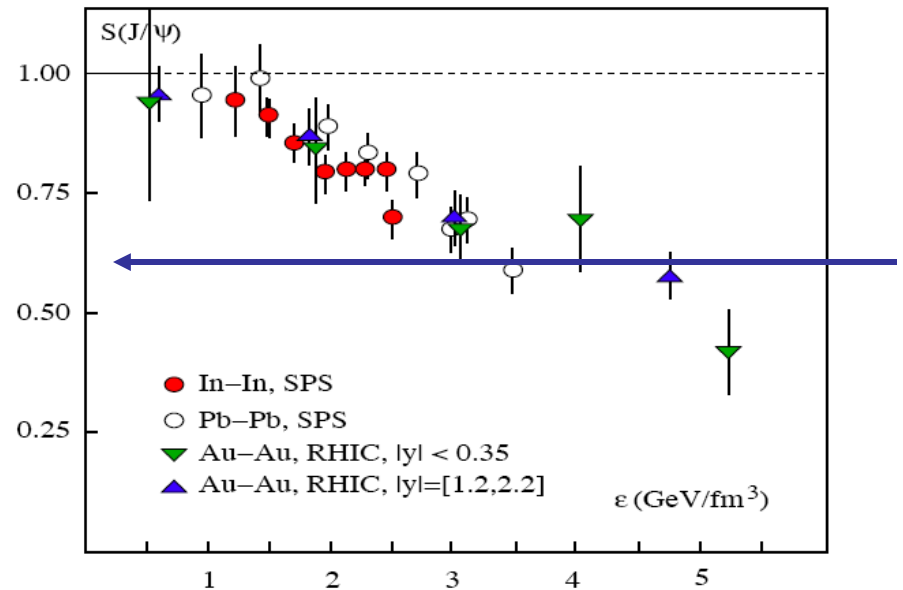
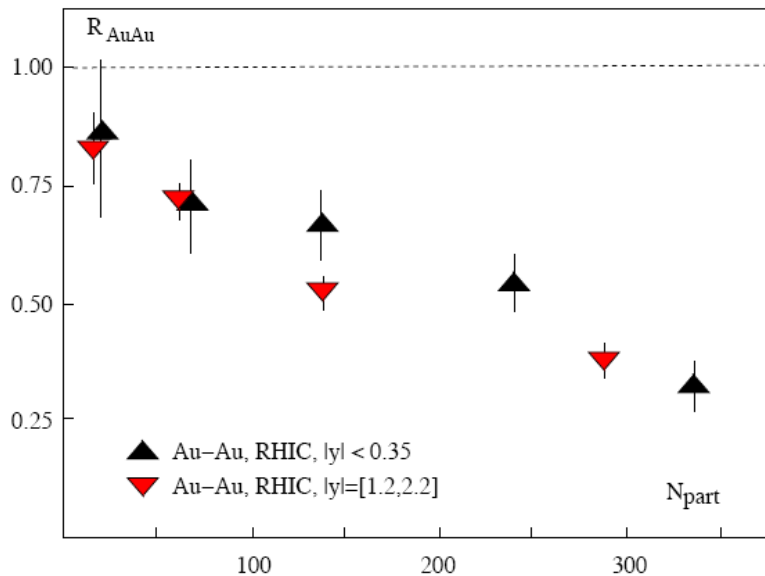
However, the two puzzles may be consistent with each other

Is there a “direct” J/ψ suppression at SPS?



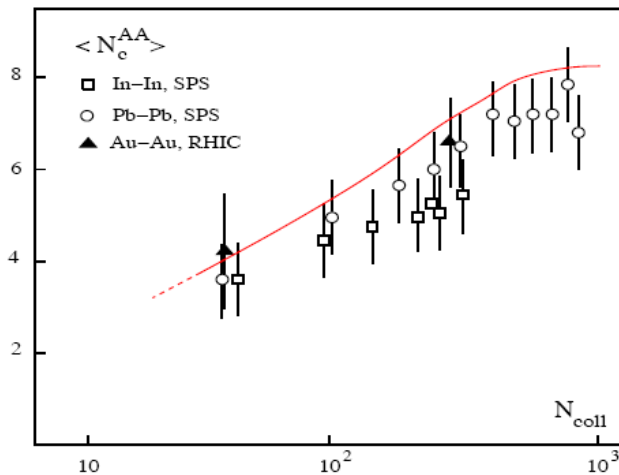
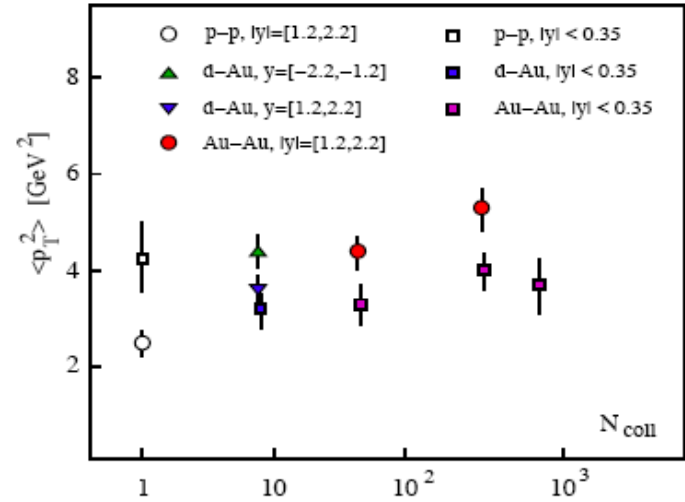
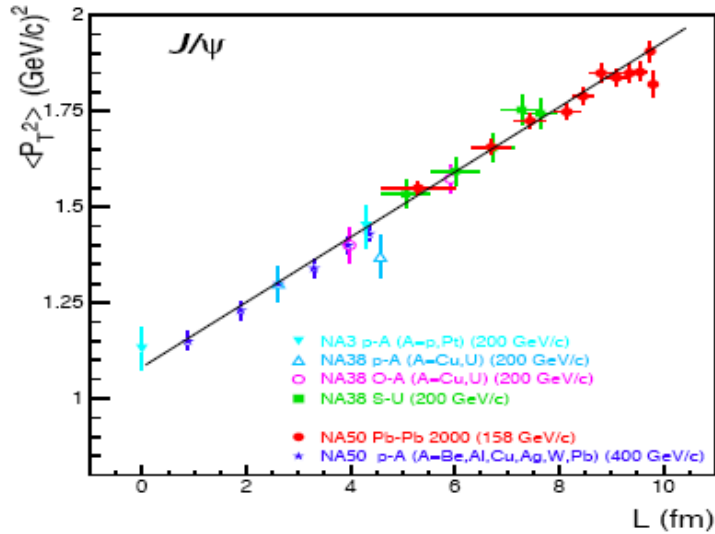
~ 40%
of observed
 J/ψ 's
originate
from χ and ψ
decays;
they should
be gone above T_c

Is there a “direct” J/ψ suppression at RHIC?



Data: PHENIX, NA50, NA60

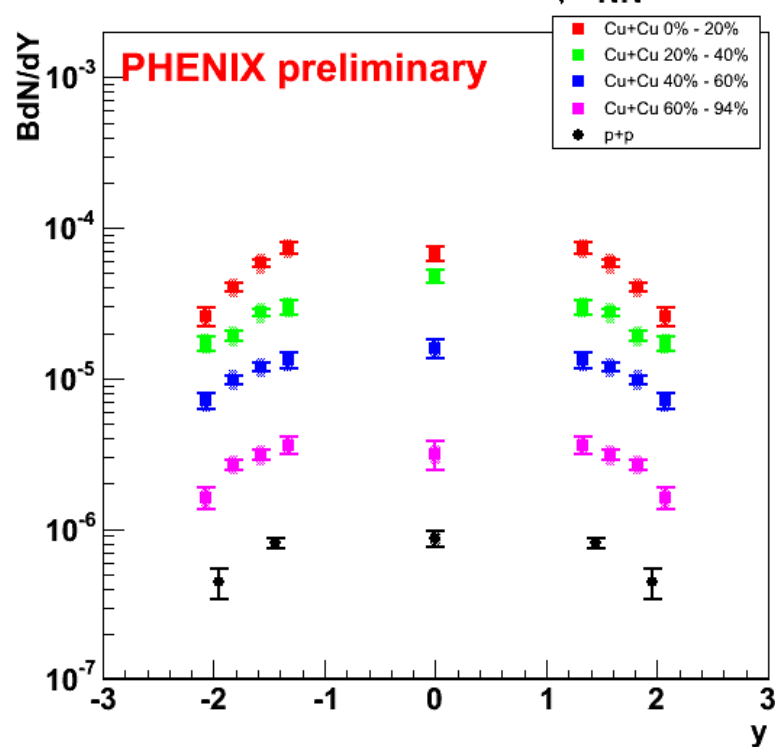
Transverse momentum distributions



Glauber model analysis

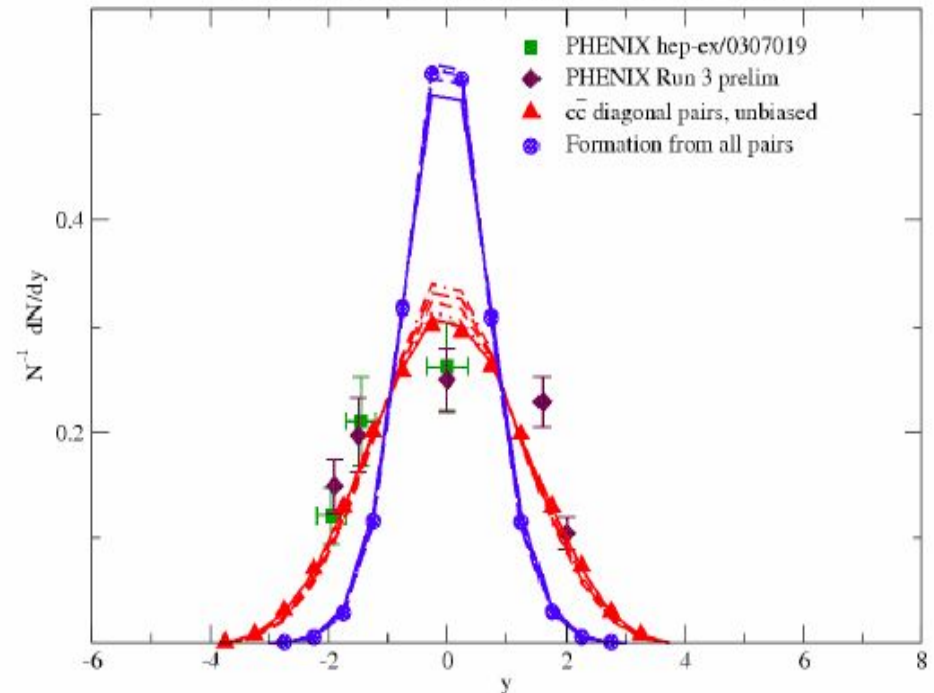
Recombination of charm quarks?

J/ψ BdN/dY - Cu+Cu @ $\sqrt{S_{NN}}=200\text{GeV}$



J/ψ Formation in AA Interactions at RHIC200

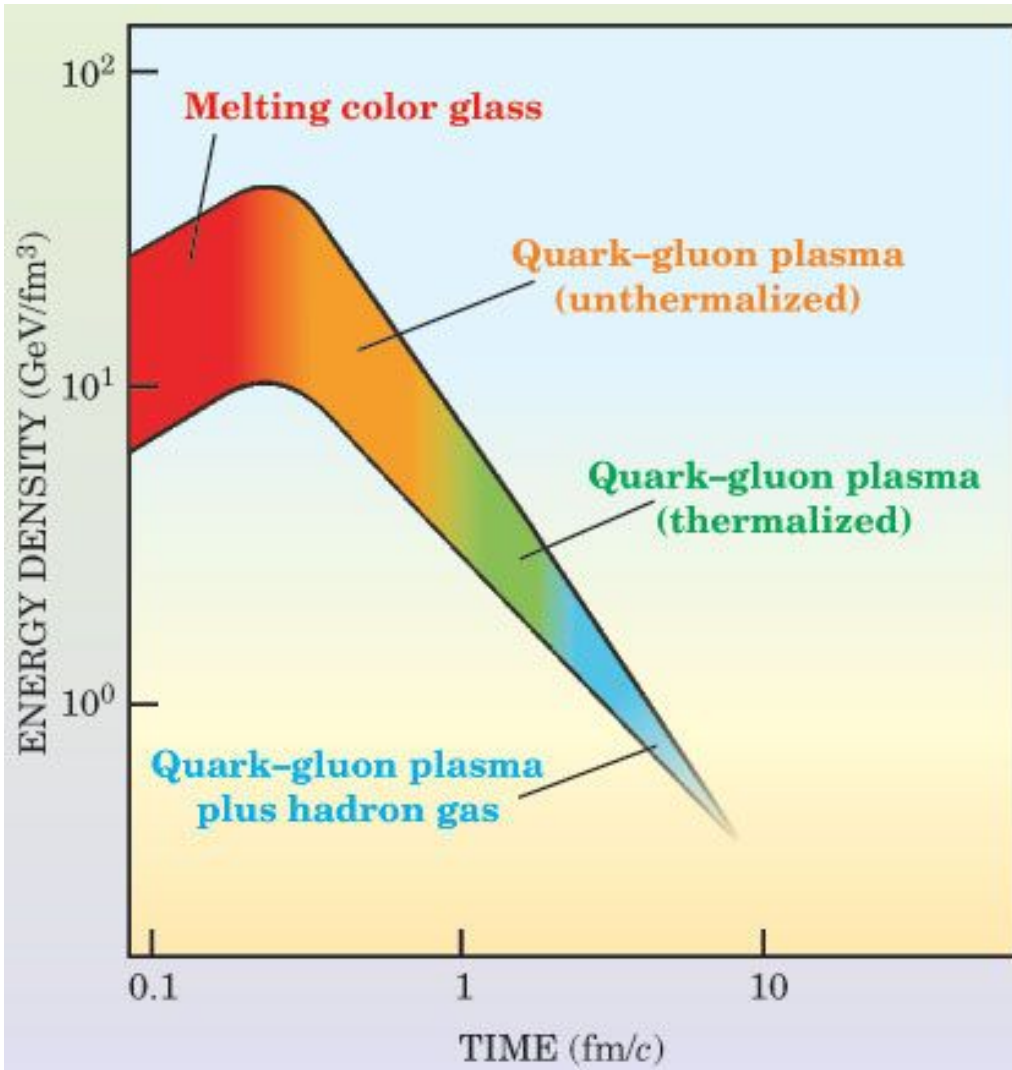
Normalized Rapidity Distributions, $10^4 \times 10^4$ NLO $c\bar{c}$ pairs



R.Thews

Recombination narrows the rapidity distribution; is this seen?
Are high p_t charmonia suppressed stronger than open charm?

The emerging picture



Big question:

How does the produced matter thermalize so fast?

Perturbation theory + Kinetic equations

→ $\tau_{therm} \sim 50 \text{ fm}$

Instabilities?

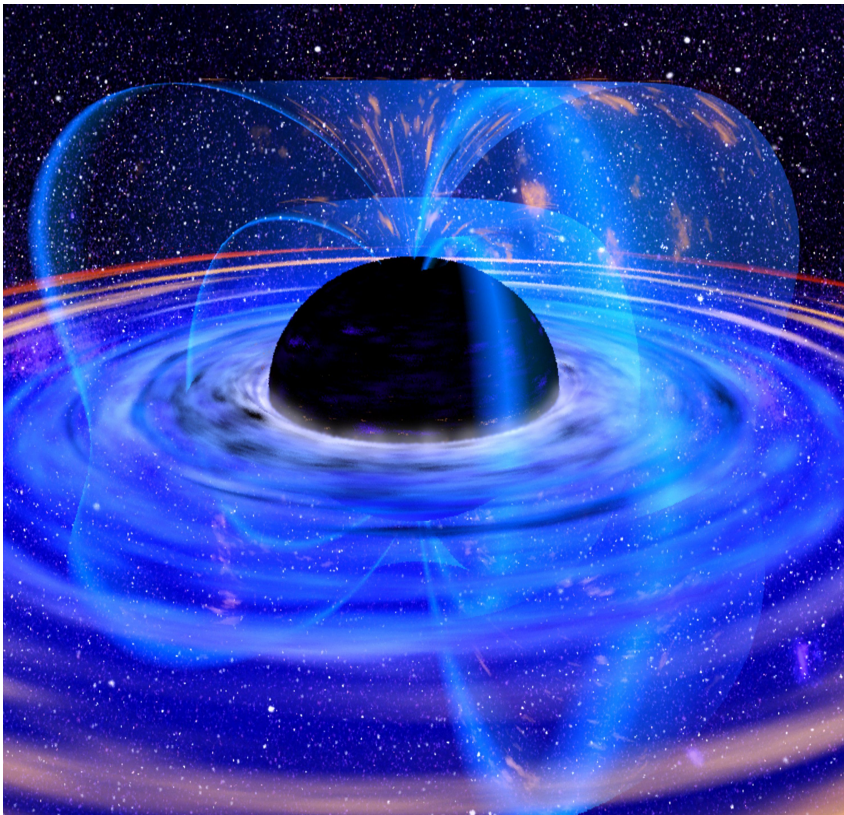
Lectures by

L. McLerran, R. Venugopalan

What is the mechanism of fast equilibration?

Hawking radiation

S.Hawking '74



Black holes emit thermal radiation with temperature

$$T = \frac{\kappa}{2\pi}$$

acceleration of gravity at the surface

Similar things happen in non-inertial frames

Einstein's Equivalence Principle:

Gravity \longleftrightarrow Acceleration in a non-inertial frame



An observer moving with an acceleration a detects
a thermal radiation with temperature

$$T = \frac{a}{2\pi}$$

W.Unruh '76

In both cases the radiation is
due to the presence of **event horizon**

Black hole: the interior is hidden from
an outside observer;
Schwarzschild metric

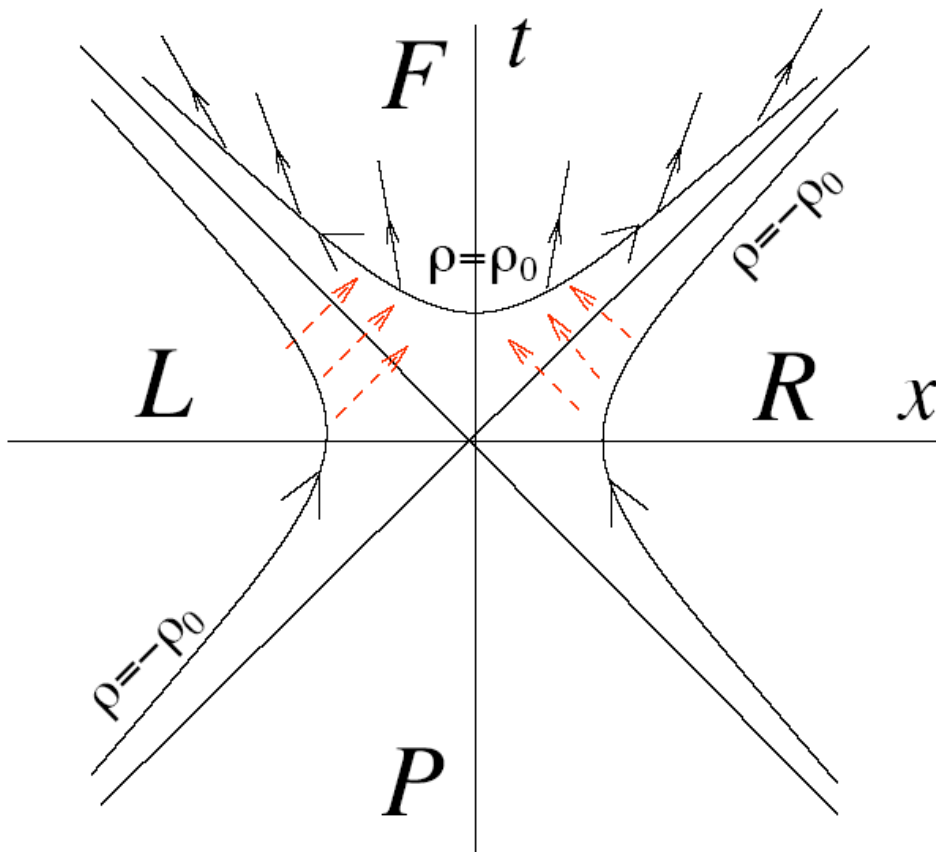
Accelerated frame: part of space-time is hidden
(causally disconnected) from
an accelerating observer;
Rindler metric

Quantum thermal radiation at RHIC

$$T = \frac{a}{2\pi} \simeq \frac{Q_s}{2\pi}$$

The event horizon emerges due to the fast deceleration $a \simeq Q_s$ of the colliding nuclei in strong color fields;

Tunneling through the event horizon leads to the thermal spectrum



Rindler and Minkowski spaces

DK, K.Tuchin;
DK, E.Levin, K.Tuchin

Rapid deceleration induces phase transitions

Nambu-
Jona-Lasinio model
(BCS - type)

Similar to phenomena in the vicinity of a large black hole:
Rindler space \approx Schwarzschild metric

Topological effects in QGP?

$$\mathcal{L} = -\frac{1}{4}F_{\alpha}^{\mu\nu}F_{\alpha\mu\nu} + \sum_f \bar{\psi}_f [i\gamma^\mu(\partial_\mu - igA_{\alpha\mu}t_\alpha) - m_f]\psi_f$$

$U_A(1)$ problem:

Invariant under chiral Left \longleftrightarrow Right

transformations in the limit of massless quarks

$U_L(N_f) \times U_R(N_f)$ chiral symmetry \implies parity doubling in
the hadron spectrum (not seen!)

If broken spontaneously, $N_f^2 = 9$ Goldstone bosons.

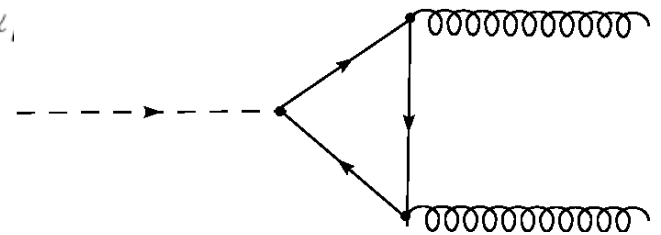
Only 8 exist; the ninth, η' is heavier than the proton!

Axial anomaly

Consider the flavor singlet current $J_{\mu 5} = \bar{\psi}_f \gamma_\mu \gamma_5 \psi_f$

It is not conserved even in the $m \rightarrow 0$ limit due to quantum effects:

$$\partial^\mu J_{\mu 5} = 2m_f i \bar{\psi}_f \gamma_5 \psi_f - \frac{N_f g^2}{16\pi^2} F_\alpha^{\mu\nu} \tilde{F}_\alpha$$



Divergence can be written down as
a surface term, and so is seemingly irrelevant:

ABJ

$$F_\alpha^{\mu\nu} \tilde{F}_{\alpha\mu\nu} = \partial_\mu K^\mu$$

Instantons and the $U_A(1)$ problem

But: sometimes, surface terms are important

Instantons: classical Euclidean solutions of QCD which map color $SU(2)$ onto the sphere S_3 ;
in Minkowski space, describe quantum tunneling between degenerate vacua with different topological Chern-Simons numbers

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

$$\nu = \int_{-\infty}^{+\infty} dt \frac{dQ_5}{dt} = 2N_f q[F]$$

$$q[F] = \frac{g^2}{32\pi^2} \int d^4x F_{\alpha}^{\mu\nu} \tilde{F}_{\alpha\mu\nu}$$

As a result, chiral charge
is no longer conserved

$$Q_5 = \int d^3x K_0$$

QCD vacuum as a Bloch crystal

“ θ - vacuum”

$$|\theta\rangle = \sum_q e^{i\theta q} |q\rangle$$

$$\langle \mathcal{O} \rangle = \sum_q f(q) \int_q D[\psi] D[\bar{\psi}] D[A] \exp(iS_{QCD}) \mathcal{O}(\psi, \bar{\psi}, A)$$

$$f(q_1 + q_2) = f(q_1)f(q_2) \longrightarrow f(q) = \exp(i\theta q)$$

“quasi-momentum” “coordinate”

The lost symmetries of QCD

The prescription

$$\langle \mathcal{O} \rangle = \sum_q f(q) \int_q D[\psi] D[\bar{\psi}] D[A] \exp(iS_{QCD}) \mathcal{O}(\psi, \bar{\psi}, A)$$

with the “Bloch” weight

$$f(q) = \exp(i\theta q)$$

is equivalent to adding to the Lagrangian a new piece

$$\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{QCD} + \mathcal{L}_\theta$$

$$\mathcal{L}_\theta = -\frac{\theta}{32\pi^2} g^2 F_\alpha^{\mu\nu} \tilde{F}_{\alpha\mu\nu}$$

which is odd under
P, T, CP symmetries !

The strong CP problem

$$\mathcal{L}_\theta = -\frac{\theta}{32\pi^2} g^2 F_\alpha^{\mu\nu} \tilde{F}_{\alpha\mu\nu}$$

Unless $\theta=0$, P, T and CP invariances are lost!

Experiment:

(e.d.m. of
the neutron)

$$\theta < 3 \times 10^{-10}$$

Why θ is so small?

Axions?

- PVLAS -recent evidence?

Will assume $\theta=0$ for the rest of the talk

The strong CP problem and the structure of QCD vacuum

Vafa-Witten theorem: P and CP cannot be broken
spontaneously in QCD

- But:
1. it does not constrain metastable states
 2. it does not apply at finite temperature,
finite baryon density,
finite isospin density

θ -vacuum in the presence of light quarks: chiral description

Non-linear σ -model with $U_A(1)$ anomaly:

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{tr}(\partial_\mu U^\dagger \partial^\mu U) + \Sigma \text{Re} [\text{tr}(\mathcal{M} U^\dagger)] - \frac{\chi}{2} (\theta + i \log \det U)^2$$

mass matrix quark condensate chiral unitary matrix $N_f \times N_f$

$$\mathcal{M} = \text{diag}(m_u, m_d, m_s)$$

$$U = \exp i\phi^a \lambda^a / f_\pi$$

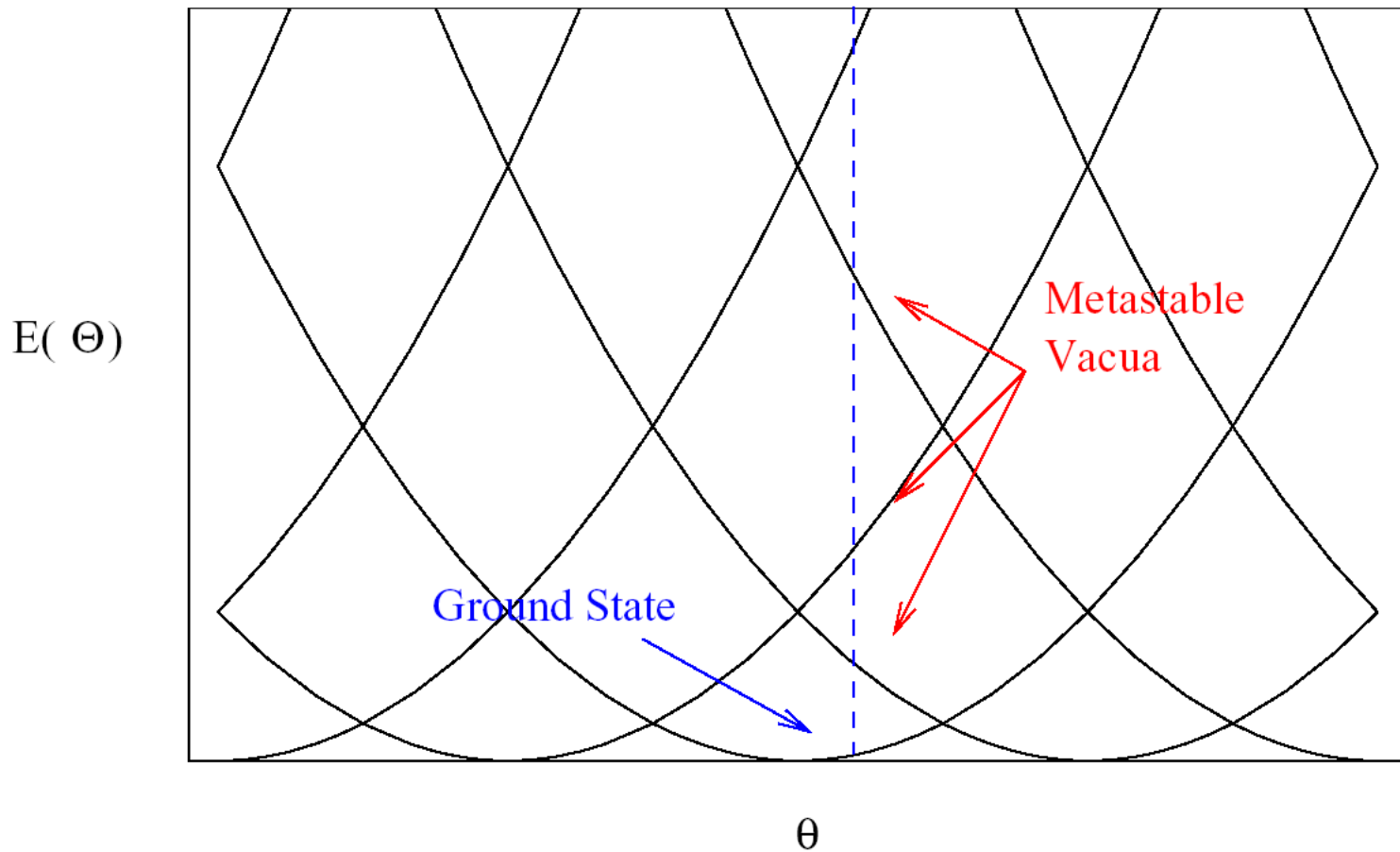
topological susceptibility

$$\chi = \int d^4x \langle q(x)q(0) \rangle$$

The effective potential:

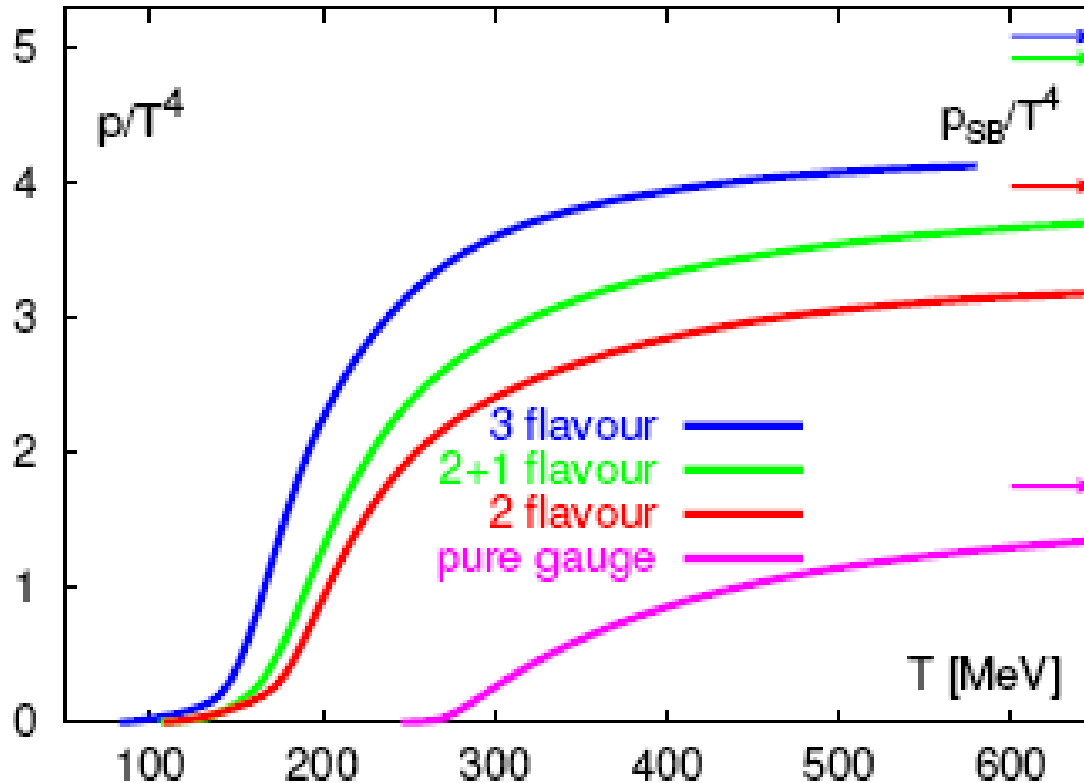
$$\mathcal{V} = - \sum_{i=u,d,s} m_i \Sigma \cos \frac{\phi_i}{f_\pi} + \frac{\chi}{2} \left(\theta - \sum_{i=u,d,s} \phi_i / f_\pi \right)^2$$

θ -vacua



Multi-valued potential \Rightarrow a family of vacuum states

What happens at high temperatures?



Phase transitions:

deconfinement

Chiral symmetry
restoration

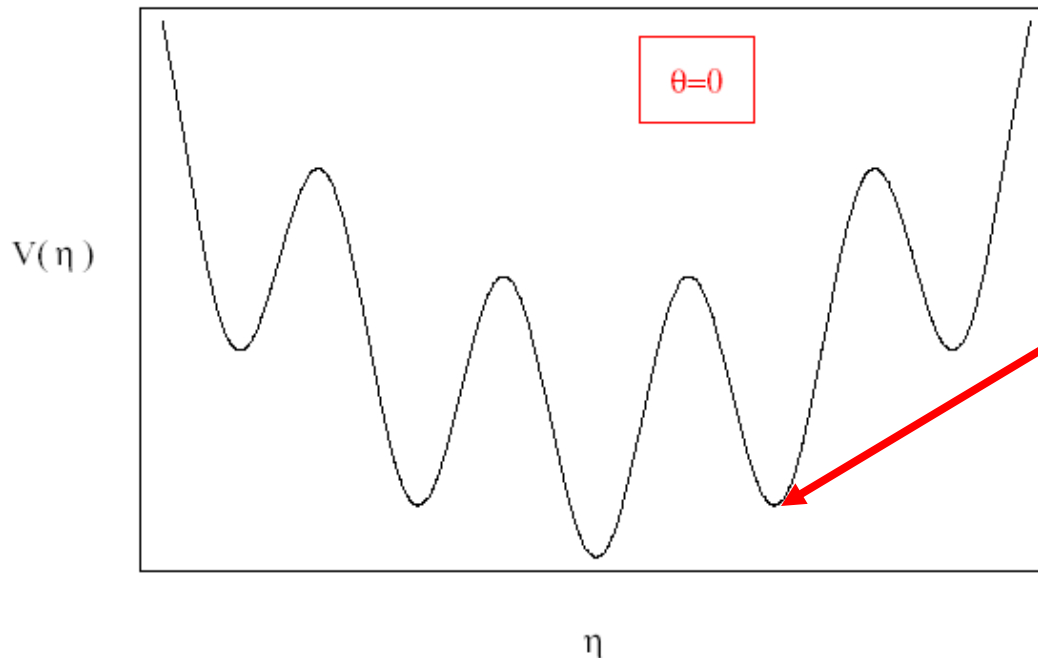
$U_A(1)$ restoration

Data from lattice QCD simulations F. Karsch et al

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

critical temperature $\sim 10^{12}$ K; cf temperature inside the Sun $\sim 10^7$ K

θ -vacuum in the presence of light quarks: chiral description



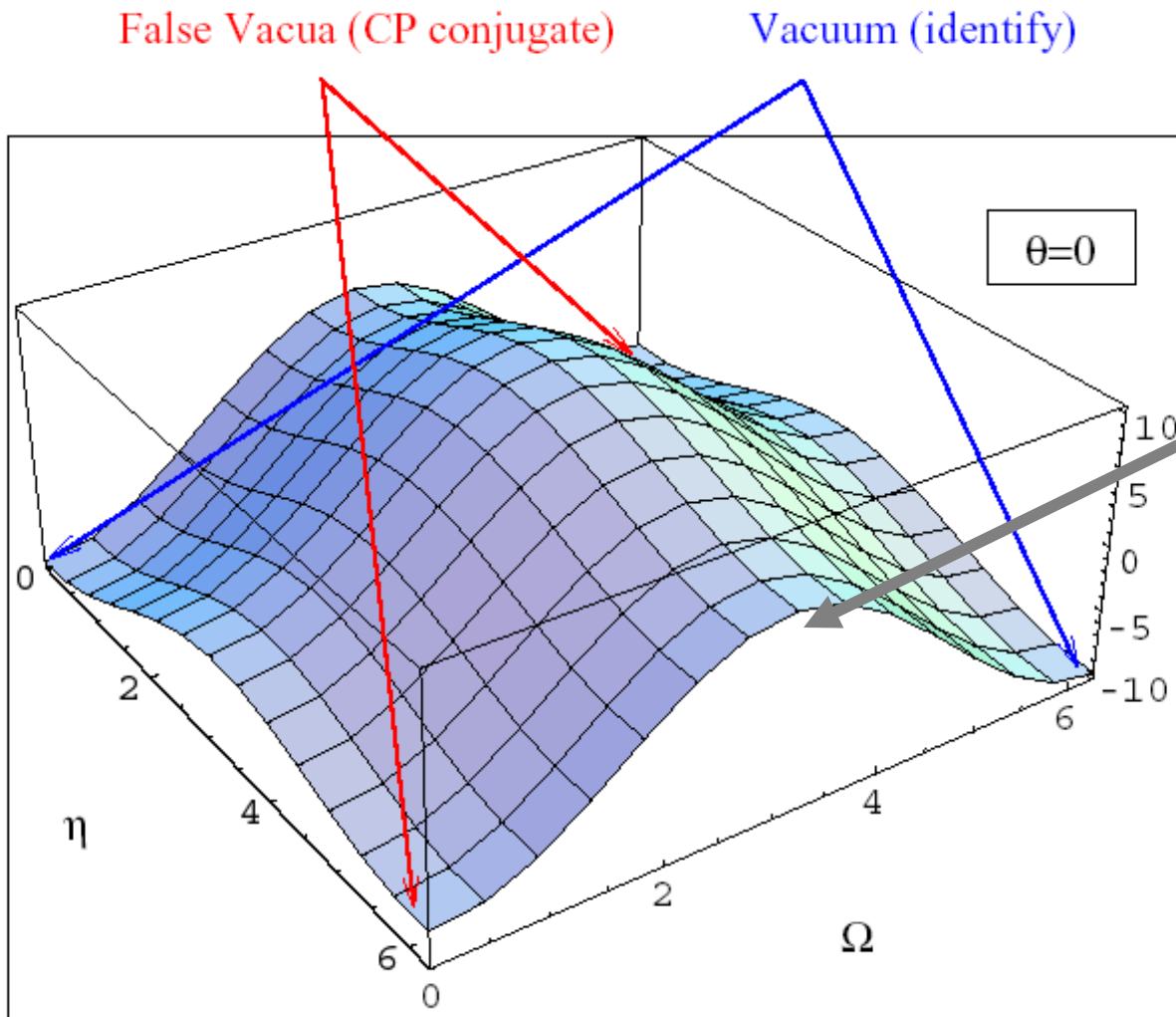
Decrease of χ results in the appearance of metastable CP-odd domains

DK, Pisarski,
Tytgat '97

$$\mathcal{V} = - \sum_{i=u,d,s} m_i \Sigma \cos \frac{\phi_i}{f_\pi} + \frac{\chi}{2} \left(\theta - \sum_{i=u,d,s} \phi_i / f_\pi \right)^2$$

v.e.v. of the η field is equivalent to non-zero θ

θ -vacuum in the presence of light quarks: chiral description

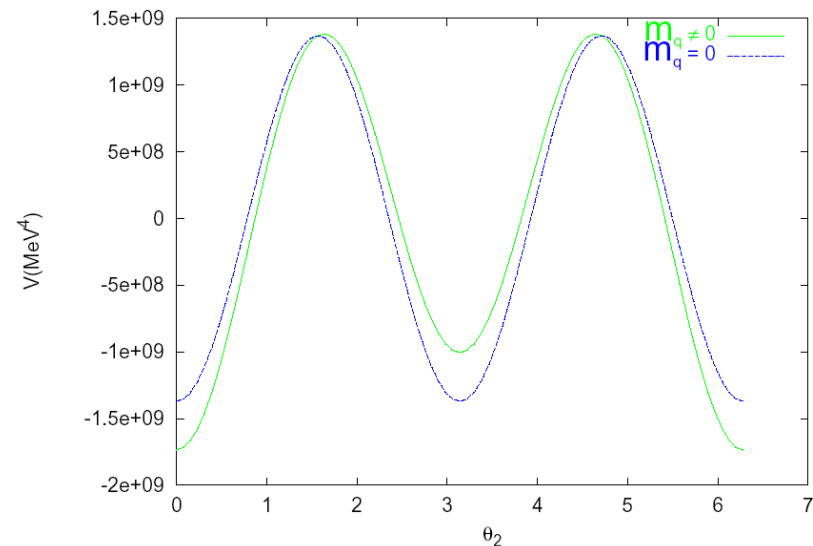
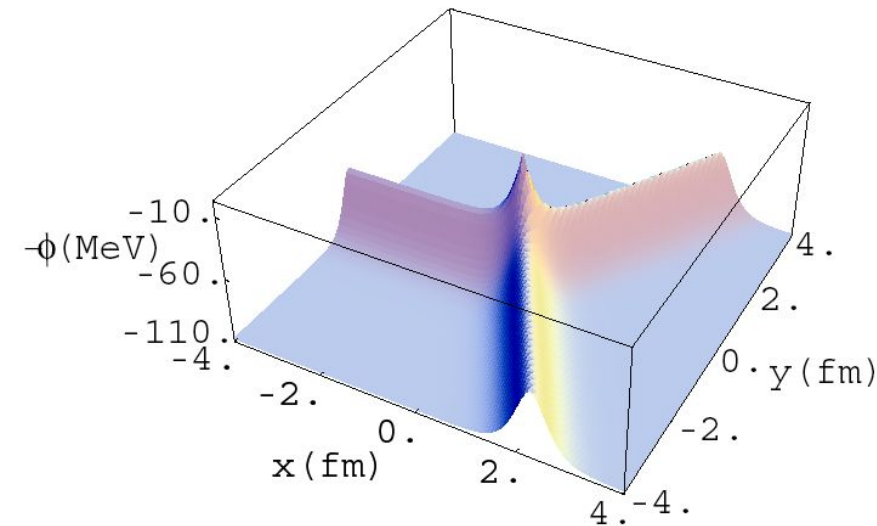


Domain wall

The structure
of domain walls:
can be studied e.g.
in SUSY QCD

θ -vacuum in the presence of light quarks: linear σ model

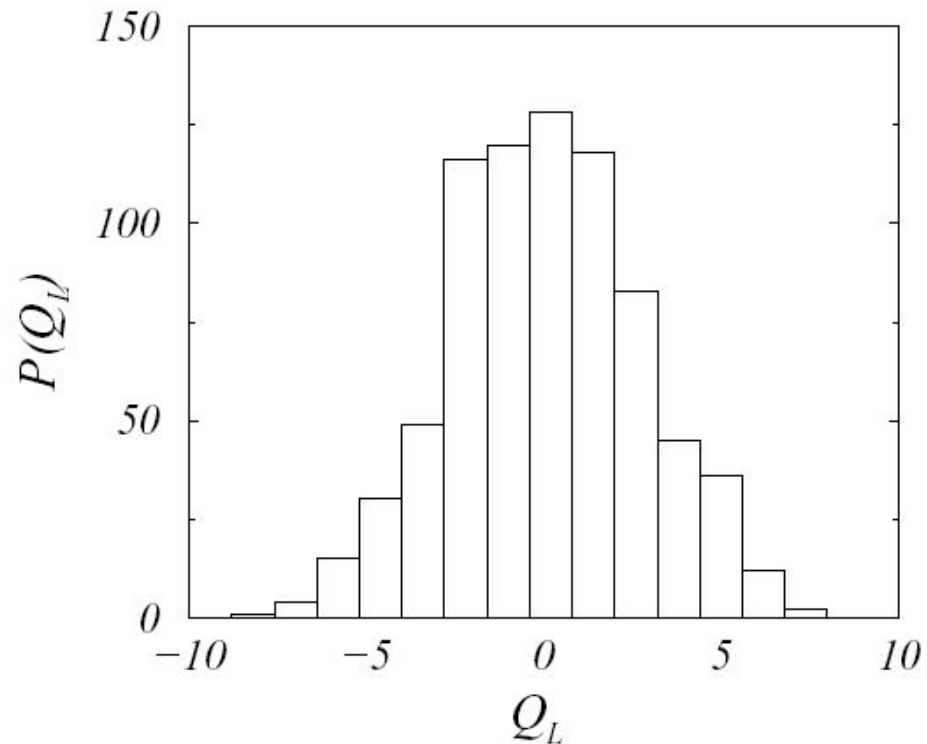
“topological strings”; similar to Abrikosov vortices, “cosmic strings”,...



$$\mathcal{L}(\Phi) = \text{Tr} \left(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi \right) - \lambda_1 \left[\text{Tr} \left(\Phi^\dagger \Phi \right) \right]^2 - \lambda_2 \text{Tr} \left(\Phi^\dagger \Phi \right)^2 + c \left[\text{Det} \left(\Phi \right) + \text{Det} \left(\Phi^\dagger \right) \right] + \text{Tr} \left[H \left(\Phi + \Phi^\dagger \right) \right] .$$

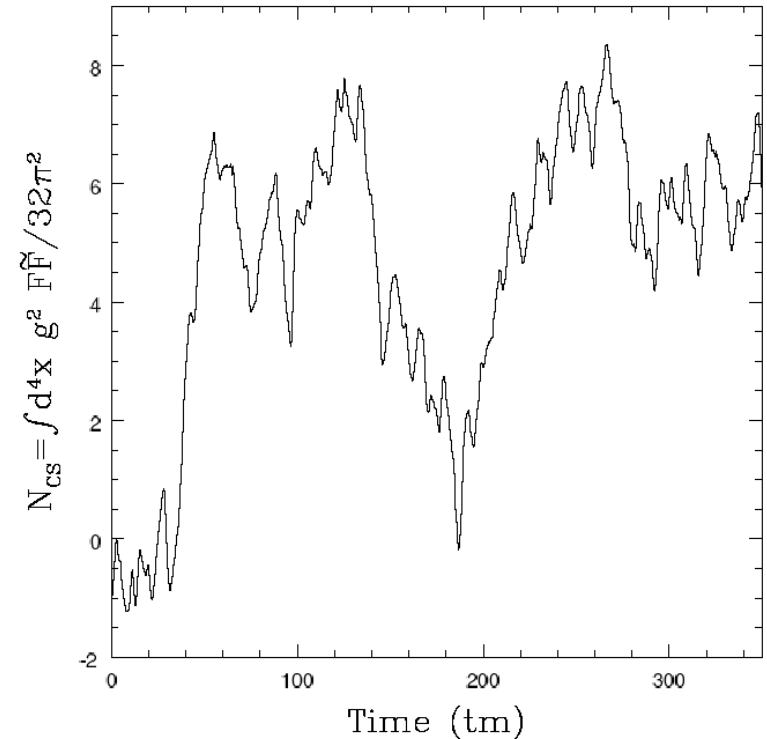
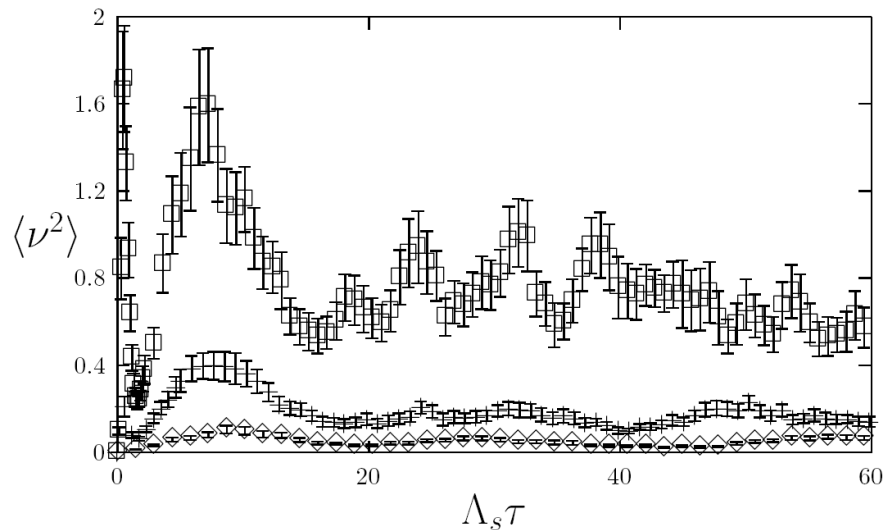
$$\Phi = T_a \phi_a = T_a (\sigma_a + i\pi_a)$$

Fluctuations of Chern-Simons number in hot QCD: numerical lattice simulations



B.Alles, M.D'Elia and A.DiGiacomo,
hep-lat/0004020

Diffusion of Chern-Simons number in QCD: real time lattice simulations

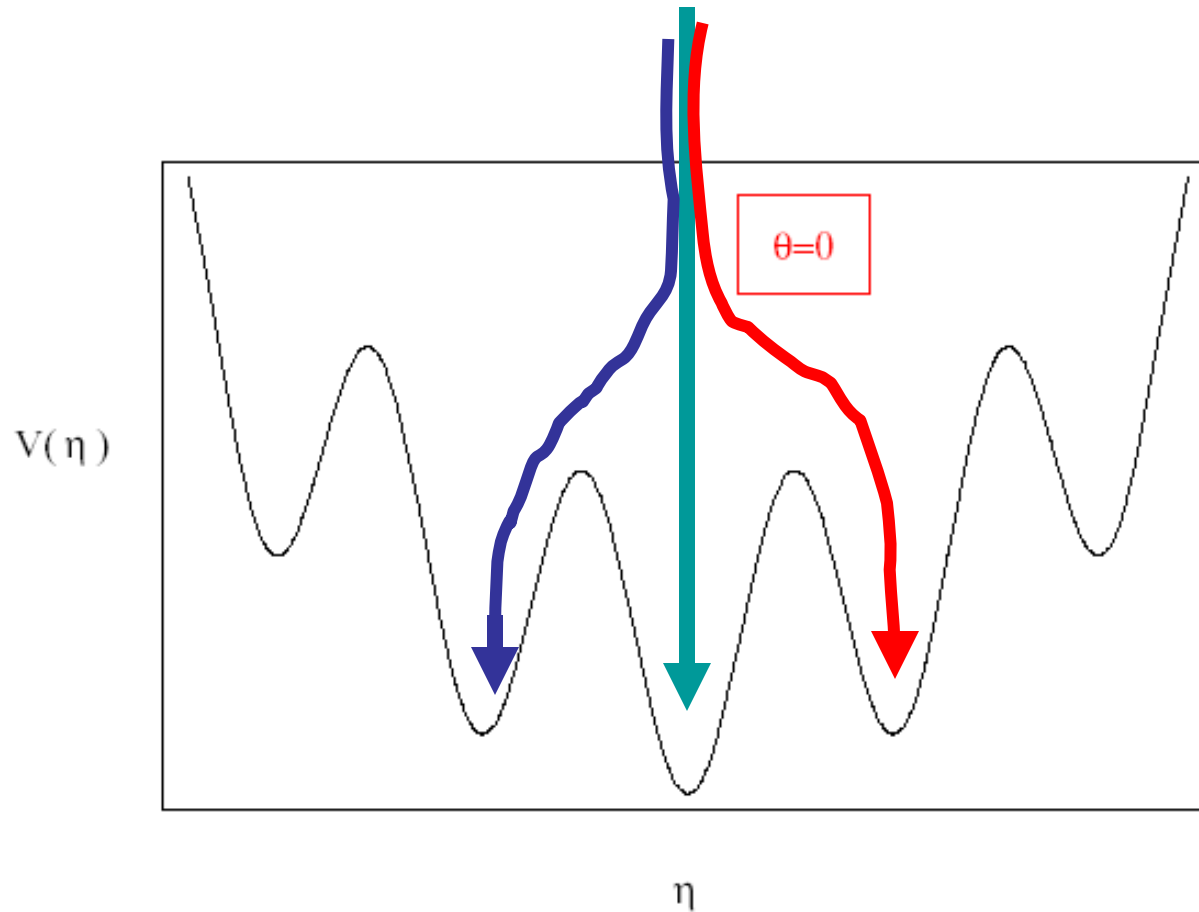


DK, A.Krasnitz and R.Venugopalan,
Phys.Lett.B545:298-306,2002

P.Arnold and G.Moore,
Phys.Rev.D73:025006,2006

What are the experimental signatures?

CP-odd domains in heavy ion collisions: how to look for them?



Similar
to DCC

v.e.v. of the η field is equivalent to non-zero θ

What are the observable signatures of strong CP violation?

rotate all CP violating phase into the quark piece of the Lagrangian:

$$\mathcal{L}_{quark} = - \sum_f (\hat{m}_f \bar{\psi}_{L,f} \psi_{R,f} + \hat{m}_f^* \bar{\psi}_{R,f} \psi_{L,f})$$

$\hat{m} = m \exp(i\theta)$ is a complex mass parameter

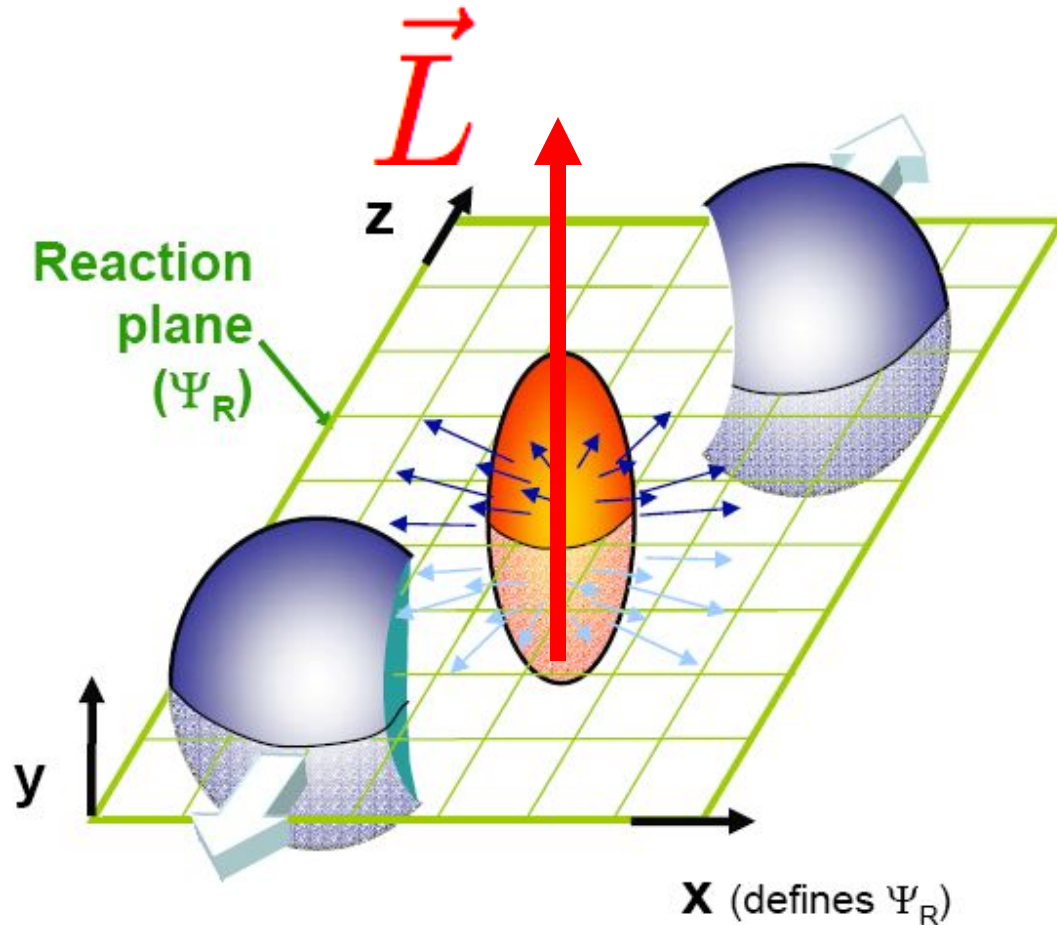
In a CP-odd domain, $\hat{m}(\mathbf{x}, t) = m \exp(i\theta(\mathbf{x}, t))$

This leads to the asymmetry between “left” and “right” quarks

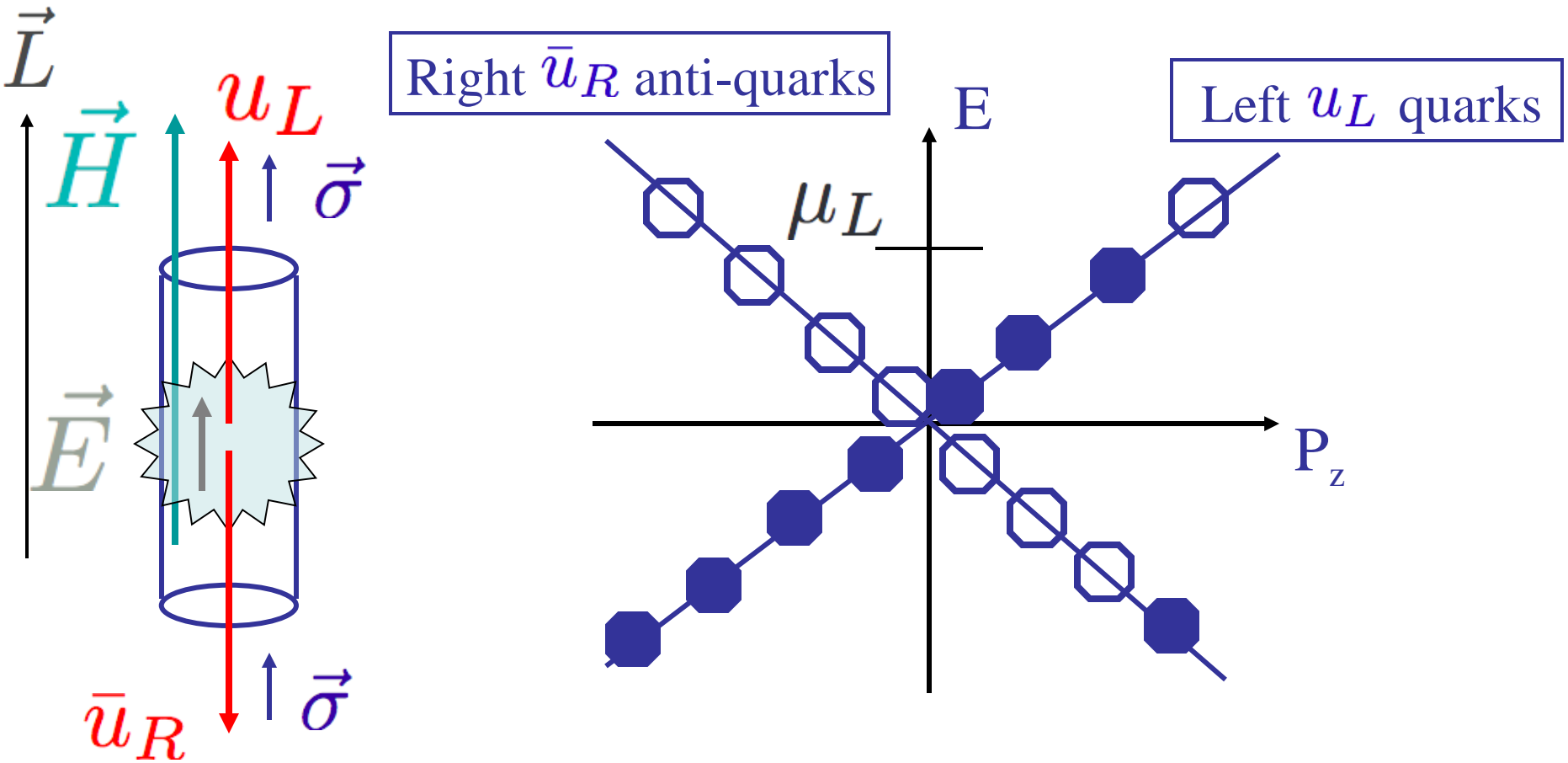
$$\mathcal{L}_\theta = -m \cos \theta (\bar{u}_L u_R + \bar{u}_R u_L) - im \sin \theta (\bar{u}_L u_R - \bar{u}_R u_L)$$

What is “left” and what is “right”? Quarks are massive, so the definition of chirality depends on the frame...

Azimuthal anisotropy = angular momentum

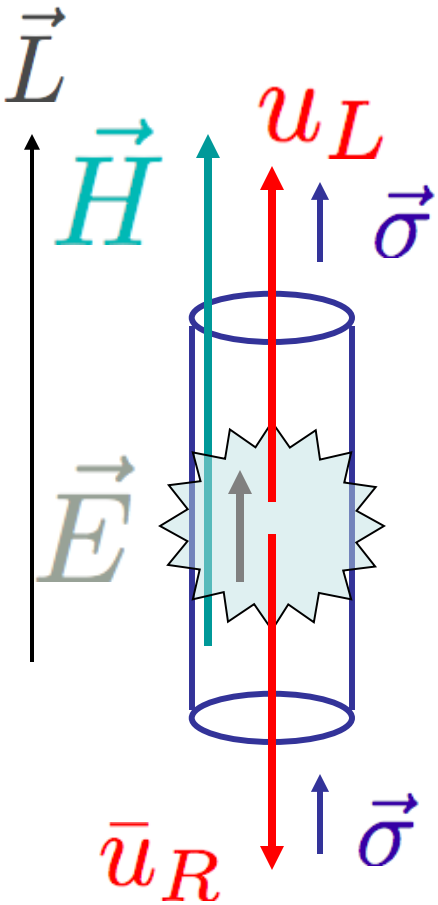


Magnetic vortices and CP violation



Fluctuation of chromo-electric field in the presence of chromo-magnetic vortex produces left quarks and right anti-quarks (level crossing at the 0th Landau level)

Magnetic vortices and CP violation



Analogous to cosmic strings, except not 1D

Left quarks: in the electric field, energy increases:

$$\Delta E_L = p_F^L = qEt$$

Decreases by the same amount for right anti-quarks.

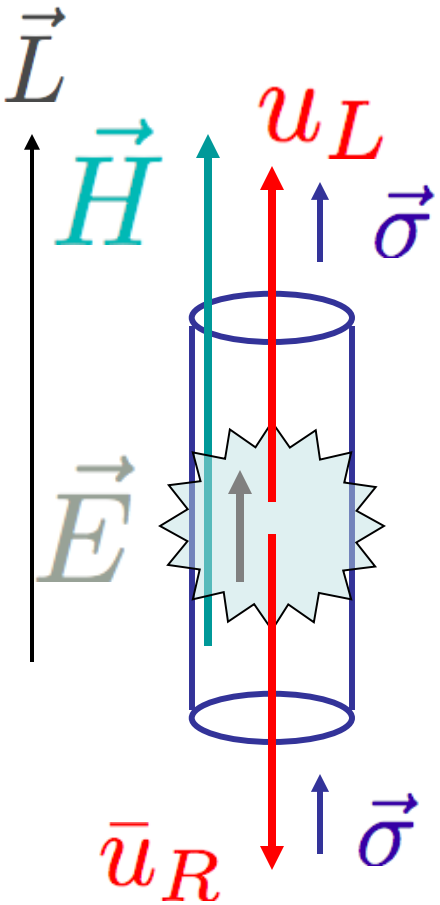
Density of states = longitudinal x transverse:

$$\frac{p_F}{2\pi} \cdot \frac{qH}{2\pi} = \frac{q^2}{4\pi^2} E \cdot H t$$

Rate of chirality generation:

$$\frac{d^4 N_L}{dt dz d^2 s} = \frac{q^2}{4\pi^2} E \cdot H$$

Magnetic vortices and CP violation



Analogs:

Dyons

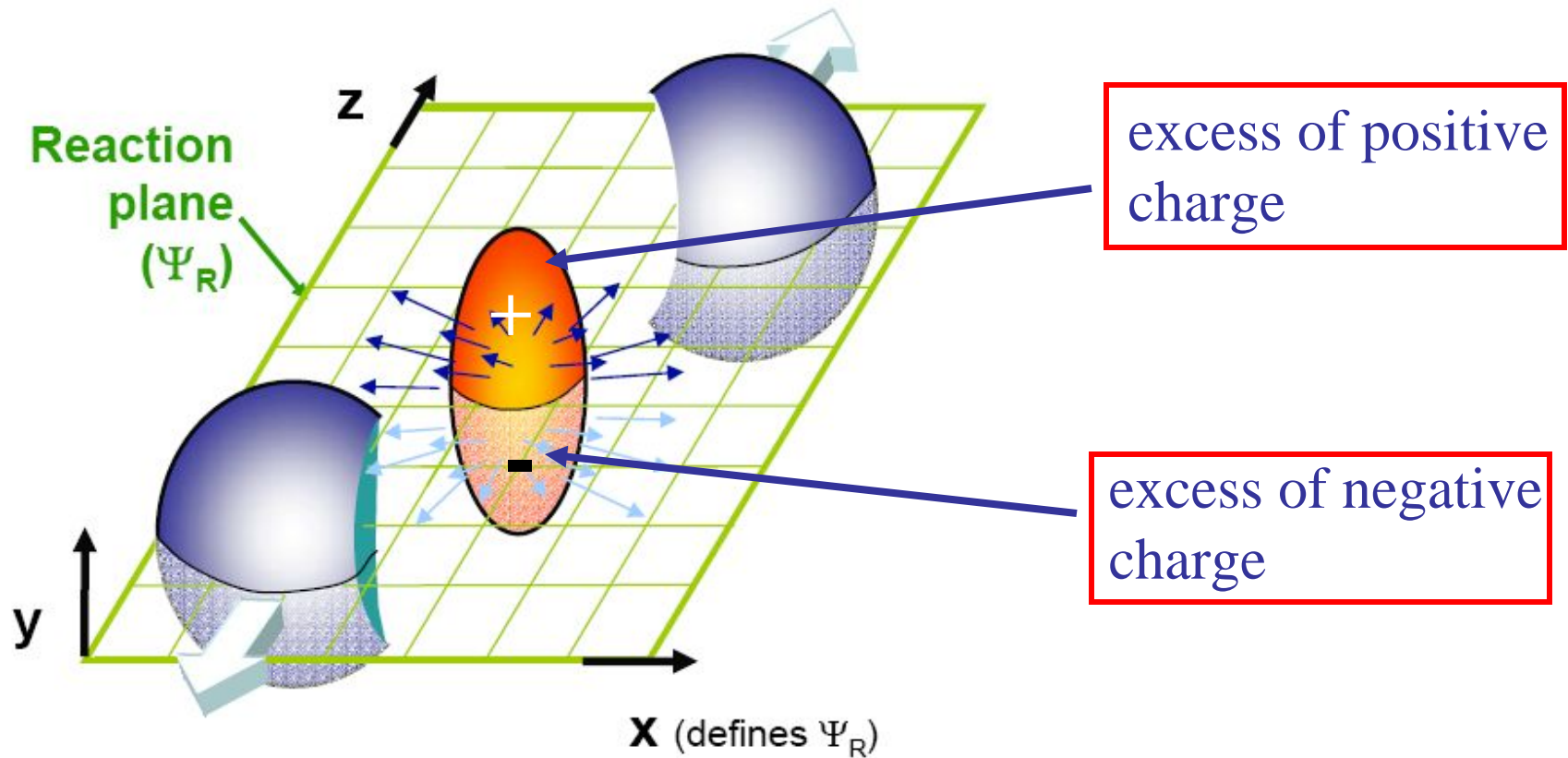
Magnetic monopole - induced baryon decay

Cosmic strings

Chirality generation in superfluid ^3He

.....

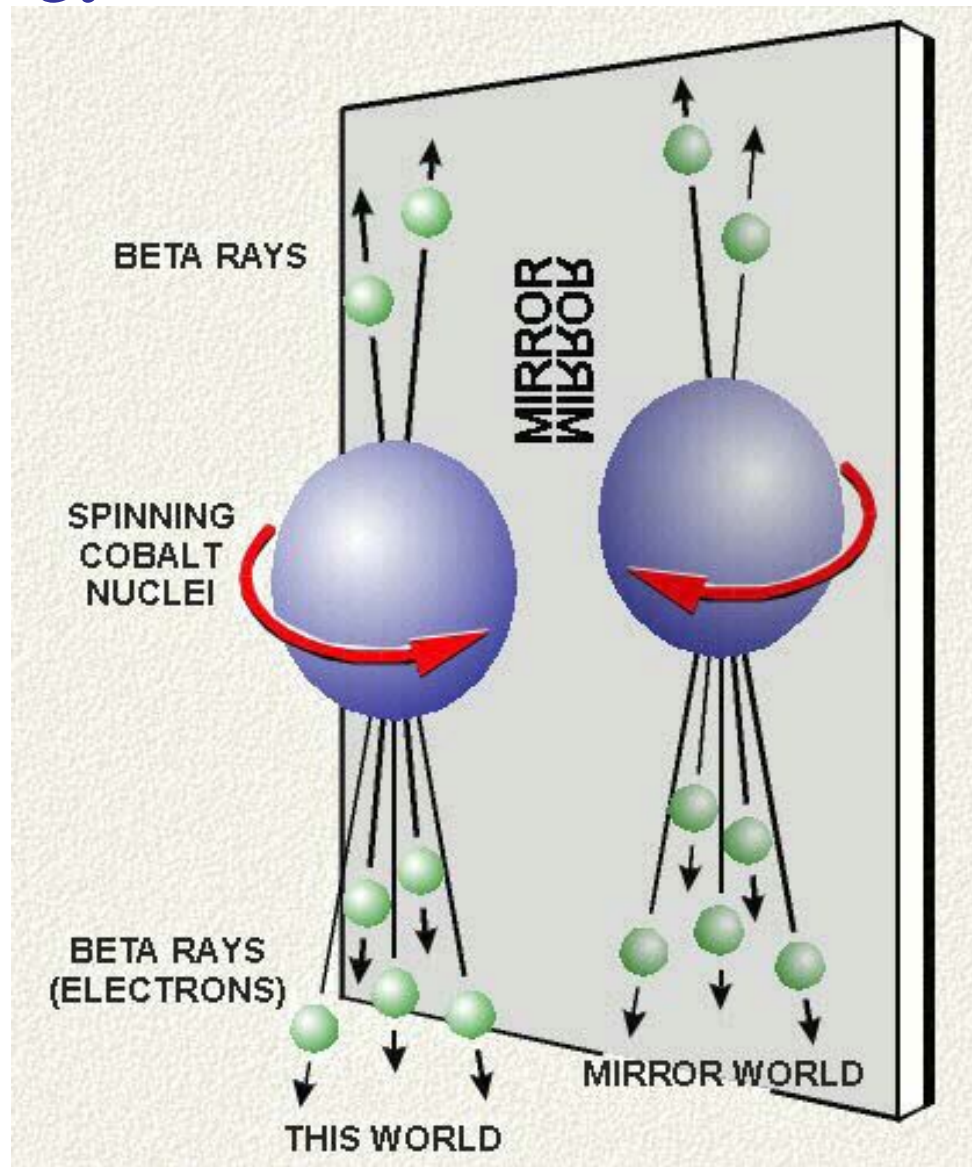
Charge asymmetry w.r.t. reaction plane as a signature of strong CP violation



Electric dipole moment of QCD matter!

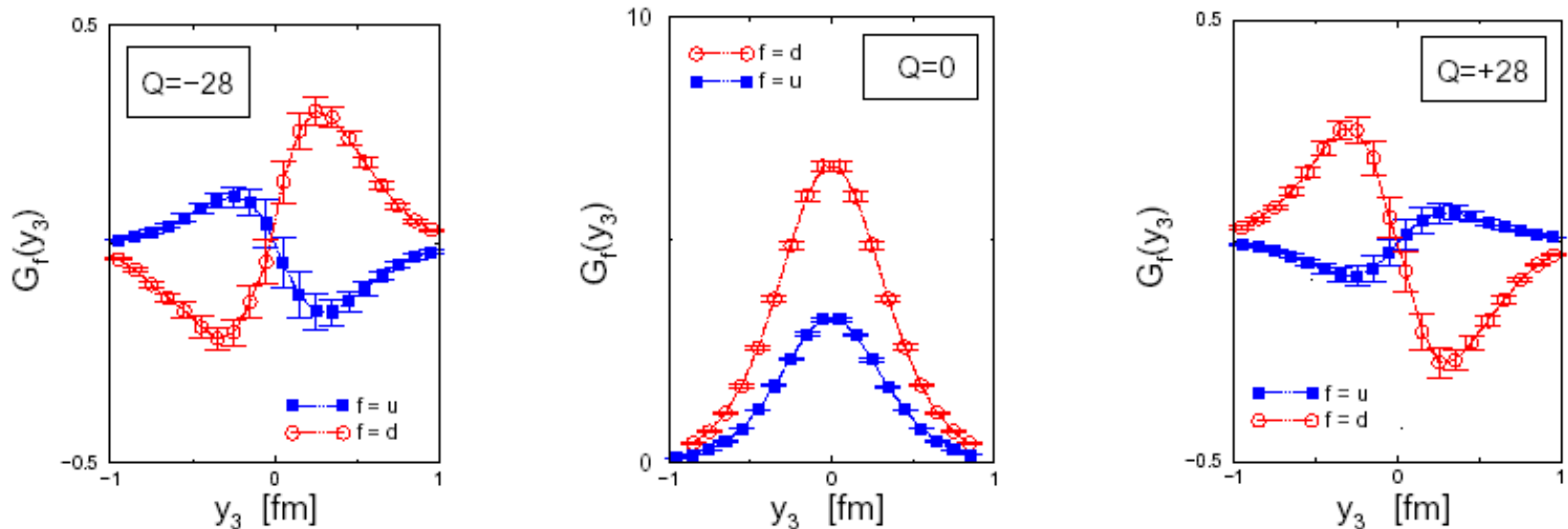
Charge asymmetry w. r.t. reaction plane
violates T, P, and (by CPT theorem) CP:

Analogy to P violation in weak interactions



Charge asymmetry w.r.t. the direction of spin and the electric dipole moment of the neutron

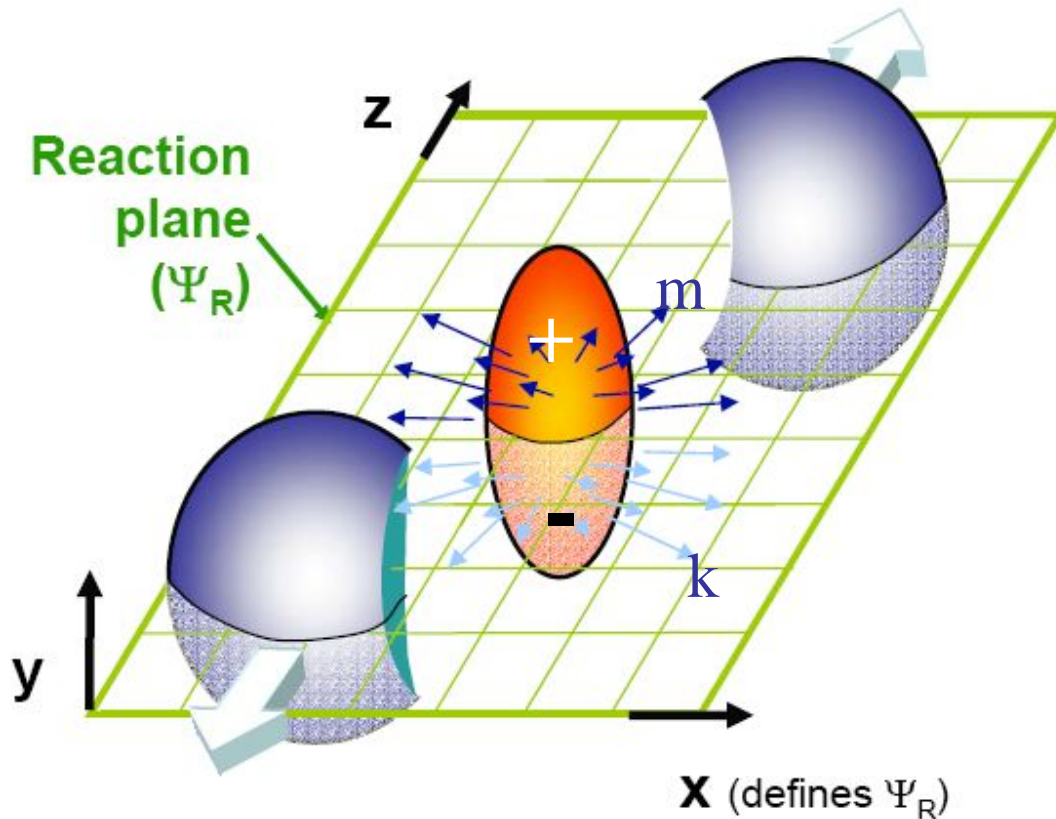
Analogy: e.d.m. of the neutron at finite θ



Lattice QCD simulation

P.Faccioli, Phys.Rev.D71:091502,2005

Charge asymmetry w.r.t. reaction plane: how to detect it?



We need
a sensitive measure
of the asymmetry

Improved method:
“mixed harmonics”

S.Voloshin, hep-ph/0406311

$$a^k a^m = \left\langle \sum_{ij} \sin(\varphi_i^k - \Psi_R) \sin(\varphi_j^m - \Psi_R) \right\rangle$$

Expect $a^+ a^+ = a^- a^- > 0$; $a^+ a^- < 0$

Strong CP violation at high T ?

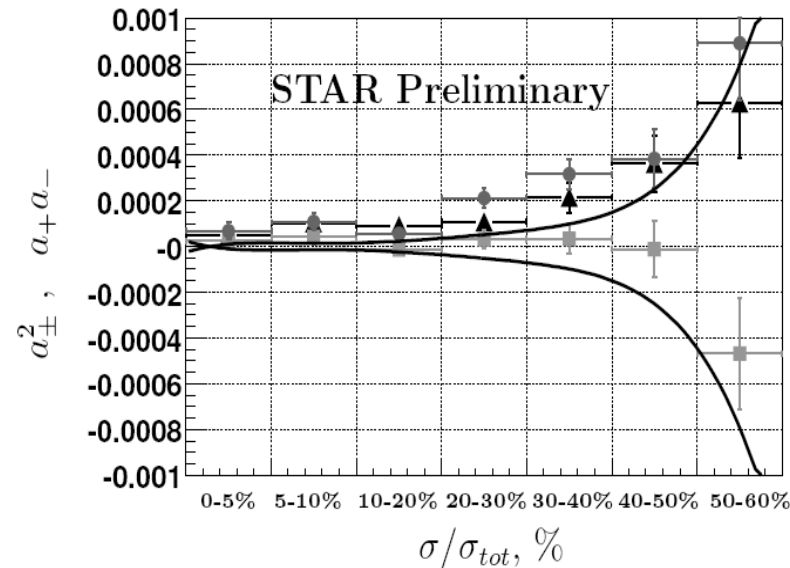
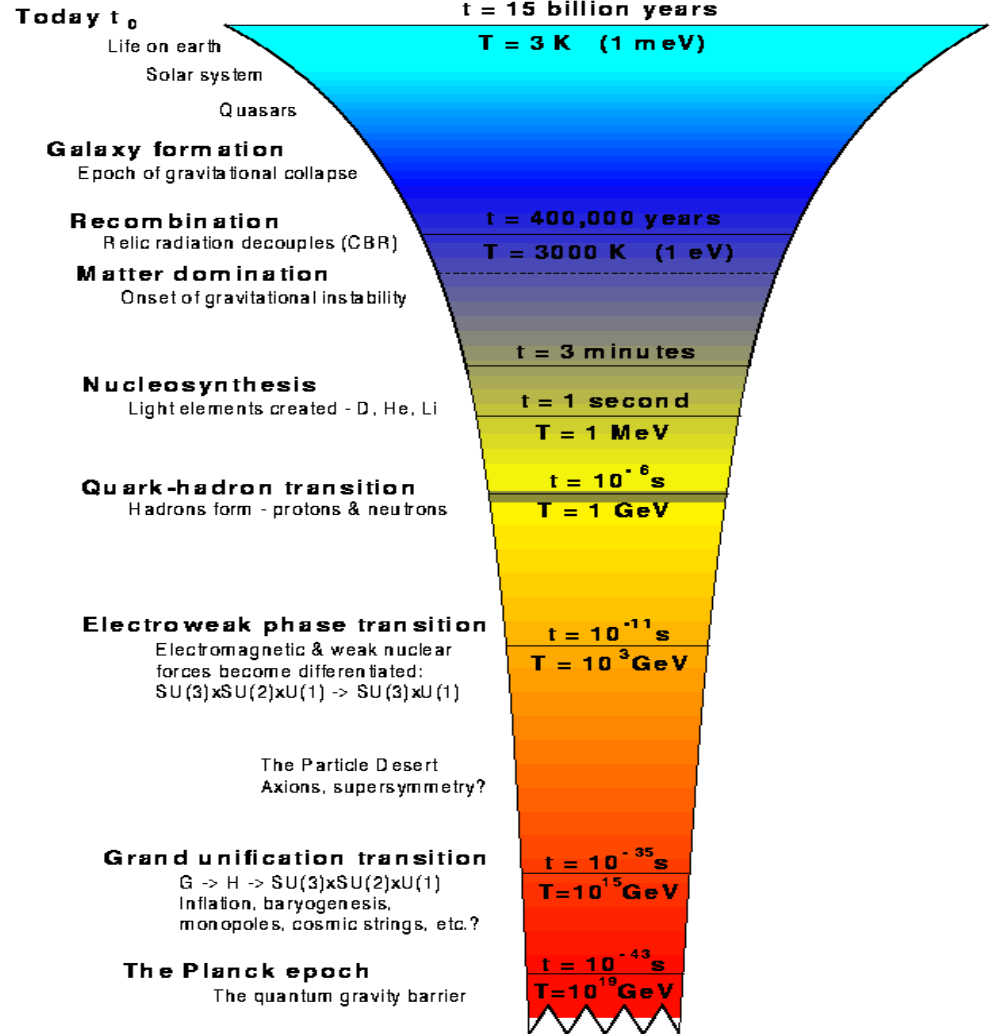
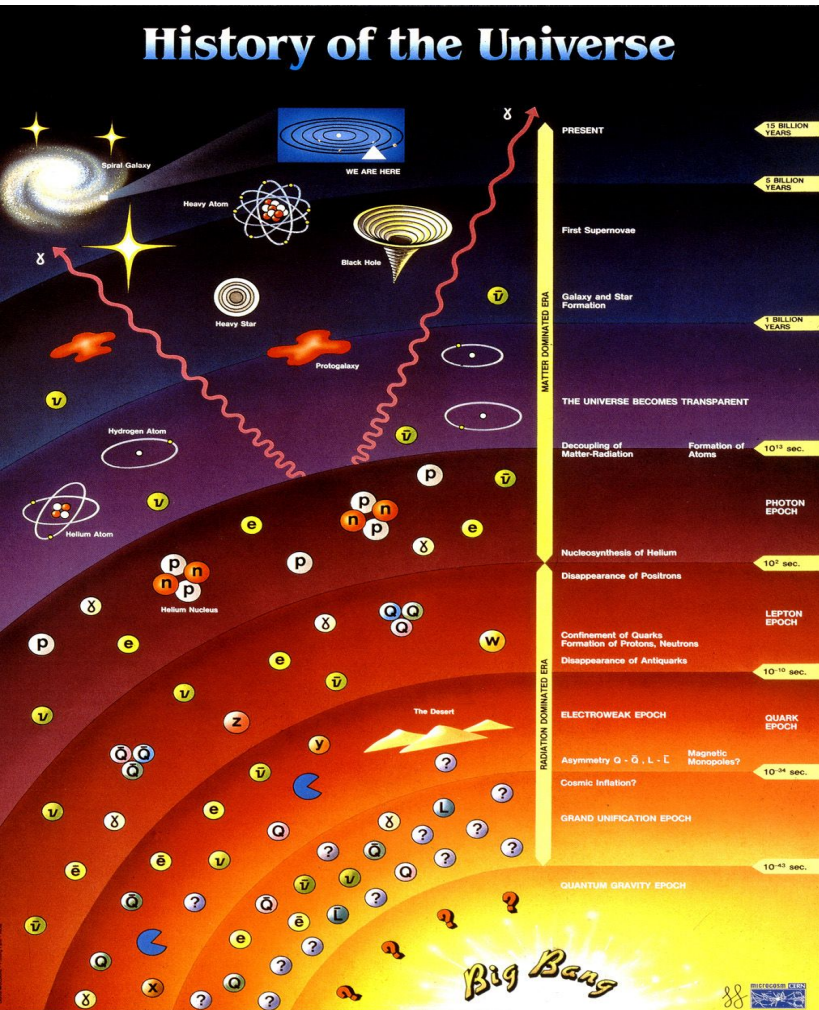


Figure 2: Charged particle asymmetry parameters as a function of standard STAR centrality bins selected on the basis of charged particle multiplicity in $|\eta| < 0.5$ region. Points are STAR preliminary data for Au+Au at $\sqrt{s_{NN}} = 62$ GeV: circles are a_+^2 , triangles are a_-^2 and squares are a_+a_- . Black lines are theoretical prediction [1] corresponding to the topological charge $|Q| = 1$.

STAR Coll., nucl-ex/0510069; October 25, 2005

Need to analyze the systematics, improve statistics - vigorous ongoing work!

What are the implications for the Early Universe?



Summary

1. Quantum Chromo-Dynamics is an established and consistent field theory of strong interactions but it's properties are far from being understood
2. High energy nuclear collisions test the predictions of **strong field QCD** and probe the properties of **super-dense matter**

Many surprises; a lot more work needs to be done!