Lecture 2: QCD Sum Rules for light mesons

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- QCD: Currents, Correlators and Spectral Densities of Real Particles.

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- Condensates and PCAC for pions in QCD.

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- QCD SRs for pion DA: NLC SRs.

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- Real case: π-meson.
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- QCD SRs for pion DA: NLC SRs.
- Comparison with Data for pion DA: CLEO data and LCSRs; JLab data and Pion FF in APT; Lattice data and renormalon model

QCD: Currents, Correlators and Spectral Densities of Real Particles

Currents related to π-mesons in QCD

Currents related to $\pi^{\pm}$ meson:

$$\text{AV:}\quad J_{\mu 5}(x) = \bar{u}(x)\gamma_\mu\gamma_5 d(x)\,; \qquad J^\dagger_{\mu 5}(x) = \bar{d}(x)\gamma_\mu\gamma_5 u(x)\,.$$

$$\text{PS:}\quad J_5(x) = i\,\bar{u}(x)\gamma_5 d(x)\,; \qquad J^\dagger_5(x) = i\,\bar{d}(x)\gamma_5 u(x)\,.$$

Note that Dirac equation $i\,\hat{D}\,q(x) = m_q\,q(x)$ gives relation:

$$\partial^\mu J_{\mu 5}(x) = (m_u + m_d)\ J_5(x)\,. \qquad (*)$$

Decay constant f_π of physical pion $\pi(P)$ is defined via

$$\langle 0 | J_{\mu 5}(0) | \pi(P)\rangle = i\,f_\pi\,P_\mu\,.$$

Eq. (*) then gives $\langle 0 | J_5(0) | \pi(P)\rangle = \dfrac{f_\pi\,m_\pi^2}{m_u + m_d}$.

Currents related to vector mesons in QCD

Currents related to $\rho^{\pm}$ meson:

$$J_\mu(x) = \bar{u}(x)\gamma_\mu d(x)\,; \quad J^{\dagger}{}_\mu(x) = \bar{d}(x)\gamma_\mu u(x)\,.$$

Decay constant f_ρ of physical $\rho^{\pm}(P,\varepsilon)$-meson with polarization ε and momentum P, satisfying $(P\,\varepsilon) = 0$ and $(\varepsilon,\varepsilon) = -1$,

$$\langle 0\big|J_\mu(0)\big|\rho(P,\varepsilon)\rangle = f_\rho\, m_\rho\, \varepsilon_\mu\,.$$

Decay $\rho^0 \to e^+e^-$ allows to measure $f_{\rho^0} = 150$ MeV, that gives $f_{\rho^{\pm}} = 210$ MeV.

Vector current correlator $\Pi_{\mu\nu}$

So, we have $\boxed{\frac{1}{\pi}\,\mathbf{Im}\,\Pi(q^2) = \rho(q^2)\theta(|q_0|) = \rho(q^2)}$, with

$$\rho(q^2)\,\theta(q_0) = \frac{-(2\pi)^3}{3q^2}\sum_{X(p)}\delta^{(4)}(q-p)\,\theta(p_0)\Big|\langle 0\big|J_\mu(0)\big|X(p)\rangle\Big|^2 .$$

Lorentz invariance and current conservation dictate

$$\langle 0\big|J^\mu(x)\big|X(p)\rangle\langle X(p)\big|J^\mu(x)\big|0\rangle = -\big|f_X\big|^2 m_X^2 \leq 0\,,$$

that gives us

$$\boxed{\rho(s) = \sum_X \big|f_X\big|^2\,\delta(s-m_X^2) \geq 0}$$

Spectral density of correlators $\Pi_{\mu\nu}$ *and* $\Pi^{+}_{\mu\nu}$

So, we have

$$\frac{1}{\pi}\,\mathbf{Im}\,\Pi(q^2) = \rho(q^2)\theta(|q_0|) = \rho(q^2)$$

If we consider correlator

$$\Pi^{+}_{\mu\nu}(q) = i\int d^4x\, e^{iqx}\langle 0\big|J^{\mu}(x)J_{\nu}(0)\big|0\rangle = \left[q_{\mu}\,q_{\nu} - g_{\mu\nu}\,q^2\right]\,\Pi^{+}(q)\,.$$

then

$$\frac{1}{\pi}\,\mathbf{Im}\,\Pi^{+}(q^2) = \rho(q^2)\theta(q_0)$$

Now we can say why we put T-product in correlators
– then spectral densities, defined only by **real particles**,
are **Lorentz invariant** and **depend only on q^2** !

Relation with cross section $e^+e^- \to$ *hadrons*

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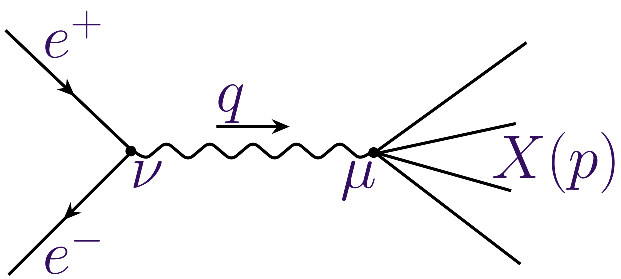
Important! This function naturally appears in 1-photon QED description of $e^+e^- \to$ hadrons:



$$\bar{u}(k)\gamma_\mu u(k')\frac{ie^2}{q^2}\langle X(p)|J_\mu(q)|0\rangle$$

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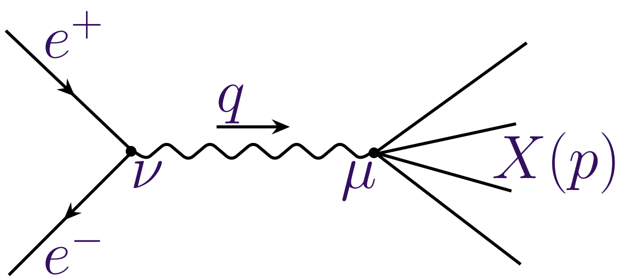
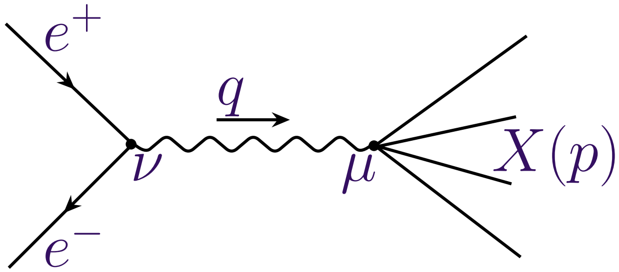


$$\sigma_{\text{had}}(s) = \frac{16\,\pi^3\,\alpha^2}{s}\,\rho(s)$$

Relation with cross section $e^+e^- \longrightarrow$ *hadrons*

In 1-photon approximation of QED:



$$\sigma_{\text{had}}(s) = \frac{16\,\pi^3\,\alpha^2}{s}\,\rho(s) = \frac{4\,\pi\,\alpha^2}{3\,s}\,R(s)$$

Here we explicitly extracted as a factor cross-section $\sigma_{\mu^+\mu^-}(s) = 4\,\pi\,\alpha^2/(3\,s)$ of the process $e^+e^- \to \mu^+\mu^-$.

Relation with cross section $e^+e^- \to$ *hadrons*

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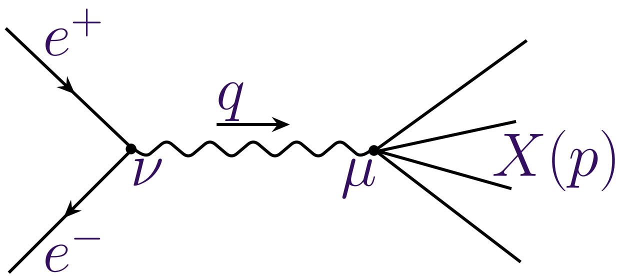

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$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}\,; \quad \rho(s) = \frac{R(s)}{12\,\pi^2}\,.$$

Quark and Gluon Condensates: What is that?

Pert. vs Non-Pert. contributions in QCD

QM oscillator: In the presence of confinement potential

$$M\left(\tau^{-1}\right) - M_0\left(\tau^{-1}\right) = \frac{m}{2\pi}\left[-\frac{1}{6}\omega^2\tau + \frac{7}{360}\omega^4\tau^3 + \ldots\right].$$

This difference vanishes at short distances $\tau \ll 1/\omega$ and one can calculate exact $M(\mu)$ perturbatively, expanding in powers of the oscillator potential.

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In QCD **confining potential** $V^{\rm conf}(r)$ is not even known.

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How to proceed further?

- to construct **perturbation expansion** in terms of **quark** and **gluon propagators**;

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How to proceed further?

- to construct **perturbation expansion** in terms of **quark** and **gluon propagators**;
- to postulate that **quark** and **gluon propagators** are **modified** by the **long-range confinement** potential;

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- to construct **perturbation expansion** in terms of **quark** and **gluon propagators**;
- to postulate that **quark** and **gluon propagators** are **modified** by the **long-range confinement** potential;
- this **modification** is soft: at $\boldsymbol{\tau \to 0}$ the difference between **exact** and **perturbative** propagators vanishes.

Condensates in QCD

We write exact propagator $\boldsymbol{\mathcal{S}^{\text{exact}}(x,0)}$ as a vacuum average in the exact vacuum Ω

$$\mathcal{S}^{\text{exact}}(x,0) = \langle \Omega |\, T(\varphi(x)\varphi(0)) | \Omega \rangle \,.$$

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$$\mathcal{S}^{\text{exact}}(x, 0) = \langle\Omega|\, T(\varphi(x)\varphi(0))|\Omega\rangle\,.$$

Using Wick theorem, one can write T-product as the sum

$$T(\varphi(x)\varphi(0)) = \underbrace{\varphi(x)\varphi(0)} + :\varphi(x)\varphi(0):$$

of the “pairing” and the “normal” product.

Condensates in QCD

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Using Wick theorem, one can write T-product as the sum

$$T(\varphi(x)\varphi(0)) = S_0(x,0) + :\varphi(x)\varphi(0):$$

Then for expectation value in the physical vacuum

$$\mathcal{S}^{\text{exact}}(x,0) = S_0(x,0) + \langle\Omega|\; :\varphi(x)\varphi(0):|\Omega\rangle$$

– the starting point to calculate power corrections in QCD.

Condensates in QCD

The examples in QCD are

- $\langle \bar{q}q \rangle$ referred to as **quark condensate**;
- $\langle \bar{q}D^2q \rangle$, characterizing **average virtuality** of the vacuum quarks;
- **gluon condensate** $\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle$, etc.

Here $D_\mu \equiv \partial_\mu - igA_\mu$ is the covariant derivative and $G_{\mu\nu} = (i/g)[D_\mu, D_\nu]$ is the gluonic field strength.

Further we will use notation $|\mathbf{0}\rangle$ for physical vacuum $|\mathbf{\Omega}\rangle$.

Condensates and PCAC for pions in QCD

We derived the relations: $\partial^\mu J_{\mu 5}(x) = (m_u + m_d)\; J_5(x)$,

$$\langle 0 \big| J_{\mu 5}(0) \big| \pi(P) \rangle = i\, f_\pi\, P_\mu\,; \quad \langle 0 \big| J_5(0) \big| \pi(P) \rangle = \frac{f_\pi\, m_\pi^2}{m_u + m_d}\,.$$

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Consider now correlator

$$\Pi_{\mu 55}(q) = i \int d^4x\, e^{iqx} \langle 0 \big| T \left[J_{\mu 5}(x) J_5^\dagger(0) \right] \big| 0 \rangle \equiv i\, q_\mu \Pi_{\rm AP}(q^2)$$

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and its contraction with q^μ

$$i\, q^2\, \Pi_{\mathrm{AP}}(q^2) = - \int d^4x\, e^{iqx} \frac{\partial}{\partial x_\mu} \langle 0 | T \left[J_{\mu 5}(x) J_5^\dagger(0) \right] |0\rangle\,.$$

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Contracting our correlator with q^μ

$$\begin{aligned} i\, q^2\, \Pi_{\rm AP}(q^2) \;\; &= \;\; - \int d^3\vec{x}\, e^{-i\vec{q}\vec{x}} \langle 0 | \left[J_{05}(0,\vec{x}); J_5^\dagger(0) \right] | 0 \rangle \\ &\quad - (m_u + m_d) \int d^4x\, e^{iqx} \langle 0 \big| T \left[J_5(x) J_5^\dagger(0) \right] | 0 \rangle \end{aligned}$$

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Convoluting our correlator with q^μ

$$\begin{aligned} i\, q^2\, \Pi_{\rm AP}(q^2) &= -\int d^3\vec{x}\, e^{-i\vec{q}\vec{x}} \langle 0 | \left[J_{05}(0,\vec{x}); J_5^\dagger(0) \right] |0\rangle \\ &\quad - (m_u + m_d) \int d^4x\, e^{iqx} \langle 0 | T \left[J_5(x) J_5^\dagger(0) \right] |0\rangle \\ &= i\, \langle \bar{u}u + \bar{d}d \rangle + i\; (m_u + m_d)\; \Pi_{55}(q^2) \end{aligned}$$

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Then for correlator $\Pi_{\mathrm{AP}}(q^2)$ we have

$$\Pi_{\mathrm{AP}}(q^2) \;\; = \;\; \frac{\langle \bar{u}u + \bar{d}d \rangle}{q^2} + (m_u + m_d)\, \frac{\Pi_{55}(q^2)}{q^2}$$

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Insert pions in between currents of $\Pi_{\rm AP}(q^2)$:

$$\Pi_{\rm AP}(q^2) \ \approx \ \frac{f_\pi\, m_\pi^2}{m_u + m_d}\, \frac{f_\pi}{m_\pi^2 - q^2} \ = \ \frac{-f_\pi^2\, m_\pi^2}{m_u + m_d}\, \frac{1}{q^2} \left[1 + O\left(\frac{m_\pi^2}{q^2} \right) \right]$$

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Comparing asymptotics $O(1/q^2)$ gives us the famous PCAC relation:

$$f_\pi^2\, m_\pi^2 \;=\; -\,\langle \bar{u}u + \bar{d}d \rangle\, (m_u + m_d) + O(m_q^2)$$

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In fact we should add other possible PS-meson states to obtain

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- $f_\pi \neq 0$, then $m_\pi \to 0 \Rightarrow$ pion is Goldstone boson;
- $m_{\pi'} \neq 0$, then $f_{\pi'} \to 0 \Rightarrow$ no decays $\pi' \to \mu\nu_\mu$!

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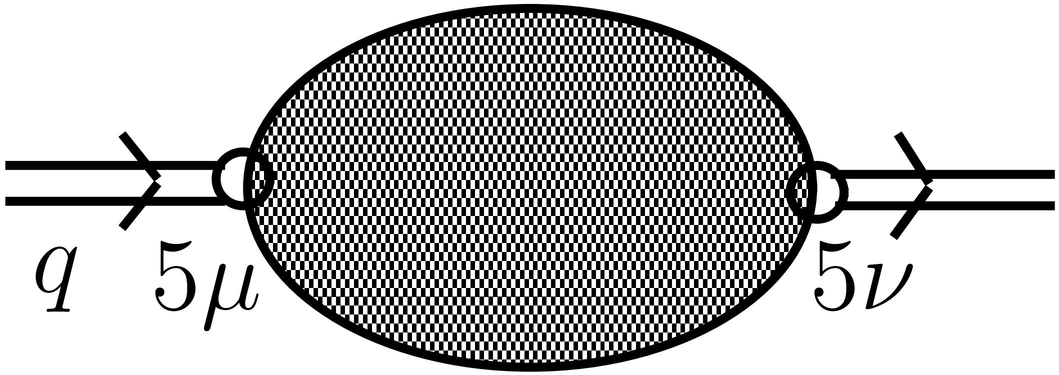
- $f_\pi \neq 0$, then $m_\pi \to 0 \Rightarrow$ pion is Goldstone boson;
- $m_\pi \approx f_\pi \approx 130$ MeV $\Rightarrow \langle \bar{q}q \rangle \approx -(260\ \text{MeV})^3$ at $m_u = m_d = 4$ MeV.

Real case: QCD SRs for π-mesons

Diagrams for axial-axial correlator $\Pi_{\mu 5;\nu 5}(q)$



$$i \int d^4x\, e^{iqx} \langle 0 | T \left[J_{\mu 5}(x) J^{\dagger}_{\nu 5}(0) \right] | 0 \rangle \equiv g_{\mu\nu} \Pi_1(q^2) + q_\mu q_\nu \Pi_2(q^2)$$

Hadronic contribution to Borel transform of $\Pi_2(q^2)$:

$$\Phi^{\rm hadr}\left(M^2\right) = B_{Q^2 \to M^2}\left[\Pi_2^{\rm hadr}(q^2)\right] = \frac{f_\pi^2}{M^2} + \frac{f_{A_1}^2}{M^2} e^{-m_{A_1}^2/M^2} + \ldots$$

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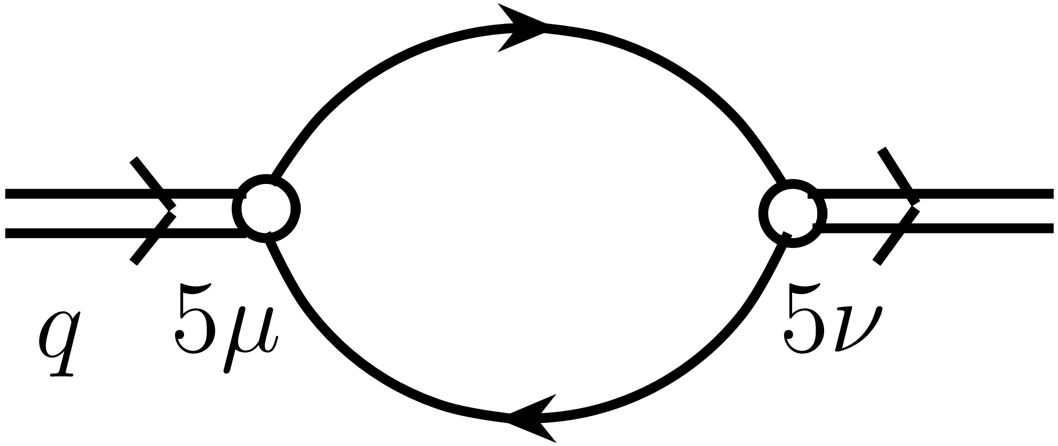

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Perturbative contribution to Borel transform of $\Pi_2(q^2)$:

$$\Phi^{\rm pert}\left(M^2\right) = \int_0^\infty \frac{1}{4\pi^2}\left[1+\frac{\alpha_s}{\pi}\right] e^{-s/M^2}\frac{ds}{M^2}$$

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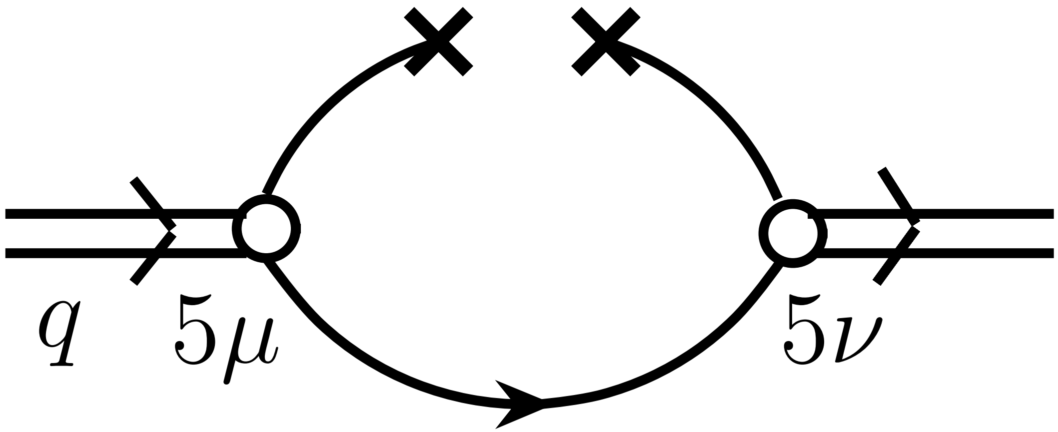

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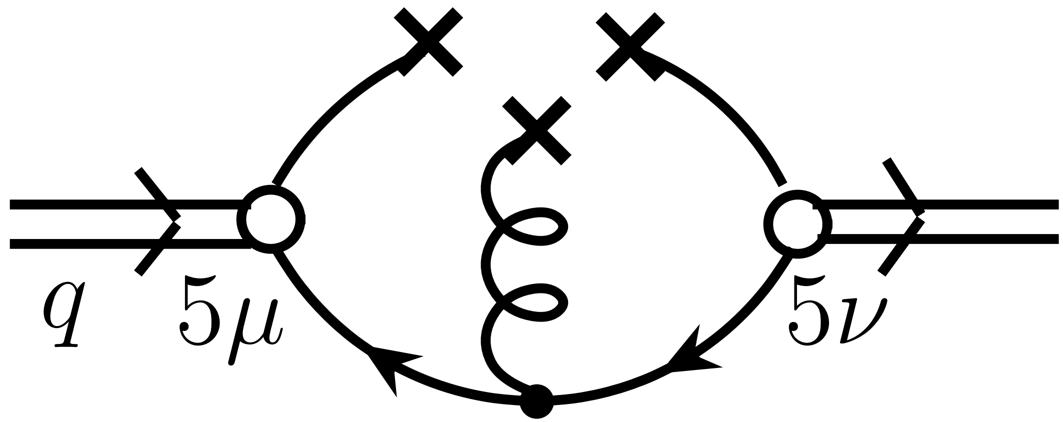
Vector QC contribution to Borel transform of $\Pi_2(q^2)$:

$$\Phi_{\rm VQC}\left(M^2\right)=\frac{16}{81}\,\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^6}$$

Diagrams for axial-axial correlator $\Pi_{\mu 5;\nu 5}(q)$



$$i\int d^4x\, e^{iqx}\langle 0|T\left[J_{\mu 5}(x)J^{\dagger}_{\nu 5}(0)\right]|0\rangle \equiv g_{\mu\nu}\Pi_1(q^2) + q_\mu q_\nu \Pi_2(q^2)$$

QGQC contribution to Borel transform of $\Pi_2(q^2)$:

$$\Phi_{\mathrm{QGQC}}\left(M^2\right) = \frac{16}{81}\,\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^6}$$

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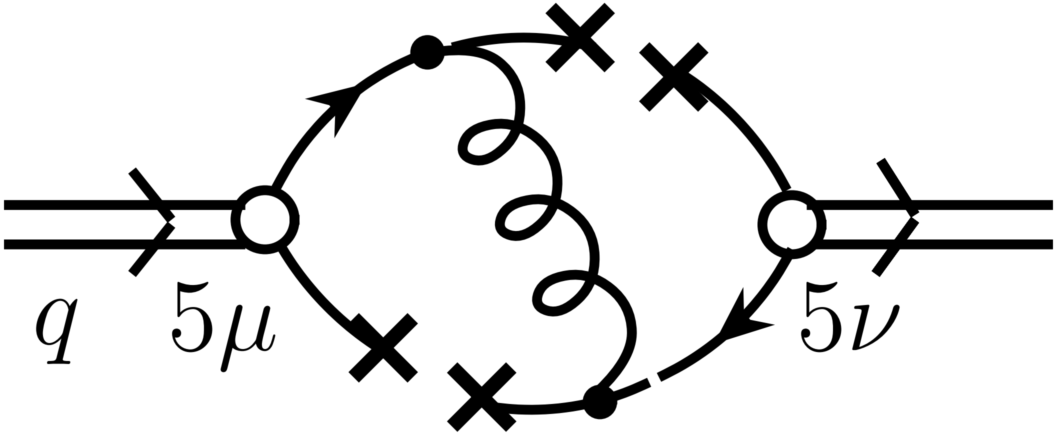

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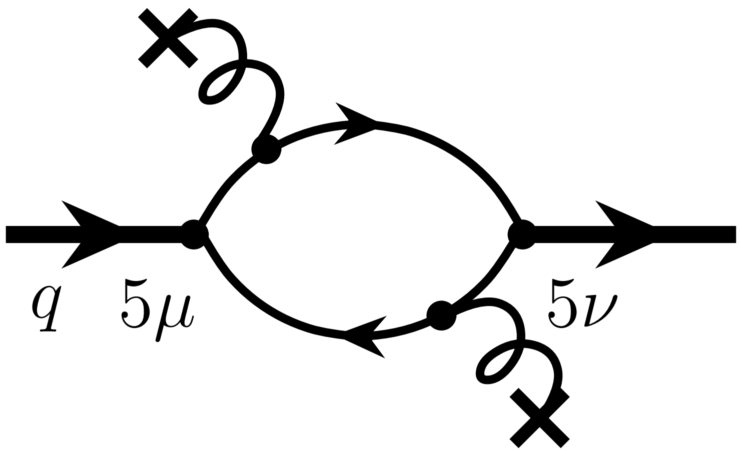
4-QC contribution to Borel transform of $\Pi_2(q^2)$:

$$\Phi_{4\text{-QC}}\left(M^2\right) = \frac{144}{81}\,\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^6}$$

Diagrams for axial-axial correlator $\Pi_{\mu 5;\nu 5}(q)$



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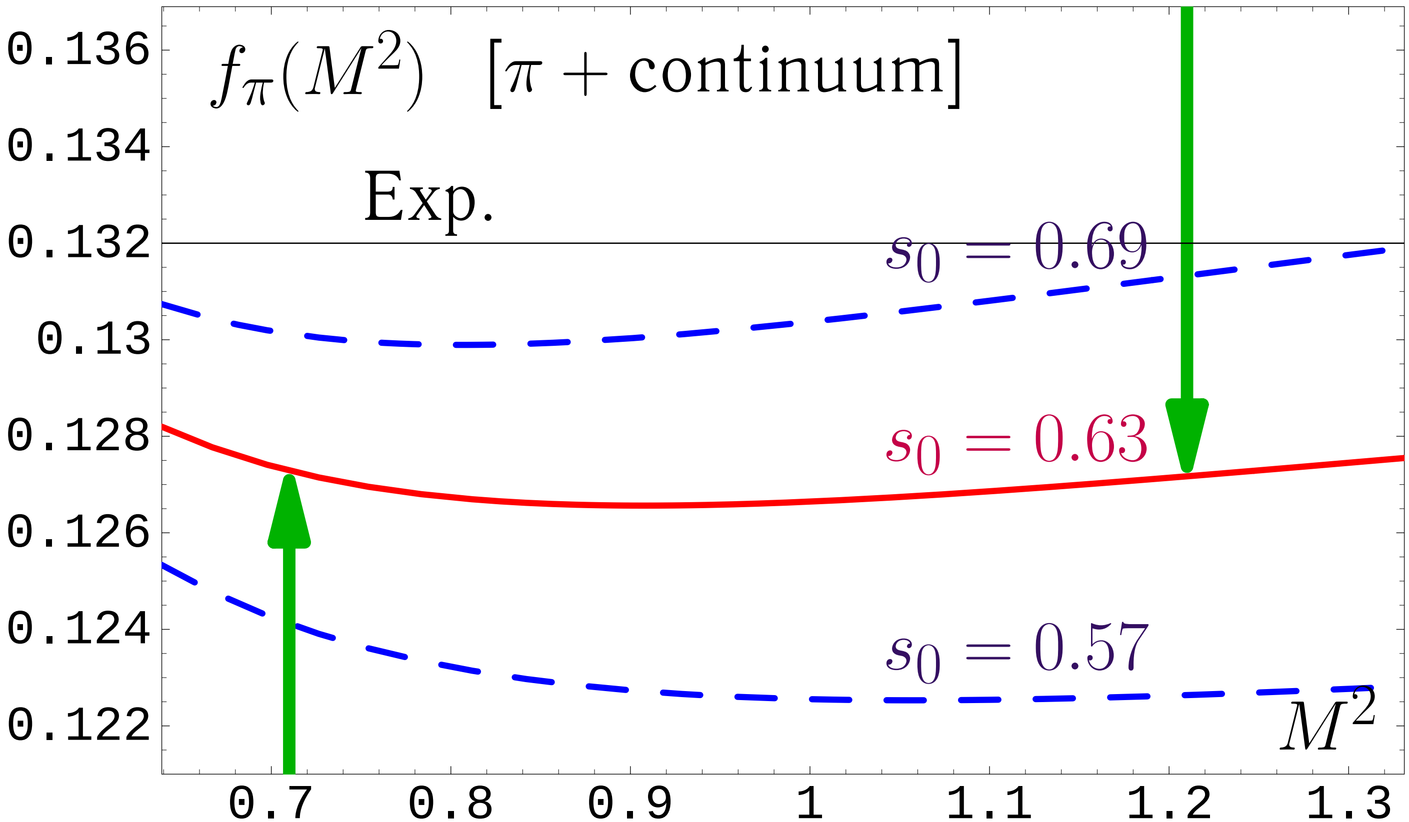
$\langle GG \rangle$ contribution to Borel transform of $\Pi_2(q^2)$:

$$\Phi_{\langle GG \rangle} \left(M^2 \right) = \frac{1}{12\,\pi} \, \frac{\langle \alpha_s \, GG \rangle}{M^4}$$

QCD SR for axial correlator $\Pi_2(q^2)$

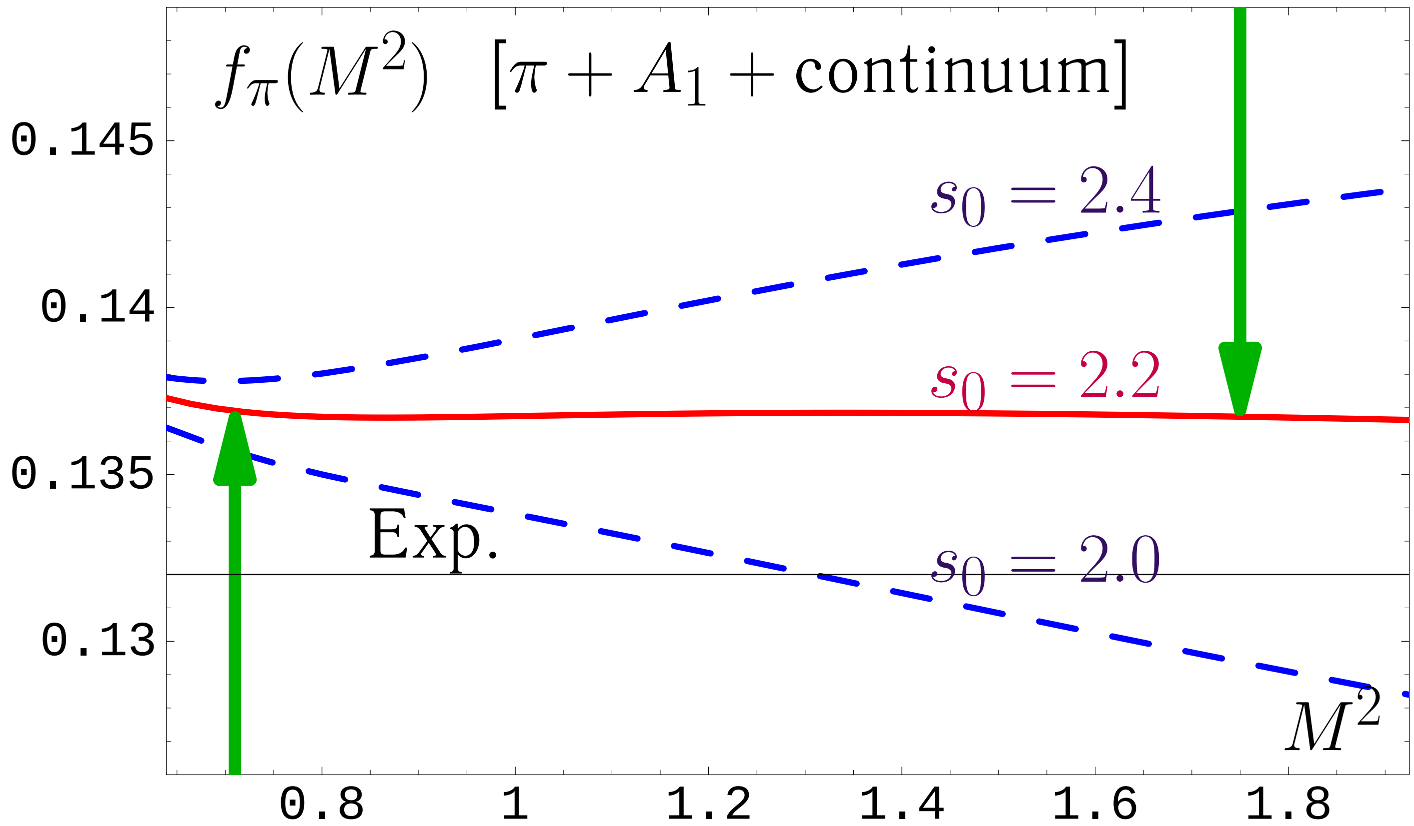
As a result we have SR for pion decay constant

$$f_\pi^2 = \frac{M^2}{4\pi^2}\left(1 - e^{-s_0/M^2}\right)\left[1 + \frac{\alpha_s}{\pi}\right] + \frac{1}{12\,\pi}\,\frac{\langle\alpha_s\,GG\rangle}{M^2} + \frac{176}{81}\,\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^4}$$



QCD SR for axial correlator $\Pi_2(q^2)$

In a model with A_1-meson we obtain slightly higher value $f_\pi = 0.137 \pm 0.13$ GeV, to be compared with $f_\pi^{\rm exp} = 0.132$ GeV.



Generalization: QCD SRs for π Distribution Amplitude

Pion distribution amplitude (DA)

- Matrix element of nonlocal axial current on light cone

$$\langle 0 \mid \bar{d}(z)\gamma_\mu\gamma_5 \boldsymbol{E(z,0)} u(0) \mid \pi(P)\rangle\Big|_{z^2=0} =$$

$$i f_\pi P_\mu \int_0^1 dx\ e^{ix(zP)}\ \varphi_\pi^{\text{Tw-2}}(x,\mu^2)$$

- **gauge-invariance** due to Fock–Schwinger string:

$$\boldsymbol{E(z,0) = \mathcal{P} e^{ig\int_0^z A_\mu(\tau)d\tau^\mu}}$$

- Physical meaning of $\varphi_\pi(x;\mu^2)$ — amplitude for transition $\pi \to u + d$

Representation of Pion DA

- It is convenient to represent the pion DA:
  $\varphi_\pi(x;\mu^2) = \varphi^{As}(x)\times$
  $\times\left[1 + a_2(\mu^2)C_2^{3/2}(2x-1) + a_4(\mu^2)C_4^{3/2}(2x-1) + \ldots\right]$
  where $C_n^{3/2}(2x-1)$ are the Gegenbauer polynomials (1-loop eigenfunctions of ER-BL kernel)

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- ER-BL solution at 2-loop level

  [Mikhailov&Radyushkin; 1986
  Müller; 1994–95
  A.B.&Stefanis; 2005]

Non-Local Condensates in QCD SR

- Illustration of
  NLC-model: $\langle \bar{q}(0) q(z) \rangle = \langle \bar{q}(0) q(0) \rangle e^{-|z^2| \lambda_q^2 / 8}$

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- A **single scale** parameter $\lambda_q^2 = \langle k^2 \rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1 \text{ GeV}^2 & [\textbf{ QCD SRs, 1987 }] \\ 0.5 \pm 0.05 \text{ GeV}^2 & [\textbf{ QCD SRs, 1991 }] \\ \approx 0.4 - 0.5 \text{ GeV}^2 & [\textbf{ Lattice, 1998-2002 }] \end{cases}$$

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- Correlation length $\lambda_q^{-1} \simeq 0.3$ Fm $\sim$ ρ-meson size
- Possible to include second ($\Lambda \simeq 450$ MeV) scale with $\langle \bar{q}(0)q(z)\rangle\Big|_{|z|\gg 1 \text{ Fm}} \sim \langle \bar{q}(0)q(0)\rangle e^{-|z|\Lambda}$ (not included here)

NLC QCD SR for Pion DA

Here is example of QCD SR with Non-Local Condensates

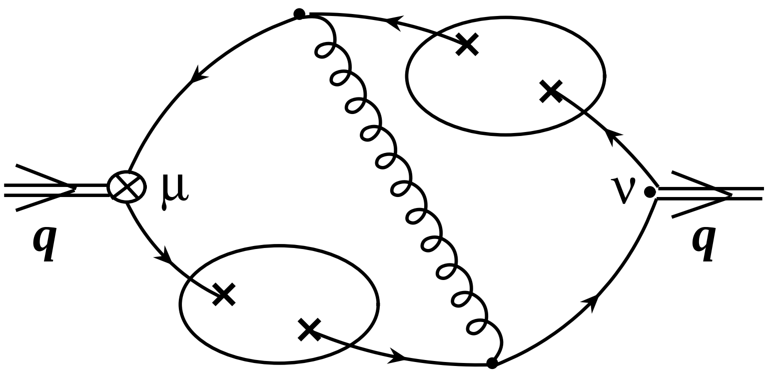
$$f_\pi^2\,\varphi_\pi(x) \;=\; \int_0^{s_{\pi 0}} \rho^{\text{pert}}(x;s)\,e^{-s/M^2}ds + \frac{\langle(\alpha_s/\pi)GG\rangle}{24M^2}\,\varphi_G(x;\Delta) + \frac{16\pi\alpha_s\langle\bar{q}q\rangle^2}{81M^4}\sum_{i=2V,3L,4Q}\varphi_i(x;\Delta)$$

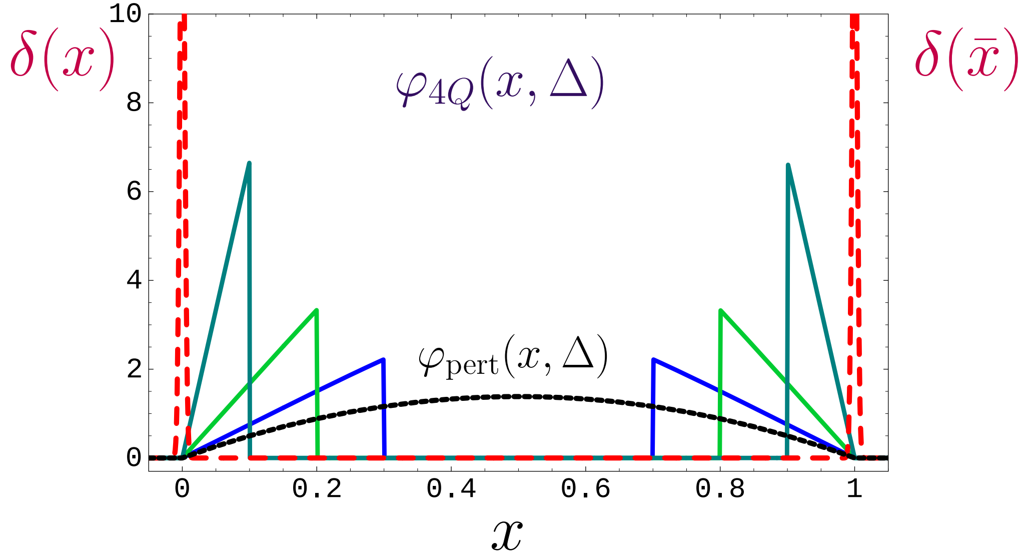
Local limit: $\lambda_q^2/M^2 \equiv \Delta \to 0$,

$$\begin{aligned}
\varphi_G(x;\Delta) &= [\delta(x)+\delta(1-x)] \\
\varphi_{2V}(x;\Delta) &= [x\delta'(\bar{x})+\bar{x}\delta'(x)] \\
\varphi_{4Q}(x;\Delta) &= 9[\delta(x)+\delta(1-x)]
\end{aligned}$$

NLC contributions to QCD SR

Examples for Gaussian NLC with a single parameter λ_q^2



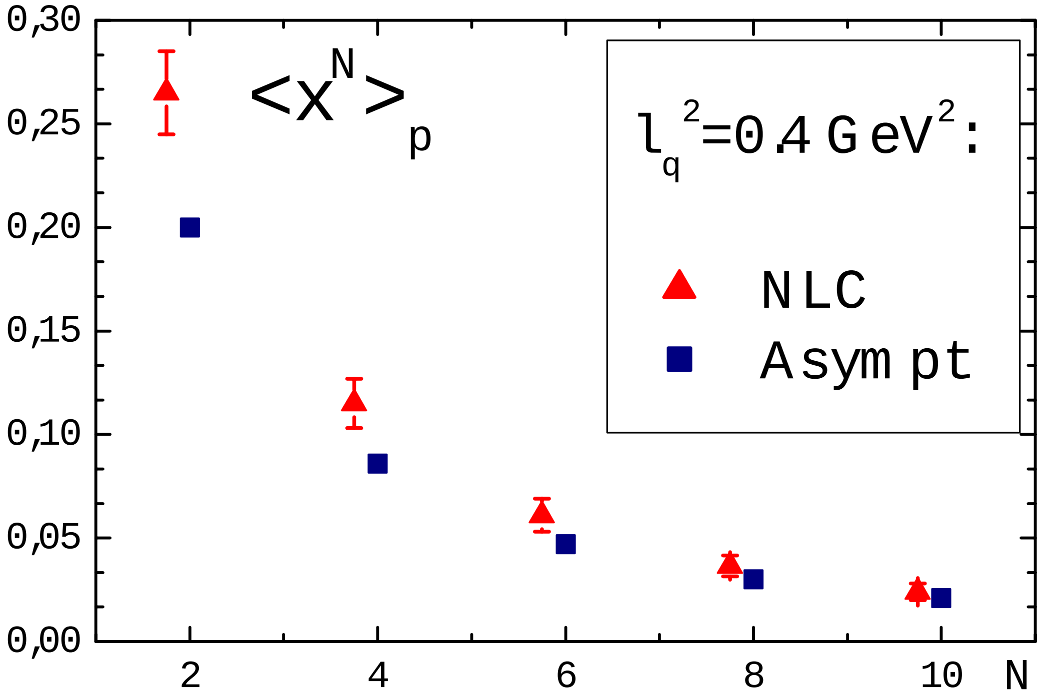


Local limit: $\lambda_q^2/M^2 \equiv \Delta \to 0$,

$$\lim_{\Delta \to 0} \varphi_{4Q}(x; \Delta) = 9[\delta(x) + \delta(1-x)]$$

NLC SRs for pion DA

Moments $\langle \xi^N \rangle_\pi = \int_0^1 \varphi_\pi(x)\,(2x-1)^N\,dx$ at $\mu^2 \approx 1$ GeV^2



from NLC SRs

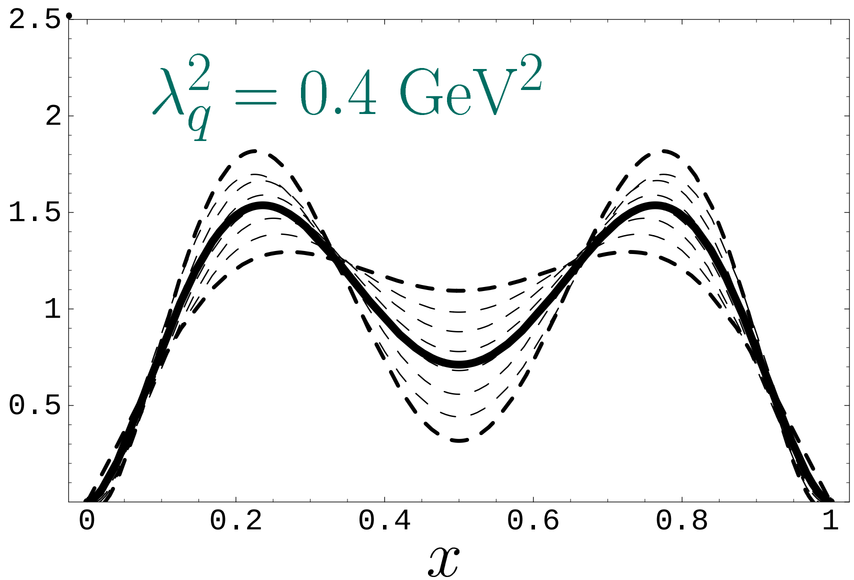
▲ **PLB 508 (2001) 279**

These $\langle \xi^N \rangle_\pi$ values allow one to **restore** DA $\varphi_\pi(x)$

NLC SRs for Pion DA

produce **bunch** of self-consistent 2-parameter models $\varphi_\pi(x)$ at $\mu^2 \simeq 1 \ \text{GeV}^2$:

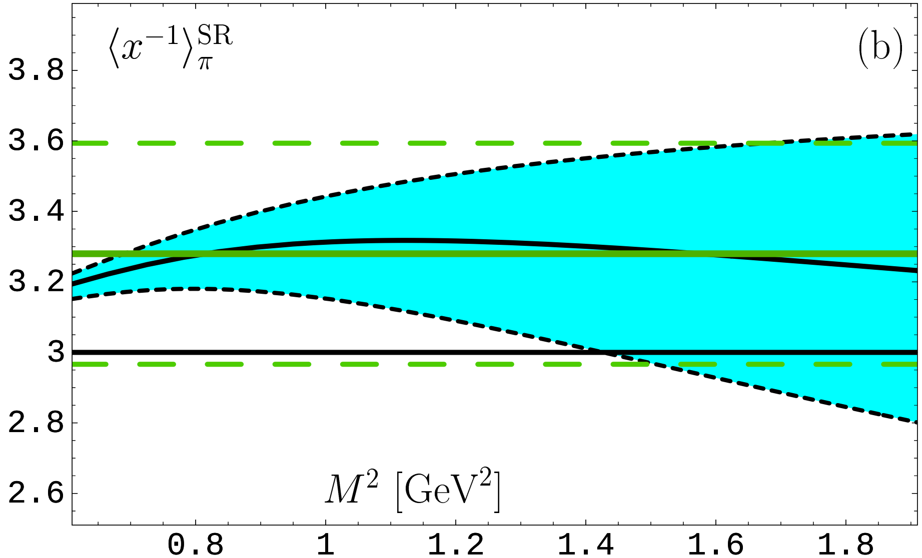
$$\varphi_\pi(x) = \varphi^{\text{as}}(x) \left[1 + a_2 \ C_2^{3/2}(2x-1) + a_4 \ C_4^{3/2}(2x-1)\right]$$



$a_2{}^{\textbf{b.f.}}$	$=$	$+0.188$
$a_4{}^{\textbf{b.f.}}$	$=$	-0.130
χ^2	$\approx$	0.001
$\langle x^{-1}\rangle^{\text{SR}}$	$=$	$3.30(30)$

NLC SR estimate of $\langle x^{-1}\rangle_\pi^{SR}$

BMS [PLB (2001)]: at $\mu^2 \simeq 1$ GeV^2



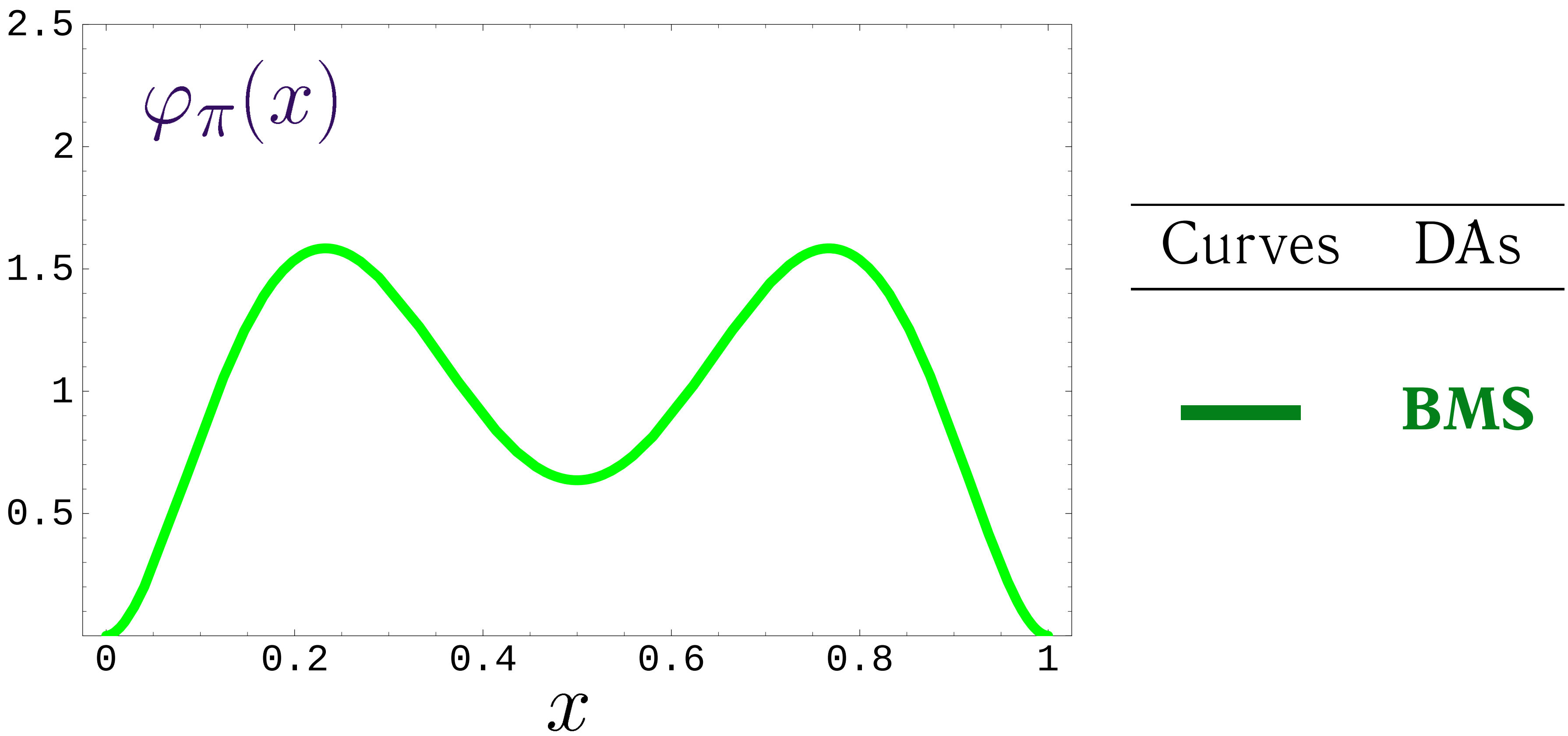
$\lambda_q^2 = 0.4$ GeV^2,

$\langle x^{-1}\rangle_\pi^{\rm SR} = 3.3 \pm 0.3$,

$\langle x^{-1}\rangle_\pi^{\bf b.f.} = 3.17$

The moment $\langle x^{-1}\rangle_\pi^{\rm SR}$ could be determined only in NLC SRs because end-point singularities absent

BMS vs CZ distribution amplitude



BMS DA is end-point suppressed!

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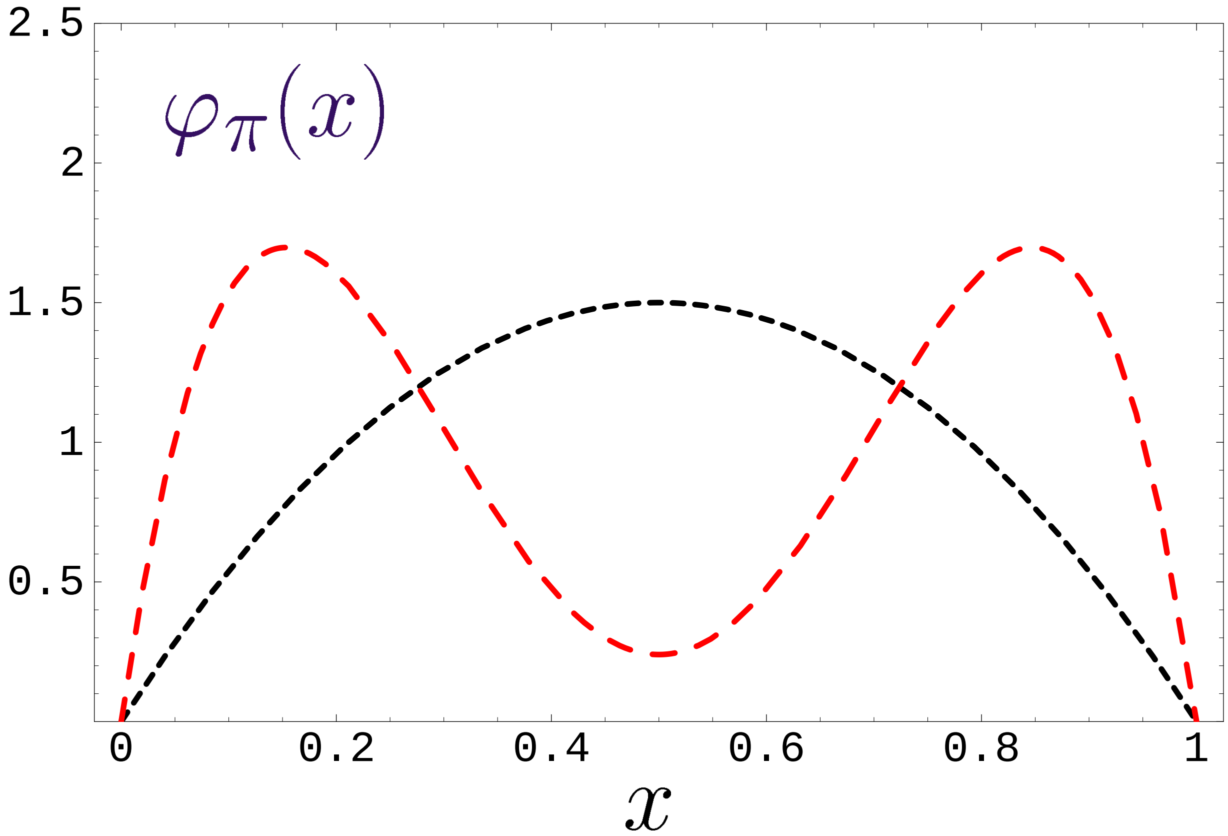
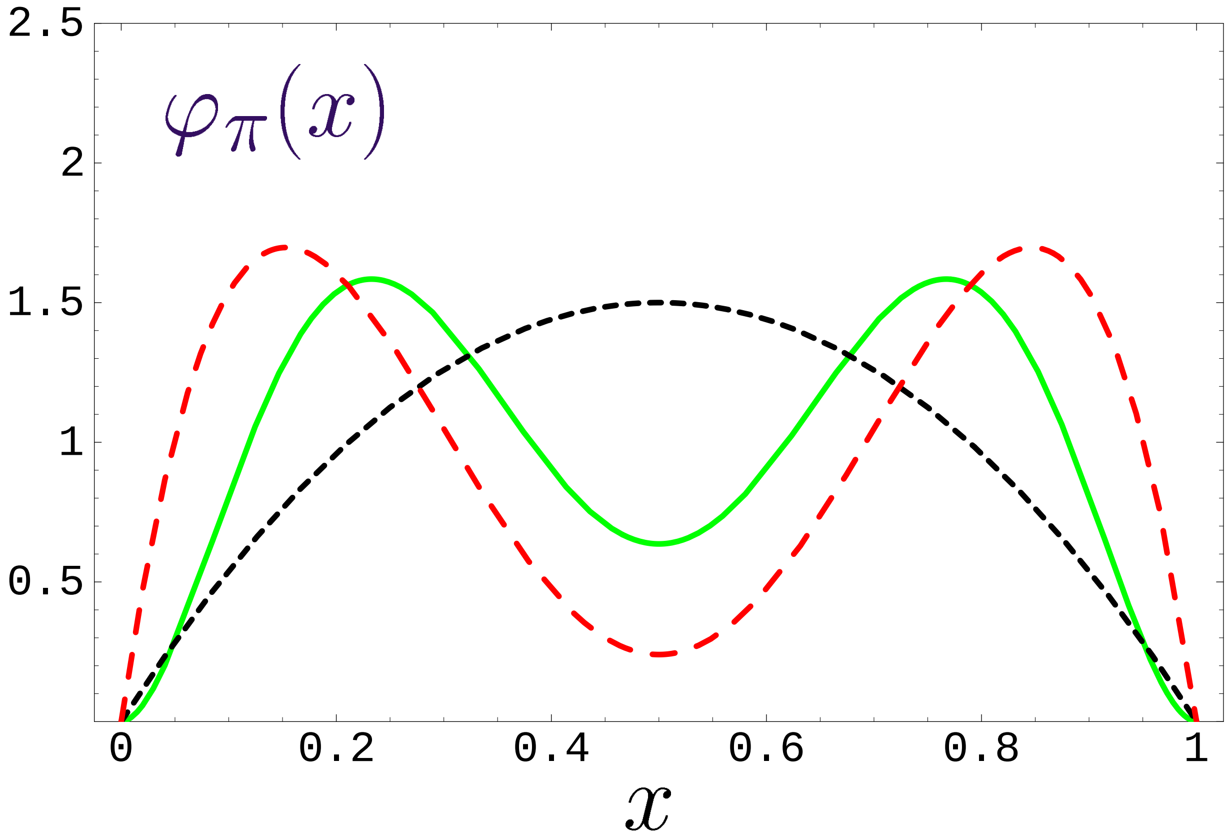


Curves	DAs
- - -	CZ
- - -	Asymp.

CZ DA: end-point enhancement

BMS vs CZ distribution amplitude



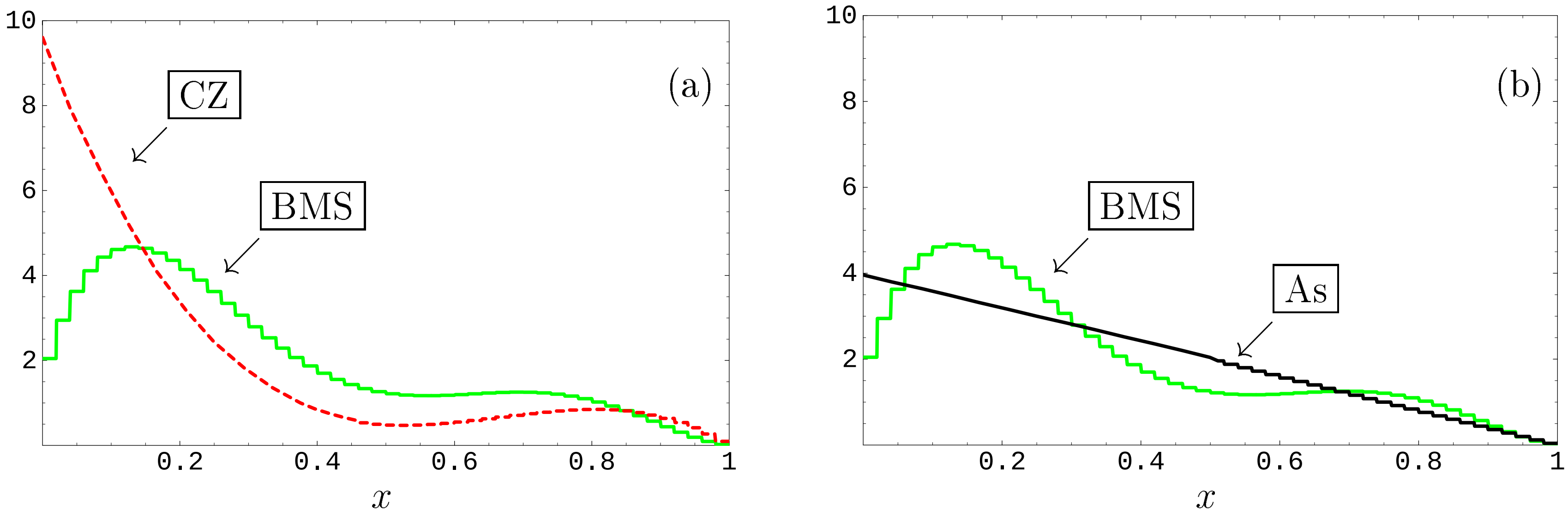
Curves	DAs
red dashed	CZ
green solid	**BMS**
black dashed	Asymp.

BMS bunch is 2-humped, but end-point suppressed!

Histograms for inverse moment $\langle x^{-1} \rangle_\pi$

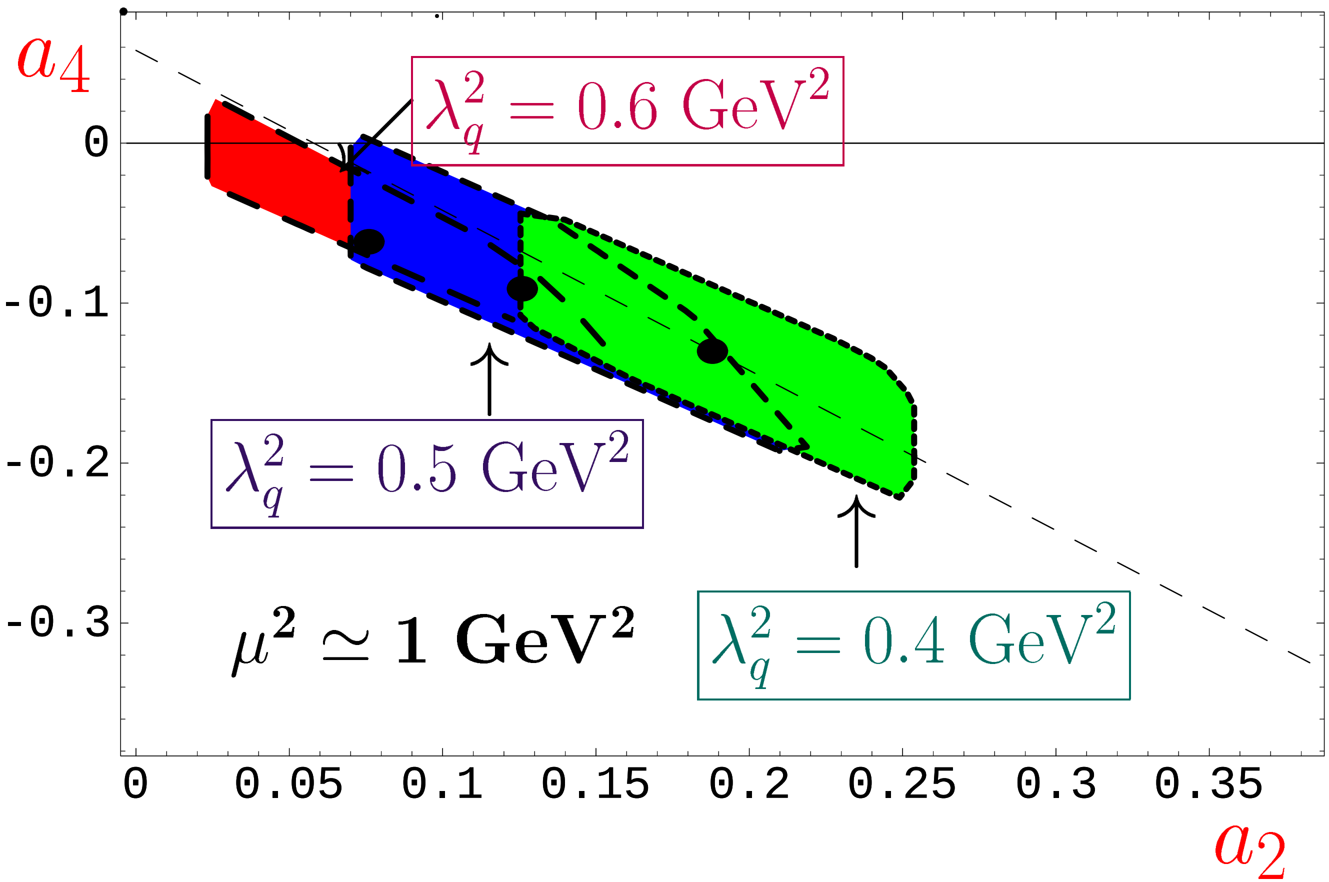
Contributions of different DAs to inverse moment $\langle x^{-1} \rangle_\pi$, calculated as $\int_x^{x+0.02} \phi(x)dx$ and normalized to 100%, for:

(a) **CZ** and **BMS** DAs; (b) Asympt. and **BMS** DAs.



In **BMS** case region $x \leq 0.1$ contributes even less than in Asymptotic DA case.

NLC SR Constraints on a_2, a_4 of Pion DA



Perturbative Part

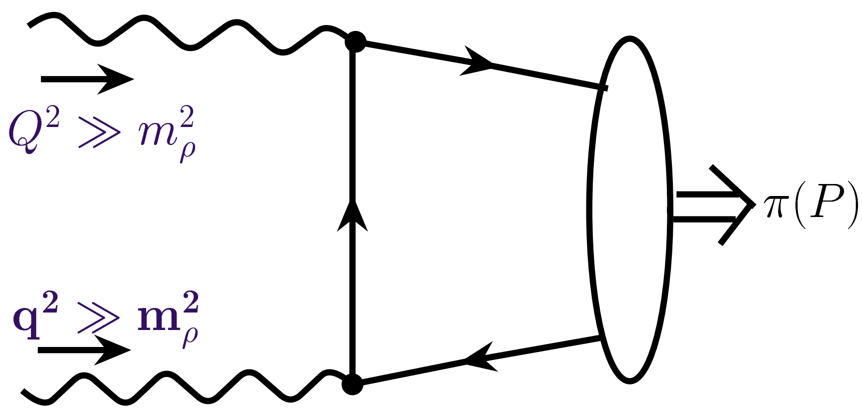
NLO Light-Cone Sum Rules $\Rightarrow$

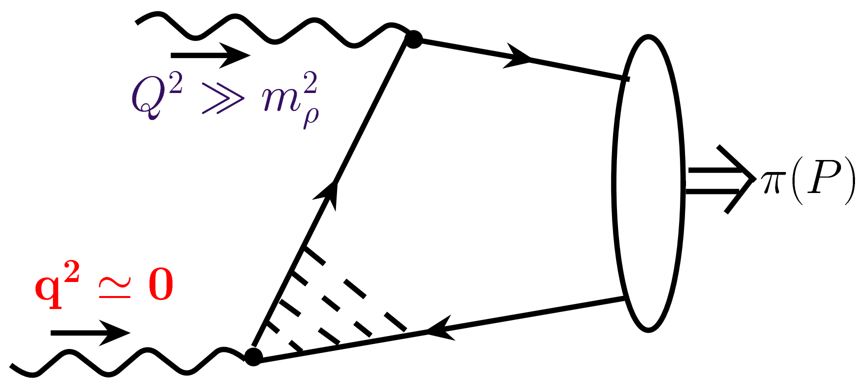
CLEO data on $F_{\gamma\gamma^*\pi}(Q^2) \Rightarrow$

Constraints on Pion DA

$\gamma^*\gamma \to \pi$: *Why Light-Cone Sum Rules?*

For $Q^2 \gg m_\rho^2$, $q^2 \ll m_\rho^2$ pQCD factorization valid only in leading twist and higher twists are of importance **[Radyushkin–Ruskov, NPB (1996)]**.
Reason: if $q^2 \to 0$ one needs to take into account interaction of real photon at long distances of order of $O(1/\sqrt{q^2})$





pQCD is OK **LCSR should be applied**

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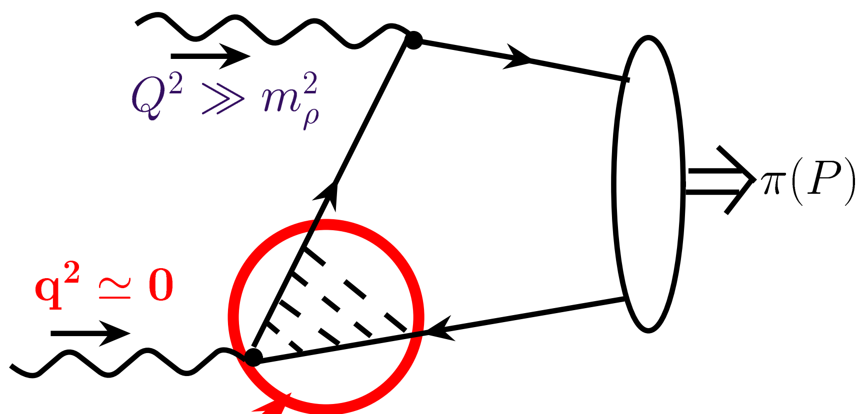

To account for long-distance effects in pQCD one needs to introduce light-cone DA of real photon

$\gamma^*\gamma \to \pi$: *Light-Cone Sum Rules!*

Khodjamirian **[EJPC (1999)]**: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in q^2

$$F_{\gamma\gamma^*\pi}(Q^2, q^2) = \frac{1}{\pi}\int_0^{s_0} \frac{\mathbf{Im} F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2, s)}{m_\rho^2 + q^2} \exp\left[\frac{m_\rho^2 - s}{M^2}\right] ds + \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\mathbf{Im} F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2, s)}{s + q^2} ds$$

$s_0 \simeq 1.5\ \text{GeV}^2$ – effective threshold in vector channel,
M^2 – Borel parameter $(0.5 - 0.9\ \text{GeV}^2)$.

Real-photon limit $\mathbf{q^2 \to 0}$ can be easily done ...

$\gamma^*\gamma \to \pi$: *Light-Cone Sum Rules!*

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... as demonstrated here.

Revision of CLEO data analysis

- Accurate NLO evolution for both $\varphi(x, Q^2_{\rm exp})$ and $\alpha_s(Q^2_{\rm exp})$, taking into account quark thresholds

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- The relation between “nonlocality"scale and twist-4 magnitude $\boxed{\delta^2_{\text{Tw-4}} \approx \lambda^2_q/2}$ was used to re-estimate $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.02$ at $\lambda^2_q = 0.4\ \text{GeV}^2$

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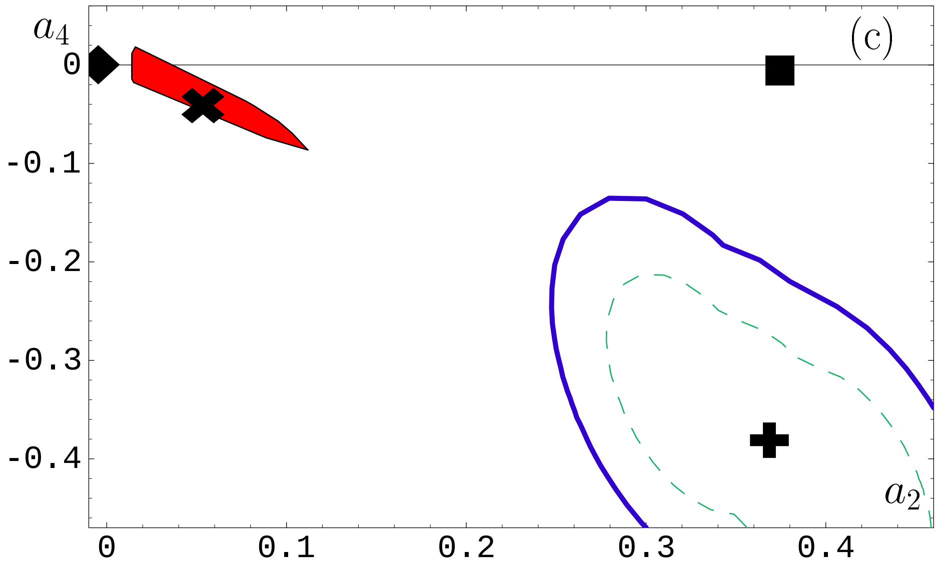
- New procedure of data processing to disentangle the statistical and theoretical uncertainties
- Constraints on $\langle x^{-1} \rangle_\pi$ from CLEO data

NLC SR Results vs NLO CLEO Constraints

[BMS, PRD 67 (2003) 074012]



$\Leftrightarrow \lambda_q^2 = 0.6\ \text{GeV}^2$,

$\delta^2_{\text{Tw-4}} = 0.28(3)\ \text{GeV}^2$

No agreement with CLEO data for $\lambda_{\mathbf{q}}^{\mathbf{2}} = 0.6\ \mathbf{GeV^2}$

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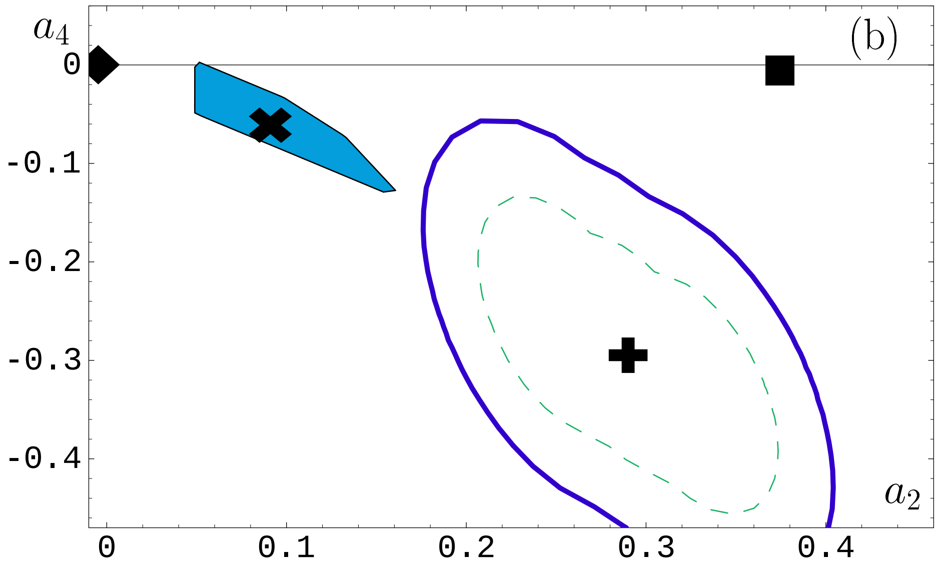
[BMS, PRD 67 (2003) 074012]

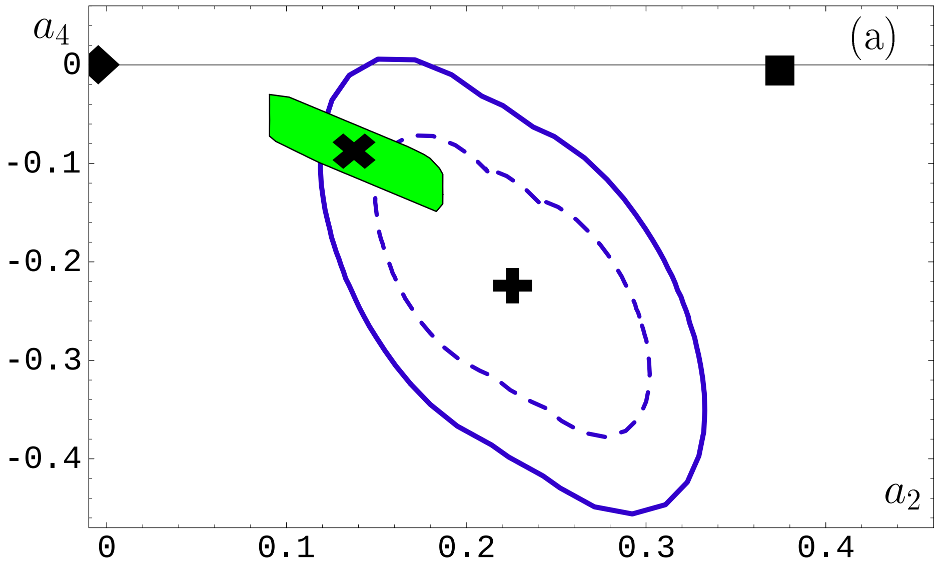


$\Leftrightarrow \lambda_q^2 = 0.5\ \text{GeV}^2$,
$\delta^2_{\text{Tw-4}} = 0.23(2)\ \text{GeV}^2$

Bad agreement with CLEO data for $\lambda_q^2 = 0.5\ \text{GeV}^2$

NLC SR Results vs NLO CLEO Constraints

[BMS, PRD 67 (2003) 074012]

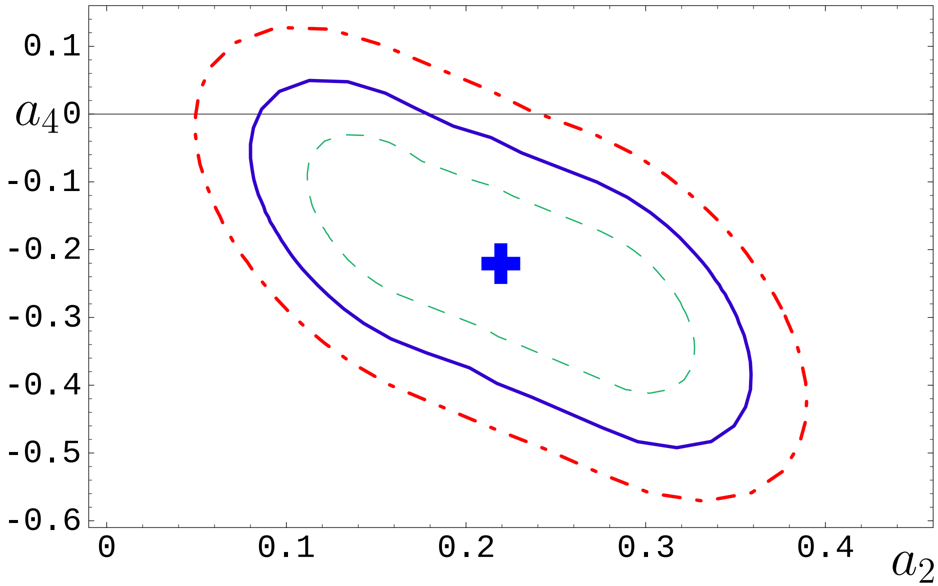


(green bar) $\Leftrightarrow \lambda_q^2 = 0.4\ \text{GeV}^2$,
$\delta^2_{\text{Tw-4}} = 0.19(2)\ \text{GeV}^2$

Reasonable agreement with CLEO data for $\boldsymbol{\lambda_q^2 = 0.4\ \text{GeV}^2}$

NLC SR Results vs Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$
with 20% uncertainty of $\delta^2_{\text{Tw-4}}$
BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ GeV^2, $\delta^2_{\text{Tw-4}} = 0.19(4)$ GeV



✚ = best-fit point

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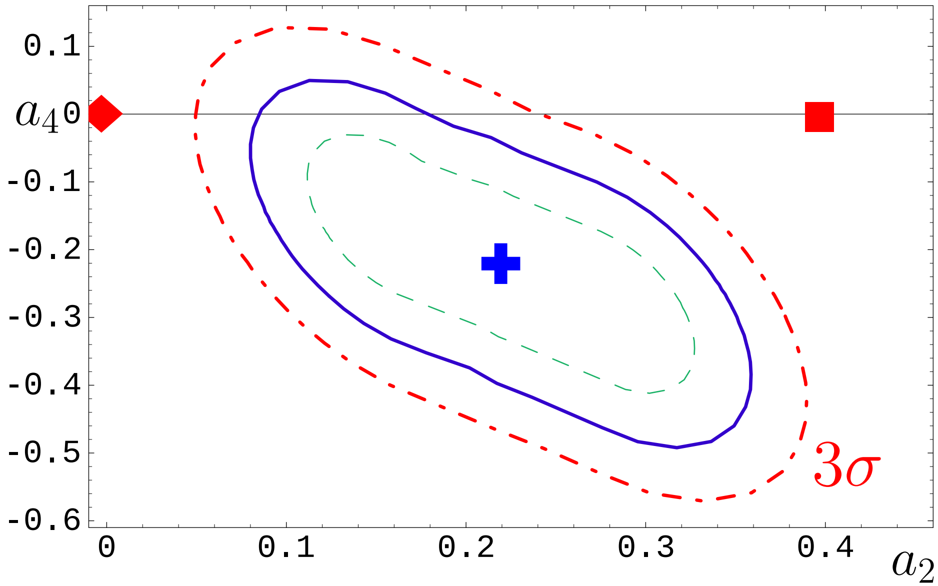

✚ = best-fit point
◆ = Asymptotic DA
■ = CZ DA

Even with 20% uncertainty in twist-4
CZ DA excluded **at least** at 4σ-level! **As** DA — at 3σ-level.

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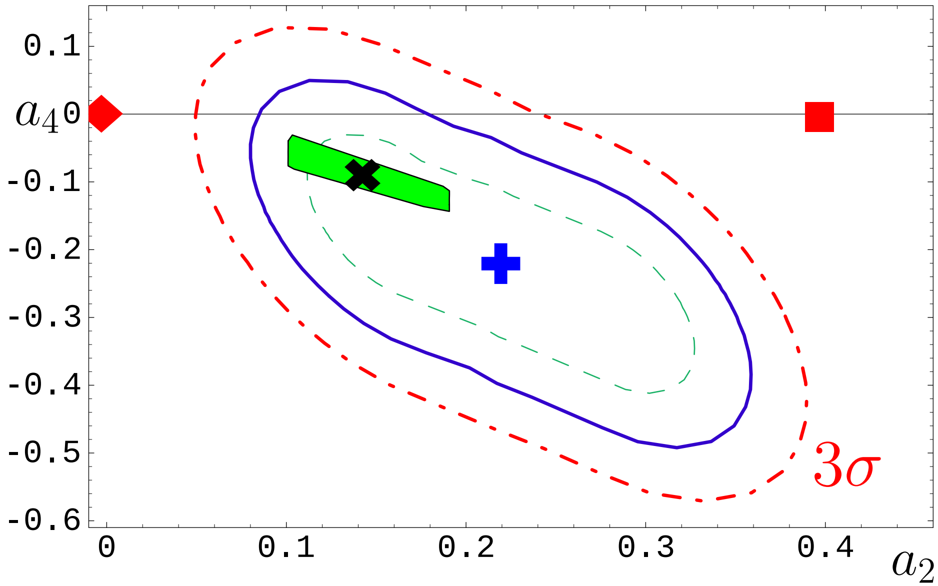

- ✚ = best-fit point
- ◆ = Asymptotic DA
- ■ = CZ DA
- ✖ = BMS model

CZ DA excluded **at least** at 4σ-level! **As** DA — at 3σ-level.
BMS DA and most of **BMS bunch** — inside 1σ-domain.

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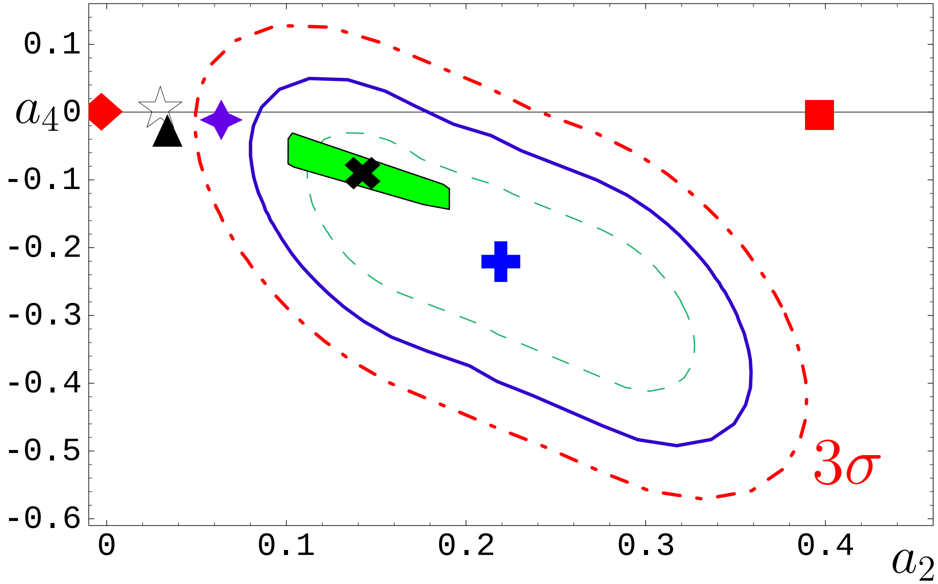

- ✚ = best-fit point
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BMS DA and most of **BMS bunch** — inside 1σ-domain.
Instanton-based models — near 3σ-boundary
(PR-model is close to 2σ-boundary).

NLC SR Results vs Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus (\mu^2 = Q^2)$
with 20% uncertainty of $\delta^2_{\text{Tw-4}}$
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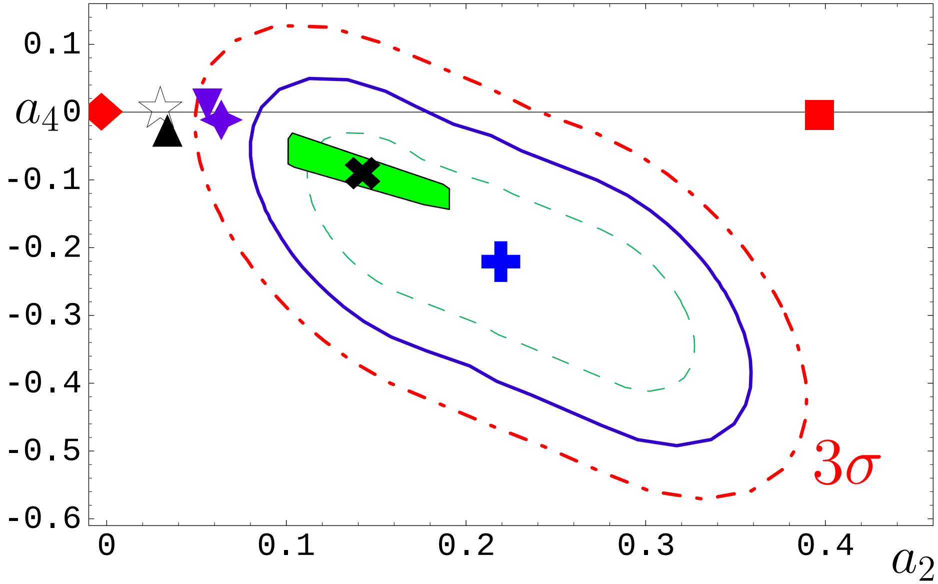

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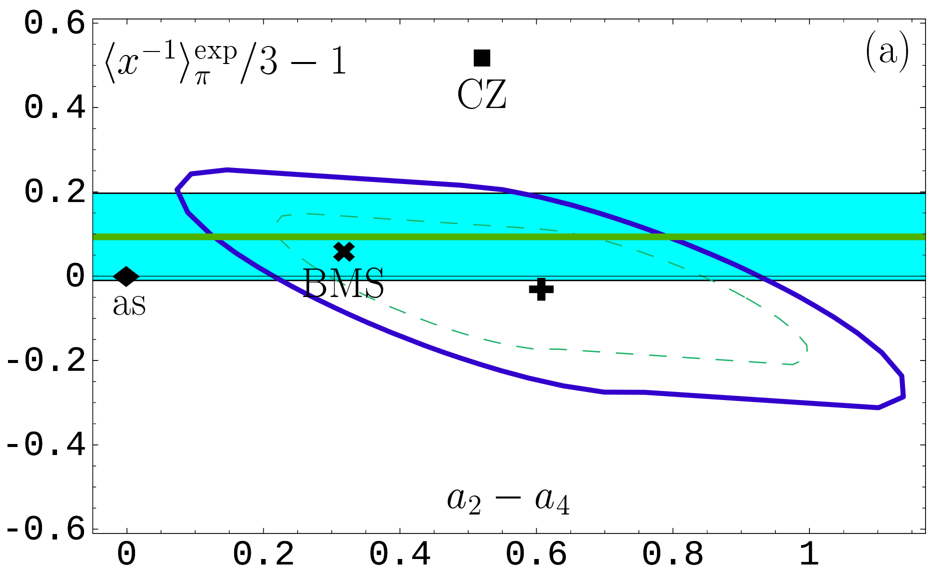
BMS DA and most of **BMS bunch** — inside 1σ-domain.
Transverse lattice model — near 3σ-boundary.

New CLEO data constraints for $\langle x^{-1}\rangle_\pi$

BMS [PLB 578 (2004) 91]: evolution to $\mu^2 = 1\ \mathrm{GeV}^2$



$\lambda_q^2 = 0.4\ \mathrm{GeV}^2$,

$\frac{1}{3}\langle x^{-1}\rangle_\pi^{\mathrm{SR}} - 1 = 0.1 \pm 0.1$ **(STRIP)**

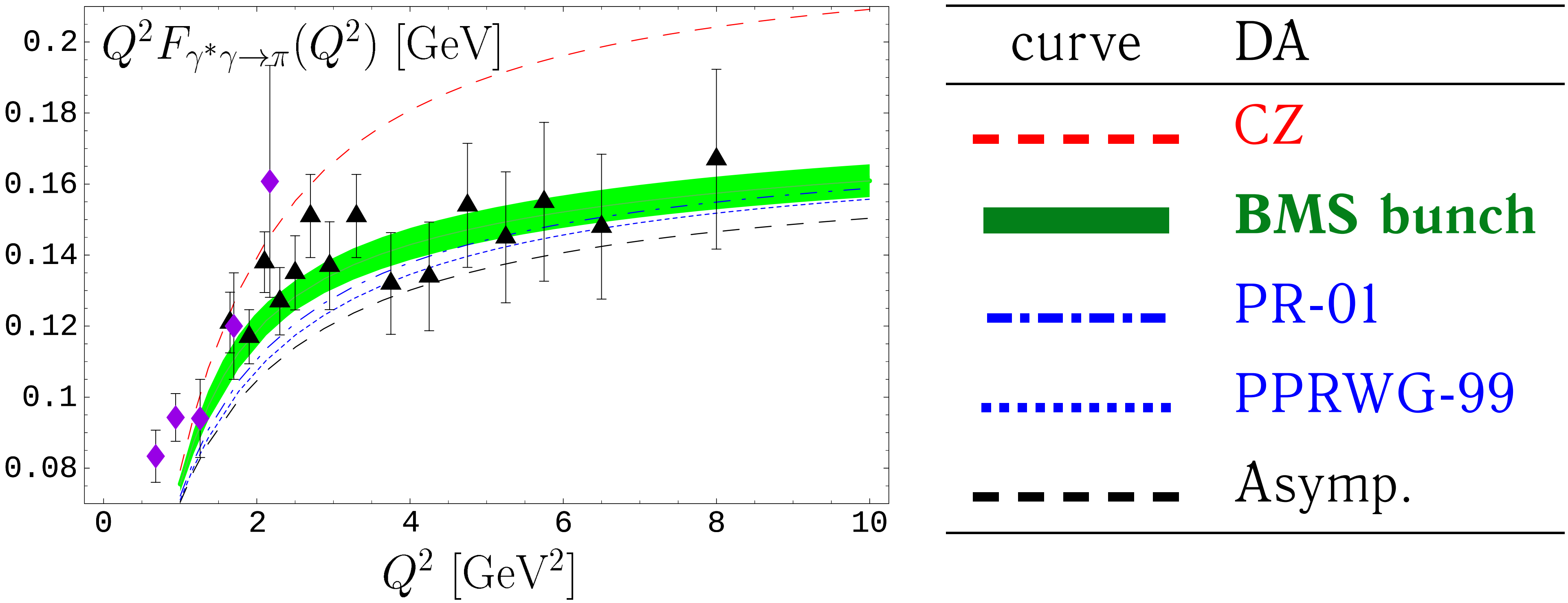
See also **Bijnens&Khodjamirian [EPJC (2002)]**:

$\frac{1}{3}\langle x^{-1}\rangle_\pi - 1 = 0.24 \pm 0.16$

Again:

Good agreement of a theoretical "tool" of different origin with CLEO data

LCSR vs. CELLO (♦) & CLEO (▲) data



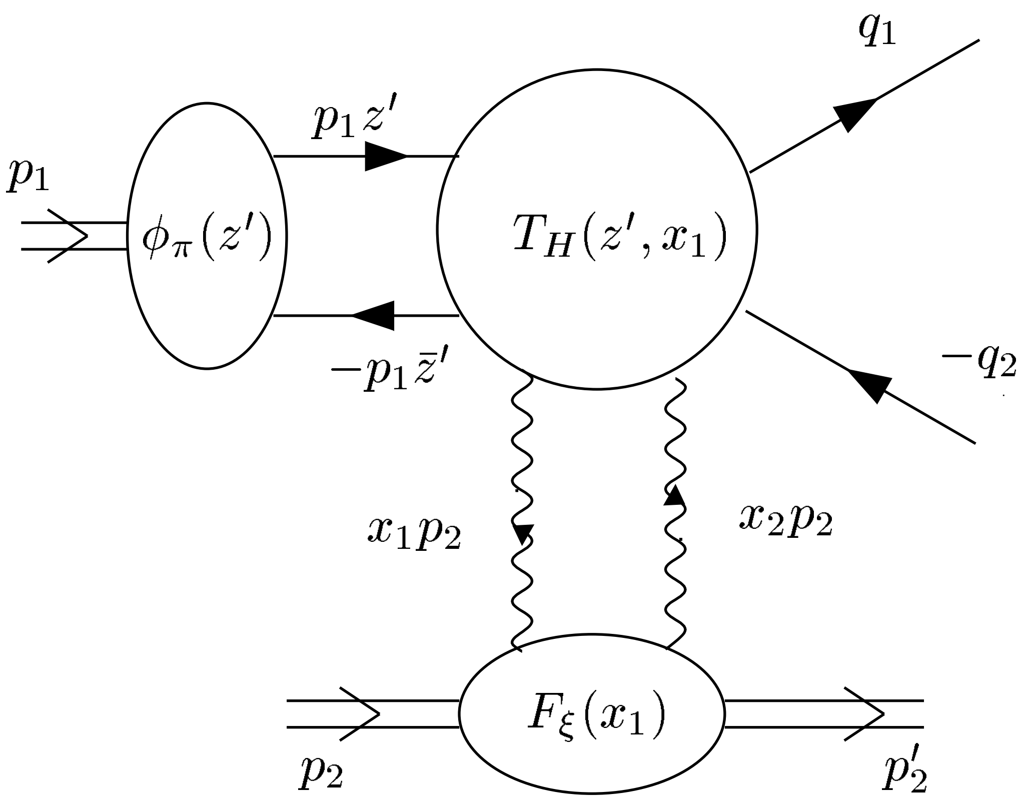
BMS bunch describes rather well all data for $Q^2 \gtrsim 1.5$ GeV^2.

Diffractive Dijet Production

What can add E791 data

E791: Diffractive dijet production

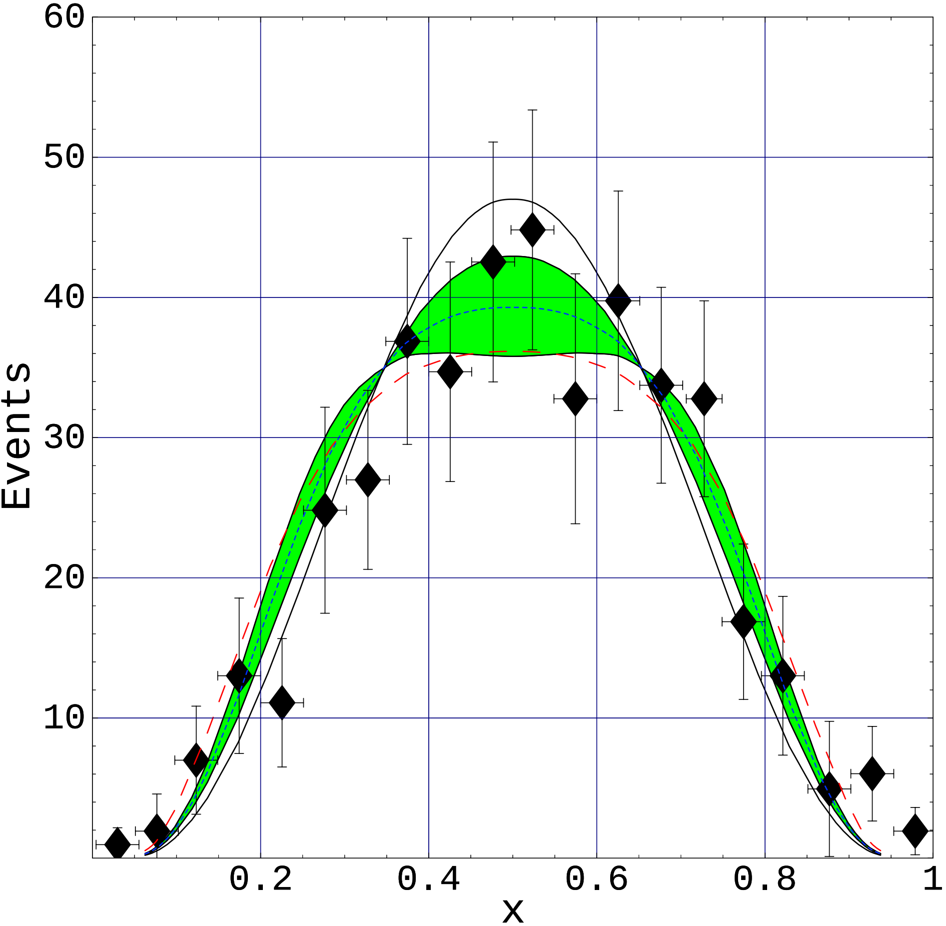
Frankfurt et al. [PLB (1993)]: Rough estimations
Braun et al. [NPB (2002)]: Account for hard GEXs



$q_\perp^2 \simeq 4$ GeV^2
$s \simeq 1000$ GeV^2

E791: Good agreement with BMS bunch

Following convolution procedure of **Braun et al.**, we found



[PLB 578 (2004) 91]

	DA	χ^2
(solid black line)	Asymp.	12.56
(green band)	**BMS bunch**	10.96
(red dashed line)	CZ	14.15

(accounting for 18 data points)

Our bunch of pion DAs has maximum uncertainty in the central region, but agrees well with E791 data!

JLab data for pion EM FF

and

Analytic NLO pQCD

Analytic Perturbation Theory

Analyticization means procedure to obtain analyticity of hadronic observables in whole Q^2 region via dispersion relations (**Radyushkin, Krasnikov&Pivovarov, Dokshitzer, Beneke&Braun, Shirkov&Solovtsov**):
Analytization combines

- **RG invariance** $\Longrightarrow$ resummation of UV logs and correct QCD asymptotics

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Analytization combines

- **RG invariance** $\Longrightarrow$ resummation of UV logs and correct QCD asymptotics
- **Causality** $\Longrightarrow$ spectral representation
  $\Longrightarrow$ **no Landau singularity**

Analytic Perturbation Theory

Analytic Perturbation Theory expresses QCD observables over non-power sequences $\{\mathcal{A}_k^{(\mathrm{L})}(Q^2)\}$ in L-loop order **[Shirkov, NPB Proc. 64 (1998) 106]**. At 1-loop:

$$\mathcal{A}_k^{(1)}(Q^2) = \frac{1}{\pi}\int_0^\infty d\sigma \frac{\rho_k^{(1)}(\sigma)}{\sigma + Q^2 - i\epsilon}\ ; \quad \rho_k^{(1)}(\sigma) = \left(\frac{4\pi}{b_0}\right)^k \Im\left(\frac{1}{\ln(-\sigma/\Lambda^2)}\right)^k$$

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- $\mathcal{A}_2^{(1)}(Q^2) = \left(\frac{4\pi}{b_0}\right)^2\left[\frac{1}{\ln^2(Q^2/\Lambda^2)} + \frac{Q^2\Lambda^2}{(\Lambda^2 - Q^2)^2}\right]$

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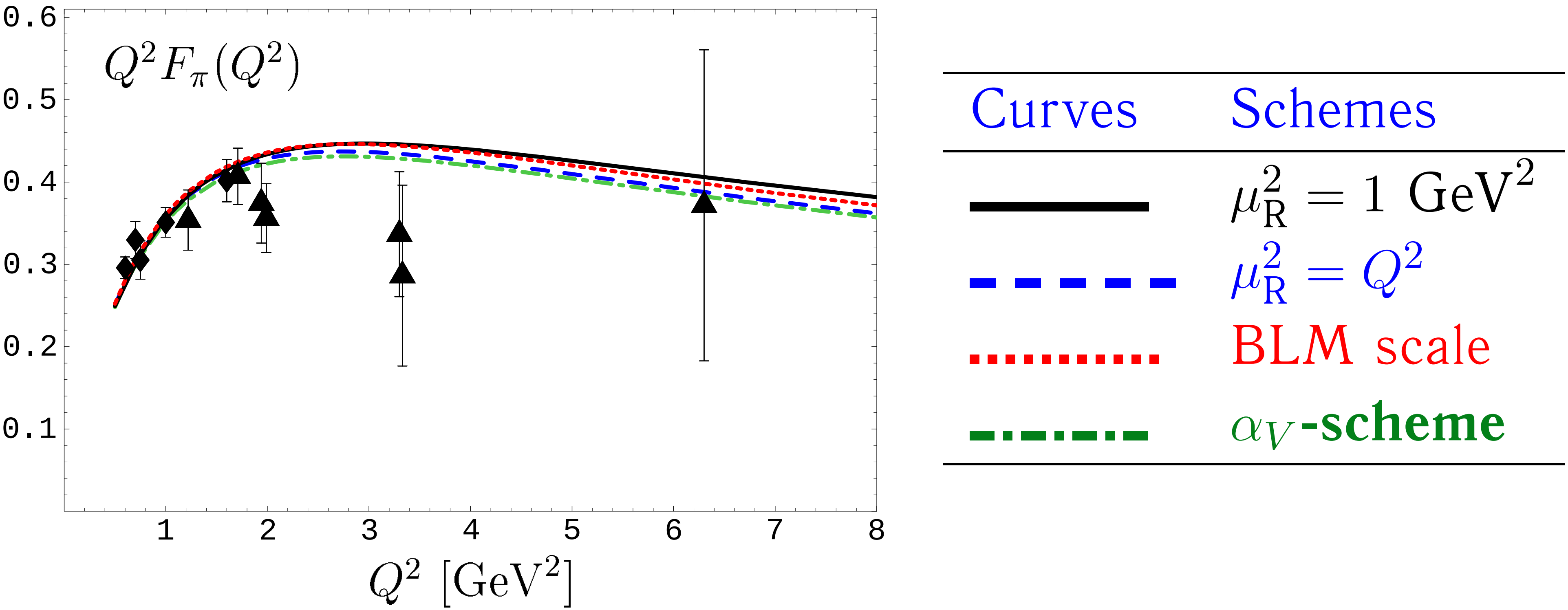
Important: $\mathcal{A}_2^{(1)}(Q^2) \neq \left[\bar{\alpha}_s^{(1)}(Q^2)\right]^2$

Pion form factor in analytic NLO pQCD

[AB-Passek-Schroers-Stefanis, PRD 70 (2004) 033014]



Practical independence on scheme/scale setting!

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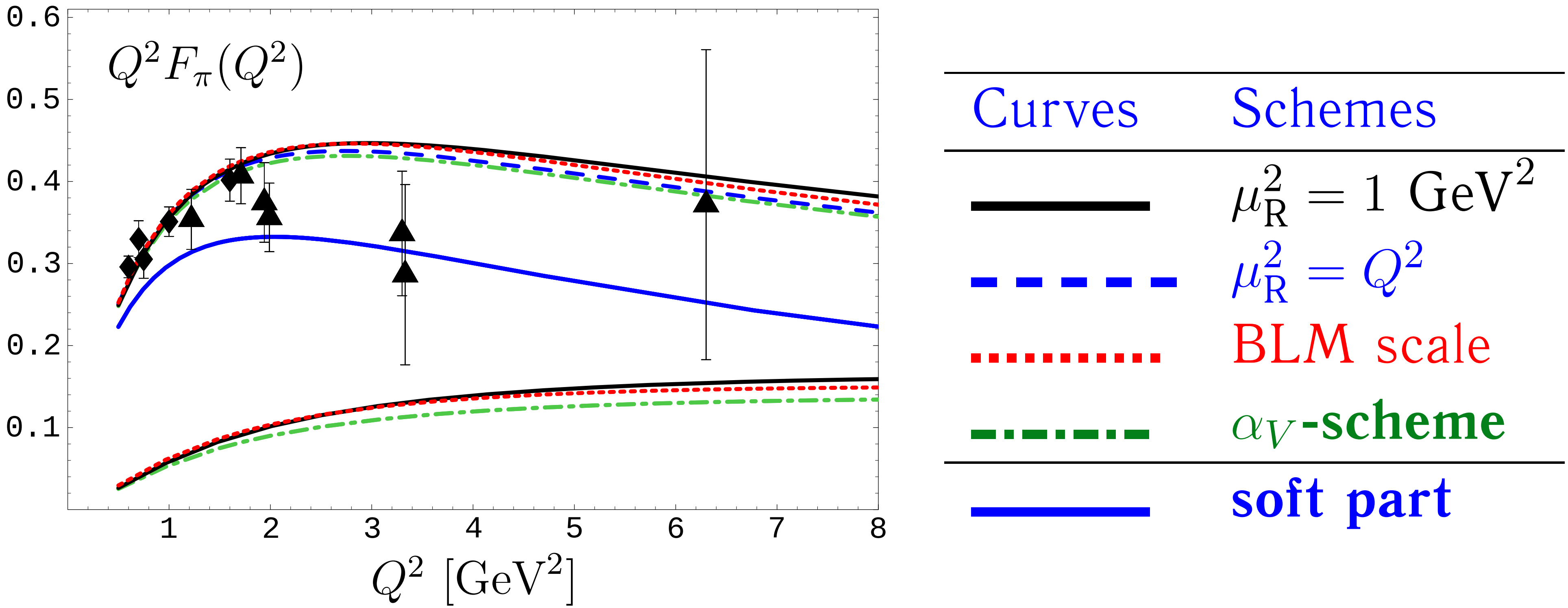
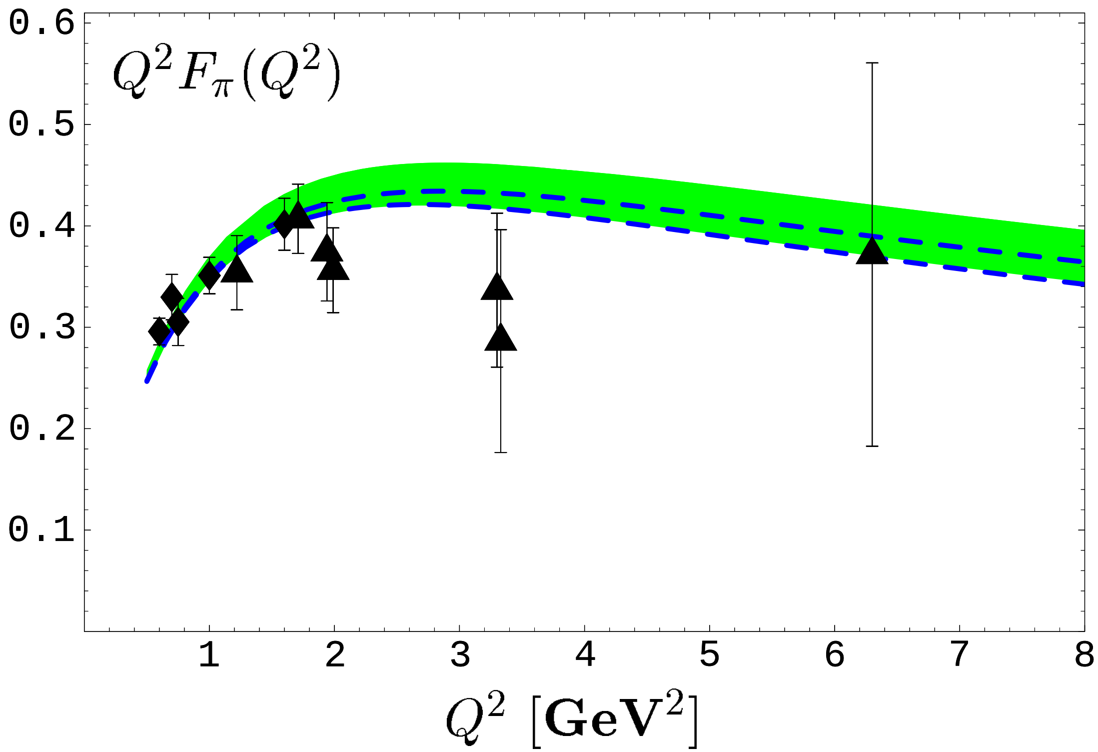


Practical independence on scheme/scale setting!

Pion FF in analytic NLO pQCD



Green strip includes

- NLC QCD SRs uncertainties **(pion DA bunch)**
- scale-setting ambiguities at NLO level

Pion EM form factor

New Lattice Data
for
pion DA

Revised CLEO Constraints and Lattice Data

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$
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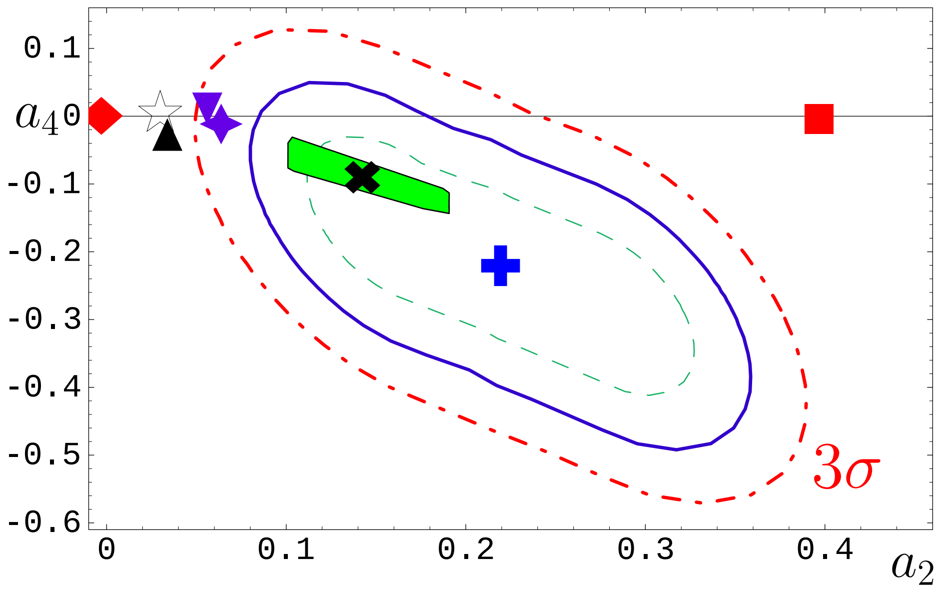

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BMS DA and most of **BMS bunch** — inside 1σ-domain.
Transverse lattice model — near 3σ-boundary.

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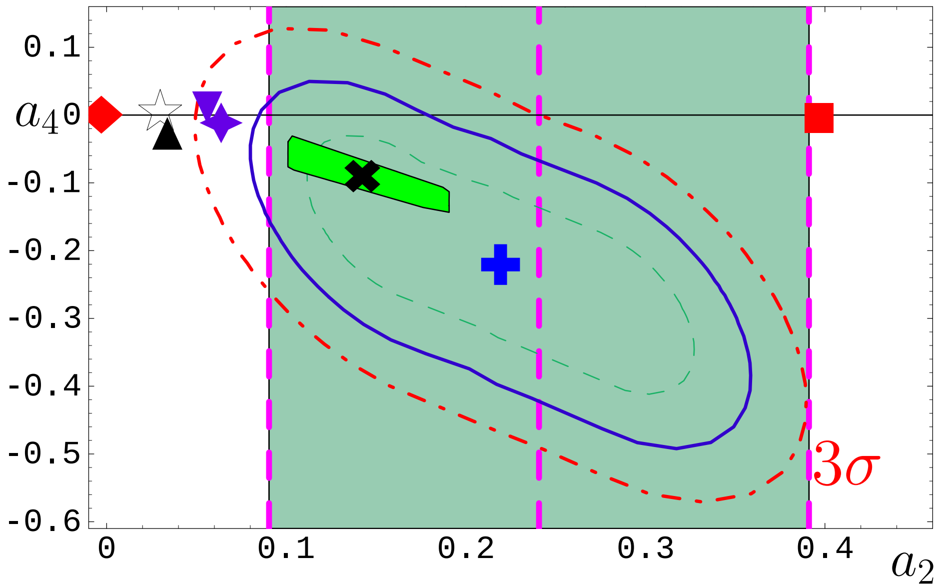
[PRD 73 (2006) 056002]: $\lambda_q^2 = 0.4\ \text{GeV}^2$, $\delta^2_{\text{Tw-4}} = 0.19(4)\ \text{GeV}$



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gray strip = lattice'04 result

BMS DA and most of **BMS bunch** — inside 1σ-domain and inside **2004 lattice strip** **[PRD 73 (2006) 056002]**.

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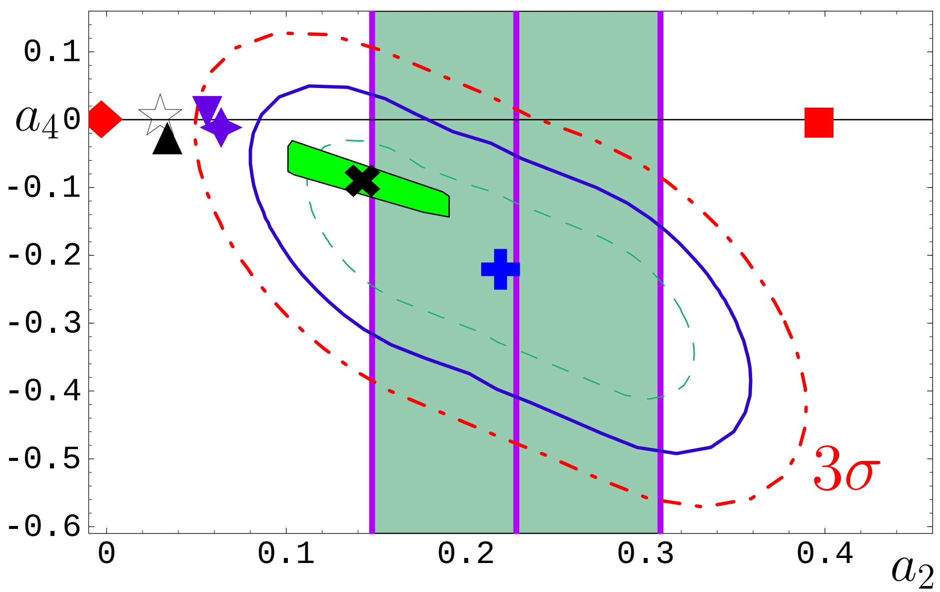

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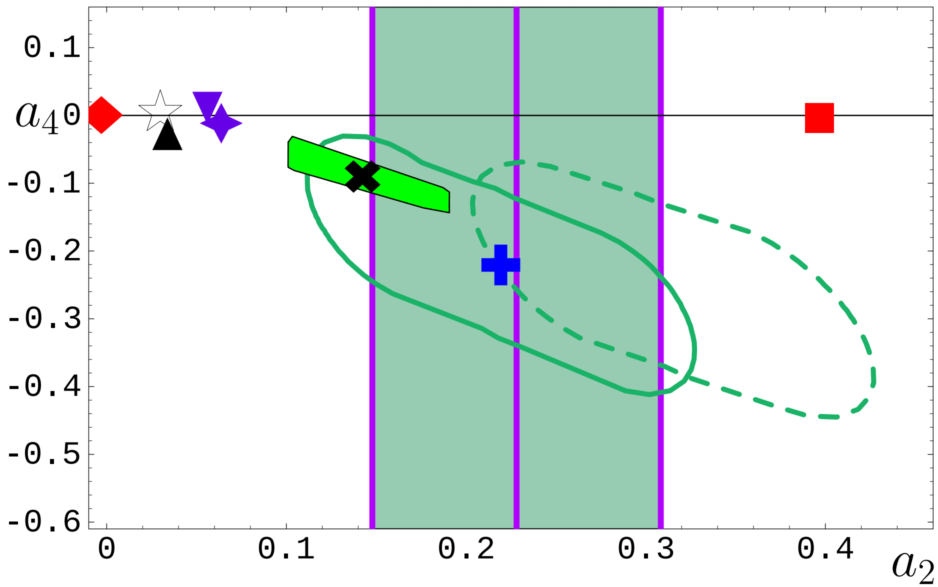
- ▼ = transverse lattice
- **gray strip** = lattice'05 result

BMS DA and most of **BMS bunch** — inside 1σ-domain and one-half inside **2005 lattice strip** **[PRD 73 (2006) 056002]**.

Revised CLEO Constraints and Lattice Data

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$
with 10% uncertainty of $\delta^2_{\text{Tw-4}}$
[PRD 73 (2006) 056002]: $\lambda_q^2 = 0.4$ GeV^2, $\delta^2_{\text{Tw-4}} = 0.19(2)$ GeV



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BMS DA and most of **BMS bunch** — inside 1σ-domain and inside **lattice strip**. **Dashed contour** = renormalon model estimation of CLEO data **[PRD 73 (2006) 056002]**.

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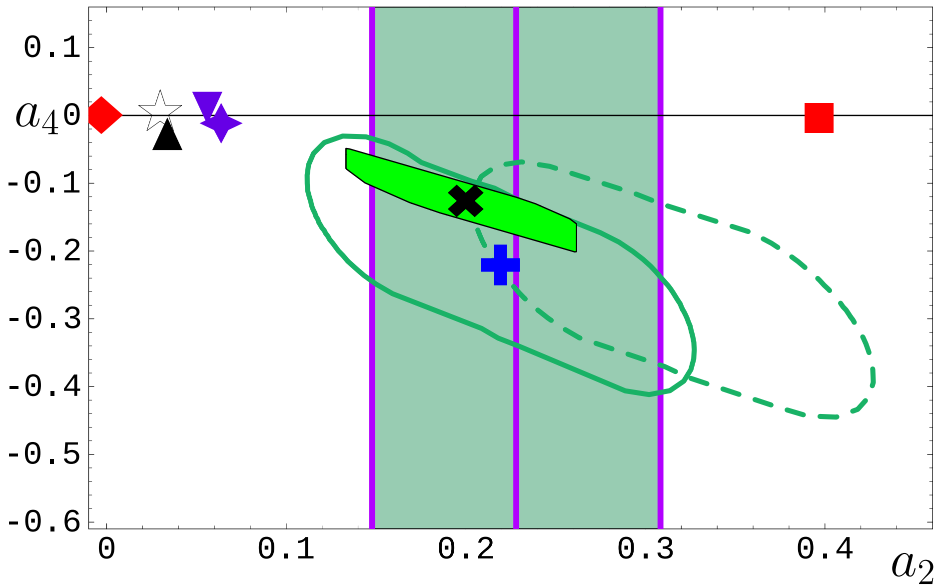

- ✚ = best-fit point
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- ✖ = our new model
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- ▼ = transverse lattice
- **gray strip** = lattice'05 result

Most of **improved BMS bunch** — inside 1σ-domain and inside **lattice strip**. **Dashed contour** = renormalon model estimation of CLEO data.

CONCLUSIONS

- **QCD SR** method with **NLC** for pion DA gives us admissible sets (**bunches**) of DAs for each λ_q value.

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- **APT with non-power NLO** for pion EM form factor diminishes scale-setting ambiguities already at NLO level.

Collaborators & Publications

Collaborators

S. Mikhailov	BLTPh, JINR, Dubna
A. Pimikov	BLTPh, JINR, Dubna
N. Stefanis	ITP-II, Ruhr-Universität Bochum
A. Karanikas	University of Athens, Athens

Publications

A.B., S.M., N.S.	**PLB**	**508**	**(2001)**	**279**
A.B., S.M., N.S.	**PRD**	**67**	**(2003)**	**074012**
A.B., S.M., N.S.	**PLB**	**578**	**(2004)**	**91**
A.B., N.S. *et al.*	**PRD**	**70**	**(2004)**	**033014**
A.B., N.S.	**NPB**	**721**	**(2005)**	**50**
A.B., A.K., N.S.	**PRD**	**72**	**(2005)**	**074015**
A.B., S.M., N.S.	**PRD**	**73**	**(2006)**	**056002**